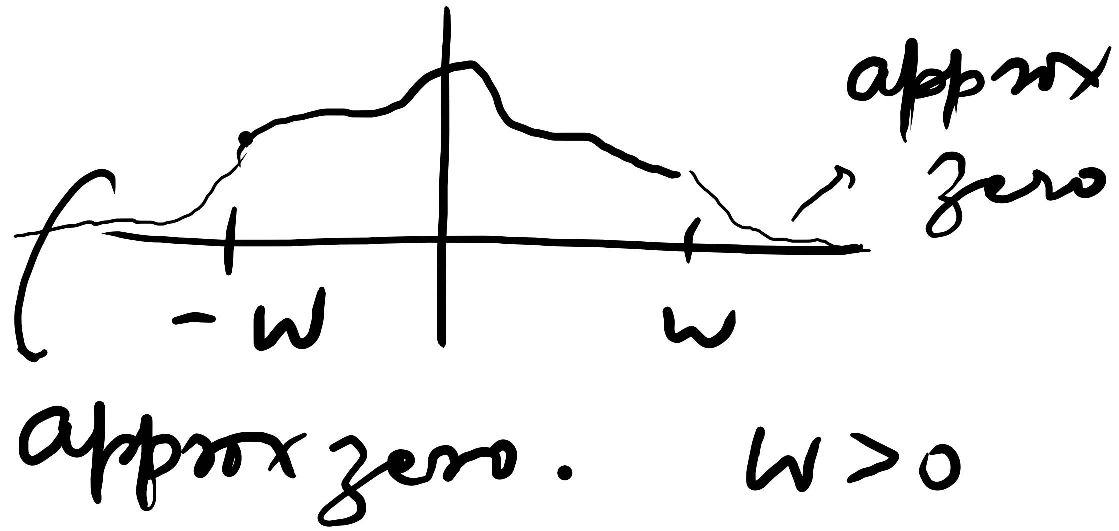
Lec-38, DC, 23_24

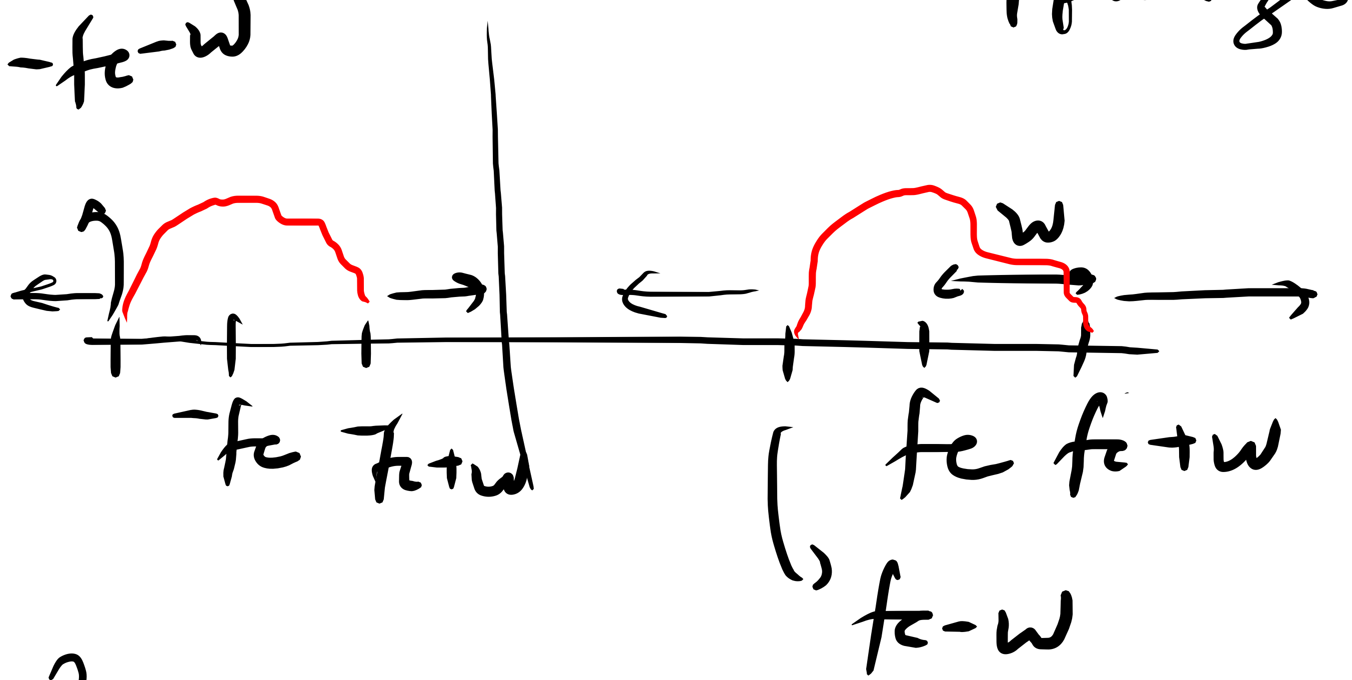
BB $U(f) \approx 0$ $\quad |f| > W$ i.e $f > W$ & $f < -W$

