CHAPTER 4: FILTERS

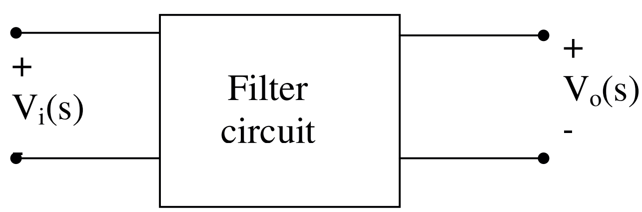
Filters are essential building blocks in many systems, particularly in communication and instrumentation systems. A filter passes one band of frequencies while rejecting another. Typically implemented in one of three technologies: passive RLC filters, active RC filters and switched-capacitor filters. Crystal and SAW filters are normally used at very high frequencies.

Passive filters work well at high frequencies, however, at low frequencies the required inductors are larges, bulky and non-ideal. Furthermore, inductors are difficult to fabricate in monolithic from and are incompatible with many modern assembly systems.

Active RC filters utilize op-amps together with resistors and capacitors and are fabricated using discrete, thick film and thin-film technologies. The performance of these filters is limited by the performance of the op-amps (e.g., frequency response, bandwidth, noise, offsets, etc.).

Switched-capacitor filters are monolithic filters which typically offer the best performance in the term of cost. Fabricated using capacitors, switched and op-amps. Generally poorer performance compared to passive LC or active RC filters.

Filters are generally linear circuits that can be represented as a two-port network:



The filter transfer function is given as follows:

$$T(j\omega) = T(s) = \frac{V_o(s)}{V_i(s)}$$

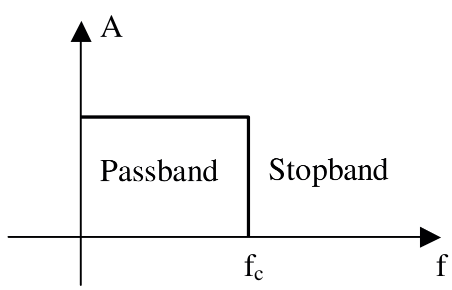
The magnitude of the transmission is often expressed in dB in terms of gain function:

$$G(\omega)_{dB}=20\log(|T(j\omega)|$$

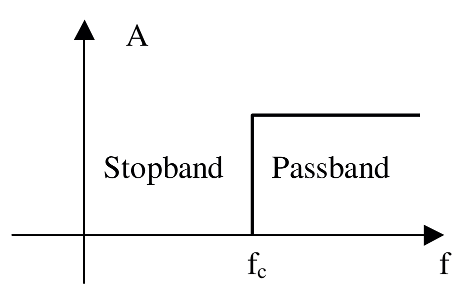
Or, alternatively, in terms of the attenuation function:

$$A(\omega)_{dB}=-20\log(|T(j\omega)|$$

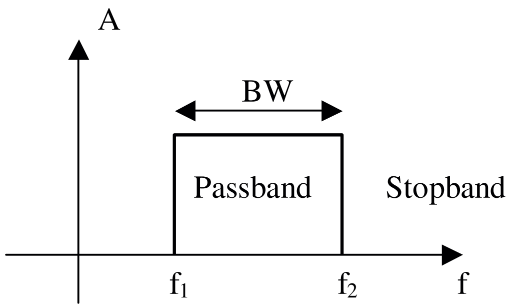
A filter shapes the frequency spectrum of the input signal, according to the magnitude of the transfer function. The phase characteristics of the signal are also modified as it passes through the filter. Filters can be classified into a number of categories based on which frequency bands are passes through and which frequency bands are stopped. Figures below show ideal responses of various filters.



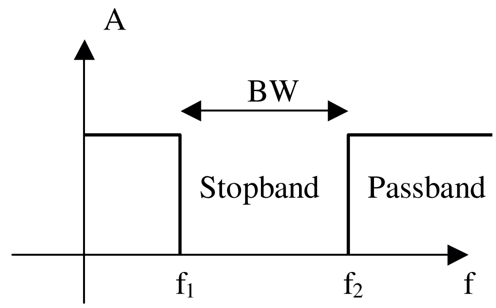
LOW-PASS FILTER RESPONSE



HIGH-PASS FILTER RESPONSE



BANDPASS FILTER RESPONSE



BANDSTOP FILTER RESPONSE

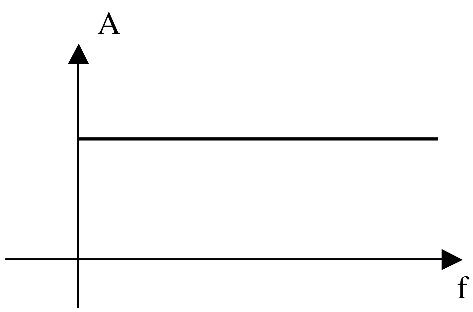
Center frequency

$$f_0 = \sqrt{f_1 f_2}$$

Quality factor Q (how fast the roll-off is)

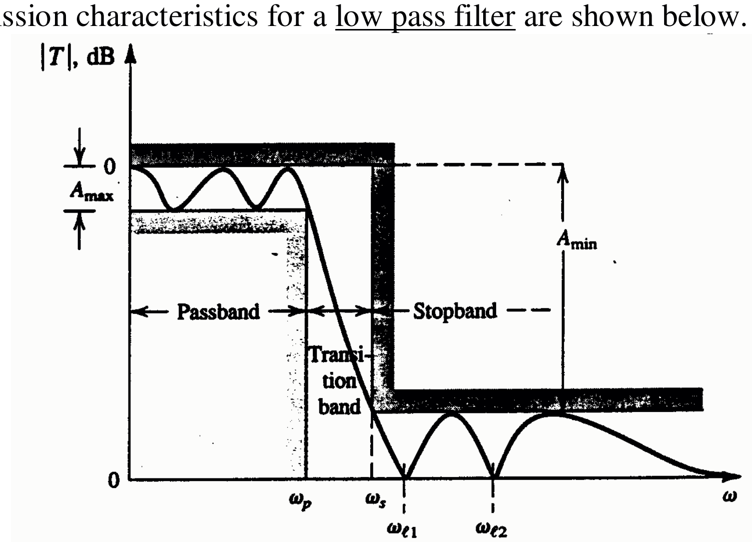
$$Q = \frac{f_0}{BW}$$

Wideband filter: Q < 1
Narrowband filter: Q > 1



ALLPASS FILTER RESPONSE

Ideal filters could not be realized using electrical circuits, therefore the actual response of the filter is not a brick wall response as shown above but increases or decreases with a roll-off factor. Realistic transmission characteristics for a low pass filter are shown below.



Transmission of a low pass filter is specified by four parameters:

- *Passband edge*, ω_p
- Maximum allowed variation in *passband transmission*, A_{max}
- *Stopband edge*, ω_s
- Minimum required *stopband attenuation*, A_{min}

The ratio ω_s/ω_p is usually used to measure the sharpness of the filter response and is called the *selectivity factor*. The more tightly one specifies a filter (i.e., lower A_{max}, higher A_{min}, ω_s/ω_p closer to unity) the resulting filter must be of higher order and thus more complex and expensive. A_{max} is commonly referred as the *passband ripple.*

The process of obtaining a transfer function that meets given specifications is known as filter approximation. Filter approximation is usually performed using computer programs or filter design tables. In simple cases, filter approximation can be performed using closed form expressions.

Figure below shows transmission specifications for a bandpass filter.

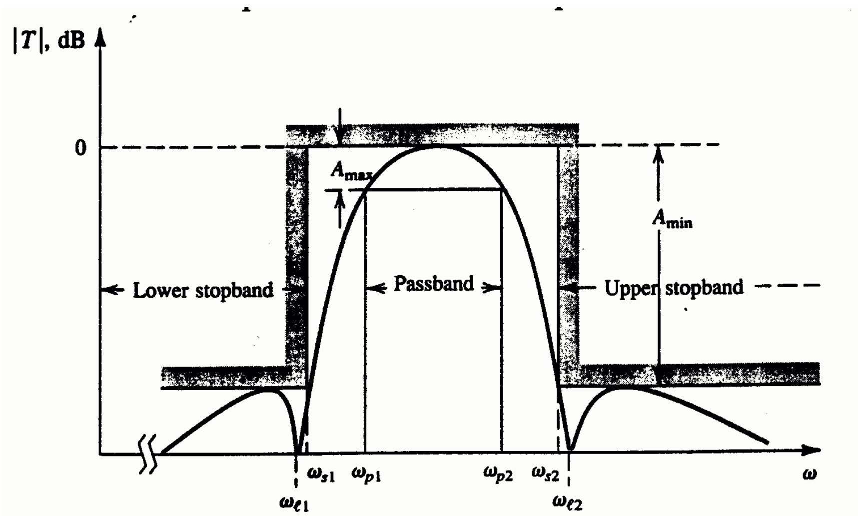
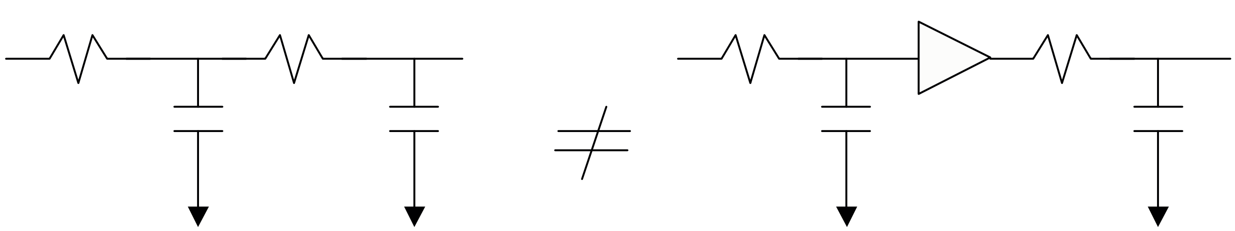


Fig. 11.4 Transmission specifications for a bandpass filter. The magnitude response of a filter that just meets specifications is also shown. Note that this particular filter has a monotonically decreasing transmission in the passband on both sides of the peak frequency.

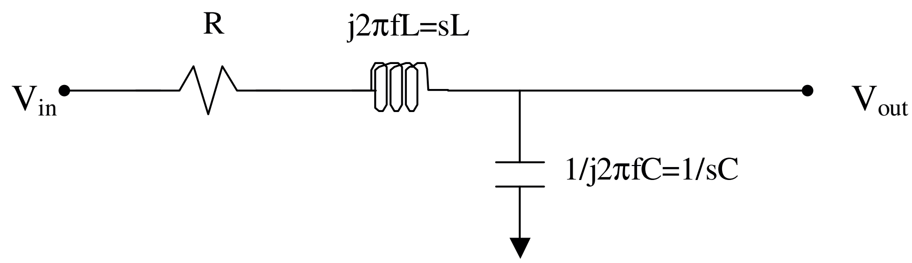
General notes:

- Each capacitive/inductive element adds a *pole* or *zero* to the frequency response.
- By adding additional poles/zeroes we can increase the roll-off of the filter response (e.g., two-pole lowpass filter would have a –40dB/decade roll-off at high frequency).
- In adding poles, we must take into account the loading of successive stages as shown



- Additional of buffer removes impedance matching effects.
- When capacitors are combined with inductors, it is possible to make circuits with very sharp response.

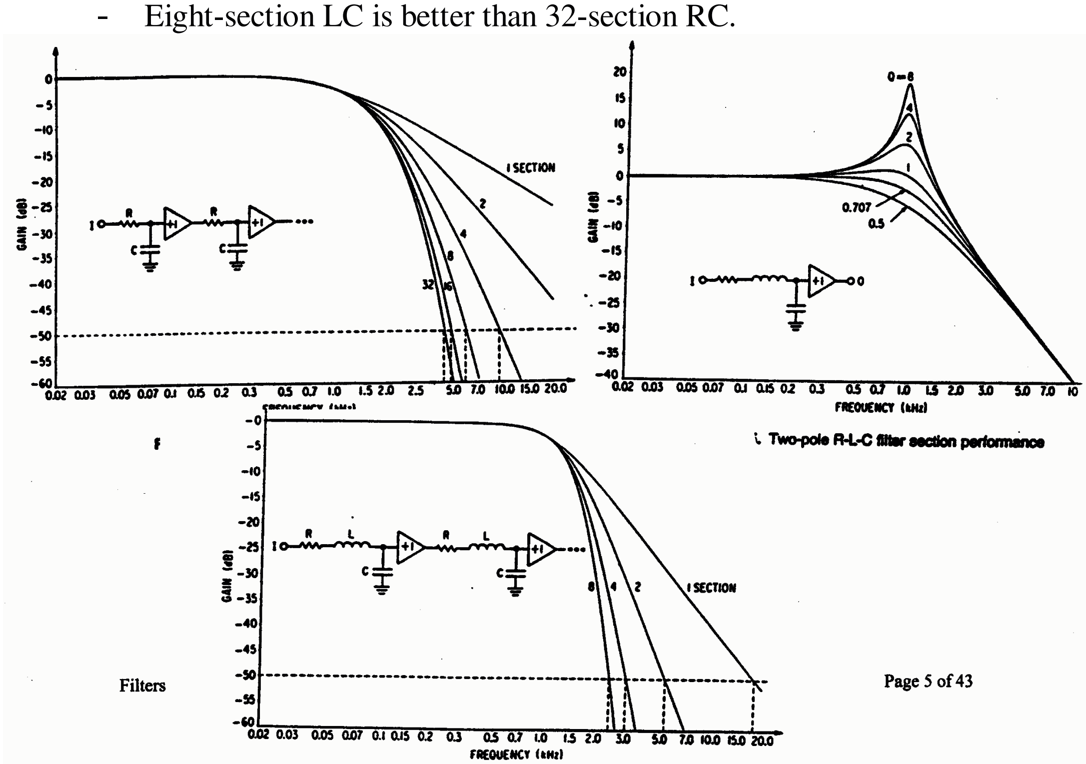
Example of a 2nd order LC filter:



$$T(s)=\frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}}=\frac{1}{s^2LC+sRC+1}=\frac{\omega_o^2}{s^2+\frac{\omega_o}{Q}s+\omega^2}$$

$$\omega_o=\frac{1}{\sqrt{LC}}\qquad \frac{\omega_0}{Q}=\frac{R}{L}\Rightarrow Q=\frac{L}{R}\omega_0=\frac{1}{R}\sqrt{\frac{L}{C}}$$

- Q of the circuit affects response.
- When Q=0.5, response is equivalent to 2 pole RC filter.
- *High Q pumps up gain below cutoff and increases slope after cut off.*
- Single RC filter has large passband ripple and shallow rolloff.
- Addition of RC stages increase rolloff, however, passband flatness not affected and using a large number of stages does not give sharp cutoff of first 50dB.
- Eight-section LC is better than 32-section RC.



- By adjusting the Q of each section of a filter, it is possible to achieve better response.
- For a two-section filter, we can increase the Q of the second stage so that the peak fills in the rounded area just beyond cutoff of the first section.
- The scheme improves the passband flatness just short of cutoff and allows us to reach ultimate slope just after cutoff. There are Q's which give optimum performance of the filter.
- How to select the Q to give optimum performance and what is the optimum response?

Butterworth response:

For a Butterworth filter, the optimum is *flat response in the passband* and steep slope soon after cutoff (maximum flat filter). This is done by combining low Q section and high Q section. The problem is similar to finding Fourier series of a square wave. The solution lies in the use of the Butterworth polynomial. Four and eight-pole Butterworth filters are common. Standard table can be used to find component values. Butterworth filters are also called maximum flat filter.

