On Two Useful Properties of Conformal-like Models of Geometric Algebra

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Abstract. Blah.

Mathematics Subject Classification (2010). Primary 14J70; Secondary 14J29.

Keywords. Algebraic Surface, Conformal Model, Conformal Transformation, Geometric Algebra.

1. Introduction

This paper begins where the first section of [1] leaves off.[1] In the said section, the function $\hat{g}$ was introduced, but its property of allowing us to combine geometries was not given any treatment. Here we rectify this glossing-over of the property by giving it an in-depth look in a some-what more general context than that of the conformal model of geometric algebra.

Another often-over-looked, and very important property of the general model laid out in the first section of [1] is that of being able to equate different characterizations of a single geometry. We will see how this property follows from what we'll call the point-fitting property, and why it is important.

2. The Point-Fitting Property

Letting $B \in \mathbb{G}$ be a k-blade, notice that if there exists a set of k points $\{x_i\}_{i=1}^k \subseteq \hat{g}(B)$ such that $\bigwedge_{i=1}^k p(x_i) \neq 0$, then there must exist a scalar $\lambda \in \mathbb{R}$ such that

$$B = \lambda \bigwedge_{i=1}^k p(x_i).$$

[1] No more than the first section of [1] need be read before reading this paper.

When there exists such a set of k points for a blade B, we will say that the geometry $\hat{g}(B)$ has the point-fitting property, and that the k points $\{x_i\}_{i=1}^k$ fit $\hat{g}(B)$, or that $\hat{g}(B)$ is fitted by these points.

3. The Reinterpretation Property

Letting $A, B \in \mathbb{G}$ both be k-blades with $\hat{g}(A) = \hat{g}(B)$, if there exists a scalar $\lambda \in \mathbb{R}$ such that $A = \lambda B$, we say that the geometries $\hat{g}(A)$ and $\hat{g}(B)$ have, collectively, the reinterpretation property.[2] It is not hard to show that if each of A and B have the ponit-fitting property, then they share the reinterpretation property.

Under the assumption that each of $\hat{g}(A)$ and $\hat{g}(B)$ have the point-fitting property, notice that if the k points $\{x_i\}_{i=1}^k$ fit $\hat{g}(A)$, then they also fit $\hat{g}(B)$. Letting $A = \alpha \bigwedge_{i=1}^k p(x_i)$ and $B = \beta \bigwedge_{i=1}^k p(x_i)$, it is clear that if $\lambda = \frac{\alpha}{\beta}$, then $A = \lambda B$.

What this property allows us to do in the model is equate one characterization of a given geometry with that of another. This may well be termed a reinterpretation of the geometry. For example, we may form a circle as the intersection of two spheres, then go on to equate this characterization of the circle with a canonical characterization, which may be that of the intersection of a plane and sphere centered on that plane. Reinterpreting the calculated geometry in terms of a canonical form, we can easily calculate the characteristics this intersection. Indeed, this property of the model, (the reinterpretation property), and not the intersection property of the model itself, is what makes the ability to do intersections in the model useful and interesting.

References

[1] S. Parkin, *The Mother Minkowski Algebra of Order m*. 2013

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[2]Clearly, if $A = \lambda B$, then $\hat{g}(A) = \hat{g}(B)$, but, depending on how p is defined, the converse of this statement may not be generally true.