

# Insertion Sort Analysis in Java

Preston Stosur-Bassett

## Abstract

### I. MOTIVATION

In order to show how an algorithm might run on a given set of hardware, and how the algorithm will perform when given large amounts of data, algorithms are analysed. More specifically, insertion sort is analysed to determine what the run time might be for sorting large amounts of data on any modern computer.

### II. BACKGROUND

A sorting algorithm is used to sort data with a natural order. One such sorting algorithm is insertion sort, which sorts by iterating through a list of data, taking the current position, and repositioning it into a more appropriate place in the list. How will this procedure perform when handling high volumes of data? How will it perform when executed on different machines? Because insertion sort is a more basic sorting algorithm, numerous papers and articles have been written answering these two questions.

### III. PROCEDURE

An insertion sort can be implemented in a multitude of languages using the pseudocode provided below.

**Insertion Sort Pre-Condition:** A is a non-empty array of data with a natural order.

**Insertion Sort Post-Condition:** A' is a permutation of A (containing all the same elements) in strictly non-decreasing order.

---

#### Algorithm 1 INSERTION-SORT(A)

---

```

1: procedure INSERTION-SORT(A)
2:   if  $A.length \leq 1$  then
3:     return A
4:   end if
5:    $i = 2$ 
6:   while  $i$  upto  $A.length$  do
7:      $key = A[i]$ 
8:      $j = i - 1$ 
9:     while  $j$  downto 1 and  $key < A[j]$  do
10:       $A[j + 1] = A[j]$ 
11:       $j = j - 1$ 
12:    end while
13:     $A[j + 1] = key$ 
14:     $i = i + 1$ 
15:  end while
16: end procedure

```

---

**Outer-Loop Invariant:** The subarray  $A'[1 \dots i - 1]$  contains all the same elements as the subarray

$A[1 \dots i - 1]$ .

**Outer-Loop Initialization:** The outer-loop invariant holds because  $A'[1 \dots i - 1]$  and  $A[1 \dots i - 1]$  both contain the same one element.

**Outer-Loop Maintenance:** The outer-loop invariant holds because  $A'[1 \dots i - 1]$  and  $A[1 \dots i - 1]$  both contain the same elements, although they may be in different orders.

**Outer-Loop Termination:** When the outer-loop terminates,  $i = A.length$ , which implies that the entire array has been traversed and the guard has been negated. The negation of the guard implies that  $A'[1 \dots i - 1]$  contains all the elements in  $A[1 \dots i - 1]$ .

**Inner-Loop Invariant:**  $A'[1 \dots j]$  is sorted in strictly non-decreasing order.

**Inner-Loop Initialization:** Before the first iteration of the loop,  $j = 1$ , meaning the subarray  $A'[1 \dots j]$  contains exactly one element, which is already sorted.

**Inner-Loop Maintenance:** At the beginning of each iteration of the loop the inner-loop invariant holds because  $j$  counts down from  $i$ , and  $A'[j+1]$  is swapped with  $A'[j]$  only if  $A'[j+1]$  is less than  $A[j]$ .

**Inner-Loop Termination:** The negation of the guard implies that  $j = A.length$  and that  $A'[1 \dots j]$  has been entirely traversed and sorted in strictly non-decreasing order, which maintains the inner-loop invariant.

**Conclusion:** The termination of both the inner and outer loops implies that the entire array has been traversed,  $A'$  is a permutation of  $A$  containing all the same elements in strictly non-decreasing order. This satisfies the post condition.

## IV. TESTING

### A. Testing Plan and Results

All arrays used in testing are Java `ArrayList<Integer>` unless otherwise specified. All times are recorded in milliseconds using a stopwatch class borrowed from [NEED CITATION]. It is important to note that the stopwatch class used takes the elapsed real-time between the start of the insertion sort algorithm and the end of the insertion sort algorithm as opposed to taking the elapsed processor-time because these tests were run on a multi-core computer. In the table below,  $A$  denotes Array. Times in the table below are given as averages out of 10 trials.

