# Chapter 4 Exercises
# Gallian's Book on Abstract Algebra

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## Problem 11

Let $G$ be a group and let $a \in G$. Prove that $\langle a^{-1} \rangle = \langle a \rangle$.

Suppose $a$ has finite order $n$. Then $a^{-1} = a^{n-1}$ and since $\gcd(n, n-1) = 1$, it follows that $\langle a^{-1} \rangle = \langle a^{n-1} \rangle = \langle a^{\gcd(n,n-1)} \rangle = \langle a \rangle$.

Suppose now that $a$ has infinite order. In this case, $a^{-1} \notin \langle a \rangle$, so the statement does not hold.