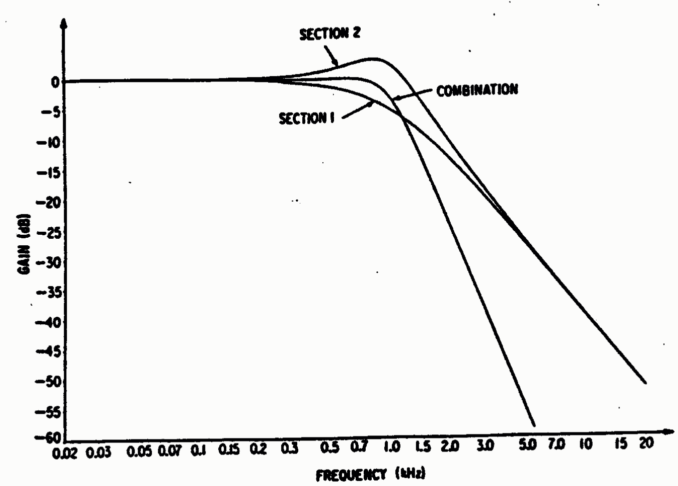


Fig. 12-17. Individual section response of two-section Butterworth

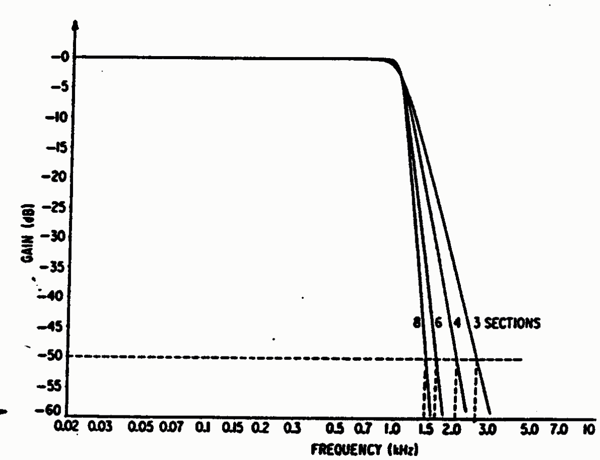


Fig. 12-18. Butterworth filter performance

Chebyshev response:

In some applications, the sharpness of the cutoff response is more important than the passband flatness. By adding higher resonant peaks, it is possible to obtain sharper cutoff at the expense of peaks in the passband.

The section cutoff frequencies and Q factors can be determined using Chebyshev polynomials. Whereas for the Butterworth filter, we only specify the number of poles or zeroes of the filter, for a Chebyshev filter, we specify the number of poles (zeroes) and passband flatness (i.e., a 0.5dB Chebyshev filter has a minimum peak 0.5dB above the minimum valley in the passband (equal–ripple filter).

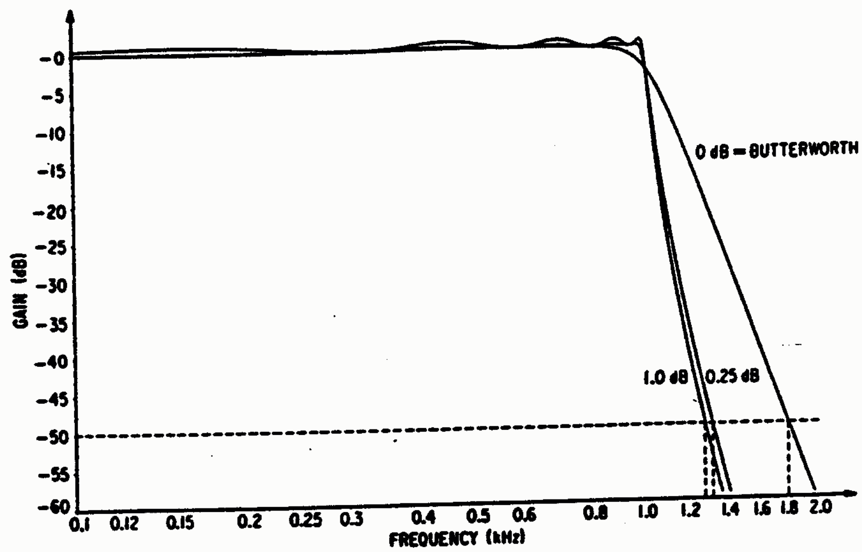


Fig. 12-19. Five-section Chebyshev filter response

Elliptical (Cauer) response:

The filter cutoff response can be improved further by following a basic low pass filter with a notch filter. The notch decreases the response just after cutoff. To be effective, notch has to be narrow and as a result, the overall response will start to increase just after the notch. To eliminate this, we add a number of notches until the original filter curve has dropped low enough. Elliptical filter is specified with three parameters: passband ripple, order and minimum stopband attenuation.

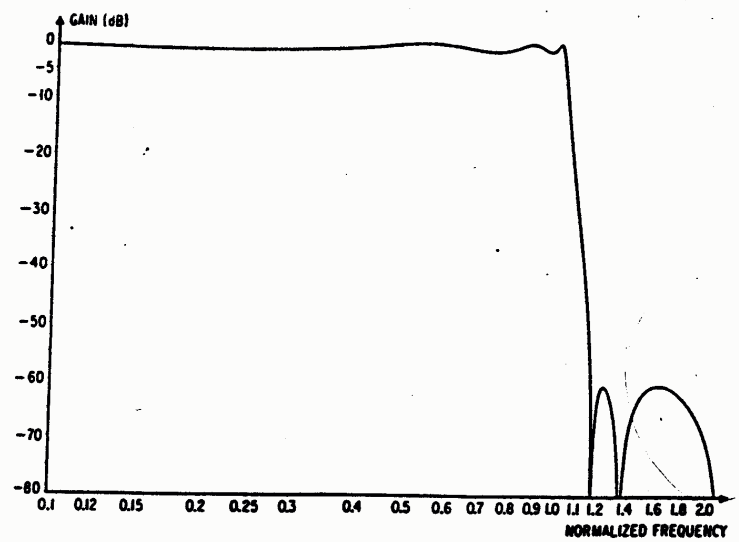


Fig. 12-20. Seventh order, 1.25-dB-ripple, 60-dB-attenuation elliptical filter amplitude response

Due to large number of options, it is difficult to find tables for Cauer filters.

Bessel (Thompson) response:

The sharper the filter cutoff, the worse the phase shift of the output signal right after cutoff (response after cutoff usually doesn't matter as the signal is attenuated). Poor phase response results in unequal delay which results in poor transient response for a low pass filter. This would normally result in output ringing when given a high frequency response. Bessel (Thompson) filter is a maximum flat filters which provides linear phase shift (i.e., equal delay). A Bessel filter can be used to compensate phase shift introduced by other parts of the system.

I. FILTER TRANSFER FUNCTION

The filter transfer function can be written as the ratio of two polynomials:

$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0}$$

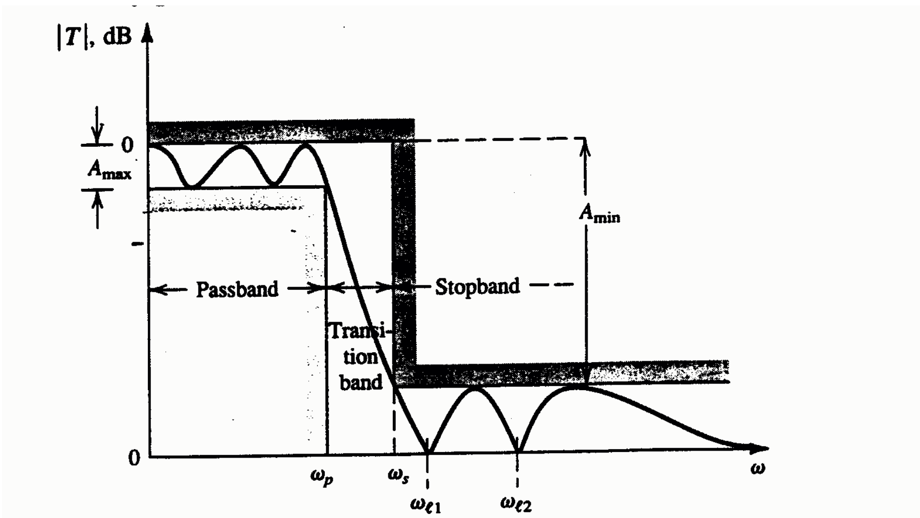
The degree of the denominator, N, is the filter order. For the filter to be stable, N≥M. The numerator and denominator coefficients are real numbers. The polynomials in the numerator and denominator can be factored and T(s) can be expressed in the form:

$$T(s) = \frac{a_M (s - Z_1)(s - Z_2) \cdots (s - Z_M)}{(s - P_1)(s - P2) \cdots (s - P_N)}$$

where the Z's are the zeroes and the P's are the poles of the filter.

Each zero and pole can be a real or complex number. Complex zeroes and poles, however, must occur in conjugate pairs. Thus if –1+j2 is a zero, then 1+j2 is also a zero.

Since in the filter stopband the transmission is required to be zero or small, the filter zeroes are usually placed on the jω axis at stopband frequencies.



This particular filter can be seen to have infinite attenuation (zero transmission) at two stopband frequencies ω_{l1} and ω_{l2}. The filter must have zeroes at s=+jω_{l1} and s=+jω_{l2}. However, since complex zeroes occur in conjugate pairs, there must also be zeroes at s=-jω_{l1} and s=-j jω_{l2}.

Thus the numerator polynomial of this filter will have the factors:

$$(s+j\omega_{l1})(s-j\omega_{l1})(s+j\omega_{l2})(s-j\omega_{l1})$$

which can be written as:

$$(s^2+\omega_{l1}^2)(s^2+\omega_{l2}^2)$$

for s=jω, the numerator becomes

$$(-\omega^2+\omega_{l1}^2)(-\omega^2+\omega_{l2}^2)$$

which is zero at $\omega=\omega_{l1}$ and $\omega=\omega_{l2}$.

It is also seen that the transmission decreases toward -∞ (i.e., 0dB) at ω approaches ∞, thus the filter must have one more zero at s=∞. In general, the number of zeroes at s=∞ is the difference between the degree of the numerator polynomial, M, and the degree of the denominator polynomial, N. This is because as s approach to ∞, T(s) approaches $\frac{a_M}{s^{N-M}}$ and thus is said to have N-M zeroes at s=∞.

For a filter circuit to be stable, _all its poles must lie in the left half of the s plane and thus all the poles must have negative real parts_. The figure below shows typical pole and zero location for the transmission function depicted in the figure above.

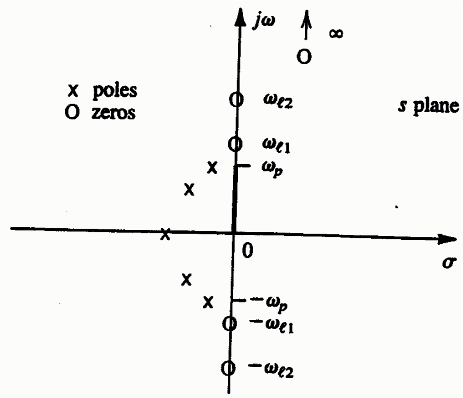


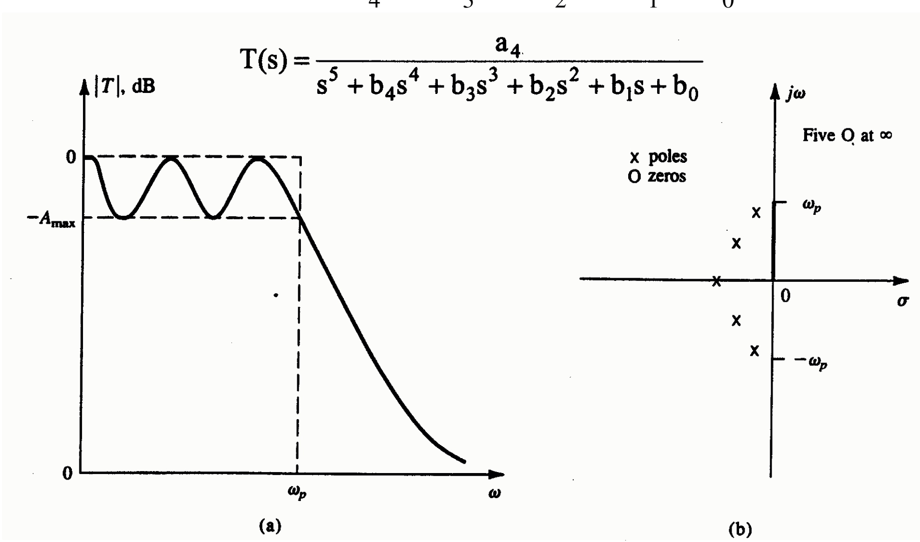
Fig. 11.5 Pole–zero pattern for the low-pass filter whose transmission is sketched in Fig. 11.3. This filter is of fifth order ($N = 5$).

This filter is a fifth order (N=5); it has two pairs of complex conjugate poles and one real-axis pole. All the poles lie in the vicinity of the passband which gives the filter its high transmission at passband frequencies. The transfer function of this filter is of the form:

$$T(s) = \frac{a_4(s^2 + \omega_{l1}^2)(s^2 + \omega_{l2}^2)}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

As a further example, consider the lowpass filter below; it is observed that there is no finite value of ω at which transmission is zero. Thus all zeroes are at s=∞. The filter transfer function is of the form

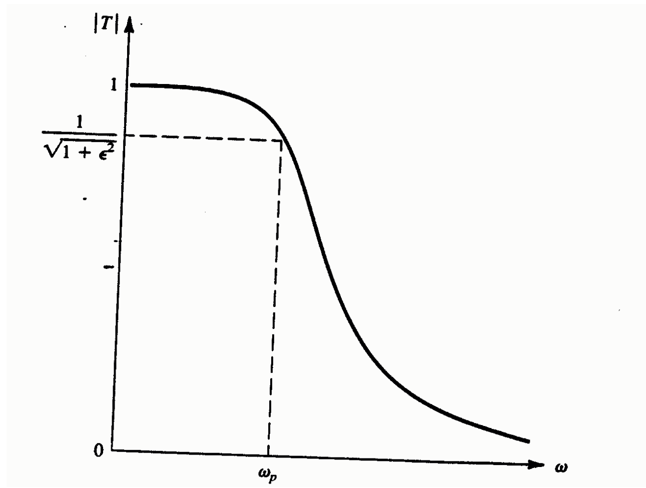
$$T(s) = \frac{a_4}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$



Such filter is known as all-pole filter.

II. <u>THE BUTTERWORTH FILTER</u>

Figure below shows the magnitude response of a Butterworth filter. The filter exhibits a monotonically decreasing transmission with all transmission zeroes at ω=∞, making it an all pole filter.



The magnitude function for an Nth-order Butterworth filter with a passband edge ω is given by:

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2(\frac{\omega}{\omega_p})^{2N}}}$$

at $\omega = \omega_p$, $$|T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}}$$

The parameter ε determines the maximum variation in passband transmission, A_{MAX} (i.e., passband ripple).

$$A_{MAX} = 20\log(\sqrt{1+\varepsilon^2} \quad \text{and} \quad \varepsilon = \sqrt{10^{\frac{A_{MAX}}{10}} - 1}$$

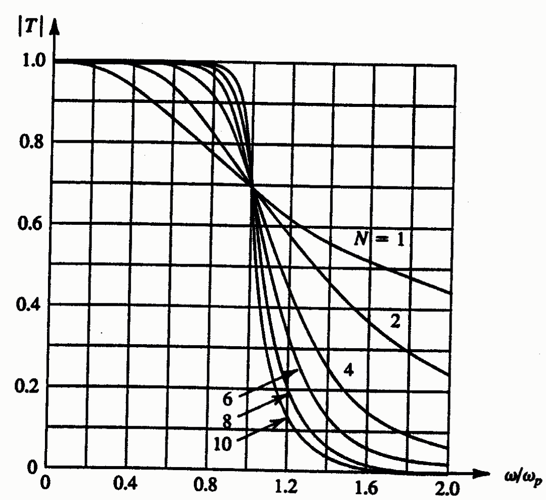


Fig. 11.9 Magnitude response for Butterworth filters of various order with ε = 1. Note that as the order increases, the response approaches the ideal brick-wall type transmission.

