Review Session 5

API-201, 10.15.21
Sophie Hill

Agenda

- Review midterm Q2 sensitivity analysis
- Sampling distributions
- Build a hypothesis test worksheet in Excel!

Midterm Q2

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$$\frac{P(\text{vax} \mid \text{COVID})}{X} = \frac{P(\text{unvax} \mid \text{COVID})}{(1 - X)}$$

$$\frac{0.74}{X} = \frac{0.26}{(1 - X)}$$

$$0.74 - 0.74X = 0.26X$$

$$0.74 = X$$

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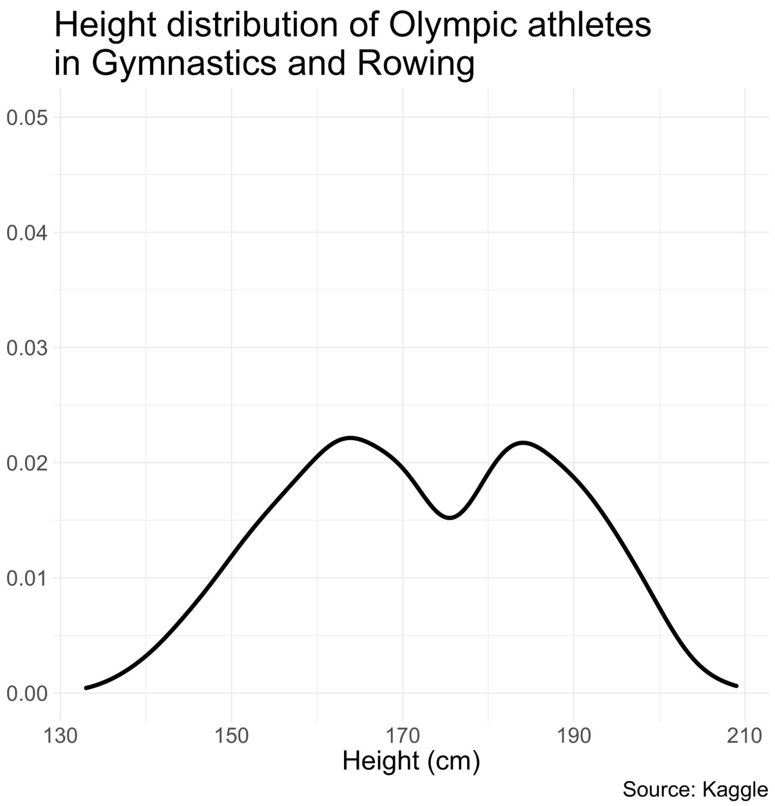
If the vaccine is effective, then the vaccinated should be a *smaller* proportion of the infected than the population… i.e., the vaccinated should be a *larger* proportion of the population than the infected.

So the vaccine is effective if the vaccination rate in the population is above 74% (and has no effect when it is equal to 74%).

Sampling distribution

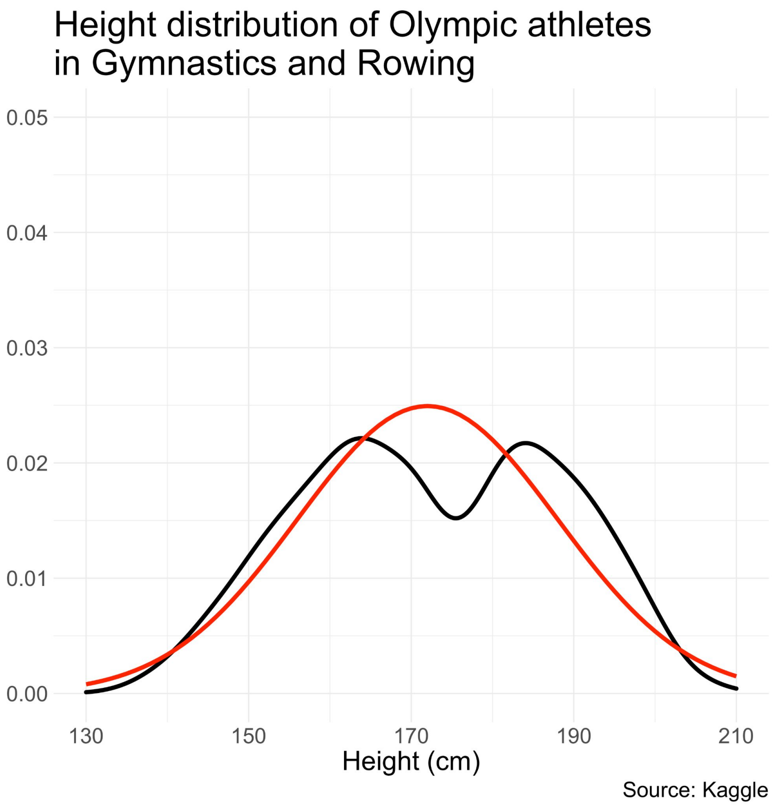
Distribution of height of Olympic athletes



Distribution of height of Olympic athletes

Not normally distributed!

