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Abstract. Abstract...

Keywords. Key words...

Definition 0.1 (Spade). An element $M \in \mathbb{G}$ is a *spade* if it can be written as the geometric product of zero or more vectors.

It follows from Definition 0.1 that all versors are spades, but not all spades are versors. Furthermore, while the set of all spades of $\mathbb{G}$ enjoys closure under the geometric product, this set, unlike the set of versors of $\mathbb{G}$, does not form a group.

Definition 0.2 (Spade Rank/Compact Spade Form). The *rank* of a spade $M \in \mathbb{G}$, denoted $\text{rank}(M)$, is the smallest number[1] of vectors for which M can be written as a geometric product of such. If a spade $M_r \in \mathbb{G}$ has factorization

$$M_r = \prod_{i=1}^{r} m_i,$$

we say that it is written in *compact form* if $\text{rank}(M_r) = r$.

Many identities involving a spade M_r hold whether or not it is written in compact form. It is even more important, however, to realize that every spade M_r can be rewritten in a compact form if not already in such a form.

Lemma 0.3. *For any given spade $M_r \in \mathbb{G}$, rank$(M_r) = r$ if and only if*

$$\langle M_r \rangle_r = \bigwedge_{i=1}^{r} m_i.$$

Proof. One direction is trivial. The other is not! □

Lemma 0.4. *For every non-zero r-blade $B_r \in \mathbb{G}$ with $r > 1$, and having factorization*

$$B_r = \bigwedge_{i=1}^{r} b_i,$$

[1] This smallest number exists by the well-ordering principle. See [].

the set of $(r-1)$-blades $\{B_r^{(i)}\}_{i=1}^r$, where the notation $B_r^{(i)}$ is given by

$$B_r^{(i)} = \bigwedge_{\substack{j=1 \\ j\neq i}}^r b_j,$$

is a linearly independent set.

Proof. Supposing to the contrary, and without loss of generality, let

$$B_{r-1} = B_r^{(r)} = \sum_{i=1}^{r-1} \alpha_i B_r^{(i)} = \left(\sum_{i=1}^{r-1} \alpha_i B_{r-1}^{(i)}\right) \wedge b_r.$$

Now notice that

$$0 \neq B_r = B_{r-1} \wedge b_r = B_r^{(r)} \wedge b_r = \left(\sum_{i=1}^{r-1} \alpha_i B_r^{(i)}\right) \wedge b_r = 0,$$

which is clearly a contradiction. □

Lemma 0.5. *Given any spade M_r, the set of all solution sets $\{\alpha_i\}_{i=1}^r$ of the equation*

$$0 = \sum_{i=1}^r \alpha_i M_r^{(i)}$$

is, for all integers $j \in [0, r]$, the intersection of all sets of solution sets of the equations

$$0 = \sum_{i=1}^r \alpha_i \langle M_r^{(i)} \rangle_j,$$

where the notation $M_r^{(i)}$ is given by

$$M_r^{(i)} = \prod_{\substack{j=1 \\ j\neq i}}^r m_j.$$

Proof. This is a simple consequence of there being no possibility of interference between elements of differing grade. □

Lemma 0.6. *For any given spade $M_r \in \mathbb{G}$ with $r > 1$, if rank$(M_r) = r$, then the set of spades $\{M_r^{(i)}\}_{i=1}^r$ is a linearly independent set.*

Proof. This follows easily in consideration of lemmas 0.3, 0.4 and 0.5. □

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