Chapter 8 Exercises
Gallian's Book on Abstract Algebra

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Understanding Theorem 8.3

We want to show that $U(st) \approx U(s) \oplus U(t)$. Let $\phi(x) = (x \bmod s, x \bmod t)$. For $x, y \in U(st)$, if $\phi(x) = \phi(y)$, then $x \equiv y \pmod{s}$ and $x \equiv y \pmod{t}$. Then, since $\gcd(s,t) = 1$, it is clear that $x \equiv y \pmod{st}$. (See Problem 15 of Chapter 0.) So ϕ is one-to-one. It is also onto since ϕ is onto-to-one and maps a finite set to another of the same cardinality. That ϕ is operation preserving is a matter of showing that for any $x, y \in U(st)$, we have

$$\begin{aligned}(xy \bmod st) \bmod m &= (x \bmod m)(y \bmod m) \bmod st \\ &= (xy \bmod m) \bmod st,\end{aligned}$$

where m is s or t. With enough thought, this is intuitive enough to warrent justification by virtue of being clear.

Problem 1

Prove that the external direct product of any finite number of groups is a group.

There is clearly an identity element. Closure is clear. Inverses are clear. Associativity is clear. I think that's it.