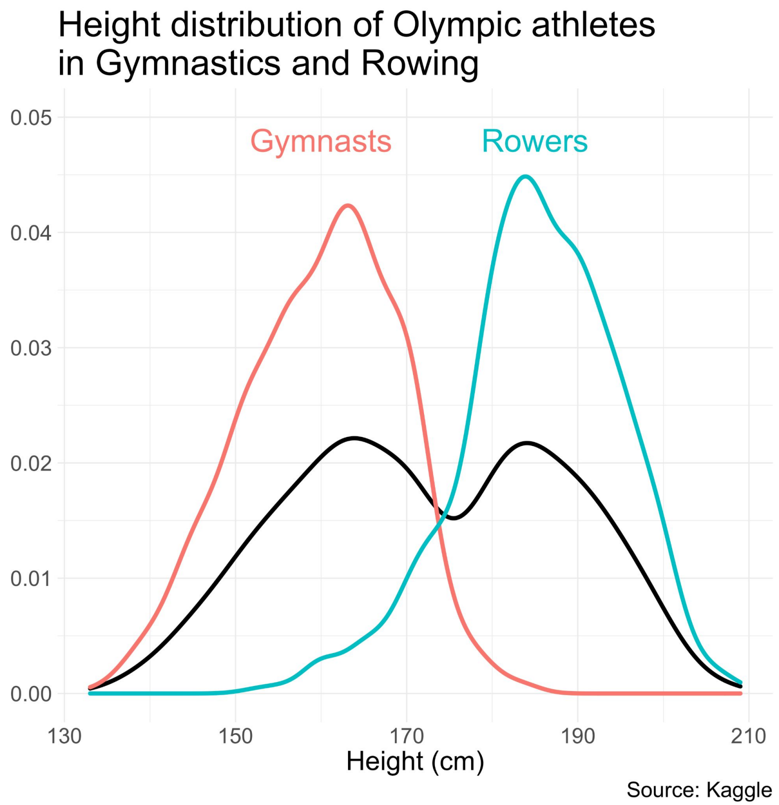
Why?



Distribution of height of Olympic athletes

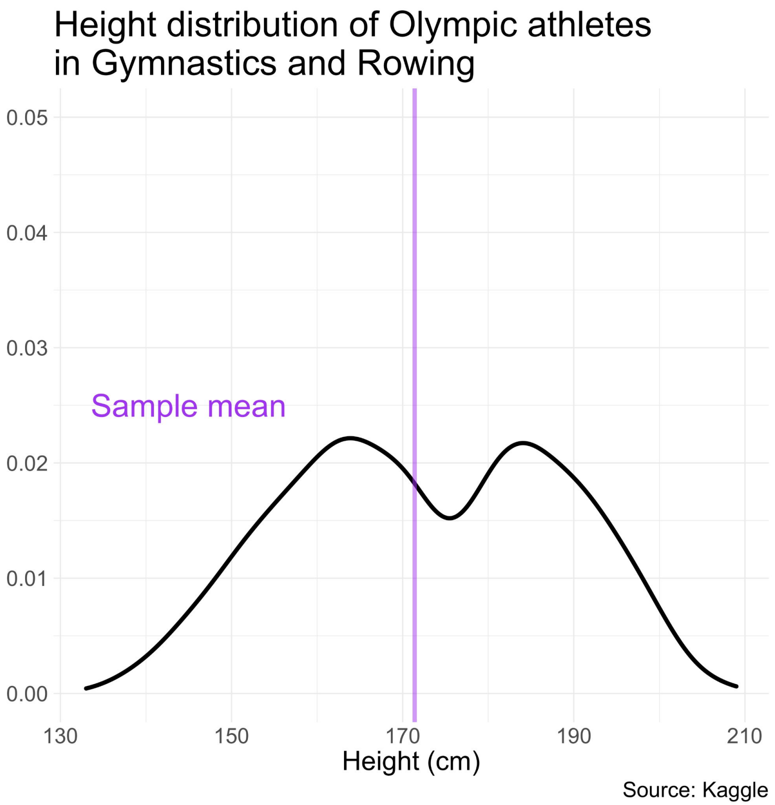
Not normally distributed!

Gymnasts tend to be much shorter than Rowers...



Distribution of height of Olympic athletes

Now let's take a random sample of 10 athletes and calculate the sample mean.