At the edge of the stopband, $\omega=\omega_s$,

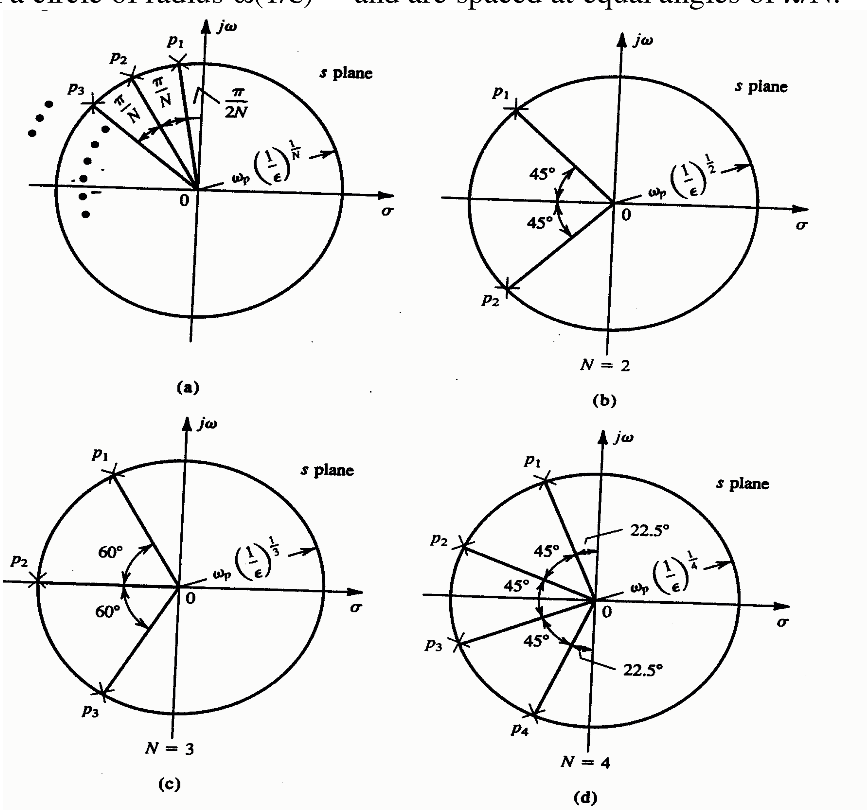
$$A(\omega_s) = -20\log\left(\frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}}}\right)$$

or

$$\boxed{A(\omega_s) = 10\log\left[1+\varepsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]}$$

<u>The last equation is used to determine the filter order required</u>, which is the lowest value of N that yields $A(\omega_s)\geq A_{MIN}$.

The poles of a Butterworth filter can be determined from the graphical construction below. The poles lie on a circle of radius $\omega(1/\varepsilon)^{1/N}$ and are spaced at equal angles of π/N.



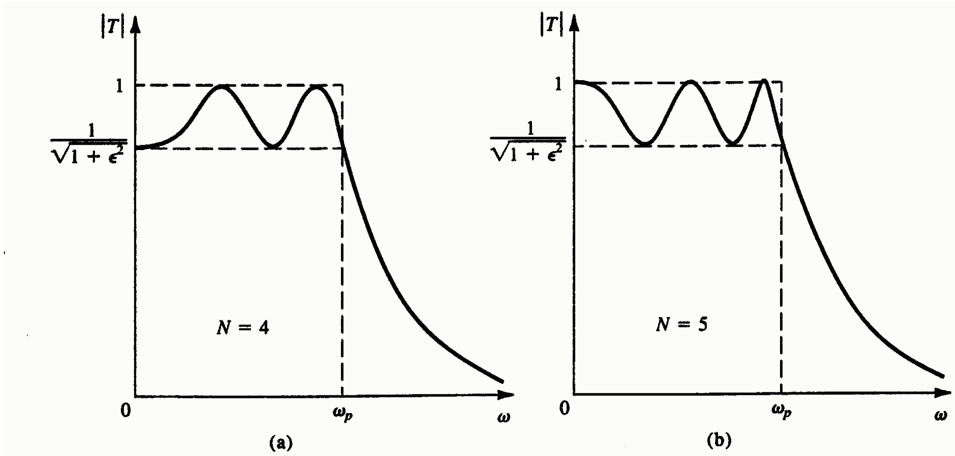
Since all poles have equal radial distance from the origin, they all have the same frequency $\omega_0=\omega_P(1/\varepsilon)^{1/N}$, i.e., all have the same Bode plot, just different Q. Once the poles are known, the transfer function can be written as:

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s=p_2)\cdots(s-p_N)}$$

where K is a constant equal to the required DC gain of the filter.

II. THE CHEBYSHEV FILTER

Figure below shows the representative transmission functions for Chebyshev filters of even (a) and odd (b) orders.



The Chebyshev filter exhibits an equal ripple response in the passband and a monotonically decreasing transmission in the stopband. *The total number of passband maxima and minima equals the order of the filter (i.e. number of poles).* All the transmission zeroes of the Chebyshev filter are at $\omega=\infty$ making it an all pole filter.

The magnitude of the transfer function of an N^{TH}-order Chebyshev filter with a passband edge ω_p is given by:

$$T(j\omega)=\frac{1}{\sqrt{1+\varepsilon^2\cos^2[N(\cos^{-1}(\frac{\omega}{\omega_p})]}}\qquad \text{where } \omega_p\geq\omega$$

and

$$T(j\omega)=\frac{1}{\sqrt{1+\varepsilon^2\cosh^2[N(\cosh^{-1}(\frac{\omega}{\omega_p})]}}\qquad \text{where } \omega\geq\omega_p$$

at the passband edge, $\omega=\omega p$, $\left|T(j\omega)\right|=\frac{1}{\sqrt{1+\varepsilon^2}}$

The parameter ε determines the passband ripple according to:

$$A_{MAX}=10\log(1+\varepsilon^2)\qquad \text{or}\qquad \varepsilon=\sqrt{10^{\frac{A_{MAX}}{10}}-1}$$

The attenuation achieved by the Chebyshev filter at the stopband edge ($\omega=\omega_s$) is:

$$\boxed{A(\omega_s)=10\log[1+\varepsilon^2\cosh^2(N\cosh^{-1}(\frac{\omega_s}{\omega_p})]}$$

This equation is used to find the order N of the Chebyshev filter for a specific ripple, ε, stopband edge, ω_s, passband edge, ω_p, and an attenuation $A(\omega_s)$ in dB.

The poles of a Chebyshev filter are given by:

$$p_k = -\omega_p \sin\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\cdot\sinh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right) + j\omega_p \cos\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\cdot\cosh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right)$$

The transfer function of the Chebyshev filter can be written as:

$$T(s) = \frac{K\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\cdots(s-p_N)}$$

As shown, the Chebyshev filter provides greater stopband attenuation than the Butterworth filter at the expense of passband ripple.

III. RLC FILTER DESIGN USING TABLE:

The following tables can be used to find the values of resistors, capacitors and inductors for various Chebyshev and Butterworth filters. The filters are design to have an impedance matching between the source and the load to obtain maximum power transfer. The given values are normalized (i.e., resistance 1Ω and frequency 1rad/sec), designers have to scale the components to fit the requirements of the system according to the following guide lines:

Let

k_m=system impedance (e.g., 50Ω, 75Ω)

$k_f=2\pi f_c$, where f_c is the cutoff frequency

R_N, L_N, C_N= normalized values from the table

Then

$$R_f = k_m R_N$$

$$L_f = \frac{k_m}{k_f} L_N$$

$$C_f = \frac{1}{k_m k_f} C_N$$

TABLE 14.3 Chebyshev lowpass element values (1-rad/s bandwidth)

n	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	R_2
				(A) Ripple width = 0.1 dB					
2	0.84304	0.62201							0.23781
3	1.03156	1.14740	1.03156						1.00000
4	1.10879	1.30618	1.77035	0.81807					0.73781
5	1.14681	1.37121	1.97500	1.37121	1.14681				1.00000
6	1.16811	1.40397	2.05621	1.51709	1.90280	0.86184			0.73781
7	1.18118	1.42281	2.09667	1.57340	2.09667	1.42281	1.18118		1.00000
8	1.18975	1.43465	2.11990	1.60101	2.16995	1.58408	1.94447	0.87781	0.73781
				(B) Ripple width = 0.5 dB					
3	1.5963	1.0967	1.5963						1.0000
5	1.7058	1.2296	2.5408	1.2296	1.7058				1.0000
7	1.7373	1.2582	2.6383	1.3443	2.6383	1.2582	1.7373		1.0000
				(C) Ripple width = 1.0 dB					
3	2.0236	0.9941	2.0236						1.0000
5	2.1349	1.0911	3.0009	1.0911	2.1349				1.0000
7	2.1666	1.1115	3.0936	1.1735	3.0936	1.1115	2.1666		1.0000
n	L_1'	C_2'	L_3'	C_4'	L_5'	C_6'	L_7'		R_2

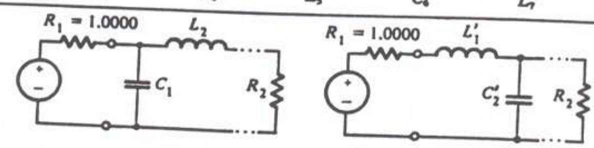
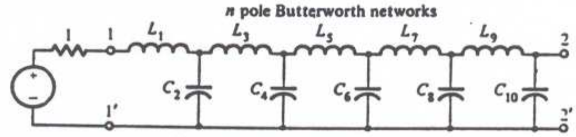


TABLE 14.5 n-pole Butterworth networks

n pole Butterworth networks



for n even, first element is L; n odd, C

n	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8	L_9	C_{10}
2	0.7071	1.414								
3	0.5000	1.333	1.500							
4	0.3827	1.802	1.577	1.531						
5	0.3090	0.8944	1.382	1.694	1.545					
6	0.2588	0.7579	1.202	1.553	1.759	1.533				
7	0.2225	0.6560	1.054	1.397	1.659	1.799	1.588			
8	0.1951	0.5776	0.9371	1.259	1.528	1.729	1.824	1.561		
9	0.1736	0.5155	0.8414	1.141	1.404	1.620	1.777	1.842	1.563	
10	0.1564	0.4654	0.7626	1.041	1.292	1.510	1.687	1.812	1.855	1.564
n	C_{10}	L_9	C_8	L_7	C_6	L_5	C_4	L_3	C_2	L_1

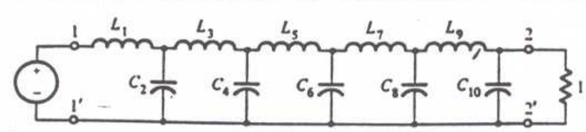


TABLE 14.7

Prototype (lowpass) elements	Highpass elements	Bandpass elements	Bandstop elements
L_p	$1/\omega_0 L_p$	L_p/bw bw/$\omega_0^2 L_p$	bwL_p/ω_0^2 1/bwL_p
C_p	$1/\omega_0 C_p$	bw/$C_p\omega_0^2$ C_p/bw	1/bwC_p bw/$C_p\omega_0^2$

will use a Butterworth response. To determine n, we make this calculation:

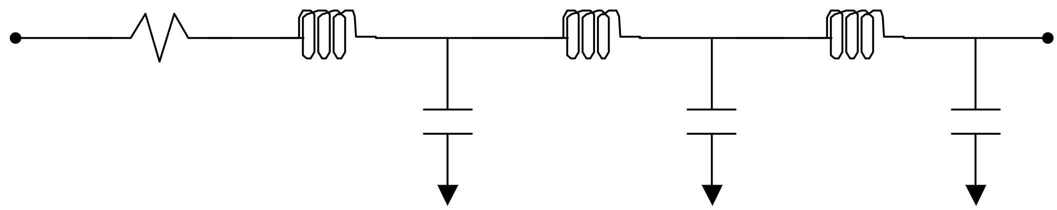
$$n = \frac{\log[(10^{20/10} - 1)/(10^{2/10} - 1)]}{2 \log 2} = 3.7 \text{ (round up to 4)} \qquad (14.54)$$

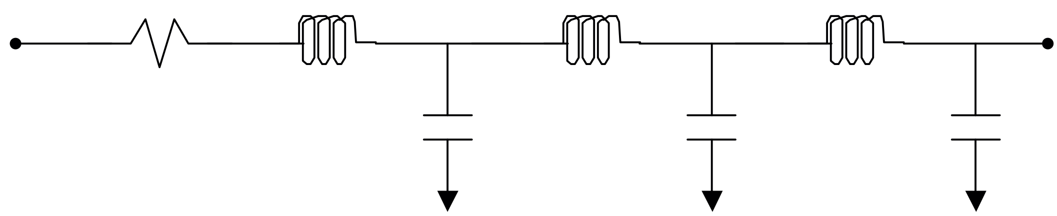
To find a suitable lowpass prototype circuit, we consult Table 14.5, which for $n = 4$ gives the circuit shown in Fig. 14.20. As given, the circuit elements are in henry (H), farads (F)

Example:

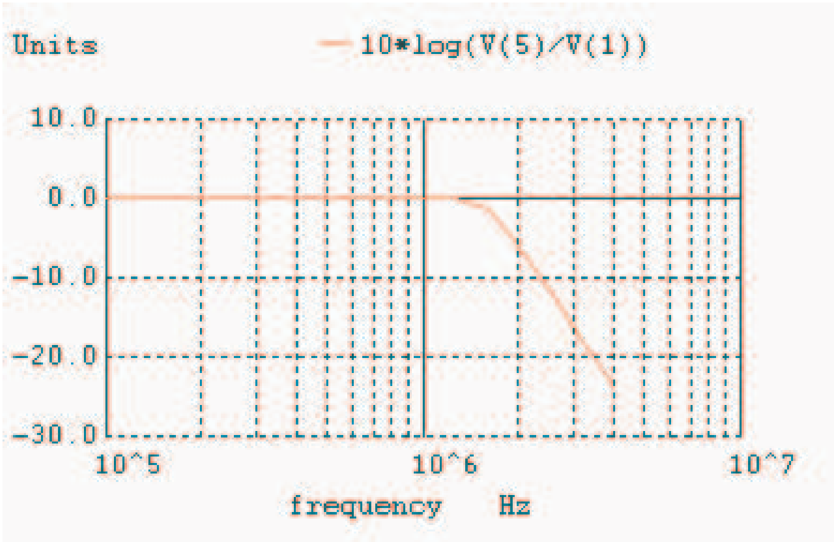
Design a RLC Butterworth filter for a wideband AM receiver using in a 50Ω system with the specifications given below:

- Stopband edge: f_s=2.8MHz.
- Passband edge, f_p,=1.6MHz.
- Attenuation $A(f_s)$=20dB
- Maximum ripple A_{MAX}=1dB.



