A Model Of Algebraic Sets Using Geometric Algebra

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Abstract. This paper is an attempt to study algebraic sets using the techniques of geometric algebra (GA). Refering to algebraic sets as geometries, conditions are found under which the outer product performs the union and intersection operations of geometries as represented by blades of a GA. This model of algebraic sets using GA is simply a generalization of the conformal model of geometric algebra (CGA) to a model capable of representing geometries that are each the zero set of one or more polynomials of any form.

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1. Introduction And Motivation

Letting $\mathbb{R}^n$ denote an n-dimensional Euclidean space, we are going to let $\mathbb{P}^n$ denote the set of all polynomials $f : \mathbb{R}^n \to \mathbb{R}$ of any degree. Then, letting $P \subseteq \mathbb{P}^n$ be any set of such polynomials, we define the zero set Z of P, denoted $Z(P)$, as the set given by

$$Z(P) = \{x \in \mathbb{R}^n | f(x) = 0 \text{ for all } f \in P\}. \tag{1.1}$$

Every subset S of $\mathbb{R}^n$ for which there exists a set $P \subseteq \mathbb{P}^n$ such that $S = Z(P)$ is what we refer to as an algebraic set. It is well known that for any algebraic set $S \subseteq \mathbb{R}^n$, there is always such a subset P of $\mathbb{P}^n$ of finite cardinality.

Given the definition in equation (1.1), it is easy to show that the subsets S of $\mathbb{R}^n$ that are the geometries of the conformal model of geometric algebra (CGA), and other similar models of geometry based upon geometric algebra (GA), are simply algebraic sets. The goal of this paper is to show that there exists a model of geometry, based upon GA, where every possible algebraic set has a representative in the form of an element of that GA. A desire to come up with such a model of GA is motivated by the admittedly fanciful dream of the German mathematician Leibniz, referred to in [] and claimed to have already been realized in the daunting book []. In any case, it would seem that a generalization of CGA or any CGA-like model to one that hosts the set of all algebraic sets in $\mathbb{R}^n$ would bring us closer to such a goal. Work to this end has already been done in [1, 2] which has brought us, the reader and writer, to the present paper.

2. Blades As Algebraic Sets

We begin with an examination of $\mathbb{P}^n$ as a linear space, observing that it is of countably infinite dimension. A set of basis vectors for this space may be taken as the set of all unit-monomials in anywhere from 1 to n of the n variable components of an arbitrary point in $\mathbb{R}^n$.[1] Considering now any mapping from the set $\mathbb{Z}^+$ of positive integers to this said set $\{g_i\}_{i=1}^{\infty}$ of unit-monomials,[2] and letting $\mathbb{V}^\infty$ denote a simple Euclidean[3] vector space of countably infinite dimension, if we define the function $p : \mathbb{R}^n \to \mathbb{V}^\infty$ as

$$p(x) = \sum_{i=1}^{\infty} g_i(x) e_i, \tag{2.1}$$

where $\{e_i\}_{i=1}^{\infty}$ is any orthonormal basis for $\mathbb{V}^\infty$, then for any polynomial $f \in \mathbb{P}^n$, there must exist a unique[4] vector $v \in \mathbb{V}^\infty$ such that

$$f(x) = p(x) \cdot v. \tag{2.2}$$

It now follows that for any subset P of $\mathbb{P}^n$, there exists a blade $B \in \mathbb{G}(\mathbb{V}^\infty)$, such that

$$Z(P) = \{x \in \mathbb{R}^n | p(x) \cdot B = 0\}. \tag{2.3}$$

Clearly, we need only consider such finite sets $P = \{f_i\}_{i=1}^{k}$ that are linearly independent sets of k polynomials. Then, for each polynomial f_i, there is an associated vector v_i by equation (2.2), and the set $\{v_i\}_{i=1}^{k}$ must clearly be a

[1] The number c_k of such monomials homogeneous of a degree k is given by

$$c_k = \sum_{i=1}^{n} \binom{n}{i} p(k, i),$$

where $p(k, i)$ is the number of partitions of size i of the integer k. The combinatorics of the matter are not important, but they are pointed out here as an added measure of the reality of the set of monomials we are talking about.

[2] For example, if $n = 2$, then we might take $g_1(x, y) = 1$, $g_2(x, y) = x$, $g_3(x, y) = y$, $g_3(x, y) = x^2$, $g_4(x, y) = xy$, $g_5(x, y) = y^2$, $g_6(x, y) = x^3$, $g_7(x, y) = x^2 y$, and so on.

