Lec-37, DC, 23-24

Prob. of error when taking a decision in favor of m_i given that $\underline{x}$ is rxd = union of events where either of $\{m_1, m_2 \dots m_M\}$ with m_i excluded being sent takes place.

$$P_e(\hat{m} = m_i \mid \underline{x}) = P[m_1 \text{ is sent} \cup m_2 \text{ is sent} \dots \cup m_M \text{ is sent (with } m_i \text{ excluded)} \mid \underline{x} \text{ is rxd}]$$

$$= P[m_i \text{ not sent} \mid \underline{x} \text{ is rxd}]$$

Starting from previous lecture, $f_{\underline{x}}(\underline{x})$ is same for all & appears as a constant in the decision rule, hence can be ignored.

Special case :- If p_k are all same i.e equiprobable symbols at Tx.

So, in this case, MAP rule reduces to,

choose $k=i$ if $f_{\underline{x}}(\underline{x} | m_k)$ is max. for $k=i$

or $l(m_k)$ is " " "

This is referred to as Maximum Likelihood (ML) rule.

& the detector is called as ML det. / decoder.

At Rx, under ML rule, obtain $l(m_1), l(m_2) \ldots$ $\{m_1 \ldots m_M\}$

$l(m_M)$ & choose the max.

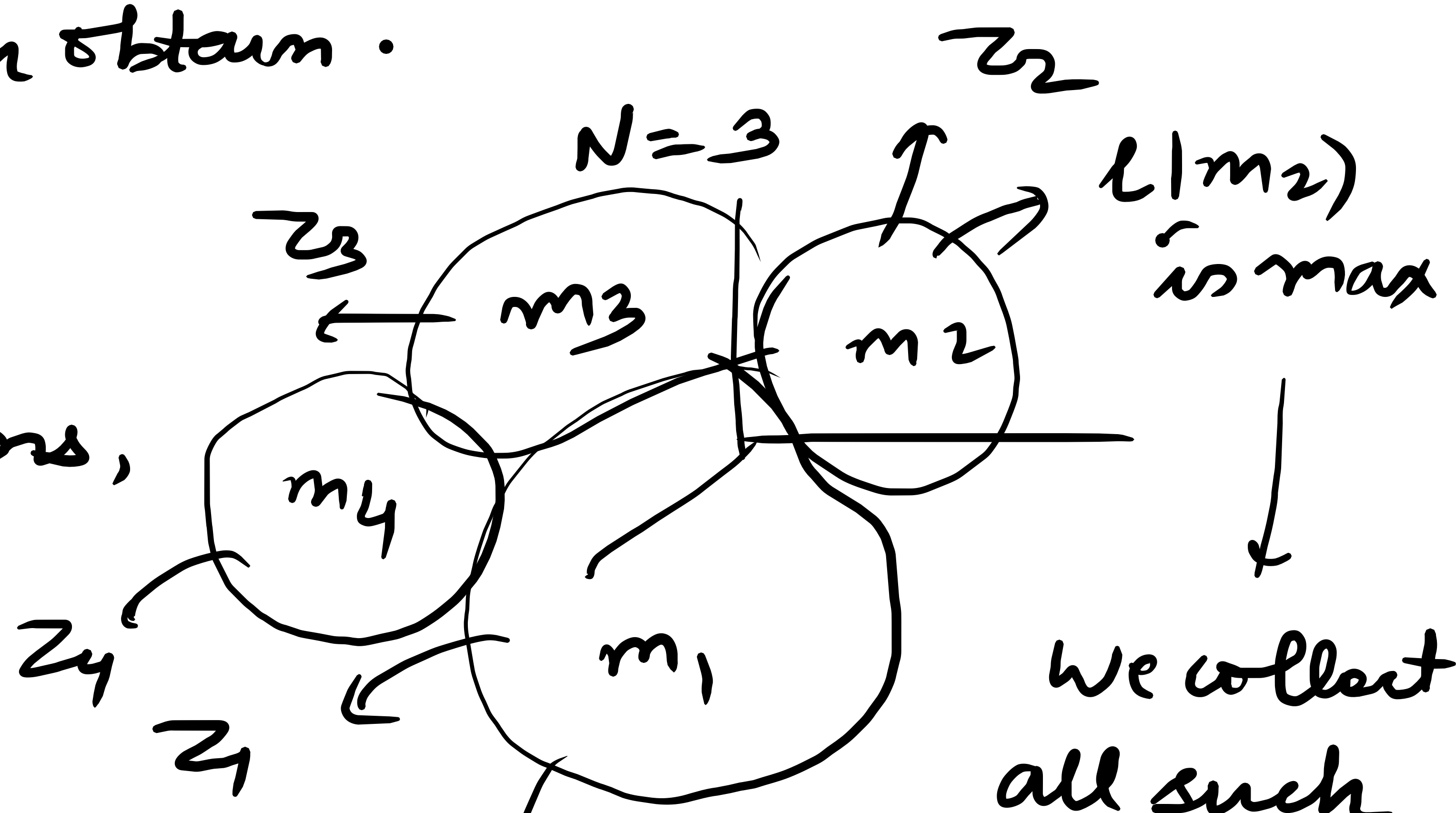
$f_{\underline{x}}(\underline{x}|m_i)$ or $l(m_i)$ is what you obtain.