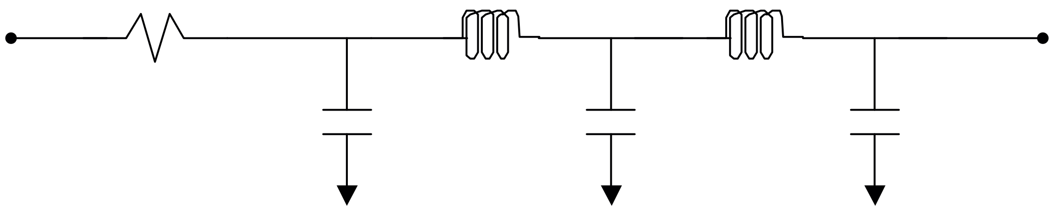


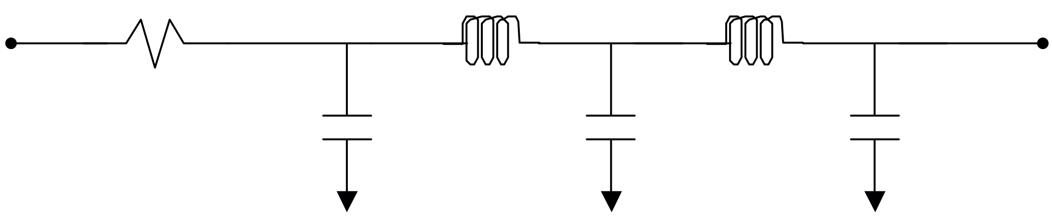
```
LPF
R1      1 2 50
L1      2 3 1.287uH
C1      3 0 1.508nF
L2      3 4 5.978uH
C2      4 0 3.089nF
L3      4 5 8.784uH
C3      5 0 3.05nF
VAC 1 0 AC 0.001
.AC DEC 10 100K 4MEG
.PRINT AC V(1) V(4) V(5) (V(5)/V(1))
.END
```



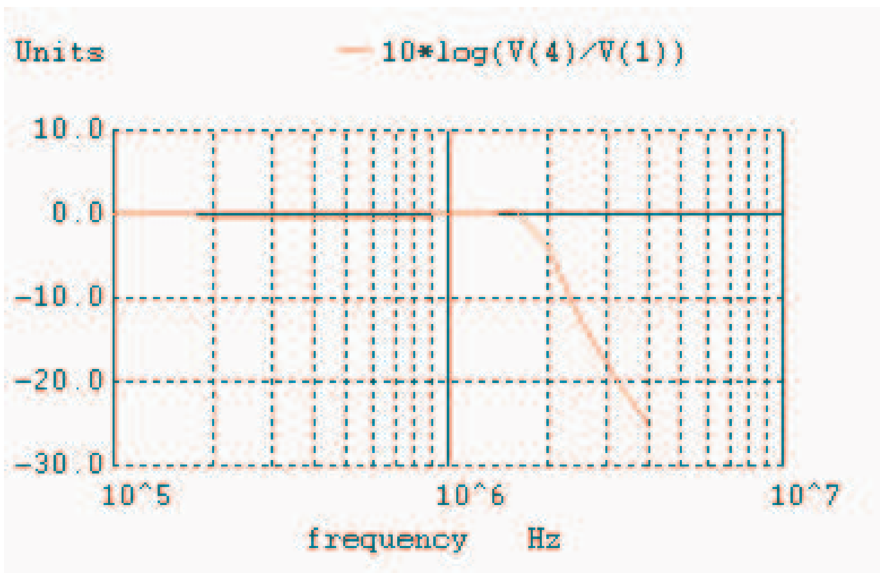
Example:

Design the above filter using Chebyshev filter, using an A_{MAX} of 0.1dB.





LPF (Chebyshev)
R1 1 2 50
L1 2 3 6.8198uH
C1 2 0 2.281nF
L2 3 4 6.8198uH
C2 3 0 3.929nF
C3 4 0 2.281nF
VAC 1 0 AC 0.001
.AC DEC 10 100K 4MEG
.PRINT AC V(1) V(4) (V(4)/V(1))
.END

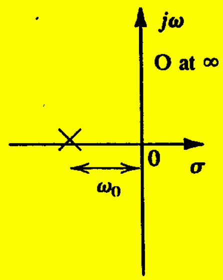
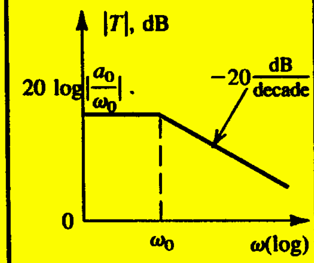
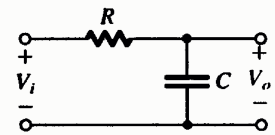
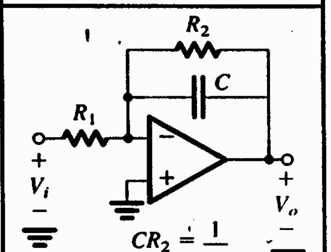
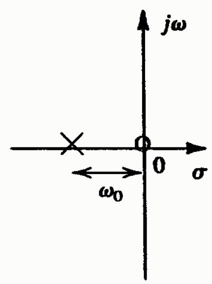
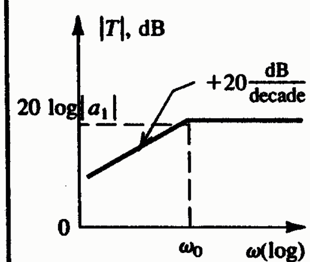
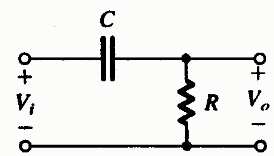
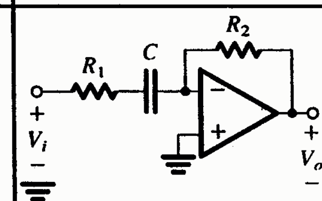
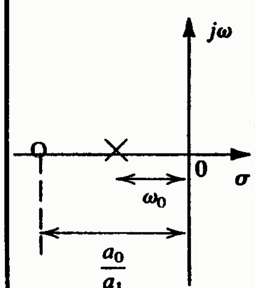
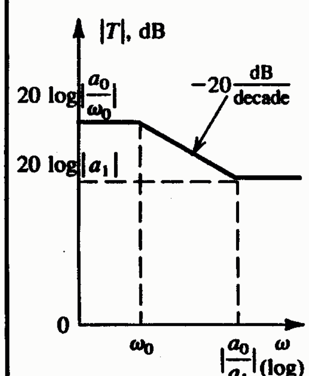
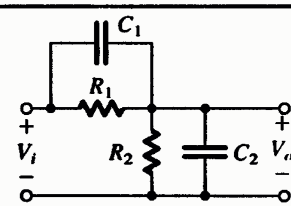
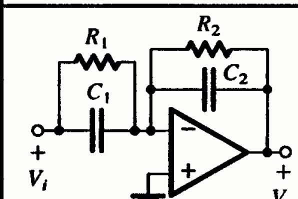


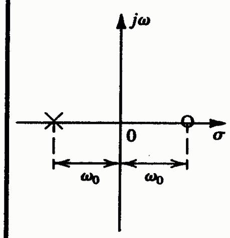
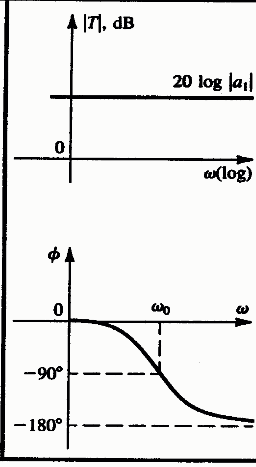
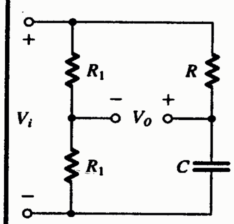
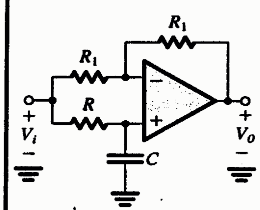
IV. FIRST ORDER FILTERS

The general first order (bilinear) transfer function is given by:

$$T(s) = \frac{a_1 s + a_0}{s + \omega_o}$$

The pole is at $s=-\omega_0$ and a zero at $s=-a_0/a_1$. The high frequency gain approached to a_1. The numerator coefficients (a_0,a_1) determine the type of filter (i.e., LP, HP, etc.).These filters are simple to design and they can be cascaded to form higher order filter. First order filters can be realized using RC or op-amp RC as shown in the figure below.

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T\rvert$	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ dc gain = 1	 $CR_2 = \dfrac{1}{\omega_0}$ dc gain $= -\dfrac{R_2}{R_1}$
(b) High-Pass (HP) $T(s) = \dfrac{a_1 s}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ High-frequency gain = 1	 $CR_1 = \dfrac{1}{\omega_0}$ High-frequency gain $= -\dfrac{R_2}{R_1}$
(c) General $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 \,//\, R_2) = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_0}{a_1}$ dc gain $= \dfrac{R_2}{R_1 + R_2}$ HF gain $= \dfrac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ dc gain $= -\dfrac{R_2}{R_1}$ HF gain $= -\dfrac{C_1}{C_2}$

$T(s)$	Singularities	$\lvert T\rvert$ and ϕ	Passive Realization	Op Amp-RC Realization
$T(s) = -a_1\dfrac{s-\omega_0}{s+\omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$

IV. SECOND ORDER FUNCTIONS

The general second order (bi-quadratic) filter transfer function is give by:

$$Ts) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

where ω_0 and Q determine the poles according to:

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1 - \frac{1}{4Q^2}}$$

We are usually interested in the case of complex conjugate poles obtained for Q>0.5.

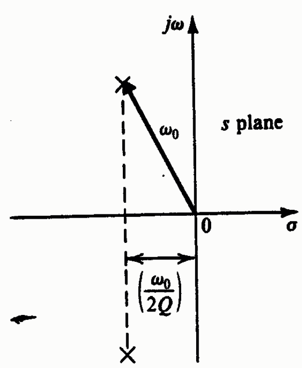


Fig. 11.15 Definition of the parameters ω_0 and Q of a pair of complex conjugate poles.

- The radial distance of the poles from the origin is the pole frequency (ω_0).
- Q is called the pole quality factor.
- The higher the value of the Q, the closer the poles are to the jω axis and the more selective (higher peak and initial roll-off) the filter response becomes.
- An infinite value of Q locates the poles on the jω axis and can yield sustained oscillations.

Followings are the transfer functions and responses of various 2^{nd} order filters.

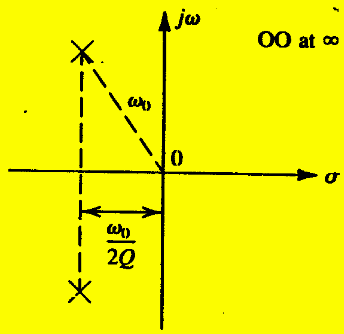
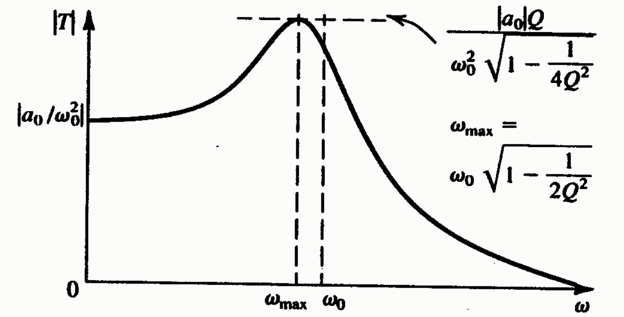
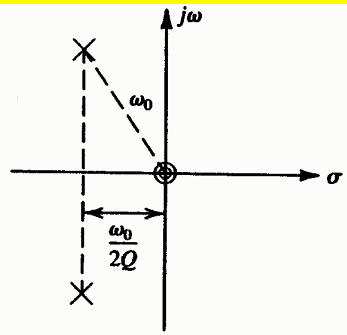
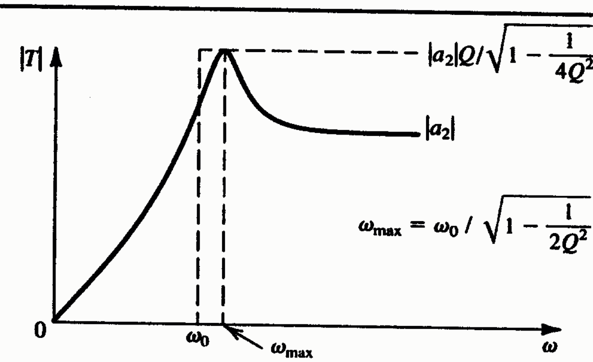
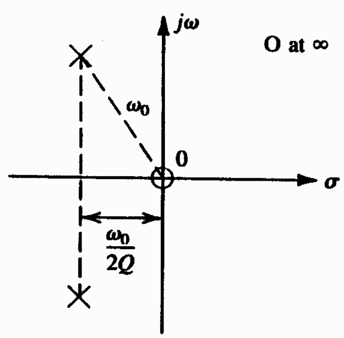
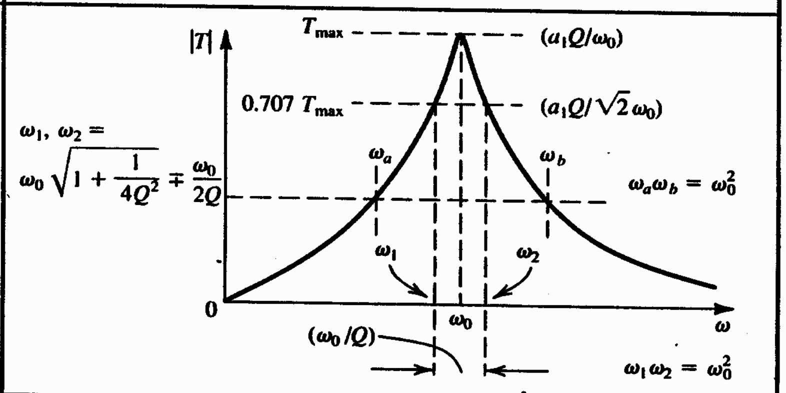
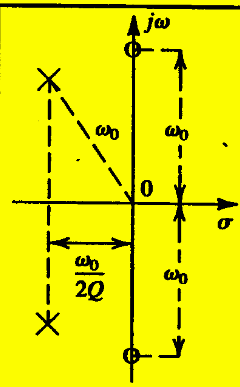
Filter Type and $T(s)$	s-Plane Singularities	$\|T\|$
(a) Low-Pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ dc gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High-Pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

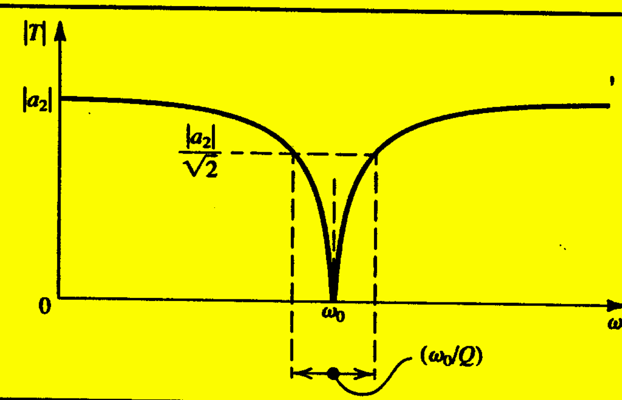
Fig. 11.16 Second-order filtering functions.