[3] A GA of Euclidean signature keeps things simpler, but the model of this paper can be made to work with any GA having a non-degenerate, non-Euclidean signature.

[4] Let $v_1, v_2 \in \mathbb{V}^\infty$ be two such vectors. Then $0 = f(x) - f(x) = p(x) \cdot (v_1 - v_2)$. Now notice that there does not exist $x \in \mathbb{R}^n$ such that $p(x) = 0$. It follows that $v_1 = v_2$.

linearly independent set of k vectors. We may then take B to be the k-blade

$$B = \bigwedge_{i=1}^{k} v_i, \tag{2.4}$$

seeing that the equation $p(x) \cdot B = 0$ becomes

$$0 = -\sum_{i=1}^{k} (-1)^i (p(x) \cdot v_i) B_i, \tag{2.5}$$

where B_i denotes the product B with v_i removed. Realize that $\{B_i\}_{i=1}^{k}$ is a linearly independent set of $(k-1)$-blades, and therefore $p(x) \cdot B = 0$ if and only if $p(x) \cdot v_i = 0$ for all $i \in \mathbb{Z}_k$. For convenience, we will let $\mathbb{Z}_k$ denote the set of k integers in $[1, k] \cap \mathbb{Z}^+$.

Already we have fulfilled the promise of the introductory section of this paper, but we will continue now with one further development that is motivated by a desire to pair our current ability to intersect geometries with an ability to take their union.

We start by letting $\{\mathbb{V}_i^\infty\}_{i=1}^{\infty}$ be a countably infinite set of vector spaces isomorphic to $\mathbb{V}^\infty$. With the exception of the zero vector, we consider these vector spaces as pair-wise disjoint, so that for any pair of non-zero vectors $v_i \in \mathbb{V}_i^\infty$ and $v_j \in \mathbb{V}_j^\infty$ with $i \neq j$, we have $v_i \cdot v_j = 0$. Let $\mathbb{V}$ simply denote the vector space spanned by the set of vectors $\cup_{i \in \mathbb{Z}^+} \{e_{ij}\}_{j=1}^{\infty}$, where for each $i \in \mathbb{Z}^+$, the set $\{e_{ij}\}_{j=1}^{\infty}$ is an orthonormal basis for the vector space $\mathbb{V}_i^\infty$. It is clear that the dimension of $\mathbb{V}$, like that of $\mathbb{V}^\infty$ above, is countably infinite as the union of countably many countable sets is countable.[5]

Definition 2.1 (The q Function). Given a blade $B \in \mathbb{G}(\mathbb{V})$, the function q is a mapping from the set of all blades in $\mathbb{G}(\mathbb{V})$ to subsets of $\mathbb{Z}^+$ so that $i \in q(B)$ if and only if there exists a vector $v \in \mathbb{V}_i^\infty$ such that $v \wedge B = 0$.

With Definition 2.1 in place, we may now proceed with the following two definitions.

Definition 2.2 (The Algebraic Set $\dot{G}$). Given a blade $B \in \mathbb{G}(\mathbb{V})$, the set $\dot{G}$ of B, denoted $\dot{G}(B)$, is the set

$$\dot{G}(B) = \left\{ x \in \mathbb{R}^n \;\middle|\; \bigwedge_{i \in q(B)} p_i(x) \cdot B = 0 \right\}, \tag{2.6}$$

where for each $i \in \mathbb{Z}^+$, we define p_i similar to equation (2.1) as

$$p_i(x) = \sum_{j=1}^{\infty} g_j(x) e_{ij}. \tag{2.7}$$

[5]This may come as some comfort to the reader. If you already thought that the dimension of $\mathbb{V}^\infty$ was ridiculously huge, the dimension of the vector space $\mathbb{V}$ is no bigger. If it would help, however, it would be reasonable to choose a positive integer $k \in \mathbb{Z}^+$ and have for all $i > k$, $g_i = 0$. This would bring $\mathbb{V}$ to finite dimension while enabling us to represent any desired subset of the set of all algebraic sets.

For any product of the form $\prod_{i\in S}$ or $\bigwedge_{i\in S}$, where S is a set of positive integers, we take the terms in the product to be in ascending order with respect to the index i. While of course the outer product is non-commutative, we will be more interested in the vector sub-spaces represented by blades in this paper rather than the handedness of those blades.