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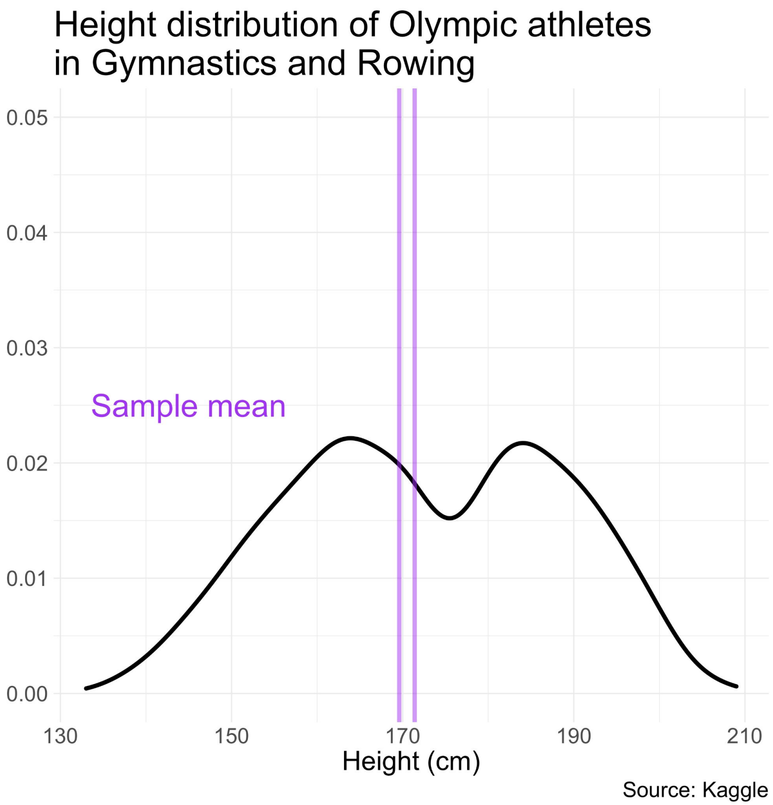

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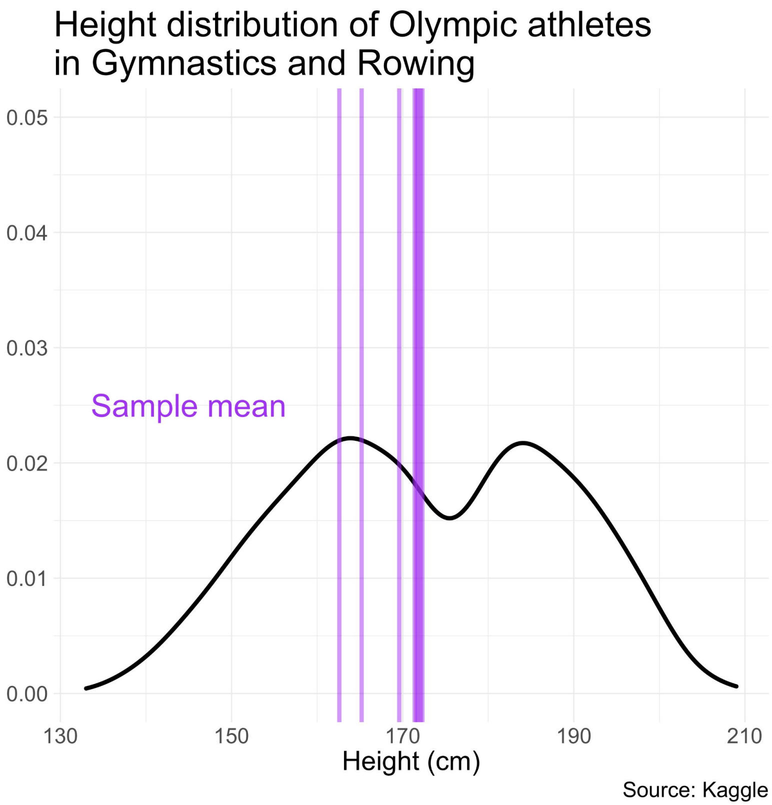