(d) Notch

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

dc gain = high-frequency gain = a_2





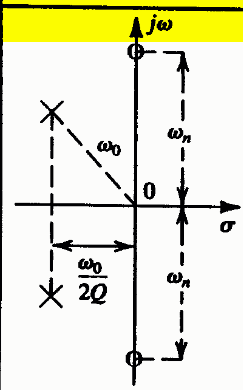
(e) Low-Pass Notch (LPN)

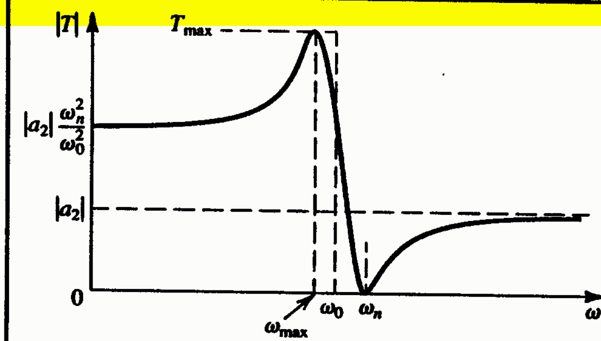
$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_n \geq \omega_0$

dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$

high-frequency gain = a_2





$$\omega_{max} = \omega_0 \sqrt{\frac{\left(\frac{\omega_n^2}{\omega_0^2}\right)\left(1 - \frac{1}{2Q^2}\right) - 1}{\left(\frac{\omega_n^2}{\omega_0^2}\right) + \frac{1}{2Q^2} - 1}}$$

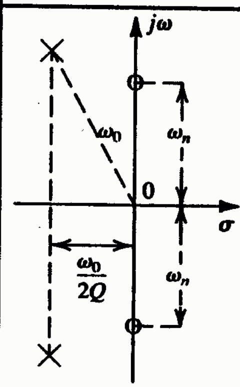
(f) High-Pass Notch (HPN)

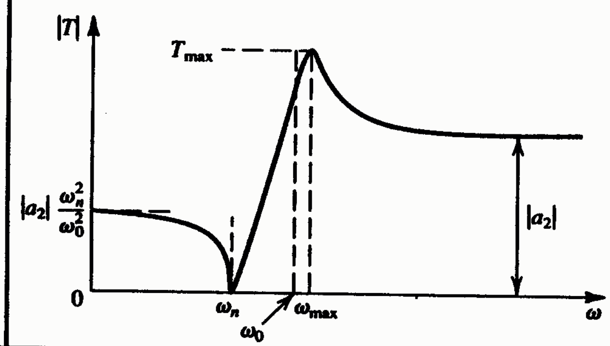
$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$\omega_n \leq \omega_0$

dc gain = $a_2 \frac{\omega_n^2}{\omega_0^2}$

high-frequency gain = a_2

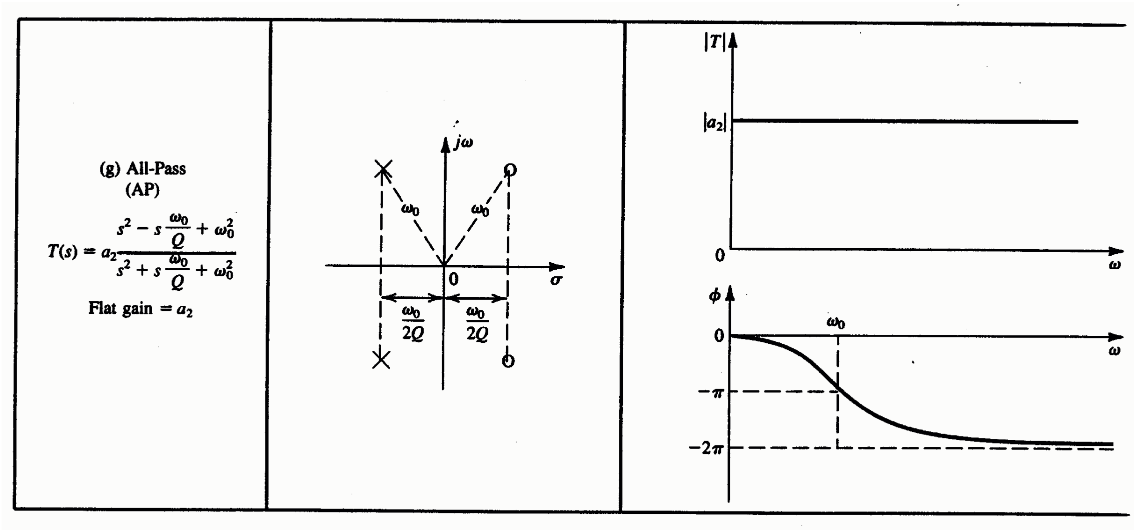




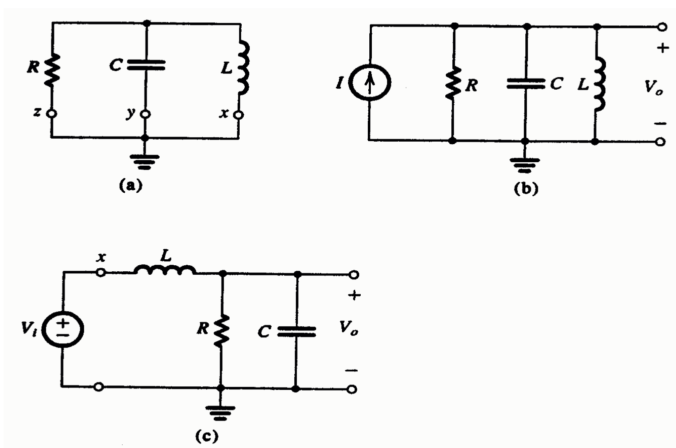
$$T_{max} = \frac{|a_2| \, |\omega_n^2 - \omega_{max}^2|}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$$

783

Fig. 11.16 (continued)



V. <u>SECOND ORDER LCR RESONATOR</u>



The resonator (a dead network) in figure (a) is excited by a current source I in figure (b), the output voltage V_o across the network is:

$$\frac{V_o}{I} = \frac{1}{Y} = \frac{1}{(\frac{1}{sL}) + sC + \frac{1}{R}} = \frac{\frac{s}{C}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

Equating the denominator to standard form of a second order response:

where $$s^2 + s\frac{\omega_o}{Q} + \omega_o^2$$

$$\omega_o^2 = \frac{1}{LC} \quad \Rightarrow \quad \omega_o = \frac{1}{\sqrt{LC}} \quad \text{and} \quad \frac{\omega_o}{Q} = \frac{1}{RC} \quad \Rightarrow \quad Q = \omega_o RC$$

The realization of various second-order filter functions using the LCR resonator is shown in figure below.

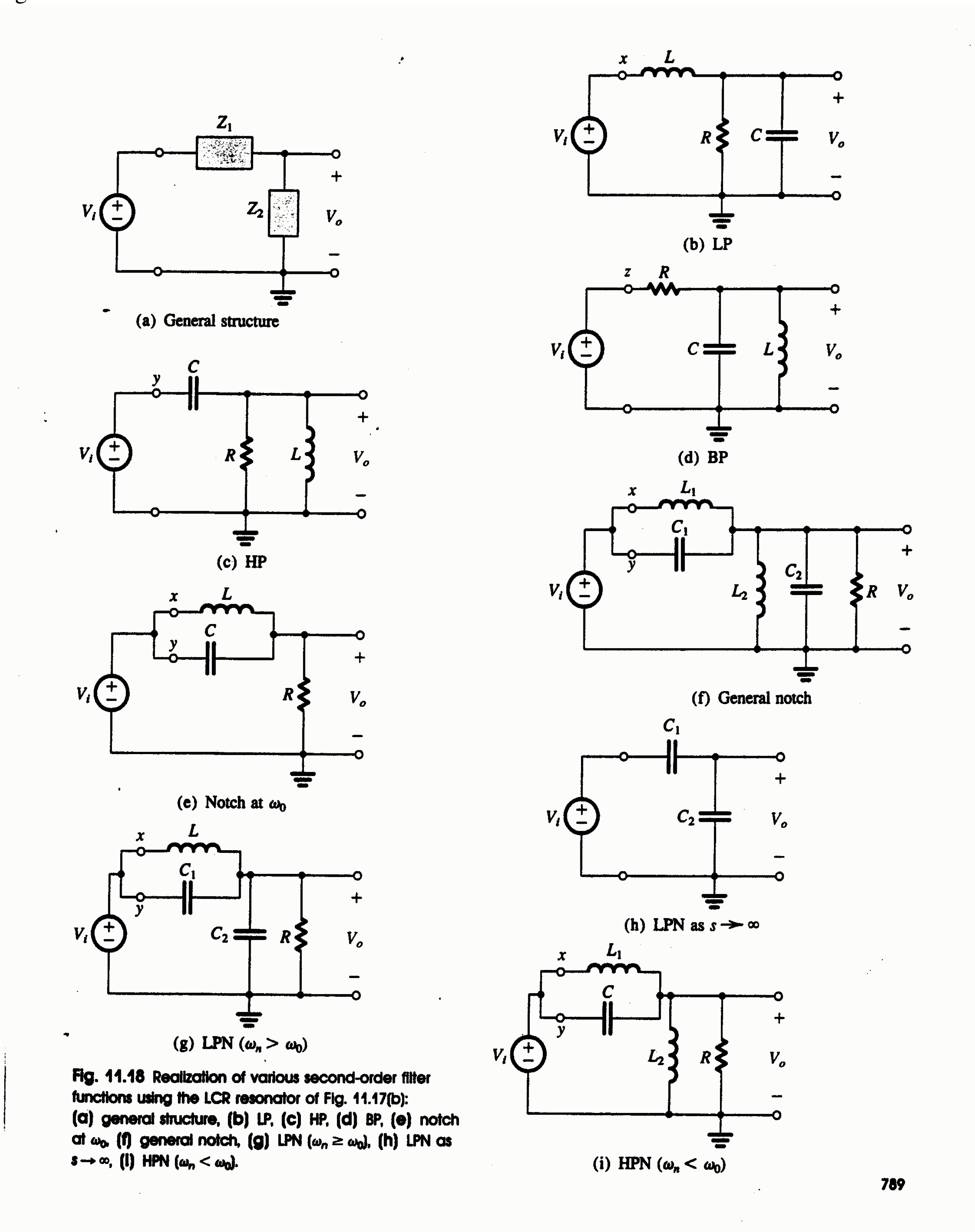
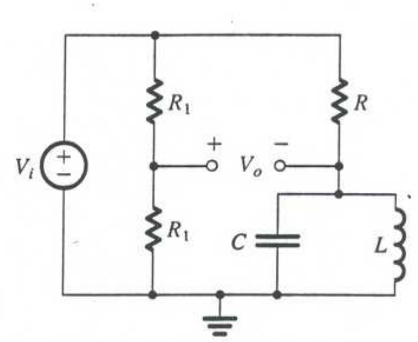


Fig. 11.18 Realization of various second-order filter functions using the LCR resonator of Fig. 11.17(b): (a) general structure, (b) LP, (c) HP, (d) BP, (e) notch at ω_0, (f) general notch, (g) LPN ($\omega_n \geq \omega_0$), (h) LPN as $s \to \infty$, (i) HPN ($\omega_n < \omega_0$).



Transfer functions of the filters can be derived as:

$$\text{LP} \quad T(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$\text{HP} \quad T(s) = \frac{s^2}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$\text{BP} \quad T(s) = \frac{s\frac{1}{RC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

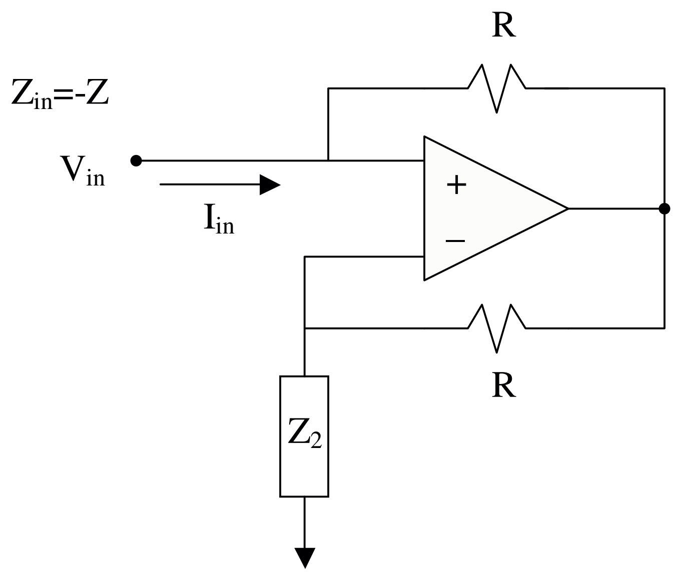
$$\text{Notch} \quad T(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$\text{AP} \quad T(s) = 0.5 - \frac{s(\frac{\omega_o}{Q})}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

VI. ACTIVE FILTERS

A family of op-amp RC circuits can realize the various second order filter functions by replacing the inductor L in the LCR resonator. These op-amp RC circuits have inductive input impedances.

Many op-amp RC circuits have been proposed for simulating the operation of an inductor. One of the simplest is the Negative Impedance Converter (NIC).



Replace Z_2 by a capacitor, this circuit converts a capacitor to a "backward" inductor.

$$Z_C = \frac{1}{j\omega C} \qquad I_{in} = -\frac{V_{in}}{Z_C} = -V_{in}\, j\omega C$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{-V_{in}\, j\omega C} = -\frac{1}{j\omega C} = \frac{j\omega}{\omega^2 C}$$

This is equivalent to an inductor of a value of $1/\omega^2 C$.

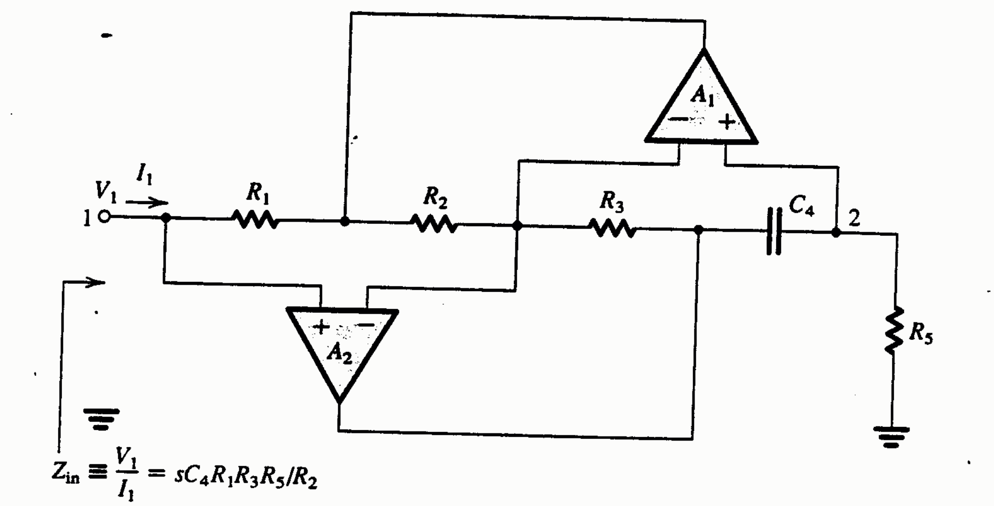
One of the best circuits for simulating an inductor is the Antoniou inductance simulation circuit. This circuit is very tolerance to the non-ideal properties of the op-amps. The circuit is shown and analyzed in the following figure.

