Applications Of
The Mother Minkowski Algebra Of Order m

Spencer T. Parkin

Abstract. It is shown that for any multi-variable function whose zero set is an algebraic surface, that the image of any line in the domain of such a function is determined entirely by all orders of its directional derivatives at any one point on the line and in the direction of the line.

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1. Introduction

That we may begin with new material immediately, this paper assumes full knowledge of the paper [], continuing where this paper ended, and regiving equations from [] only where necessary.

2. A Refinements Of Notation

Notation is important as it facilitates understanding and ease of algebraic manipulation. We are going to keep the notation of [], but think of it somewhat differently. In this paper we are going to work exclusively in $\mathbb{G}_1^n$ and transform multivectors from this sub-algebra to their counter-parts in $\mathbb{G}_{i>1}^n$ through a use of sub-script notation. To denote the sub-algebra in which an element B resides, (for any integer $i \in [1, m]$, we will never use a single letter to denote an element that cannot be placed in $\mathbb{G}_i$), we will use a sub-script, writing B_i, to show that $B \in \mathbb{G}_i$. If the sub-script is absent, we can assume $B = B_1$.

As shown in the appendix of [], there is an outermorphism that lets us transform elements in one of these sub-algebras to its counter-part in any other. And indeed, to make our notation more concrete, we may let the subscription of an otherwise unscripted letter or expression denote the application of this outermorphism in the case $i > 1$, i being the subscript in question. Doing so, we see that for any two vectors $a, b \in \mathbb{V}_1$ and any integer $i \in [1, m]$, we have the following properties.

$$(a)_i = a_i$$
$$(a + b)_i = a_i + b_i$$
$$(a \wedge b)_i = a_i \wedge b_i$$
$$a \cdot b = a_i \cdot b_i$$

The last property here applies to outermorphisms generally, but is especially obvious with the particular outermorphism we're using here. It should also be mentioned that for all $i \neq j$, we have $a_i \cdot a_j = 0$, yet $a_i \wedge a_j \neq 0$, a property that has been specifically exploited in the use of the mother algebra to represent algebraic surfaces. It should be noted that if $i > 1$, we have $(a_i)_i = -a$.

Thinking of these subscripts as an outermorphism may aide us in making algebraic manipulation and in the way we think about what we're doing.

3. Intersecting Rays With Algebraic Surfaces

In [] it was shown that an m-vector B with $\infty \cdot B = 0$ may be representative of any algebraic surface of up to degree m as the set of all points $x \in \mathbb{R}^n_1$ such that $F(x) = 0$, where $F : \mathbb{R}^n_1 \to \mathbb{R}$ is given by

$$F(x) = \bigwedge_{i=1}^{m} p_i(x) \cdot B,$$

the function $p_i : \mathbb{R}^n_1 \to \mathbb{V}_i$ being given by

$$p_i(x) = o_i + x_i + \frac{1}{2} x_i^2 \infty_i,$$

having its origins in the paper [].

What we wish to do in this section is, given a point $x \in \mathbb{R}^n_1$ and a direction $v \in \mathbb{R}^n_1$, find the scalar $\lambda \in \mathbb{R}$, if any, such that $F(x + \lambda v) = 0$. Attempting to do so, we easily find[1] that

$$F(x + \lambda v) = \bigwedge_{i=1}^{m} (p_i(x) + \lambda v_i) \cdot B,$$

the expansion of which gives us a polynomial of degree m in the scalar λ. We can then show that this polynomial can be written in terms of the directional derivative of F in the direction of v as

$$F(x + \lambda v) = F(x) + \sum_{i=1}^{m} \frac{\lambda^i}{2^{i-1}} \nabla_v^i F(x)$$

[1]Notice that since $\infty \cdot B = 0$, we can pretend that we have $p_i(x) = o_i + x_i$, ignoring the $\frac{1}{2} x_i^2 \infty_i$ term.

where j is any integer in $[1, m]$. Here, $\nabla_v^i F$ is the i^{th} order directional derivative of F in the direction of v. It is the directional derivative of the directional derivative, and so on, i times.

What we see now is that if all orders of the gradient of F are available to us, we can use them to find the coefficients of the polynomial whose roots we need to find in the problem intersecting a ray with the surface. Interestingly, this also shows how one might numerically integrate the function F.

(This result has to exist somewhere already. Find it.)

Spencer T. Parkin
102 W. 500 S., Salt Lake City, UT 84101
e-mail: `spencerparkin@outlook.com`