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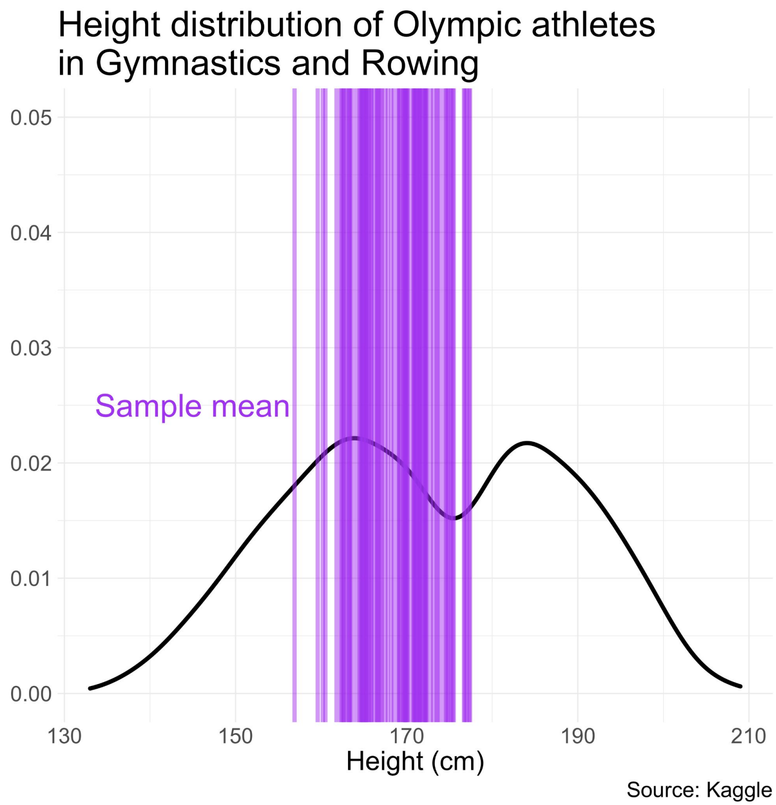
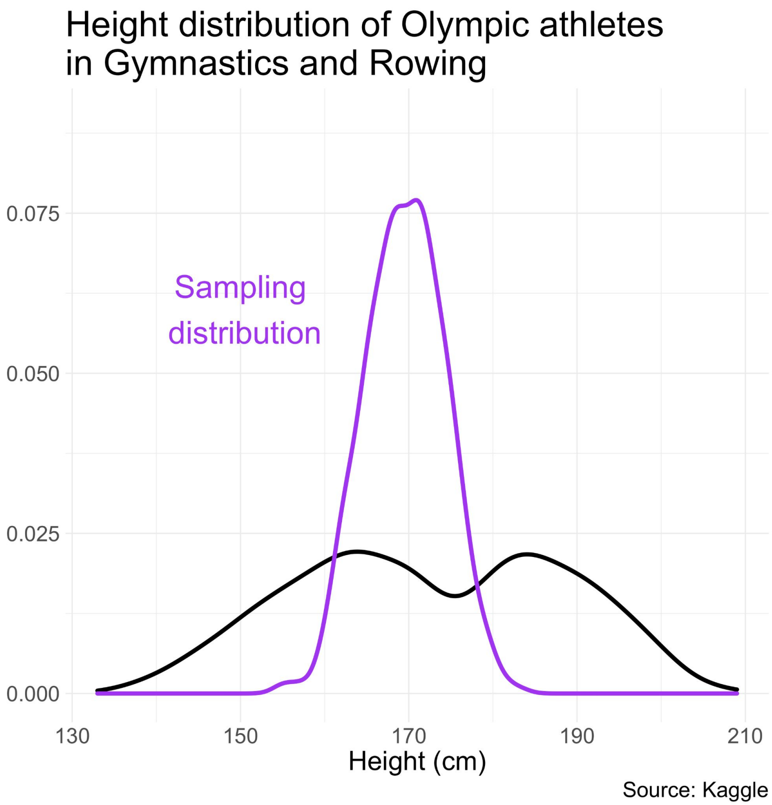


Distribution of height of Olympic athletes

If we did this lots of times, we could plot the PDF of the sample means, i.e., the sampling distribution.

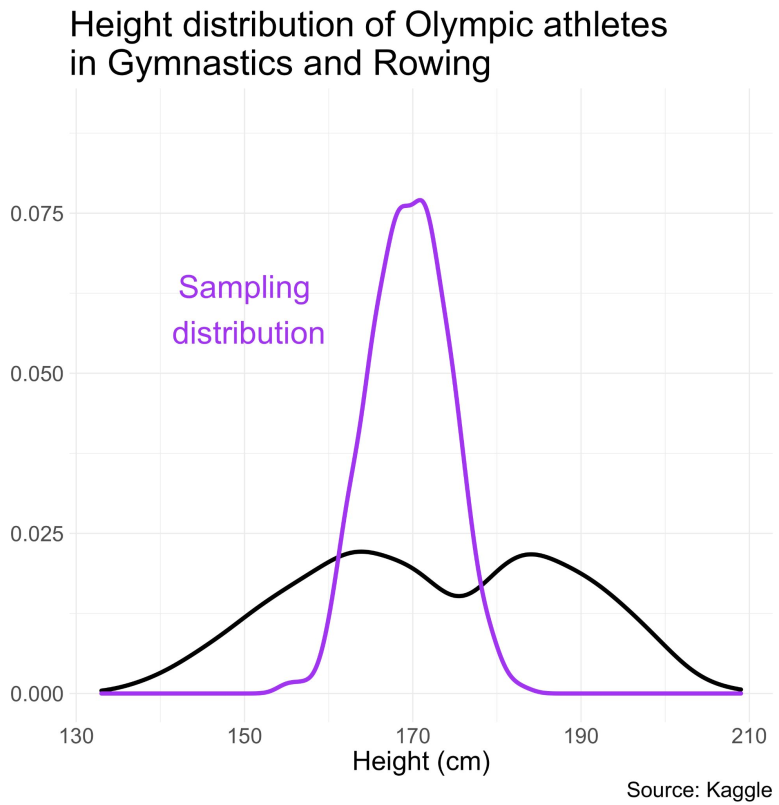
What do you notice about its shape, compared to the PDF of the raw data?



Distribution of height of Olympic athletes

Key takeaway:
The sampling distribution is approximately normal, even though the raw distribution is *not*.

This is the magic of the CLT!!



Hypothesis tests

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Let's make a hypothesis test worksheet in Excel (link to template on Canvas).

HYPOTHESIS TESTING FOR PROPORTIONS

Inputs		
Null hypothesis	q =	0.50
Significance level	alpha =	0.05
Sample estimate	q-hat =	0.47
Sample size	n =	800

Outputs		
Does the CLT apply?		Yes
Standard error	SE =	0.0177
Z-score	Z =	-1.6971
p-value	p =	0.0897
Is p-value < alpha?		FALSE
Do we reject the null?		Fail to reject

FAQs

Why does the formula for the p-value look like that??