Table I  
TEST RESULTS

Tested Input	Expected Results	Actual Results	Time
Empty A	Empty A	Empty A	0.0003
A of 1000 Strings	Sorted A 1000 Strings	Sorted A 1000 Strings	0.021
A 1 Element	Original A	Original A	0.0003
A 10 Elements	Sorted A 10 Elements	Sorted A 10 Elements	0.0005
A 100 Elements	Sorted A 100 Elements	Sorted A 100 Elements	0.0021
A 1000 Elements	Sorted A 1000 Elements	Sorted A 1000 Elements	0.019
A 10000 Elements	Sorted A 10000 Elements	Sorted A 10000 Elements	0.129
A 100000 Elements	Sorted A 100000 Elements	Sorted A 100000 Elements	6.4923
A 1000000 Elements	Sorted A 1000000 Elements	Sorted A 1000000 Elements	2135.5007
A 10000000 Elements	Sorted A 10000000 Elements	OS Crash	N/A
A 1000 Identical Elements	Original Array	Original Array	0.0052

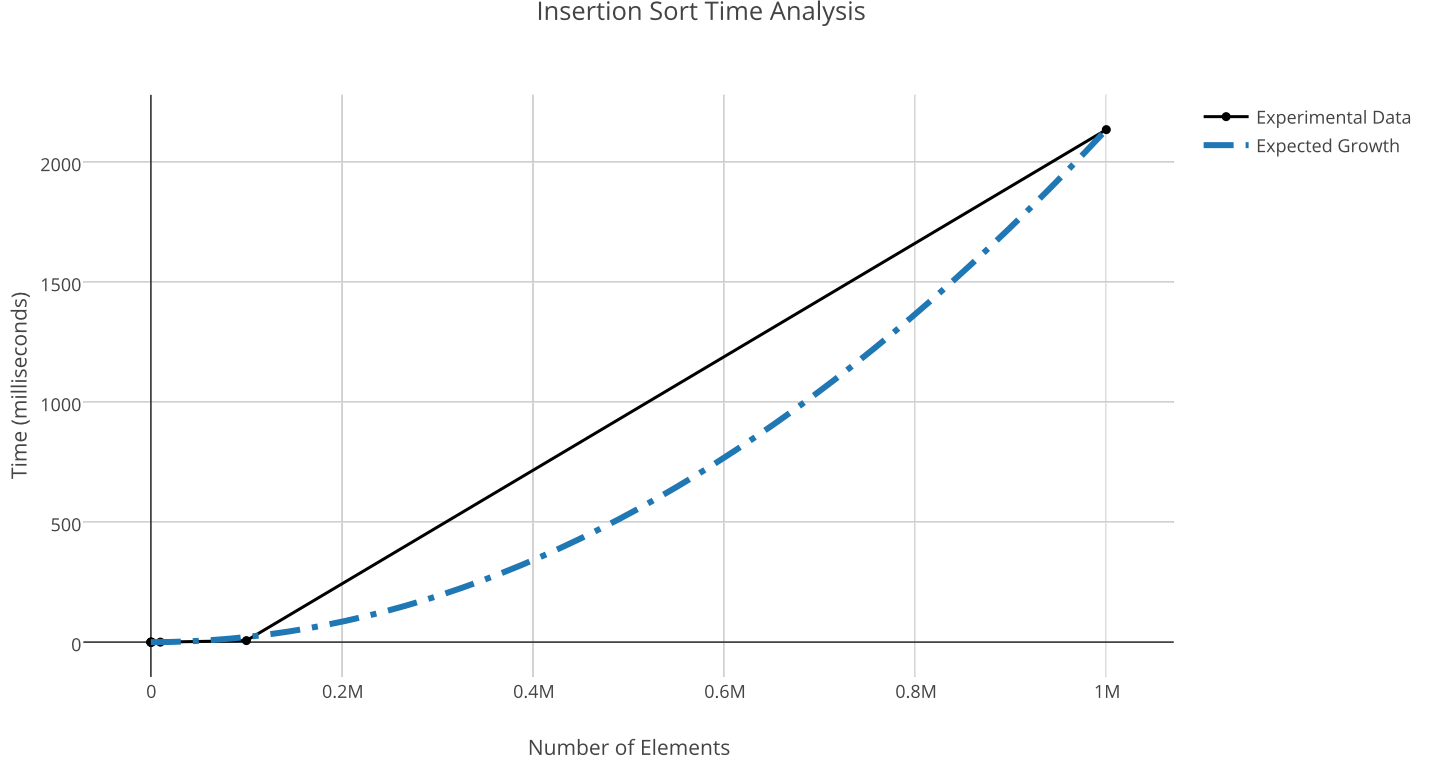


Figure 1. Insertion Sort Time Analysis

### B. Problems Encountered

One major issue encountered during the development of this insertion sort was that after completing the sort,  $A'$  was sorted properly except for the first element in the array. No matter what value the first element of  $A$  had, it did not change position in  $A'$ . For example, if  $A[5, 6, 3, 4, 7]$  was passed to the insertion sort algorithm, the returned array would look like  $A'[5, 3, 4, 6, 7]$ . Changing the guard for the inner for loop (see Algorithm 1 line 6) from *key* <  $A[j]$  and *j* *downto* 1 to *j* *downto* 1 and *key* <  $A[j]$  corrected this issue.

Another issue encountered during the development of this program was executing Java *assert* statements. When assertions were wrapped in parenthesis, they evaluated to a boolean, and the *assert* would throw an *AssertionError* with the explanation 'cannot compare to boolean'. Removing the parenthesis wrapping the assertion fixed this issue.

The last issue encountered during the development process of this program was generic programming. The program would not compile because not all Objects have a *compareTo* method. Adding the statement *< T extends Comparable < T >>* fixed this issue.

