3. Linear Algebra

1. (a) Angle between $\vec{u}$ & $\vec{v}$ is 90°.

Hence, $\|u \times v\| = |u| \cdot |v|$

$= 3 \times 5 = \underline{\underline{15}}$

$\vec{u} \times \vec{v}$ would lie in the second quadrant of the x-y plane as per the right ~~left~~ hand rule for cross products.

Hence,

(b) $<$

(c) $>$

(d) $=$

2. (a) Given $|u| = |v| = 2\sqrt{2} = |u - v|$

$|u-v|^2 = |u|^2 + |v|^2 - 2|u||v|\cos\theta$

$8 = 8 + 8 - 2 \times 8 \times \cos\theta$

$\Rightarrow 16(1-\cos\theta) = 8$

$1 - \cos\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$, $\underline{\underline{\theta = \frac{\pi}{3}}}$

$|u+v| = \sqrt{8+8+16 \times \frac{1}{2}} = \underline{\underline{2\sqrt{6}}}$

(b) $\theta = \pi/3$

3. Matrix $\rightarrow$ $\begin{bmatrix} 1 & 3 & 2 \\ a & 6 & 2 \\ 0 & 9 & 5 \end{bmatrix}$

(a) I do the operation : $R_2 - aR_1 = R_2$

This gives me, $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 6-3a & 2-2a \\ 0 & 9 & 5 \end{bmatrix}$

I need $6-3a$ to be ~~[illegible]~~ non 0.

If $6-3a$ is 0, I can not make it 1 by scaling and hence would require a row interchange.

Hence, $6-3a = 0$

or, $\underline{a = 2}$

(b) Det (matrix) = 0

So, $1(12) - 3(5a) + 2(9a) = 0$

or, $12 + 3a = 0$

$\underline{a = -4}$