The effective inductance of this circuit is:

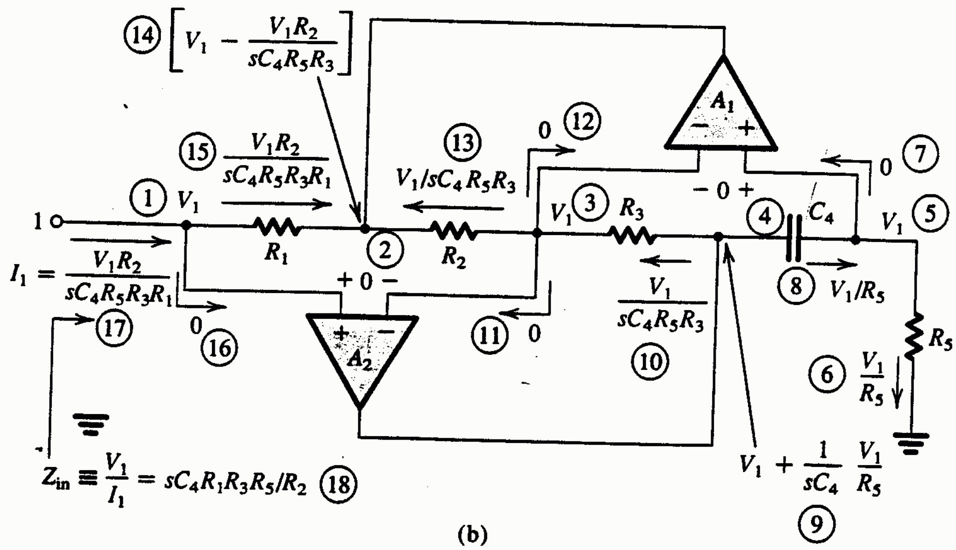
$$L = \frac{C_4 R_1 R_3 R_5}{R_2}$$

Typically, R_1= R_2= R_3= R_5=R and C_4=C, then

$$L=CR^2$$



(a)



(b)

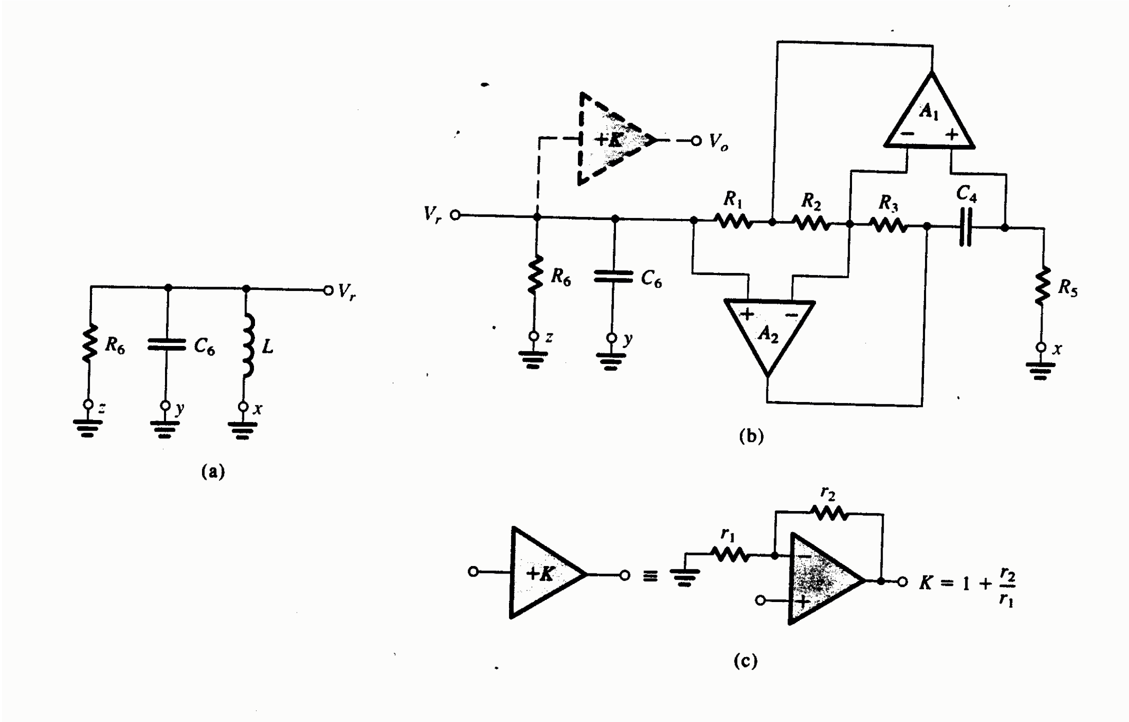
The Antoniou inductor simulating circuit is replacing the inductor in the LCR resonator as shown in figure below and the circuit parameter can be derived as:

$$\omega_o = \frac{1}{\sqrt{LC_6}} = \frac{1}{\sqrt{\frac{C_4C_6R_1R_3R_5}{R_2}}}$$

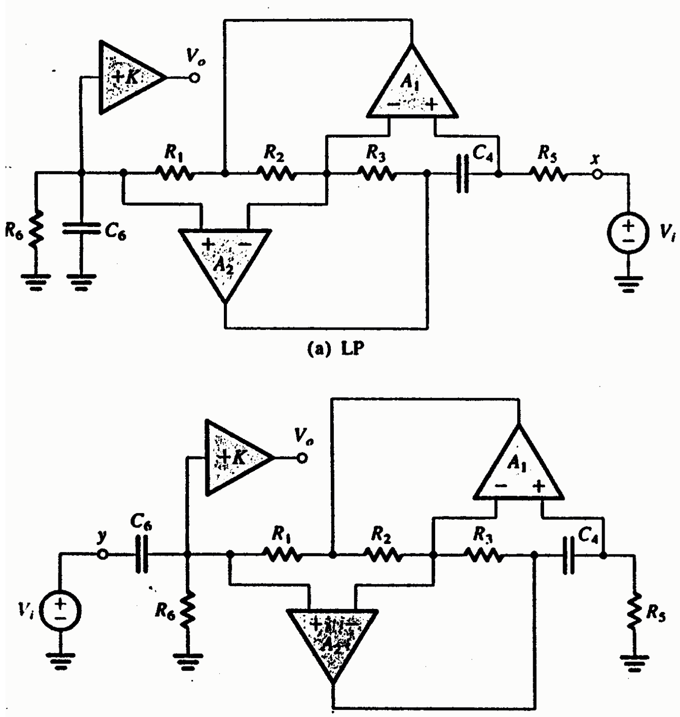
$$Q = \omega_o C_6 R_6 = R_6\sqrt{\frac{C_6}{C_4}\cdot\frac{R_2}{R_1R_3R_5}}$$

$$\text{If} \quad C_4 = C_6 = C \quad R_1 = R_3 = R_5 = R$$

$$\text{then} \quad \omega_o = \frac{1}{RC} \quad Q = \frac{R_6}{R}$$



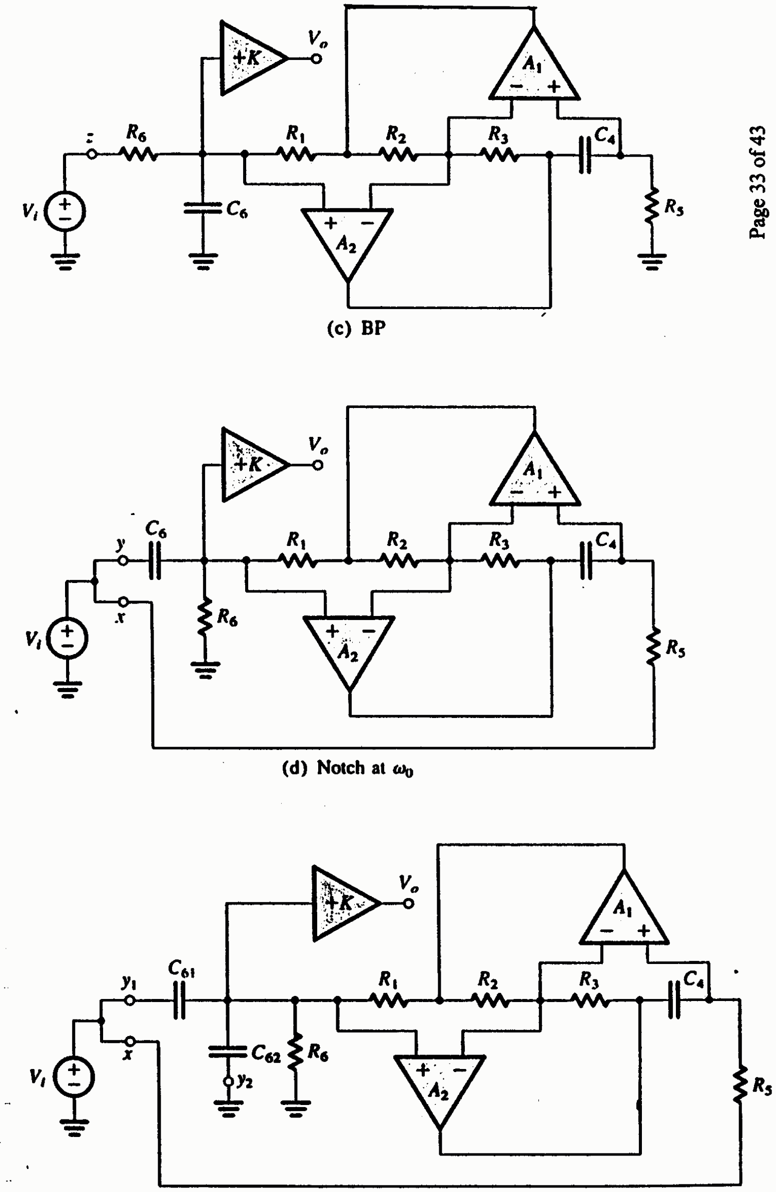
Various filters using LCR second order resonator realization using the inductor simulating circuits with design data are shown in the followings.



(a) LP

(b) HP

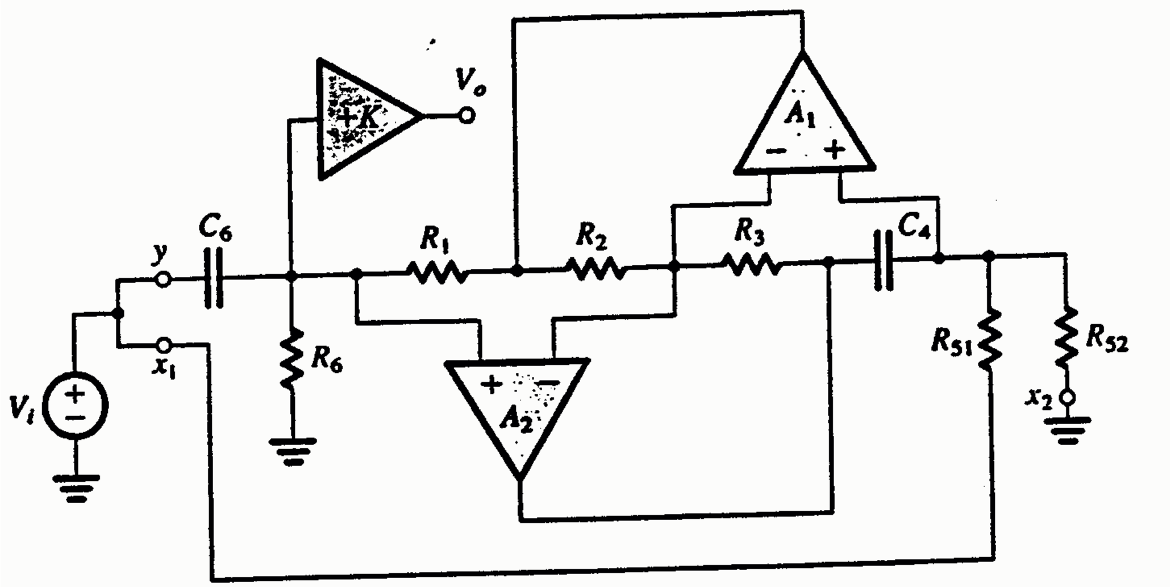
Fig. 11.22 Realizations for the various second-order filter functions using the op amp–RC resonator of Fig. 11.21(b). (a) LP; (b) HP; (c) BP, (d) notch at ω_0; (e) LPN, $\omega_n \geq \omega_0$; (f) HPN, $\omega_n \leq \omega_0$; (g) all-pass. The circuits are based on the LCR circuits in Fig. 11.18. Design equations are given in Table 11.1.