$\underline{x} = (x_1, x_2 \ldots x_N)$

ML rule leads to M decision regions, denoted as $Z_1, Z_2 \ldots Z_M$.

Basis for forming the regions is

$\underline{x}$ lies in region Z_i if $l(m_k)$ is max. for $k = i$



We collect all $\underline{x}$ for whom $l(m_1)$ is max

We collect all such $\underline{x}$ for whom $l(m_2)$ is max

Q. What happens if $\arg\max_{k} l(m_k) = k_1, k_2$? m_{k_1}, m_{k_2} – select any of m_{k_1} & m_{k_2} randomly.

It won't affect the prob. of error. → Q- why? H.W.

over AWGN, $l(m_k) = -\frac{1}{N_0} \sum_{j=1}^{N} (x_j - s_{kj})^2$

We note that $l(m_k)$ attains its $\boxed{\text{max.}}$ value when the summation term $\sum_{j=1}^{N} (x_j - s_{kj})^2$ is minimized by the choice $k=i$

While deriving the MAP rule & ML rule we have not used the AWGN assumption. This theory is valid in general

We know, $\sum_{j=1}^{N} (x_j - s_{kj})^2 = \| \underline{x} - \underline{s}_k \|^2$

$\underline{s}_k = (s_{k1}, s_{k2} \dots s_{kN})^T$

$\underline{x} = (x_1, x_2 \dots x_N)^T$

Observation vector $\underline{x}$ lies in reg. Z_i if the Euclidean distance $\| \underline{x} - \underline{s}_k \|$ is min. for $k=i$

$$\sum_{j=1}^{N}(x_j - s_{kj})^2 = \underbrace{\sum_{j=1}^{N} x_j^2}_{} - 2\sum_{j=1}^{N} x_j s_{kj} + \sum_{j=1}^{N} s_{kj}^2$$

does it depend on k? — No

Observ^n vector $\underline{x}$ lies in region Z_i if

$$\sum_{j=1}^{N} x_j s_{kj} - \frac{E_k}{2} \text{ is max. for } k = i$$

where E_k is the energy of the txd. sig. $s_k(t)$

$$E_k = \sum_{j=1}^{N} s_{kj}^2$$

(see fig 5.8 & 5.9 from Haykin's TB)

H.W. "Equivalence of correlation & Matched filter receivers"

Pg 327-328 Simon Haykin's TB.

Avg. prob. of symbol error P_e

$$\sum_{i=1}^{M} P_i \, P(\underline{x} \text{ does not lie in } Z_i \mid m_i \text{ sent})$$

equiprob. case?

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} P(\underline{x} \text{ lies in } Z_i \mid m_i \text{ sent})$$

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i} f_{\underline{x}}(\underline{x} \mid m_i)\, d\underline{x}$$

Passband systems

(BB) Baseband signal

(PB) Passband signal

A signal $u(t)$ is said to be BB if the signal energy is concentrated in a band around DC, and $U(f) \approx 0$, $|f| > W$ for some $W > 0$.

A signal $u(t)$ is said to be PB if its energy is concentrated in a band away from DC, with

$$U(f) \approx 0 \quad |f \pm f_c| > W$$

where $f_c > W > 0$