

Chapter 9 Exercises

Gallian's Book on Abstract Algebra

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Exercise 61

Suppose that H is a normal subgroup of a finite group G . If G/H has an element of order n , show that H has an element of order n . Show, by example, that the assumption that G is finite is necessary.

The case $n = 1$ is trivial, so let $n > 1$. Let $a \in G$ such that $|aH| = n$. Clearly $a \neq e$. It follows that the mapping $\phi : H \rightarrow H$, given by $\phi(h) = a^n h$ is a non-trivial permutation of the elements of H and so ϕ is a member of the group of permutations of H . We then see that $a^{|\phi|n} = e$. But it is easy to see that for all integers $i \in [1, |\phi|n - 1]$, we have $a^i \neq e$. So $|a^{|\phi|}| = n$.