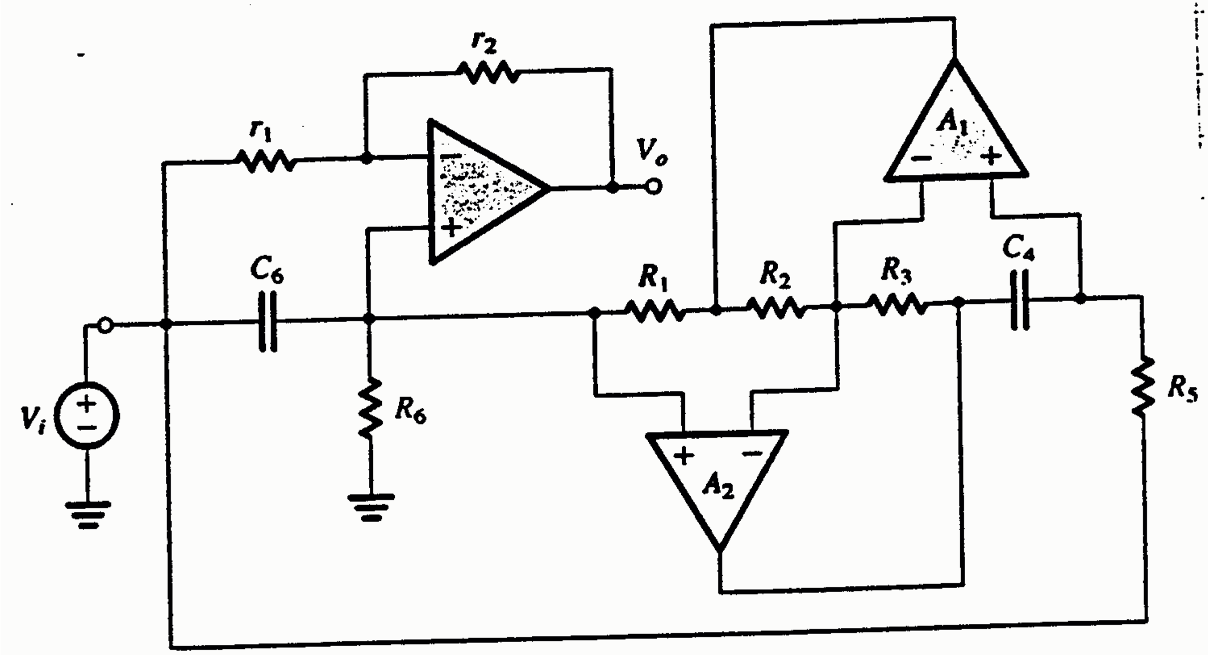
(c) BP

(d) Notch at ω_0

(e) LPN, $\omega_n \geq \omega_0$



(f) HPN, $\omega_n \leq \omega_0$



(g) All-pass

Table 11.1 DESIGN DATA FOR THE CIRCUITS OF FIG. 11.22

CIRCUIT	TRANSFER FUNCTION AND OTHER PARAMETERS	DESIGN EQUATIONS
Resonator Fig. 11.21(b)	$\omega_0 = 1/\sqrt{C_4C_6R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}}$	$C_4 = C_6 = C$ (practical value) $R_1 = R_2 = R_3 = R_5 = 1/\omega_0 C$ $R_6 = Q/\omega_0 C$
Low-pass (LP) Fig. 11.22(a)	$T(s) = \dfrac{KR_2/C_4C_6R_1R_3R_5}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = dc gain
High-pass (HP) Fig. 11.22(b)	$T(s) = \dfrac{Ks^2}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = High-frequency gain
Bandpass (BP) Fig. 11.22(c)	$T(s) = \dfrac{Ks/C_6R_6}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = Center-frequency gain
Regular notch (N) Fig. 11.22(d)	$T(s) = \dfrac{K[s^2 + (R_2/C_4C_6R_1R_3R_5)]}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$	K = Low- and high-frequency gain
Low-pass notch (LPN) Fig. 11.22(e)	$T(s) = K\dfrac{C_{61}}{C_{61} + C_{62}}$ $\times \dfrac{s^2 + (R_2/C_4C_{61}R_1R_3R_5)}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4(C_{61} + C_{62})R_1R_3R_5}}$ $\omega_n = 1/\sqrt{C_4C_{61}R_1R_3R_5/R_2}$ $\omega_0 = 1/\sqrt{C_4(C_{61} + C_{62})R_1R_3R_5/R_2}$ $Q = R_6\sqrt{\frac{C_{61} + C_{62}}{C_4}\frac{R_2}{R_1R_3R_5}}$	K = dc gain $C_{61} + C_{62} = C_6 = C$ $C_{61} = C(\omega_0/\omega_n)^2$ $C_{62} = C - C_{61}$
High-pass notch (HPN) Fig. 11.22(f)	$T(s) = K\dfrac{s^2 + (R_2/C_4C_6R_1R_3R_{51})}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$ $\omega_n = 1/\sqrt{C_4C_6R_1R_3R_{51}/R_2}$ $\omega_0 = \sqrt{\frac{R_2}{C_4C_6R_1R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$ $Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$	K = High frequency gain $\frac{1}{R_{51}} + \frac{1}{R_{52}} = \frac{1}{R_5} = \omega_0 C$ $R_{51} = R_5(\omega_0/\omega_n)^2$ $R_{52} = R_5/[1 - (\omega_n/\omega_o)^2]$
All-pass (AP) Fig. 11.22(g)	$T(s) = \dfrac{s^2 - s\frac{1}{C_6R_6}\frac{r_2}{r_1} + \frac{R_2}{C_4C_6R_1R_3R_5}}{s^2 + s\frac{1}{C_6R_6} + \frac{R_2}{C_4C_6R_1R_3R_5}}$ $\omega_z = \omega_0 \quad Q_z = Q(r_1/r_2)$ Flat gain = 1	$r_1 = r_2 = r$ (arbitrary) Adjust r_2 to make $Q_z = Q$.

VII. ACTIVE FILTER BASED ON TWO-LOOP INTEGRATOR (THE BIQUAD)

This is an op-amp RC circuit that realizes second order filter functions based on the use of two integrators connected in cascade in an overall feedback loop.

Consider a second order HPF,

$$T(s) = \frac{V_{HP}}{V_i} = \frac{Ks^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

where K is the high frequency gain. Rearranging the equation gives:

$$V_{HP} + \frac{1}{Q}\left(\frac{\omega_o}{s}V_{HP}\right) + \left(\frac{\omega_o^2}{s^2}V_{HP}\right) = KV_i$$

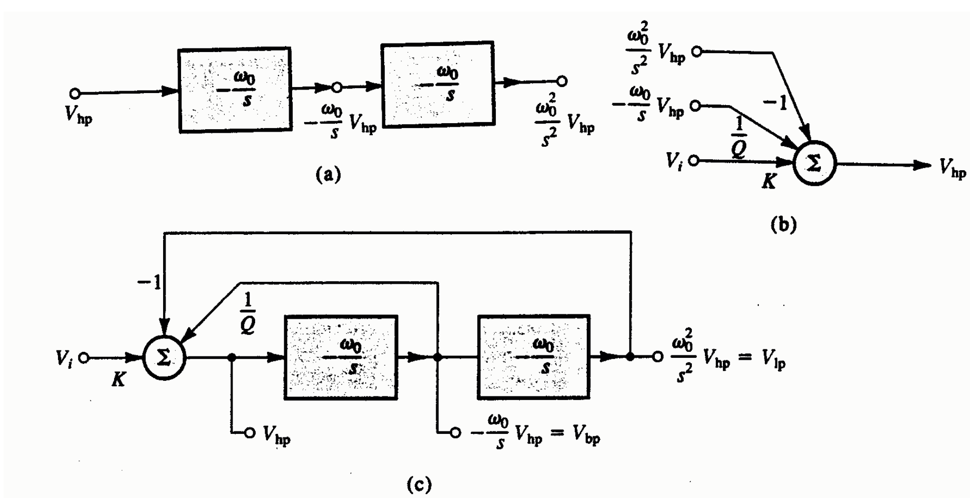
The signal,

$$\frac{\omega_o}{s}V_{HP}$$

can be obtained by passing V_{HP} through an integrator with a time constant equal to $1/\omega_0$. Passing the resulting signal through another identical integrator generate:

$$\left(\frac{\omega_o^2}{s^2}\right)V_{HP}$$

Then the output signal V_{HP} can be generated as the feedback configuration as shown below:



$$V_{HP} = KV_i - \frac{1}{Q}\cdot\frac{\omega_o}{s}V_{HP} - \frac{\omega_o^2}{s^2}V_{HP}$$

The term $\left(\dfrac{-\frac{\omega_0}{s}V_{HP}}{V_i}\right)$ is the signal at the output of the first integrator which is a bandpass function.

$$T_{BP}(s) = \frac{-\frac{\omega_0}{s}V_{HP}}{V_i} = -K\omega_0 \frac{s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Similarly, the output of the second integrator is a low pass function.

$$T_{LP}(s) = \frac{\frac{\omega_0^2}{s^2}V_{HP}}{V_i} = K\omega^2 \frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

The two-integrator loop biquad realizes three basic second order filter functions LP, BP and HP simultaneously. This circuit is very popular and is commonly called the *universal active filter* (the Kirwin-Huelsman-Newcomb = KHN biquad).

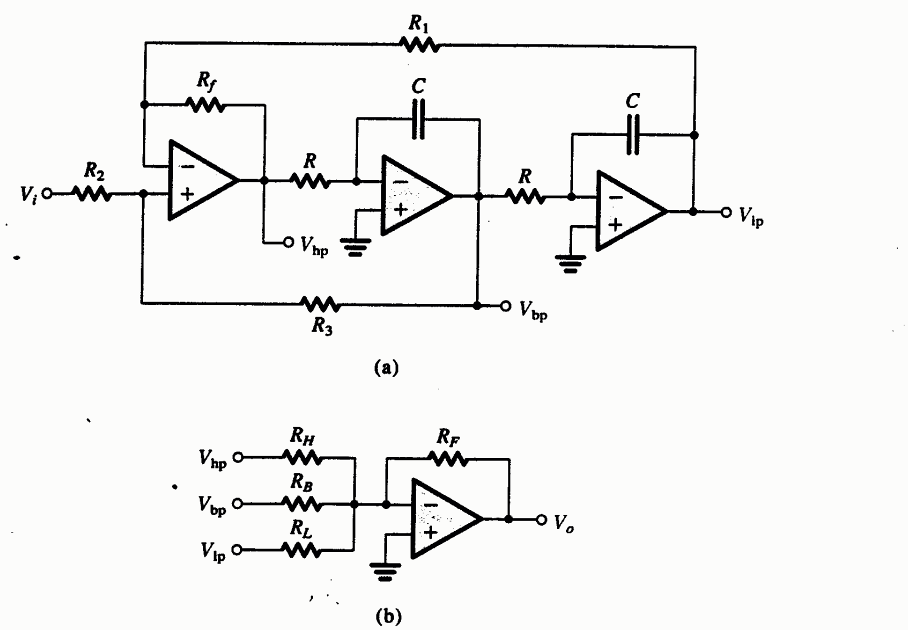


Fig. 11.24 (a) The KHN biquad circuit, obtained as a direct implementation of the block diagram of Fig. 11.23(c). The three basic filtering functions, HP, BP, and LP, are simultaneously realized. **(b)** To obtain notch and all-pass functions, the three outputs are summed with appropriate weights using this op amp summer.

In the design, ω_0, K and Q are given.

If $\dfrac{R_f}{R_1} = 1$, and $RC = \dfrac{1}{\omega_0}$ $\quad \dfrac{R_3}{R_2} = 2Q - 1 \quad K = 2 - \dfrac{1}{Q}$ and the gain parameter K is fixed to this value.

By summing the LP, BP and HP outputs, the overall transfer function of the KHN biquad and the summer in figure (b) is:

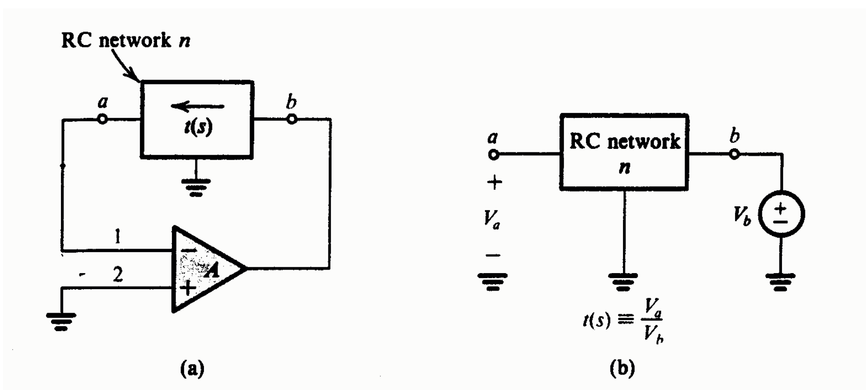
$$\frac{V_0}{V_i} = -K \frac{\frac{R_F}{R_H} s^2 - \frac{R_F}{R_B} \cdot \omega_0 s + \frac{R_F}{R_L} \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Although the two-integrator loop biquads are versatile and easy to design, their performance is adversely affected by the finite bandwidth of the op-amps.

VIII. SINGLE AMPLIFER BIQUAD FILTERS

Second order filter functions can also be implemented with a single amplifier. These minimal realizations are low power and low cost, however, they suffer from greater dependence on op-amp gain and bandwidth and are generally more sensitive to tolerances in the resistors and capacitors. The single amplifier biquads (SABs) are therefore generally limited to less stringent filter specifications (Q<10).

Consider the circuit below.



$$t(s) = \frac{N(s)}{D(s)} \quad \text{and} \quad \text{Loop\_Gain} \quad L(s) = At(s) = \frac{AN(s)}{D(s)}$$

Refer this to the negative feedback configuration, the feedback loop gain is:

$A_{FB} = \frac{A_0}{1 + A_0\beta}$, set the loop gain 1+L(s)=0, which results in the poles s_p of the closed-loop circuit at:

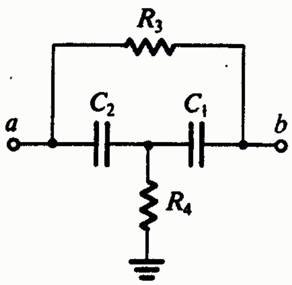
$$t(s_p) = -\frac{1}{A}$$

assume an ideal op-amp with an infinity gain A, the poles are obtained from:

$$N(s_p) = 0$$

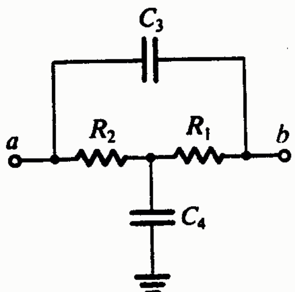
That is, the poles are identical to the zeroes of the RC network above.

Since our objective is to realize a pair of complex conjugate poles, we should select an RC network that has complex conjugate zeroes. The simplest such networks are bridge-T networks.



$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4}\right) + \frac{1}{C_1C_2R_3R_4}}$$

(a)



$$t(s) = \frac{s^2 + s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C_4} + \frac{1}{C_3C_4R_1R_2}}{s^2 + s\left(\frac{1}{C_4R_1} + \frac{1}{C_4R_2} + \frac{1}{C_3R_2}\right) + \frac{1}{C_3C_4R_1R_2}}$$

The pole polynomial of the active filter circuit will be equal to the numerator polynomial of the bridge-T network.

$$s \;\; + \frac{\omega_0}{Q}s + \omega_0^2 = s^2 + s\cdot\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\cdot\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}$$

and

$$\omega_0 = \frac{1}{\sqrt{C_1C_2R_3R_4}} \quad \text{and} \quad Q = \left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]^{-1}$$

Common implementation,

$C_1=C_2=C$
$R_3=R$
$R_4=R/m$

where $m = 4Q^2$ and $CR = \frac{2Q}{\omega_0}$

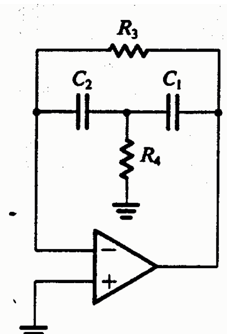
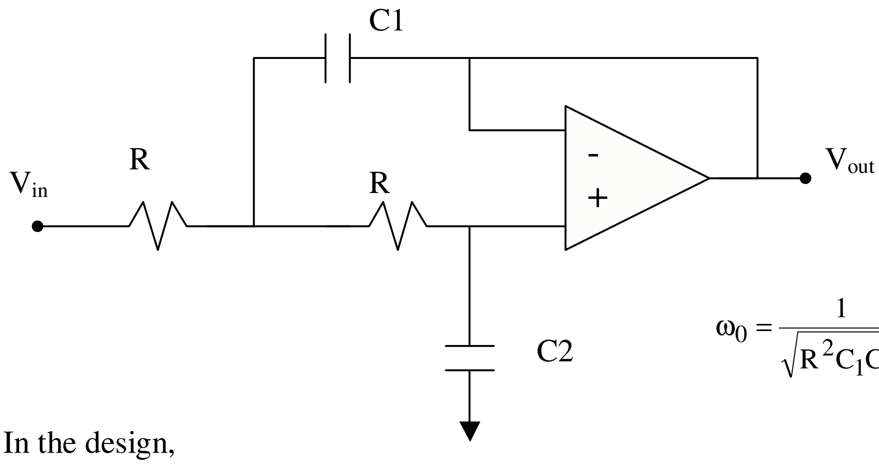


Fig. 11.29 An active-filter feedback loop generated using the bridged-T network of Fig. 11.28(a).