Definition 2.3 (The Algebraic Set $\hat{G}$). Given a blade $B \in \mathbb{G}(\mathbb{V})$, the set $\hat{G}$ of B, denoted $\hat{G}(B)$, is the set

$$\hat{G}(B) = \left\{ x \in \mathbb{R}^n \,\middle|\, \bigwedge_{i\in q(B)} p_i(x) \wedge B = 0 \right\}. \tag{2.8}$$

We now proceed to investigate the consequences of Definition 2.2 and that of Definition 2.3.

Fatal error! Definition 2.3 is wrong!!! I don't know how to fix it. For almost all blades B, $p_i(x) \wedge B \neq 0$ for any x and any i. Also, we didn't need the q function for Definition 2.2.

3. The Algebraic Set $\dot{G}$

As promised, there now exist conditions under which we are able to take the union of any two geometries represented by blades in $\mathbb{G}(\mathbb{V})$. The result is as follows.

Lemma 3.1 (The Union Of Geometries). *For any two blades $A, B \in \mathbb{G}(\mathbb{V})$ with the property that $q(A) \cap q(B)$ is empty, we have*

$$\dot{G}(A) \cup \dot{G}(B) = \dot{G}(A \wedge B). \tag{3.1}$$

Proof. Without loss of generality, we may write $C = A \wedge B$ as $C = \bigwedge_{i=1}^{k} C_k$, where for each $i \in \mathbb{Z}_k$, we have $C_i \in \mathbb{G}(V_i^\infty)$. (Notice that $q(C) = \mathbb{Z}_k$.) It is now clear that

$$\bigwedge_{i\in\mathbb{Z}_k} p_i(x) \cdot C = \pm \bigwedge_{i\in\mathbb{Z}_{k-1}} p_i(x) \cdot (p_k(x) \cdot C_k) \wedge \bigwedge_{i\in\mathbb{Z}_{k-1}} C_i \tag{3.2}$$

$$= \pm \bigwedge_{i\in\mathbb{Z}_k} p_i(x) \cdot C_i. \tag{3.3}$$

(Notice that the outer product in (3.3) becomes a scalar product in the case that every C_i is a vector.) It is now clear that $\dot{G}(C)$ is indeed the union of $\dot{G}(A)$ and $\dot{G}(B)$. □

It is not hard to show that for any blade $B \in \mathbb{G}(\mathbb{V})$, there exists a rotor R, such that B', given by $B' = RBR^{-1}$, has the property $\dot{G}(B') = \dot{G}(B)$ while $q(B')$ is mapped to any other set of integers of size $|q(B)|$. This fact can be used to adjust any given pair of blades $A, B \in \mathbb{G}(\mathbb{V})$, if needed, so that $q(A) \cap q(B)$ is empty. Admittedly, the need for any such adjustment prior to a union operation seems to detract from our dream of an algebra where the geometric elements would combine effortlessly in products that perform desired geometric operations. Unfortunately, things don't get any better as the next lemma shows.

Lemma 3.2 (The Intersection Of Geometries). *For any two blades $A, B \in \mathbb{G}(\mathbb{V})$ such that $q(A) = q(B)$ with $|q(A)| = |q(B)| = 1$, we have*

$$\dot{G}(A) \cap \dot{G}(B) = \dot{G}(A \wedge B). \tag{3.4}$$

Proof. Revisit the conversation of this paper surrounding equation (2.5). □

In this case, a simple rotor adjustment to one of A and B will only help in the case that $|q(A)| = |q(B)| = 1$. If either one of $|q(A)|$ or $|q(B)|$ is greater than one, however, more adjustments are needed before an intersection can be taken. The following equation illustrates why.

$$\bigcup_{i \in q(A)} \dot{G}(A_i) \cap \bigcup_{i \in q(B)} \dot{G}(B_i) = \bigcup_{\substack{i \in q(A) \\ j \in q(B)}} \dot{G}(A_i) \cap \dot{G}(B_j), \tag{3.5}$$

Here we have considered A as the blade $\bigwedge_{i \in q(A)} A_i$, where for each $i \in q(A)$, we have $A_i \in \mathbb{G}(\mathbb{V}_i^\infty)$. We have similiarly considered B in terms of the blades $\{B_i\}_{i \in q(B)}$. It is now easy to see that, by the right-hand side of equation (3.5), there exists[6] a blade C with the property that $\dot{G}(C) = \dot{G}(A) \cap \dot{G}(B)$, but we do not necessarily have $\text{grade}(C) = \text{grade}(A \wedge B)$, showing that a rotor adjustment, which is grade preserving, to one or both of A and B, cannot be of general help. We need the factorizations of A and B to formulate C. One possible saving grace taunts us with the following lemma.