## V. EXPERIMENTAL ANALYSIS

The insertion sort demonstrated in Algorithm 1 was implemented in Java and executed on an HP SproctreXT TouchSmart with 4 Core Intel i7 processor clocked at 1.9GHz running Ubuntu Gnome 14.10 64-bit.

The expected growth of insertion sort as the number of elements ( $n$ ) grows large can be represented as  $\theta(n^2)$  where  $\theta()$  represents the asymptotically tightly bound running time. At  $n_0$  the algorithm took 0.0003 milliseconds to complete. At  $n_1$  the algorithm also took 0.0003 milliseconds to complete.

For both these values of  $n$ , the algorithm runs at a constant time because no for-loops are executed. As  $n$  grows larger the time to complete the experimental data correlates quite accurately with the expected growth. For  $n_{1*10^6}$  the data matches up perfectly. This is to be expected, because as  $n$  grows larger the constants and lower orders of the actual running time of the insertion sort start to affect the running time less and less as the highest order of  $n^2$  is so large. It is expected for lower values of  $n$  to not correlate well with  $\theta(n^2)$  because the system running the algorithm may be executing superfluous commands such as checking for system. For the data graphed above (see Fig. 1) this was in fact the case. To aid this issue, averages of 10 trails were taken and used in Figure 1.

## VI. CONCLUSIONS

Insertion sort is not the best choice for sorting large amounts of data because as  $n$  grows large, it takes insertion sort  $\theta(n^2)$  time to complete on modern day computers. Obviously, the faster the system executing insertion sort, the faster the algorithm will complete, however, with large enough values of  $n$ , it can take powerful systems quite a long amount of time to complete, as shown in Figure 1.

