

Chapter 4 Exercises

Algebraic Geometry

A Problem Solving Approach

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April 7, 2014

Exercise 4.2.9

Let I be the ideal in $k[x_1, \dots, x_n]$ generated by a set $S \subset k[x_1, \dots, x_n]$. Show that $V(S) = V(I)$. Thus every algebraic set is defined by an ideal.

Let $(a_1, \dots, a_n) \in V(S)$ and consider $f \in I$. Notice that f has the form

$$f = \sum_{s \in S} s f_s,$$

where, for each $s \in S$, f_s is any polynomial taken from $k[x_1, \dots, x_n]$. From this it is clear that $f(a_1, \dots, a_n) = 0 \implies (a_1, \dots, a_n) \in V(I)$.

Now let $(a_1, \dots, a_n) \in V(I)$ and consider $f \in S$. Clearly $f(a_1, \dots, a_n) = 0$, because $S \subseteq I$. So $f \in V(I)$.