We told you that you can calculate a p-value from a Z-score in Excel using this formula:

$$= 2 * (1 - NORM.S.DIST(ABS(Z),TRUE))$$

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But what does it actually mean?? Let's break it down step-by-step.

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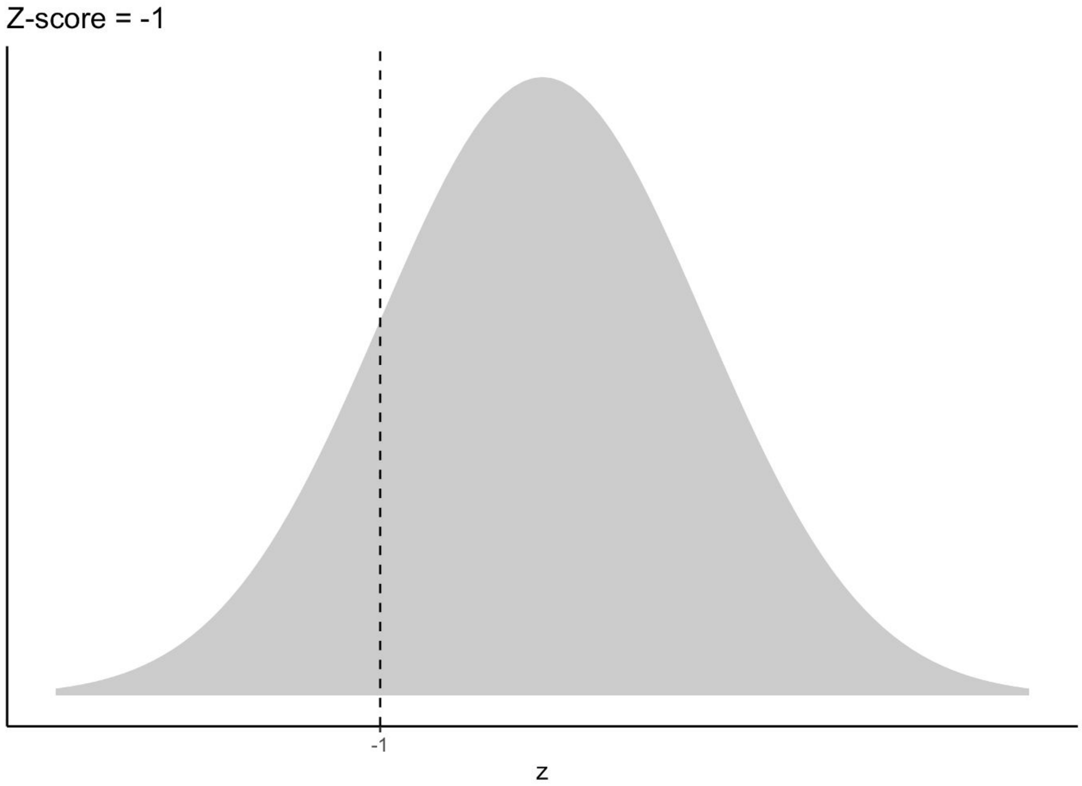

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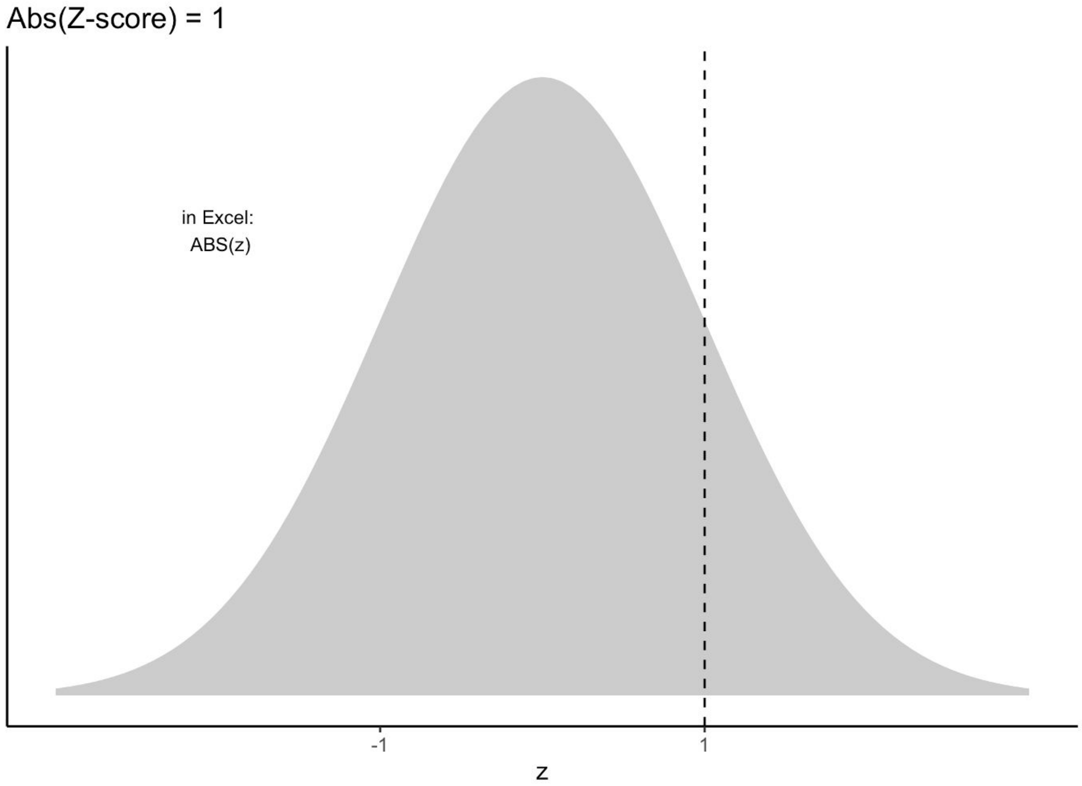