## REFERENCES

@Booktextbook, author = "Thomas H. Cormen and Charles E. Leiserson and Ronald L. Rivest and Clifford Stein", title = "Introduction to Algorithms", year = "2009", edition = "3", publisher = "The MIT Press", address = "Cambridge, Massachusetts"

@miscerik, author = "Erik Hartig", howpublished = "Conversation", month = "January", note = "Generic Programming", year = "2015"

@miscstopwatch, author = "Robert Sedgewick and Kevin Wayne", howpublished = "Java Class", title = "Stopwatch"

## APPENDIX

Listing 1  
TESTDRIVER

```
/*
 *      @Author Preston Stosur-Bassett
 *      @Date Jan 25, 2015
 *      @Class TestDriver
 *      @Description this class will create test data to run through and
 *      test the other classes in this directory.
 */

import java.util.ArrayList;

public class TestDriver {

    /*
     *      @Description This serves as the Driver function for this
     *      program, run this class to execute the program
     */
    public static void main(String args[]) {
        //Turn on debugger
        Debug debugger = new Debug();
        debugger.turnOn();

        //Test debug
        debugger.print("Debug is on.");

        //Create test data for sorts class
        //Note that this Array is for testing purposes only, the
        //Algorithm can handle all Comparable Generic Types
        ArrayList<Integer> testList = new ArrayList<Integer>();
        testList = DummyData.runArrayList(10, 0, 10, testList);
        //testList = DummyData.identicalElement(1000, 10,
        //testList);
        //ArrayList<String> testList = new ArrayList<String>();
        //testList = DummyData.runArrayList(1000, testList);

        System.out.println("Un-Sorted List");
        System.out.println(testList);

        //Test sort List
        Sort sorter = new Sort<Integer>();
        //Sort sorter = new Sort<String>();
        Stopwatch watchStopper = new Stopwatch();
        testList = sorter.insertion(testList);

        //Print out the results.
        System.out.println("Sorted List");
        System.out.println(testList);
    }
}
```

```

        System.out.println("Time to complete: "+watchStopper.
            elapsedTime());
    }
}

```

Listing 2  
DEBUG

```

/*
 *      @Author Preston Stosur-Bassett
 *      @Date Jan 21, 2015
 *      @Class Debug
 *      @Description This class will help debugging by being able to turn
 *      on and turn off debug messages easily
 */

import java.util.List;

public class Debug<T> {
    boolean debugOn; //Variable to keep track of whether or not debug
        is on

    /*
     *      @Description constructor method that sets the default
     *      value of debugOn to false so that debug statements will not
     *      automatically print
     */
    public void Debug() {
        debugOn = false;
    }

    /*
     *      @Description turn on debugging print statements
     */
    public void turnOn() {
        debugOn = true;
    }

    /*
     *      @Description turn off debugging print statements
     */
    public void turnOff() {
        debugOn = false;
    }

    /*
     *      @Description will print messages only when debugOn
     *      boolean is set to true
     *      @param String message the string to print when debugging
     *      is turned on
     */
}

```

```

public void print(T message) {
    if(debugOn == true) {
        System.out.println(message);
    }
}

/*
 *      @Pre-Condition <code>T expected</code> and <code>T actual
</code> are both of the same type T
 *      @Post-Condition If <code>T expected</code> and <code>T
actual</code> are found to be equal, the program moves on,
otherwise the program halts with <code>AssertionError</code>
is thrown
 *      @Description runs an assert statement against an expected
value and the actual value that are passed as parameters only
when <code>debugOn == true</code>
 *      @param T expected the expected value to assert against
the actual value
 *      @param T actual the actual value to assert against the
expected value
 */
public void assertEquals(T expected, T actual) {
    if(debugOn == true) {
        assert actual.equals(expected);
    }
}

/*
 * @Pre-Condition: <code>List<Integer> actual</code> is a iterable
list of Integer objects
 * @Post-Conditions: If the List of Integer objects is in
strictly non-decreasing order, the program moves on normally,
if not, the program halts with an <code>AssertionError</code>
 * @Description: runs an assertion statement against a list of
Integer objects to ensure that for <code>k = actual.size(); A[
k - 2] <= A[k - 1];</code>
 * @param List<Integer> actual the list to assert is in
strictly non-decreasing order
 */
public void assertOrder(List<Integer> actual) {
    if(debugOn == true) {
        int i = actual.size();
        while(i > 1) {
            assert actual.get(i - 1).compareTo(actual
                .get(i - 2)) >= 0;

            i--;
        }
    }
}