Lemma 3.3. *For any blade $B \in \mathbb{G}(\mathbb{V})$, there exists a blade $B' \in \mathbb{G}(\mathbb{V})$ with $|q(B')| = 1$ such that $\dot{G}(B') = \dot{G}(B)$.*

Proof. It is clear that $\dot{G}(B)$ is an algebraic set as finite unions and arbitrary intersections of such sets are algebraic. Now simply see that the set of all algebraic sets is covered by the set

$$\{\dot{G}(B') | B' \in \mathbb{G}(\mathbb{V}) \text{ and } q(B') = \{1\}\}. \tag{3.6}$$

□

Lemma 3.3 proves the existance of a blade with a desired property, but does not give us any clue to a means of calculating it. Providing such a means in geometric algebra, as we'll see, is possible, but doesn't seem to come naturally.

Letting $A, B \in \mathbb{G}(\mathbb{V})$ be two blades with $q(A) \cap q(B)$ empty, we know that $A \wedge B$ is the union of two geometries. Suppose now that $|q(A)| = |q(B)| =$

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[6] For each $(i, j) \in q(A) \times q(B)$, choose blades A'_i and B'_j such that $\dot{G}(A'_i) = \dot{G}(A_i)$, that $\dot{G}(B'_i) = \dot{G}(B_i)$, and that Lemma 3.2 may be applied to get $\dot{G}(A'_i) \cap \dot{G}(B'_j) = \dot{G}(A'_i \wedge B'_j)$. (This may be done using rotor adjustments of A_i and B_j.) Furthermore, make these choices so that for all $(i, j) \neq (k, l)$, Lemma 3.1 may be applied to get

$$\dot{G}(A'_i \wedge B'_j) \cup \dot{G}(A'_k \wedge B'_l) = \dot{G}(A'_i \wedge B'_j \wedge A'_k \wedge B'_l).$$

The blade C may now be taken as an outer product of all $A'_i \wedge B'_j$ over all $(i, j) \in q(A) \times q(B)$.

1 and that we want a blade $C \in \mathbb{G}(\mathbb{V})$ with $\dot{G}(C) = \dot{G}(A \wedge B)$, where $|q(C)| = 1$. Well, writing the k-blade A as $\bigwedge_{i=1}^k a_i$ and the l-blade B as $\bigwedge_{i=1}^l b_i$, we see that

$$\bigcap_{i=1}^{k} \dot{G}(a_i) \cup \bigcap_{i=1}^{l} \dot{G}(b_i) = \bigcap_{i=1}^{k} \bigcap_{j=1}^{l} \dot{G}(a_i \wedge b_j), \tag{3.7}$$

showing that

$$C = \bigwedge_{i=1}^{k} \bigwedge_{j=1}^{l} r(a_i \wedge b_j), \tag{3.8}$$

where the function $r : \mathbb{G}(\mathbb{V}) \to \mathbb{V}$ is a linear function that maps a basis 2-blade to the appropriate basis vector by the associations between basis vectors and unit-monomials implied by equation (2.7). For example, we have e_{ij} associated with g_j and e_{kl} with g_l, therefore, we have, for $i \neq k$, $r(e_{ij} \wedge e_{kl})$ mapped to the basis vector e_m for the integer m where $g_j g_l = g_m$. This isn't pretty, but it is a well defined function.

Such a mapping r could easily be extended to map blades of any grade, but it cannot be extended to an outermorphism so that $C = r(A \wedge B)$. Notice that grade(C) is not necessarily grade$(A \wedge B)$. It would seem that geometric algebra doesn't go to work for us in this case without the need to add more machinary.

4. The Algebraic Set $\hat{G}$

Interestingly, the condition of Lemma 3.1 is the same as the following lemma.