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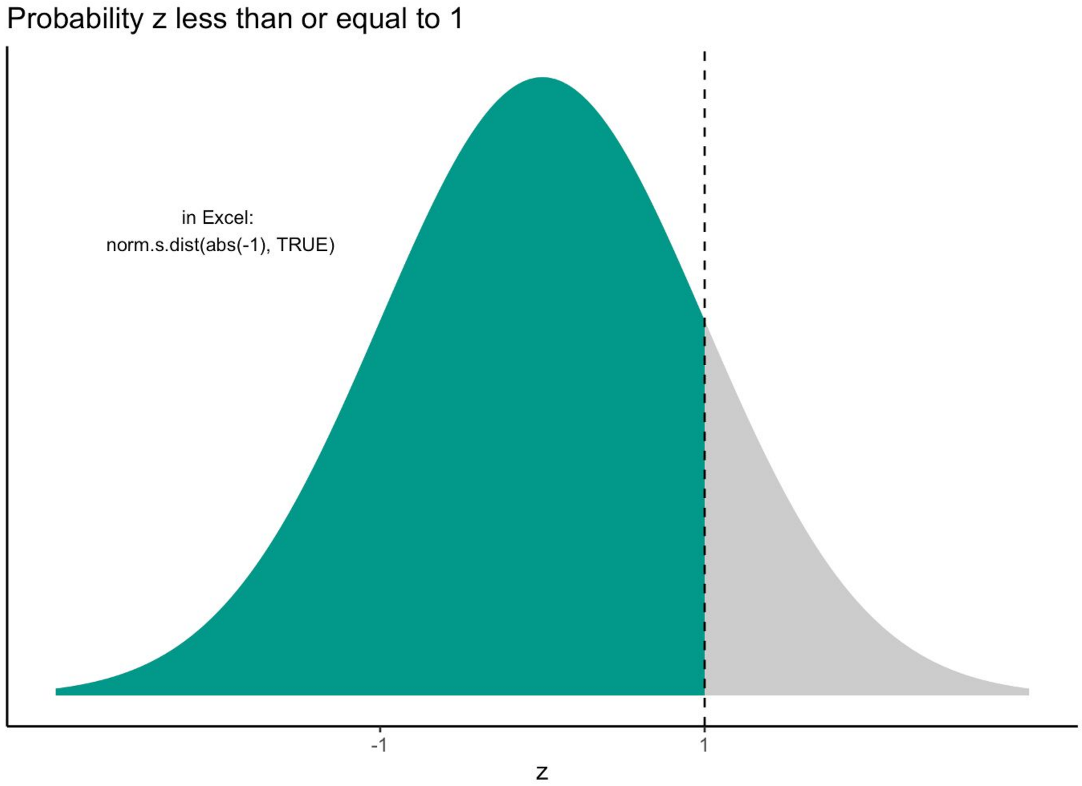

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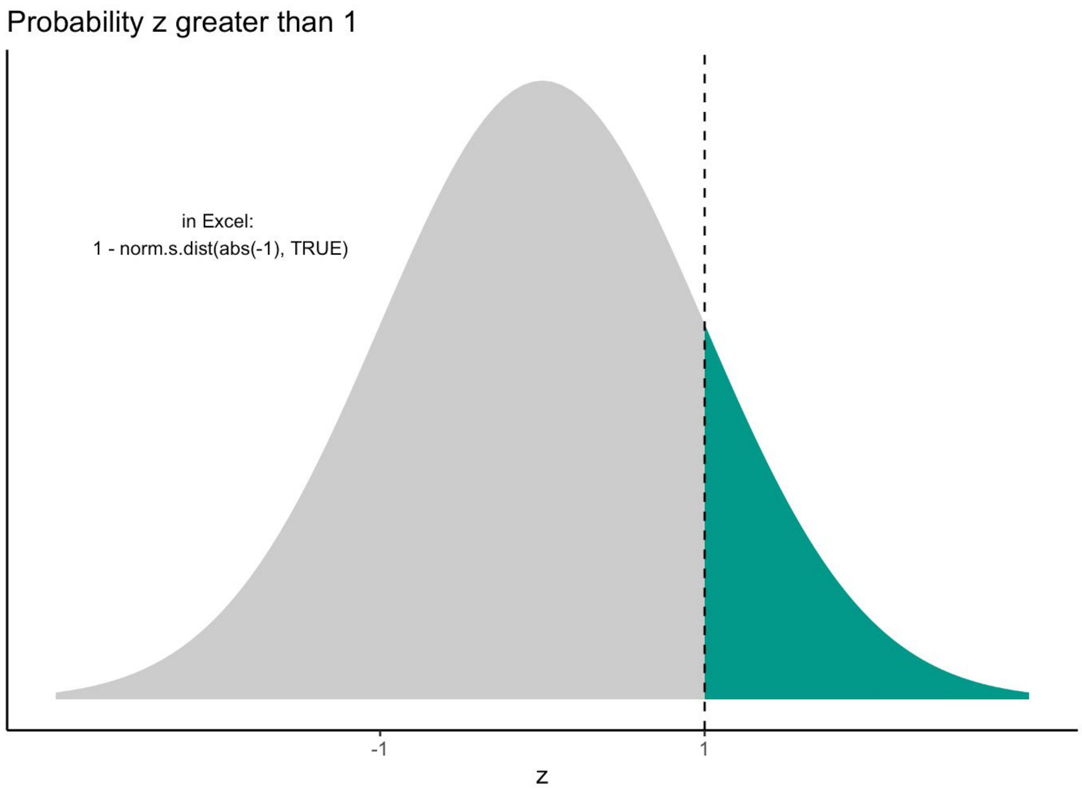