```



```
/*
 * @Description: asserts that the first arguement is stricly
 *               greator than the second arguement
 *       @param int large an integer primitive value to assert is
 *               strictly greator than the second arguement
 *       @param int small an integer primitive value to assert the
 *               first arguement is strictly greator than.
 */
public void assertStrictGreat(int large, int small) {
    if(debugOn == true) {
        assert large > small;
    }
}

/*
 *       @Description: asserts that the first arguement is
 *               strictly less than the second arguement
 *       @param int small an integer primitive value to assert is
 *               stricly less than the second arguement
 *       @param int large an integer primitive value to assert the
 *               first arguement is strictly less than.
 */
public void assertStrictLess(int small, int large) {
    if(debugOn == true) {
        assert small < large;
    }
}

/*
 *       @Description: asserts that the first arguement is greator
 *               than or equal to the second arguement
 *       @param int large an integer primitive value to assert is
 *               greator than or equal to the second arguement
 *       @param int small an integer primitive value to assert the
 *               first arguement is greator than or equal to.
 */
public void assertGreatEqual(int large, int small) {
    if(debugOn == true) {
        assert large >= small;
    }
}

/*
 *       @Description: asserts that the first arguement is less
 *               than or equal to the second arguement
 *       @param int small an integer primitive value to assert is
 *               less than or equal to the second arguement
 *       @param int large an integer primitive value to assert the
 *               first arguement is less than or equal to.
 */
```

```

    */
    public void assertLessEqual(int small, int large) {
        if(debugOn == true) {
            assert small <= large;
        }
    }
}

```

Listing 3  
DUMMYDATA

```

/*
 *      @Author Preston Stosur-Bassett
 *      @Date Jan 25, 2015
 *      @Class DummyData
 *      @Description This class contains methods to generate dummy data
 *      given a set of parameters.
 */

import java.util.ArrayList;
import java.util.Random;

public class DummyData {

    /*
     *      @Description runArrayList<Integer> will take an ArrayList
     *      of Integer Objects and add a given amount of values to it
     *      @param int end the ending value to denote when to stop
     *      adding to the array list
     *      @param int min the minimum value of the randomly
     *      generated data.
     *      @param int max the maximum value of the randomly
     *      generated data.
     *      @param ArrayList<Integer> list the list to add value to
     *      and return
     *      @return ArrayList<Integer> the list after it has been
     *      updated with the randomly generated data
     */
    public static ArrayList<Integer> runArrayList(int end, int min,
        int max, ArrayList<Integer> list) {
        Random random = new Random();
        Debug debugger = new Debug();
        int start = 0;
        // INVARIANT: A.length >= start
        // INITIALIZATION: start = 0, A.length can be longer than
        // 0 when initially passed, but not smaller, so our
        // invariant holds
        debugger.assertGreaterEqual(list.size(), start);
        while(start < end) {
            // MAINTANANCE: At the beginning of each
            // iteration, one element was added to A and

```

```

        start was increased by one, therefore, our
        invariant holds true.
        debugger.assertGreatEquals(list.size(), start);
        Integer intToAdd = new Integer(random.nextInt((
            max - min + 1) + min));
        list.add(intToAdd);

        //Count up on the iterator
        start++;
    }
    /*TERMINATION: The negation of the guard implies that (
        end - start) number of elements have been added to A,
        since start is initialized as 0 at the beginning of
        the method and is
            incremented by 1 each iteration of the loop,
            which means that start amount of elements have
            been added to A, and so our invariant holds
            true.    */
    debugger.assertGreatEquals(list.size(), start);

    return list;
}

/*
 *      @Description: runArrayList<String> will take an ArrayList
 *      of String Objects and add a given amount of String numerical
 *      values to it
 *      @param int end the ending value to denote when to stop
 *      adding to the array list
 *      @param ArrayList<String> list the list to add String
 *      values to and return
 *      @return ArrayList<String> the list after it has been
 *      updated with the randomly generated numerical String values
 */
public static ArrayList<String> runArrayList(int end, ArrayList<
    String> list) {
    Random random = new Random();
    Debug debugger = new Debug();
    int start = 0;
    // INVARIANT: A.length >= start
    // INITIALIZATION: Before the first iteration of the loop
    , start = 0 and A.length cannot be less than 0, so our
    invariant holds true
    debugger.assertGreatEquals(list.size(), start);
    while(start < end) {
        // MAINTENANCE: At the beginning of each
        iteration of the loop our invariant holds
        because for each iteration of the loop one
        element is added to A and start is incremented
        by 1