A common implementation of the single amplifier biquad is the Salen-Key filter. A LP filter using **Salen-Key circuit** is shown below.



$$\omega_0 = \frac{1}{\sqrt{R^2 C_1 C_2}} \quad \text{and} \quad Q = 0.5\sqrt{\frac{C_1}{C_2}}$$

In the design,

Given ...C_2, f, Q	Given....R, f, Q
$C_1 = 4Q^2 C_2$ $R = \dfrac{1}{2\pi f\sqrt{C_1 C_2}}$	$C_1 = \dfrac{Q}{\pi f R}$ $C_2 = \dfrac{C_1}{4Q^2}$

Table 12-1. Design Data for Chebyshev Filters

Ripple = 0 dB (Butterworth) Cutoff frequency = section frequency = 1.0

Number of sections	F_{50}/F_3	*Q Sct 1*	*Q Sct 2*	*Q Sct 3*	*Q Sct 4*	*Q Sct 5*	*Q Sct 6*	*Q Sct 7*	*Q Sct 8*
1	17.79	0.7071							
2	4.22	0.5411	1.305						
3	2.61	0.5176	0.7071	1.932					
4	2.05	0.5098	0.6014	0.8999	2.563				
5	1.78	0.5062	0.5612	0.7071	1.101	3.196			
6	1.61	0.5043	0.5412	0.6302	0.8213	1.307	3.831		
7	1.51	0.5032	0.5297	0.5905	0.7071	0.9401	1.514	4.466	
8	1.43	0.5024	0.5225	0.5669	0.6468	0.7882	1.061	1.722	5.101

Ripple = 0.1 dB Cutoff frequency = 1.0

Number of sections	F_{50}/F_3	*F Sct 1* *Q Sct 1*	*F Sct 2* *Q Sct 2*	*F Sct 3* *Q Sct 3*	*F Sct 4* *Q Sct 4*	*F Sct 5* *Q Sct 5*	*F Sct 6* *Q Sct 6*	*F Sct 7* *Q Sct 7*	*F Sct 8* *Q Sct 8*
1	16.59	0.9321							
		0.7674							
2	3.36	0.6491	0.9491						
		0.6190	2.185						
3	1.95	0.4688	0.7628	0.9717					
		0.5997	1.333	4.639					
4	1.52	0.3623	0.6129	0.8493	0.9828				
		0.5934	1.184	2.456	8.092				
5	1.32	0.2940	0.5065	0.7292	0.8984	0.9887			
		0.5906	1.128	2.046	3.926	12.54			
6	1.22	0.2469	0.4296	0.6314	0.8038	0.9275	0.9920		
		0.5890	1.100	1.883	3.123	5.733	17.98		
7	1.16	0.2126	0.3723	0.5539	0.7187	0.8523	0.9459	0.9941	
		0.5881	1.084	1.798	2.794	4.403	7.871	24.40	
8	1.12	0.1866	0.3280	0.4920	0.6463	0.7796	0.8852	0.9582	0.9955
		0.5875	1.074	1.748	2.619	3.850	5.883	10.34	34.82

Ripple = 0.25 dB Cutoff frequency = 1.0

Number of sections	F_{50}/F_3	F Sct 1 / Q Sct 1	F Sct 2 / Q Sct 2	F Sct 3 / Q Sct 3	F Sct 4 / Q Sct 4	F Sct 5 / Q Sct 5	F Sct 6 / Q Sct 6	F Sct 7 / Q Sct 7	F Sct 8 / Q Sct 8
1	16.00	0.8993							
		0.8093							
2	3.20	0.5893	0.9424						
		0.6575	2.539						
3	1.88	0.4174	0.7467	0.9700					
		0.6373	1.557	5.527					
4	1.48	0.3201	0.5958	0.8438	0.9822				
		0.6307	1.385	2.935	9.729				
5	1.30	0.2588	0.4906	0.7224	0.8961	0.9884			
		0.6277	1.319	2.447	4.729	15.14			
6	1.21	0.2169	0.4153	0.6243	0.8005	0.9264	0.9919		
		0.6261	1.287	2.252	3.763	6.928	21.749		
7	1.15	0.1865	0.3594	0.5470	0.7151	0.8052	0.9453	0.9941	
		0.6251	1.268	2.151	3.367	5.323	9.531	29.56	
8	1.11	0.1636	0.3164	0.4855	0.6425	0.7775	0.8842	0.9578	0.9955
		0.6245	1.256	2.091	3.158	4.656	7.125	12.53	38.58

Ripple = 0.5 dB Cutoff frequency = 1.0

Number of sections	F_{50}/F_3	F Sct 1 / Q Sct 1	F Sct 2 / Q Sct 2	F Sct 3 / Q Sct 3	F Sct 4 / Q Sct 4	F Sct 5 / Q Sct 5	F Sct 6 / Q Sct 6	F Sct 7 / Q Sct 7	F Sct 8 / Q Sct 8
1	15.44	0.8672							
		0.8637							
2	3.08	0.5425	0.9376						
		0.7055	2.944						
3	1.82	0.3793	0.7357	0.9689					
		0.6839	1.812	6.520					
4	1.45	0.2894	0.5844	0.8403	0.9819				
		0.6769	1.612	3.469	11.54				
5	1.28	0.2334	0.4801	0.7179	0.8946	0.9882			
		0.6737	1.536	2.894	5.618	19.01			
6	1.19	0.1954	0.4060	0.6198	0.7984	0.9257	0.9918		
		0.6720	1.498	2.664	4.472	8.249	25.91		
7	1.14	0.1679	0.3510	0.5426	0.7127	0.8494	0.9449	0.9940	
		0.6710	1.477	2.545	4.002	6.340	11.36	35.25	
8	1.11	0.1472	0.3089	0.4813	0.6401	0.7762	0.8835	0.9576	0.9955
		0.6703	1.463	2.474	3.753	5.546	8.495	14.95	46.03

Ripple = 1.0 dB Cutoff frequency = 1.0

Number of sections	F_{50}/F_3	F Sct 1 / Q Sct 1	F Sct 2 / Q Sct 2	F Sct 3 / Q Sct 3	F Sct 4 / Q Sct 4	F Sct 5 / Q Sct 5	F Sct 6 / Q Sct 6	F Sct 7 / Q Sct 7	F Sct 8 / Q Sct 8
1	14.77	0.8295							
		0.9563							
2	2.95	0.4964	0.9332						
		0.7850	3.562						
3	1.77	0.3432	0.7261	0.9679					
		0.7613	2.200	8.012					
4	1.42	0.2608	0.5746	0.8373	0.9815				
		0.7535	1.958	4.270	14.26				
5	1.26	0.2099	0.4712	0.7142	0.8934	0.9881			
		0.7499	1.867	3.564	6.944	22.29			
6	1.18	0.1755	0.3960	0.6160	0.7966	0.9251	0.9918		
		0.7480	1.820	3.281	5.530	10.22	32.11		
7	1.13	0.1507	0.3439	0.5389	0.7108	0.8485	0.9446	0.9940	
		0.7469	1.794	3.134	4.949	7.853	14.08	43.71	
8	1.10	0.1321	0.3025	0.4779	0.6381	0.7751	0.8830	0.9574	0.9954
		0.7461	1.777	3.047	4.642	6.870	10.53	18.54	57.10

IX. SENSITIVITY

Because of tolerances in component values and because of the finite op-amp gain, the response of the actual filter will deviate from the ideal response. As a means of predicting such deviations, filter designers employs the concept of sensitivity.

For second order filters, one is normally interested in finding how sensitive their poles are relative to variations (both initial tolerances and future changes) in RC component values and amplifier gain. This is important because positions of the poles in s-plane determine stability of the circuits.

These sensitivities can be quantified using the classical sensitivity function:

$$S_X^Y = \lim_{\Delta X \to 0} \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{\delta Y}{\delta X}\frac{X}{Y}$$

Here the X denotes the values of a component and Y denotes a circuit parameter of interest (e.g., ω_0, Q, …), for small changes, the approximation below is used:

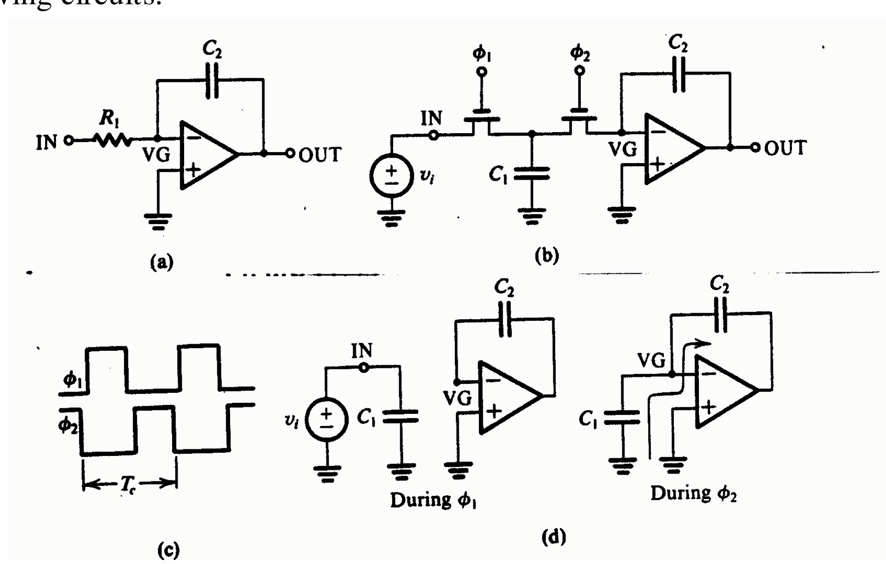
$$S_X^Y = \frac{\Delta X / Y}{\Delta Y / X}$$

Thus we use the value of sensitivity S to determine the per unit change in Y due to a given per unit change in X. For example, if the sensitivity of Q relative to a particular resistance R1 is 5, then 1% increase in R1 results in a 5% increase in the value of Q.

X. SWITCHED-CAPACITOR FILTERS

Active RC filters are difficult to implement totally on an Integrated Circuit (IC) due to the requirements of large-valued capacitors and accurate RC time constants. The switched-capacitor filter technique is based on the realization that a capacitor switched between two circuit nodes that a sufficiently high rate is equivalent to a resistor connecting these two nodes.

Consider the following circuits.



From the circuit, we see that during each clock period, T_C, an amount of charge $q_{C1}=C_1V_i$ is subtracted from the input source and supplied to the integrator capacitor C_2. The average current flow between the input node and virtual ground (V_G) is

$$i_{av} = \frac{C_1 v_i}{T_C} = \frac{\text{charge}}{\text{cycle}}$$

If T_C is sufficiently short, one can think of the process as continuous and define an equivalent resistance R_{EQ} that is an effect present between nodes in and V_G:

$$R_{EQ} = \frac{v_i}{i_{av}} = \frac{T_C}{C_1}$$

The time constant for the integrator can be calculated as:

$$\text{Time} \cdot \text{constant} = C_2 R_{EQ} = T_C \frac{C_2}{C_1}$$

Thus the time constant that determines the frequency response of the filter is determined by the clock period T_C and the capacitor ratio C_2/C_1. Both of these parameters can be well controlled in an IC fabrication process.

Note:

- The dependence is on *capacitor ratio* rather than capacitor absolute values. The accuracy of capacitor ratio in MOS technology is on the order of 0.1%.
- For reasonable clock frequencies (100KHz) and not to large capacitor ratios (10) one can obtain relatively large time constants (10^{-4}s).
- The clock frequency must be higher than any frequency component of the signal (typically 100x).
- The filter cut off frequency can be easily programmed by changing the clock frequency.
- Some of clock signal will feedthrough to output, signals near the clock frequency can be aliased into the passband, overall increases in the noise floor (due to the on-off switching of the clock).
- Switched-capacitor filter IC offers a low cost high order filter on a single IC.

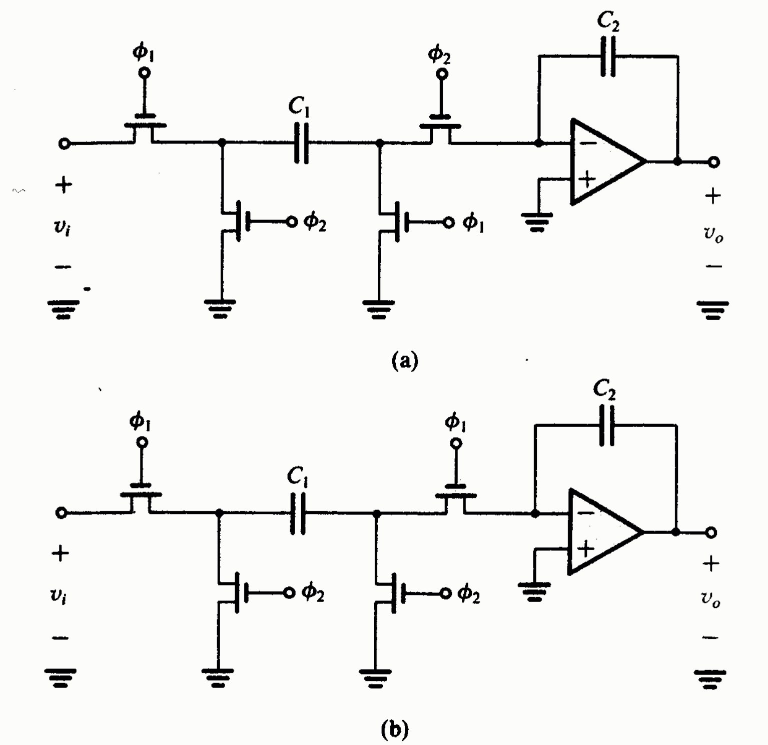


Fig. 11.36 A pair of complementary stray-insensitive switched-capacitor integrators. **(a)** Noninverting switched-capacitor integrator. **(b)** Inverting switched-capacitor integrator.

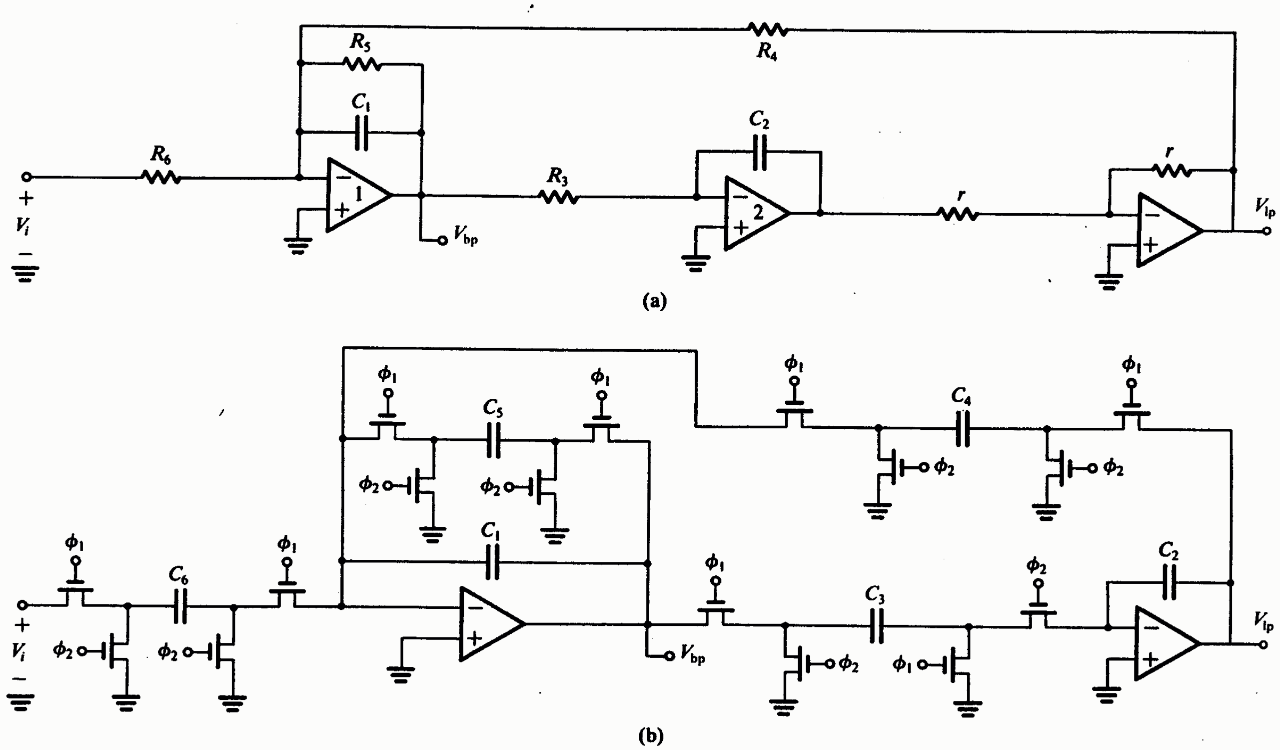


Fig. 11.37 A two-integrator-loop active-RC biquad and its switched-capacitor counterpart.