Fun Problems

Spencer T. Parkin

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1 Rotating One Vector Into Another

Let a and b be two non-zero vectors with the property that if there exists a scalar λ such that $a = \lambda b$, then $\lambda > 0$. (These vectors are not necessarily parallel, but if they are parallel, they point in the same direction.) Given two such vectors a and b, find a rotor R such that

$$b = RaR^{-1}. \tag{1}$$

Do not use trigonometric functions.

2 The Magnitude Of A Rotor

For any two vectors a and b, show that

$$|a||b| = |ab|. \tag{2}$$

Hint: You need to know that for any element E of a geometric algebra generated by an n-dimensional vector space, we define

$$|E|^2 = \sum_{i=0}^{n} |\langle E \rangle_i|^2. \tag{3}$$

3 The Rotor ab

Given any two unit-length vectors a and b with $a \neq b$, and any non-zero vector c, show that the vector c', given by

$$c' = RcR^{-1}, \tag{4}$$

where $R = ab$, is the rotation of c through an angle 2θ in a plane parallel to the plane containing a and b, where θ is the angle between a and b.

Bonus: Find the polar decomposition of R. That is, write R in terms of e, the base of the natural logarithm.