```

```

        debugger.assertGreatEquals(list.size(), start);
        Integer intToString = new Integer(random.nextInt
            ((1000000 - 1) + 1));
        String intString = String.valueOf(intToString);
        list.add(intString);

        //Count up on the iterator
        start++;
    }
    /* TERMINATION: The negation of the guard implies that (
        end - start) number of elements have been added to A,
        since start is initialized as 0 at the beginning of
        the method and is
            incremented by 1 each iteration of the
            loop, which means that start amount of
            elements have been added to A, and so
            our invariant holds true. */
    debugger.assertGreatEquals(list.size(), start);

    return list;
}

/*
 *      @Description: identicalElement will take an element and
 *      add it to the ArrayList<Integer> for a given amount of times
 *      @param int end the ending value to denote when to stop
 *      adding elements to the array
 *      @param int element the element to add over and over again
 *      to the array
 *      @param ArrayList<Integer> list the list to add elements
 *      to
 *      @return ArrayList<Integer> the list after it has been
 *      updated with the given data
 */
public static ArrayList<Integer> identicalElement(int end, int
    element, ArrayList<Integer> list) {
    // INVARIANT: A.length >= start
    int start = 0;
    Debug debugger = new Debug();
    //The element to add over and over again
    Integer iden = new Integer(element);
    // INITIALIZATION: Before the first iteration of the loop
    , start = 0 and A.length cannot equal anything less
    than 0, so our invariant holds true
    debugger.assertGreatEquals(list.size(), start);
    while(start < end) {
        // MAINTENANCE: At the beginning of each
        iteration of the loop our invariant holds
        because for each iteration of the loop one
        element is added to A and start is incremented

```

```

        by 1
        debugger.assertGreatEquals(list.size(), start);
        list.add(iden);

        //Count up on the iterator
        start++;
    }
    /* TERMINATION: The negation of the gaurd implies that (
       end - start) number of elements hav ebeen added to A,
       since start is initialied as 0 at the beginning of the
       method and is
           incremented by 1 each iteration of the
           loop, which means that start amount of
           elements have been added to A, and so
           our invariant holds true */
    debugger.assertGreatEquals(list.size(), start);

    return list;
}
}

```

The class Stopwatch has not been altered from its original form.

Listing 4  
STOPWATCH

```

/*****
 * Compilation:  javac Stopwatch.java
 *
 *
 *****/

/**
 * The <tt>Stopwatch</tt> data type is for measuring
 * the time that elapses between the start and end of a
 * programming task (wall-clock time).
 *
 * See {@link StopwatchCPU} for a version that measures CPU time.
 *
 * @author Robert Sedgewick
 * @author Kevin Wayne
 */

public class Stopwatch {

    private final long start;

    /**
     * Initialize a stopwatch object.
     */
}

```

```

public Stopwatch() {
    start = System.currentTimeMillis();
}

/**
 * Returns the elapsed time (in seconds) since this object was created.
 */
public double elapsedTime() {
    long now = System.currentTimeMillis();
    return (now - start) / 1000.0;
}
}