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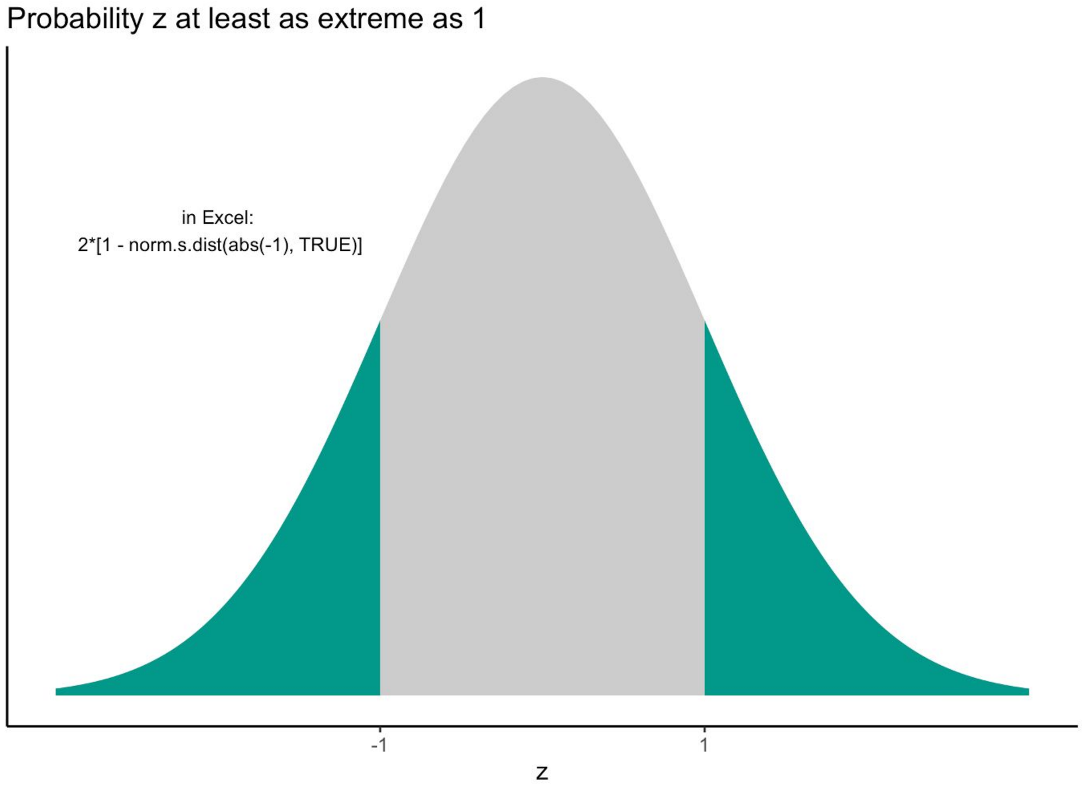

