"Machine Learning and Computational Statistics"

1st Homework

Exercise 1 (optional):

Name a classifier whose associate *f*(.) is of (a) **parametric** and (b) **non parametric** nature.

Exercise 2:

(a) Define the parametric set of the **quadratic** functions $f_{\vartheta}: R \rightarrow R$ and give two instances of it. What is the dimensionality of $\boldsymbol{\theta}$?

(b) Define the parametric set of the **3rd degree polynomials** $f_{\vartheta}: R^2 \rightarrow R$ and give two instances of it. What is the dimensionality of $\boldsymbol{\theta}$?

(c) Define the parametric set of the **3rd degree polynomials** $f_{\vartheta}: R^3 \rightarrow R$ and give two instances of it. What is the dimensionality of $\boldsymbol{\theta}$?

(d) Consider the function $f_{\boldsymbol{\theta}}(\boldsymbol{x}): R^5 \rightarrow R$, $f_{\theta}(\boldsymbol{x}) = \frac{1}{1+\exp(-\boldsymbol{\theta}^T\boldsymbol{x})}$. Define the associated parametric set and give two instances of it. What is the dimensionality of $\boldsymbol{\theta}$?

(e) In which of the above cases $f_{\boldsymbol{\theta}}$ is linear with respect to $\boldsymbol{\theta}$?

Exercise 3:

Verify that for two l-dimensional column vectors $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_l]^T$ and $\boldsymbol{x} = [x_1, x_2, \dots, x_l]^T$ it holds: $(\boldsymbol{\theta}^T\boldsymbol{x})\boldsymbol{x} = (\boldsymbol{x}\,\boldsymbol{x}^T)\boldsymbol{\theta}$.

Exercise 4:

Consider the vectors $\boldsymbol{x}_n = [x_{n1}, x_{n2}, \dots, x_{nl}]^T, n = 1, \dots, N$. Define the Nxl matrix X and N-dimensional column vector $\boldsymbol{y}$ as follows:

$$X = \begin{bmatrix} \boldsymbol{x}_1{}^T \\ \boldsymbol{x}_2{}^T \\ \vdots \\ \boldsymbol{x}_N{}^T \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1l} \\ x_{21} & x_{22} & \cdots & x_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{Nl} \end{bmatrix} \text{ and } \boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

(Note that the rows of X are the vectors $\boldsymbol{x}_n, n = 1, \dots, N$).

Verify the following identities:

$X^T X = \sum_{n=1}^{N} \boldsymbol{x}_n \boldsymbol{x}_n{}^T$ and $X^T \boldsymbol{y} = \sum_{n=1}^{N} y_n \boldsymbol{x}_n$.