

Another Way To Represent Geometric Sets

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Abstract. Abstract...

Keywords. Key words...

1. Blades And Spades

As stated in the abstract of [2], blades are a powerful tool for representing geometry. They may be thought of as doing so under the definition of a *geometric set* as given in [4], or as doing so under an isomorphism as given in [3]. In [5], another way to represent geometry was presented that uses vectors homogeneous of a grade equal to the degree of the polynomial whose zero set is the geometry in question. In this paper, yet another way to represent geometry in a geometric algebra is presented that is very much similar to that given in [4, 3, 1], and certainly related, as we'll see; but different enough, and exhibiting enough desirable characteristics, that it may warrant attention. Its conception begins with a new term as found in Table 1 among its traditional counter-parts.

TABLE 1. A few terms used in GA

Term	Definition
Blade	An outer product of zero or more linearly-independent vectors.
Versor	A geometric product of zero or more invertible vectors, not necessarily forming a linearly-independent set.
Spade	A geometric product of zero or more vectors, not necessarily forming a linearly-independent set.

The difference here between “versor” and “spade” may be subtle, but justifies the new term as it must be emphasized that each vector in the product being invertible will not be a requirement for us, nor would we want it to be. No where in the derivation of any identity of this paper involving spades, nor in many proofs involving them, will we rely upon invertibility with

respect to the geometric product. It is this kind of invertibility, however, that may make spades, as apposed to blades, in the cases where invertible vector factors do appear, a desirable alternative to representing what we'll see are the exact same classes of geometry; namely, geometric sets.

Similar to the concept of grade, that of rank will be introduced in this paper with respect to spades. As an r -blade refers to a blade of grade r , we will let an r -spade refer to a spade of rank r . If an element of a geomtric algebra can be written as any geometric product of vectors, then it is a spade. The rank of that spade is then the smallest possible number of vectors for which it can be written as such a product.¹ Note that blades of grade zero are indistinguishable from spades of the same rank as each denotes the set of all scalars.

2. Representation By Blades

The representation of geometric sets by spades is best appreciated when viewed in contrast to what may be its complementary representation by blades. In any case, it may go beyond instructive to give the latter representation before the former as each are so closely related.

3. Representation By Spades

References

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¹While the correctness of many identities of this paper do not require a spade to be written in the most compact form, the concept of rank would be ill-defined without its consideration.