```

Listing 5  
SORT

```

/*
 * @Author: Preston Stosur-Bassett
 * @Date: Jan 24, 2015
 * @Class: Sort
 * @Description: This class will contain many methods that will sort
 * generic data types using common sorting algorithms.
 */

import java.util.ArrayList;
import java.util.List;

public class Sort<T extends Comparable<T>> {
    /*
     * @Pre-Condition: ArrayList<T> unsorted is an unsorted
     * ArrayList of a comparable data type that is non-empty
     * @Post-Condition: ArrayList<T> will return a permutation
     * of <code>unsorted</code> that will be in increasing order
     * @Description: Insertion will sort an ArrayList of generic
     * type T in increasing order using an insertion sort
     * @param ArrayList<T> unsorted is a non-empty unsorted
     * array list of T, where T is a comparable type
     * @return sorted is a permutation of <code>unsorted</code>
     * where all the elements are sorted in increasing order
     */
    // INVARIANT (Outer-Loop): The pre condition implies that sorted
    // [0 ... i - 1] will contain all the same data as unsorted[0 ...
    // i - 1].
    // INVARIANT (Inner-Loop): sorted[0 ... j] is sorted in strictly
    // non-decreasing order.
    public ArrayList<T> insertion(ArrayList<T> unsorted) {
        Debug debugger = new Debug<List<T>>();
        debugger.turnOn();
        ArrayList<T> sorted = unsorted;
    }
}

```

```

if(sorted.size() > 1) {
    int i = 1;
    /* INITIALIZATION (Outer-Loop): The invariant
       holds because i = 1, and there is one element
       in the subarray of sorted[0 ... i - 1] and
       unsorted[0 ... i - 1], */
    List<T> subSortedOI = sorted.subList(0, i - 1);
    List<T> subUnsortedOI = unsorted.subList(0, i -
        1);
    debugger.assertEquals(subUnsortedOI, subSortedOI)
        ;

    while(i < sorted.size()) {
        /* MAINTENANCE (Outer-Loop): At the
           beginning of each iteration of the
           loop, the loop invariant is maintained
           because the subarray of sorted[0 ...
           i - 1] contains all the same elements
           as
                unsorted[0 ... i - 1] */
        List<T> subSortedOM = sorted.subList(0, i
            - 1);
        List<T> subUnsortedOM = unsorted.subList
            (0, i - 1);
        debugger.assertEquals(subUnsortedOM,
            subSortedOM);

        T value = sorted.get(i);
        int j = i - 1;
        // INITIALIZATION (Inner-Loop): Before
           the first iteration of the loop, j =
           0, the subarray of sorted[0 ... 0]
           contains one elements and therefore
           the invariants holds vacuously.
        List subSortedII = sorted.subList(0, j);
        debugger.assertOrder(subSortedII);

        while(j >= 0 && (value.compareTo(sorted.
            get(j)) < 0)) {
            // MAINTENANCE: (Inner-Loop): At
               the beginning of each
               iteration sorted[0 ... j] is
               sorted in stricly non-
               decreasing order
            List subSortedIM = sorted.subList
                (0, j);
            debugger.assertOrder(subSortedIM)
                ;

            sorted.set(j+1, sorted.get(j));

```

```

        j--;
    }
    sorted.set(j+1, value);
    // TERMINATION (Inner-Loop): The negation
    // of the guard implies that the sorted
    // [0 ... j] has been traversed and is
    // stricly non-decreasing order.
    List subSortedIT = sorted.subList(0, j+1)
    ;
    debugger.assertOrder(subSortedIT);

    //Count up on the iterator
    i++;
}
/* TERMINATION (Outer-Loop): When the loop
   terminates, i is equal to sorted.size()
   meaning the entire array has been traversed
   and that the guard has been negated.
   The negation of the guard implies that
   sorted[0 ... i - 1] contains all the
   elements of unsorted[0 ... i - 1] */
List subSortedOT = sorted.subList(0, i - 1);
debugger.assertOrder(subSortedOT);
Integer integerI = new Integer(i);
Integer sortedSizeO = new Integer(sorted.size());
debugger.assertEquals(sortedSizeO, integerI);
debugger.assertEquals(unsorted, sorted);
}
return sorted;
}
}

```