Lemma 4.1 (The Union Of Geometries). *For any two blades $A, B \in \mathbb{G}(\mathbb{V})$ with the property that $q(A) \cap q(B)$ is empty, we have*

$$\hat{G}(A) \cup \hat{G}(B) = \hat{G}(A \wedge B). \tag{4.1}$$

Proof. Without loss of generality, we may write $C = A \wedge B$ as $C = \bigwedge_{i=1}^k C_k$, where for each $i \in \mathbb{Z}_k$, we have $C_i \in \mathbb{G}(\mathbb{V}_i^\infty)$. (Notice again that $q(C) = \mathbb{Z}_k$.) It is now clear that

$$\bigwedge_{i \in \mathbb{Z}_k} p_i(x) \wedge C = \pm \bigwedge_{i \in \mathbb{Z}_k} p_i(x) \wedge C_i, \tag{4.2}$$

and therefore, $\hat{G}(C)$ is indeed the union of $\hat{G}(A)$ and $\hat{G}(B)$. □

Applying the condition of Lemma 3.2, we get the following lemma.

Lemma 4.2. *For any two blades $A, B \in \mathbb{G}(\mathbb{V})$ such that $q(A) = q(B)$ with $|q(A)| = |q(B)| = 1$, we have*

$$\hat{G}(A) \cup \mathbb{G}(B) \subseteq \hat{G}(A \wedge B). \tag{4.3}$$

Proof. Letting $i \in \mathbb{Z}^+$ be the integer such that $\{i\} = q(A) = q(B)$, realize that while

$$p_i(x) \wedge A = 0 \text{ or } p_i(x) \wedge B = 0 \implies p_i(x) \wedge A \wedge B = 0 \tag{4.4}$$

is a true statement, the converse of this statement is not true. □

Lemma 4.2 gives us the exciting prospect of fitting geometries to a set of points as the next lemma shows.

Lemma 4.3 (Point-Fitting Geometries). *Letting $i \in \mathbb{Z}^+$ be a fixed integer, $S \subset \mathbb{R}^n$ a finite set of points, and $B = \bigwedge_{x \in S} p_i(x)$, if we have $B \neq 0$, then for all $x \in S$, we have $x \in \hat{G}(B)$; in other words, $S \subseteq \hat{G}(B)$.*

This begs the following definition.

Definition 4.4 (Point-Fit-Able Geometries). For any k-blade $B \in \mathbb{G}(\mathbb{V})$, write it as $B = \bigwedge_{i \in q(B)} B_i$. Then, if for every $i \in q(B)$, there exists a scalar $\lambda \in \mathbb{R}$ and a set $S \subset \mathbb{R}^n$ of points such that

$$B_i = \lambda \bigwedge_{x \in S} p_i(x), \tag{4.5}$$

then we will refer to B as point-fit-able.

It is not hard to show[7] that not all blade $B \in \mathbb{G}(\mathbb{V})$ are point-fit-able.

Lemma 4.2 is unsatisfactory, however, because, unlike Lemma 3.2, it does not tell us exactly what geometry we get out of $A \wedge B$ in terms of A and B. Interestingly, we can apply Lemma 3.2 to solve this problem, if we use it in addition to the following lemma.

Lemma 4.5 (Dual Geometries). *Let $i \in \mathbb{Z}^+$ be a fixed integer and let $B \in \mathbb{G}(\mathbb{V}_i^\infty)$ be a blade. Now let I_i denote the unit-psuedo-scalar of any vector sub-space $\mathbb{V}_i$ of $\mathbb{V}_i^\infty$ of finite dimension such that $B \in \mathbb{G}(\mathbb{V}_i)$ and B is not a psuedo-scalar. It then follows that*

$$\hat{G}(B) = \dot{G}(BI_i). \tag{4.6}$$

Proof. Simply note that

$$p_i(x) \wedge B = 0 \text{ iff } (p_i(x) \cdot BI_i)I_i = 0 \text{ iff } p_i(x) \cdot BI_i = 0. \tag{4.7}$$

□

The solution to the dilemma presented by Lemma 4.2 can now be addressed as follows. Let $i \in \mathbb{Z}_k$ be a fixed integer, and let us assume that for every $c \in \mathbb{V}_i^\infty$, we know what geometry we get from $\dot{G}(c)$. It now follows by Lemma 3.2 that for any blade $C \in \mathbb{V}_i^\infty$, we know what geometry we get from $\dot{G}(C)$. Then, for any two blades A and B of Lemma 4.2, we can apply Lemma 4.5 to show that

$$\hat{G}(A \wedge B) = \dot{G}((A \wedge B)I_i). \tag{4.8}$$

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[7]To see this, simply consider $xye_1 + xze_2 + yze_3 = e_1 + e_2 - e_3$. This creates the system of equations $xy = 1$, $xz = 1$ and $yz = -1$, which has no solution in $\mathbb{R}^3$.