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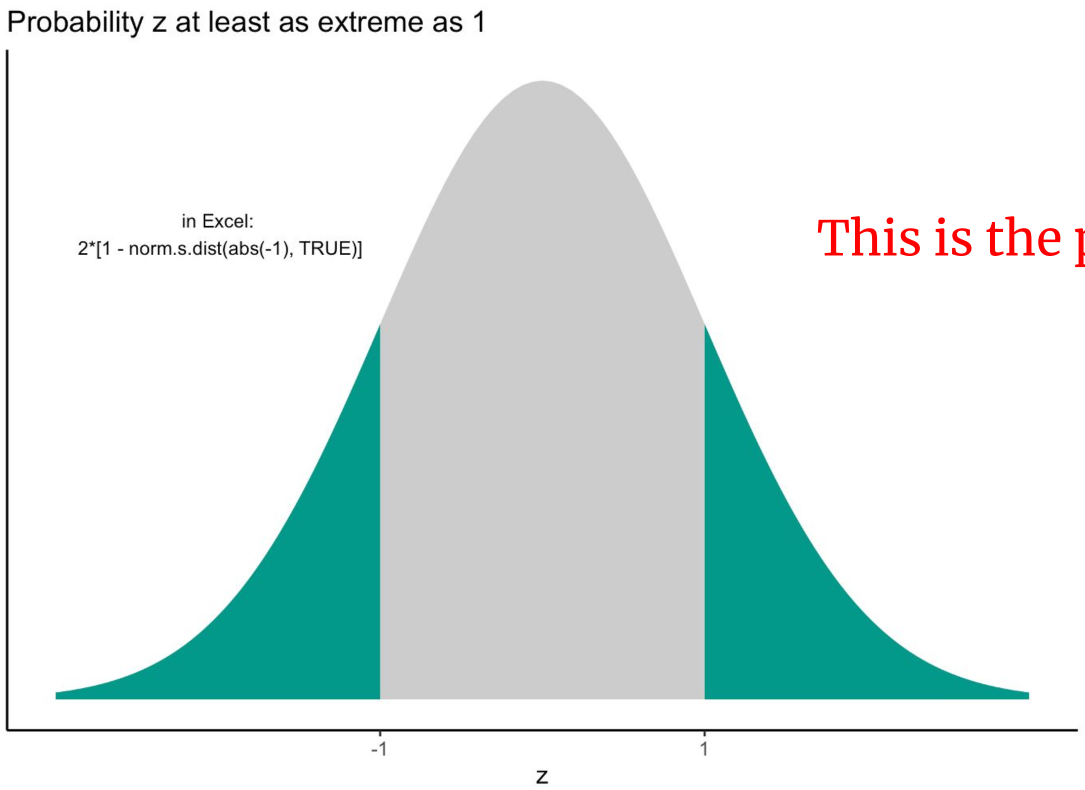

This is the p-value!!

How does the proportion and sample size affect the SE?

$$SE = \sqrt{\frac{q * (1 - q)}{n}}$$

What happens to SE as n increases (holding q fixed)?

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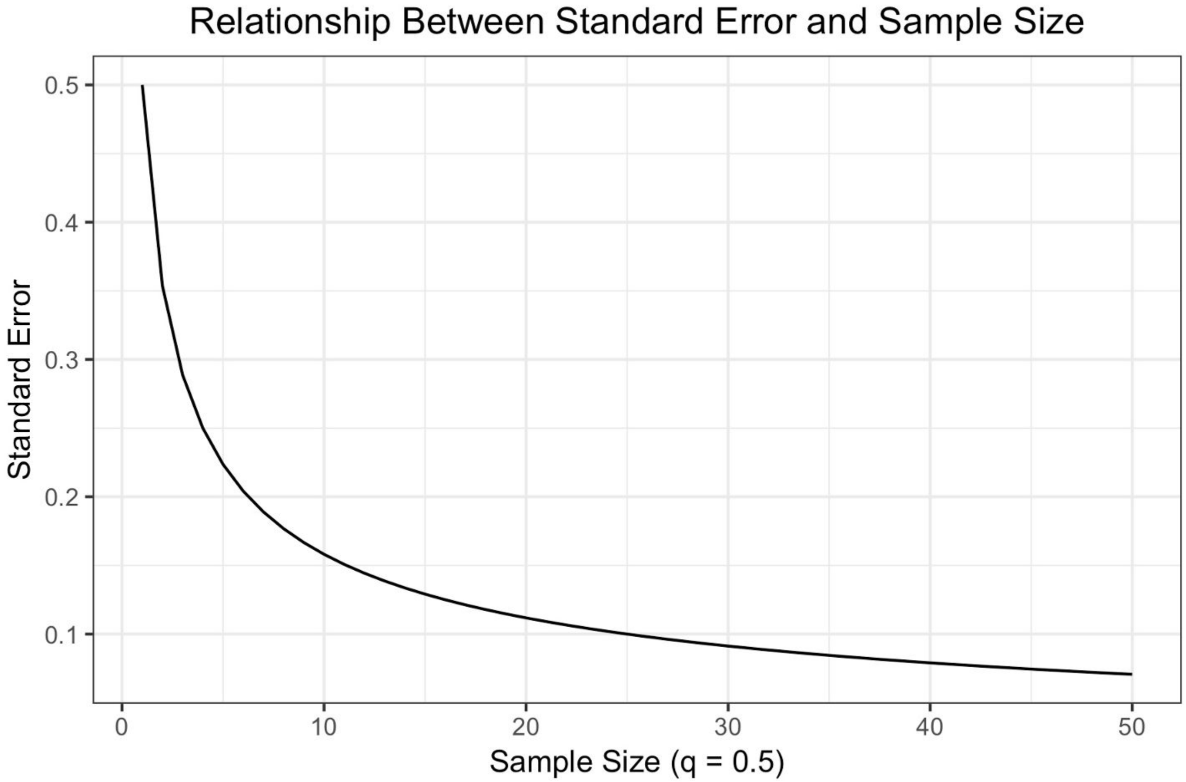

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