Further Treatment Of Geometric Sets

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Abstract. Abstract goes here...

1. Introduction

We let $P(\mathbb{V})$ denote the set of all possible vector sub-spaces of $\mathbb{V}$, not the power-set of $\mathbb{V}$.

2. The Meet And Join Of Vector Sub-Spaces

An excellent treatment of *meet* and *join* is given in []. Unlike [], however, here we will not, as they said, abuse language by referring to any blade as a vector sub-space.[1] In an attempt to be as rigorous as possible, we will introduce a function $\overline{B} : \mathbb{B} \to P(\mathbb{V})$, and, for any blade B, only refer to $\overline{B}(B)$ as a vector sub-space. For convenience in notation, we write $\overline{B}$ instead of $\overline{B}(B)$.

Definition 2.1. For any blade $B \in \mathbb{B}$, we define the function $\overline{B} : \mathbb{B} \to P(\mathbb{V})$ as

$$\overline{B} = \{x \in \mathbb{V} | x \wedge B = 0\}. \tag{2.1}$$

In light of Definition **??**, it is important to realize that for any two blades $A, B \in \mathbb{B}$, and a vector sub-space $V \subseteq \mathbb{V}$, if we have $\overline{A} = V$ and $\overline{B} = V$, then there exists a scalar $\lambda \in \mathbb{R}$ such that $A = \lambda B$.

As operations (or well defined functions), *meet* and *join* operate on vector spaces, not blades. We therefore do not speak of taking the *meet* or *join* of two blades. Rather, the blades of a geometric algebra will help us calculate the *meet* and *join* of vector sub-spaces represented by those blades.

Definition 2.2 (The *meet* of two vector sub-spaces). For any two blades $A, B \in \mathbb{B}$, the *meet* M of $\overline{A}$ and $\overline{B}$ is the vector space given by

$$M = \overline{A} \cap \overline{B} = \{x \in \mathbb{V} | x \in \overline{A} \text{ and } x \in \overline{B}\}. \tag{2.2}$$

[1] The present author has abused language in this way in other writings, but repents of this, at least in the present paper.

From Defintion 2.2, we see that the *meet* of two vector subspaces is the largest common sub-space. Notice that *meet* is clearly communtative.

Definition 2.3 (The *join* of two vector sub-spaces). For any two blades $A, B \in \mathbb{B}$, the *join* J of $\overline{A}$ and $\overline{B}$ is the vector space given by

$$J = \overline{A} + \overline{B} = \{x_1 + x_2 \in \mathbb{V} | x_1 \in \overline{A} \text{ and } x_2 \in \overline{B}\}. \tag{2.3}$$

From Definition 2.3, we see that the *join* of two vector spaces is the smallest common super-space. Notice here too the commutativity of *join* .

Note that some care should be taken with Definition 2.2 and Definition 2.3 by easily verifying that M and J each satisfy the necessary properties of a vector space. Also note that any pair of blades, each representative of the same *meet* or *join* , are clearly scalar multiples of one another.

At this point, a natural question arrises. Given two blades $A, B \in \mathbb{B}$, how do we find a blade $C \in \mathbb{B}$ such that $\overline{C} = \overline{A} \cap \overline{B}$ and $\overline{C} = \overline{A} + \overline{B}$? Do this in terms of orthogonal complement...define that first...

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