It follows now that we know what geometry we get from $A \wedge B$ in terms of its dual $\pm(A \wedge B)I_i$. So the answer is that we can understand the geometry of such blades through a geometric interpretation of their duals.

To give a concrete example of this, it is not at all obvious that the outer product of the points of CGA can produce the rounds and flats of that model until we relate those blades with their duals, the geometry of which may be thought of as the set of all possible intersections of rounds and flats.

Returning to the idea of point-fitting in Lemma 4.3, one way to understand what geometry we get in our generalized model of algebraic sets from the fitting of all points in a set S is to analyze a dual of the blade B. This may require finding a factorization of BI_i.

5. Transforming The Geometries Of $\mathbb{G}(\mathbb{V})$

Admittedly, up this point in the paper, nothing insightful or new about algebraic sets has been revealed through our use of GA to model such sets. Perhaps the only advantage of using such a model is that, by virtue of using blades to represent algebraic sets, this lends itself well to the use of versors as transformations applicable to any geometry of the model. As can be seen in CGA, this reveals an interesting relationship between transformations as versors and geometries as blades. It is that many geometries (blades) are in fact transformations (versors). When reinterpreted as a geometry, the transformation performed by a given versor often has geometric significance with respect to that geometry. For example, spheres of CGA, as versors, perform inversions about a sphere. The planes of CGA, as versors, perform reflections about a plane.

Fixing an integer $i \in \mathbb{Z}^+$, consider the set of all versors $V \in \mathbb{G}(\mathbb{V}_i^\infty)$ such that for any point $x \in \mathbb{R}^n$, there exists a scalar $\lambda \in \mathbb{R}$ and point $y \in \mathbb{R}^n$ such that $V^{-1}p_i(x)V = \lambda p_i(y)$. Given such a versor, it is easy to see that the problem of determining how V transforms a given blade $B \in \mathbb{G}(\mathbb{V}_i^\infty)$ representative of a point-fit-able geometry simply reduces to the problem of determining how, for any point $x \in \mathbb{R}^n$, the versor V transforms $p_i(x)$. This is because VBV^{-1} is also point-fit-able. For non-point-fit-able geometries, however, it is not so obvious that this same reduction of the problem applies. To see that it does, begin by factoring the k-blade B using the set of k vectors in $\{b_i\}_{i=1}^k$ as $B = \bigwedge_{i=1}^k b_i$, and write

$$V^{-1}p_i(x)V \cdot B = -\sum_{j=1}^{k}(-1)^j(p_i(x) \cdot Vb_jV^{-1})B_j, \tag{5.1}$$

where again B_j denotes the product B with b_j removed. Cleary, we have $V^{-1}p_i(x)V \cdot B = 0$ if and only if for all $j \in \mathbb{Z}_k$, we have $p_i(x) \cdot Vb_jV^{-1} = 0$. Now notice that for all $j \in \mathbb{Z}_k$, we have $p_i(x) \cdot Vb_jV^{-1} = 0$ if and only if

$p_i(x) \cdot VBV^{-1} = 0$, since

$$p_i(x) \cdot VBV^{-1} = -\sum_{j=1}^{k}(-1)^j (p_i(x) \cdot Vb_j V^{-1}) VB_j V^{-1}. \tag{5.2}$$

Notice that the linear independence of the set $\{B_i\}_{i=1}^k$ remains invariant under an application of the versor V to get the set $\{VB_iV^{-1}\}_{i=1}^k$.

6. Comments And Criticisms

Unfortunately, the conclusion that must be reached at the end of this paper is that there does not appear to be any tangible benefit to the now presented model of algebraic sets using geometric algebra in terms of the union and intersection operations. The outer product does not appear to calculate for us anything beyond what we can already do by simply multiplying polynomial equations together, or storing them in a set.

The only possible redeeming quality of the approach given is perhaps that, by virtue of using blades to represent algebraic sets, it lends itself well to the use of versors in the desire to transform such sets.

Given a blade $C \in \mathbb{V}^\infty$, we may think of $\dot{G}(C)$ and $\hat{G}(C)$ as two different geometric interpretations of C. If at any point we find one of these sets empty, (i.e., if we intersect two non-intersecting geometries), then we may find the other set as non-empty. Furthermore, there is always geometric significance in the comparison of the geometries $\dot{G}(C)$ and $\hat{G}(C)$. For example, in CGA, every imaginary dual round, when reinterpreted as a direct geometry, is a real round.

References

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