A Model Of Algebraic Sets Using Geometric Algebra

Spencer T. Parkin

Abstract. Blah.

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1. Introduction And Motivation

Letting $\mathbb{R}^n$ denote an n-dimensional Euclidean space, we are going to let $\mathbb{P}^n$ denote the set of all polynomials $p : \mathbb{R}^n \to \mathbb{R}$ of any degree. Then, letting $P \subseteq \mathbb{P}^n$ be any set of such polynomials, we then define the zero set Z of P, denoted $Z(P)$, as the set given by

$$Z(P) = \{x \in \mathbb{R}^n | p(x) = 0 \text{ for all } p \in P\}. \tag{1.1}$$

Every subset S of $\mathbb{R}^n$ for which there exists a set $P \subseteq \mathbb{P}^n$ such that $S = Z(P)$ is what we refer to as an algebraic set. It is well known that for any algebraic set $S \subseteq \mathbb{R}^n$, there is always such a subset P of $\mathbb{P}^n$ of finite cardinality.

Given the definition in equation (1.1), it is easy to show that the subsets S of $\mathbb{R}^n$ that are the geometries of CGA, and other similar models of geometry based upon geometric algebra, are simply algebraic sets. The goal of this paper is to show that there exists a model of geometry, based upon geometric algebra, where every possible algebraic set has a representative in the form of an element of that geometric algebra. A desire to come up with such a model of geometric algebra is motivated by the admittedly fanciful dream of the German mathematician Leibniz, referred to in [] and claimed to have already been realized in []. In any case, it would seem that a generalization of CGA or any CGA-like model to one that hosts the set of all algebraic sets in $\mathbb{R}^n$ would bring us closer to such a goal. Work to this end has already been done in [1, 2] which has brought us, the reader and writer, to the present paper.

2. Blah

We begin with an examination of $\mathbb{P}^n$ as a linear space, observing that it is of countably infinite dimension. A set of basis vectors for this space may be taken as the set of all unit monomials in the n variables of an arbitrary point in $\mathbb{R}^n$. The number c_k of such monomials homogeneous of a degree k is given by

$$c_k = \sum_{i=1}^{n} \binom{n}{i} p(k,i), \tag{2.1}$$

where $p(k,i)$ is the number of partitions of k of size i.

References

1. S. Parkin, *A model for quadric surfaces using geometric algebra*, Advances in Applied Clifford Algebras **?** (2013), ?–?
2. ———, *A variation of the quadric model of geometric algebra*, Advances in Applied Clifford Algebras **?** (2013), ?–?

Spencer T. Parkin
e-mail: `spencer.parkin@gmail.com`