P.I.D. equivalent of optimal regulator

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P.I.D. EQUIVALENT OF OPTIMAL REGULATOR

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In this work an optimal regulator for linear systems has been realised indirectly in terms of the proportional, integral and derivative of the measurable outputs of the system.

To avoid high cost as well as complexity of realisation of simple optimal regulators through a complete state feedback arrangement, an indirect method of realisation of the same is suggested here. This method simplifies the one developed by Calovic and Cuk[1] and is superior to it because of the need for a smaller number of measurable outputs of the system.

Consider the linear system described as

$$\dot{x} = Ax + Bu \qquad \text{with } x(0) = x_0 \tag{1}$$

$$y = Cx \tag{2}$$

where x is an n-vector state of the system, u is an m-vector input to the system, y is an l-vector output of the system and A, B and C are constant-coefficient matrices of appropriate dimensions.

The controlled input u has been assumed to be the output of a 3-term controller with y, $\int y\,dt$, $\dot{y}$ as its inputs. Their relationship in forming u may be expressed as

$$u = -K_p y - K_i \int_0^t y\,dt - K_d \dot{y} \qquad \text{for } u(0) = 0 \tag{3}$$

where K_p, K_i and K_d are proportional, integral and derivative feedback gain matrices of size ~~$m \times p$, $m \times i$, $m \times d$, respectively~~ $m \times l$.

With the help of eqns. 1 and 2, eqn. 3 may be rewritten as

$$u = -K_p Cx - K_i \int_0^t y\,dt - K_d C(Ax + Bu)$$

or

$$u = -\bar{K}_p x - \bar{K}_i \int_0^t y\,dt \tag{4}$$

where the new gains $\bar{K}_p$ and $\bar{K}_i$ are defined as

$$\bar{K}_p = (I_m + K_d CB)^{-1}(K_p C + K_d CA) \tag{5}$$

$$\bar{K}_i = (I_m + K_d CB)^{-1} K_i \tag{6}$$

(with I_m as the mth order unity matrix).

Expressing $\int y\,dt$ in terms of a new set of variables z, it may be written

$$z = \int_0^t y\,dt \quad \text{for} \quad z(0) = 0 \tag{7}$$

therefore

$$\dot{z} = y = Cx \tag{8}$$

Defining augmented states of the system as

$$\bar{x} = [xz]^T \tag{9}$$

the system may now be described as

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \tag{10}$$

where the augmented system matrix $\bar{A}$ and control matrix $\bar{B}$ are expressible as

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

With $\bar{Q}$, R as augmented state and input weighting matrices to minimise the performance index of the form

$$J = \tfrac{1}{2} \int_0^\infty (\bar{x}^T \bar{Q} \bar{x} + u^T R u)\,dt \tag{11}$$

the desired optimal control u^* may be evaluated as (provided the relevant conditions of controllability and observability are satisfied[1])

$$u^* = -R^{-1} \bar{B}^T \bar{P} \bar{x} \tag{12}$$

where the matrix $\bar{P}$ is the positive-definite solution of the nonlinear algebraic matrix Riccati equation

$$\bar{P}\bar{A} + \bar{A}^T \bar{P} - \bar{P}\bar{B}R^{-1}\bar{B}^T + \bar{Q} = 0$$

Comparison of eqns. 4 and 12 gives

$$\bar{K}_p = R^{-1} B^T P_{11} \quad \text{and} \quad \bar{K}_i = R^{-1} B^T P_{12} \tag{13}$$

where P_{11}, P_{12} are the submatrices of dimensions $n \times n$ and $n \times l$, respectively of the $(n+l) \times (n+l)$ matrix $\bar{P}$ given by

$$\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$$

After evaluation of these new gain matrices $\bar{K}_p$ and $\bar{K}_d$, proportional, integral and derivative gains K_p, K_i and K_d may be evaluated from the eqns. 5 and 6 which may be rewritten in the forms

$$[K_p\, K_d] = \bar{K}_p \bar{C}^{-1} \quad \text{where} \quad \bar{C} = \begin{bmatrix} C \\ CA - CB\bar{K}_p \end{bmatrix} \tag{14}$$

and

$$K_i = (I_m + K_d CB)\bar{K}_i \tag{15}$$

As the matrix $\bar{C}$ is of size $2l \times n$, for the existence of its inverse l should ideally be equal to $n/2$. This condition is superior to Calovic and Cuk's p.i.d. controller which requires $l \geqslant n$. Otherwise if $l \neq n/2$, for an approximate solution the generalised inverse of $\bar{C}$ may be computed. Applications of the proposed controller with systems of 3rd- and 4th-order have been reported in another paper.[2]

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