PB $U(f) \approx 0$, $|f \pm f_c| > W$

where $f_c > W > 0$



$W > 0$



$f + f_c > W$ & $f + f_c < -W$

$f - f_c > W$ & $f - f_c < -W$

$$\left.\begin{aligned} f &> W - f_c \\ f &> W + f_c \end{aligned}\right\} \qquad \left.\begin{aligned} f &< -W - f_c \\ f &< f_c - W \end{aligned}\right\}$$

$$\boxed{\begin{aligned} f &> -f_c + W \qquad & f &> f_c + W \\ f &< -f_c - W \qquad & f &< f_c - W \end{aligned}}$$

Information sources typically emit BB signals. ex- analog audio signal → signifi: Band from DC to 20KHz

Consider a real-valued BB message signal $m(t)$ of BW W. to be sent over a PB channel centered around f_c.

$$m(t)\ \sin(\ )/\cos(\ ) \rightarrow \boxed{M(f) * \left[\delta(f-f_c) \pm \delta(f+f_c)\right] \times \frac{1}{2}}$$

$$= \frac{M(f-f_c) \pm M(f+f_c)}{2}$$

We can translate the message to PB simply by $\times$ it by a sinusoidal at f_c

$$u_p(t) = m(t)\cos(2\pi f_c t) \leftrightarrow U_p(f) = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

Instead of cosine, we can use sin also.

$$v_p(t) = m(t)\sin(2\pi f_c t) \leftrightarrow V_p(f) = \frac{1}{2j}\left[M(f-f_c) - M(f+f_c)\right]$$

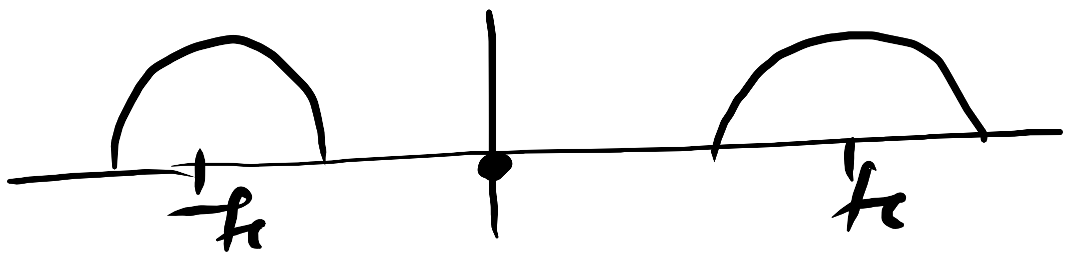
$|U_p(f)|$ & $|V_p(f)|$ have freq. content in a band around f_c,
& are PB signals (no DC component)

Are $u_p(t)$ & $v_p(t)$ orthogonal? Inner prod. of $u_p(t)$ & $v_p(t)$ should be zero.

$$\int_{-\infty}^{\infty} u_p(t)\, v_p(t)\, dt = 0.$$

$$\int_{-\infty}^{\infty} m(t)\, m(t) \sin(2\pi f_c t) \cos(2\pi f_c t)\, dt = \frac{1}{2}\int_{-\infty}^{\infty} m^2(t) \sin(4\pi f_c t)\, dt$$

If I assume that $m^2(t)$ is a BB signal & $m^2(t) \times \sin(4\pi f_c t)$ is a PB signal, it implies that DC component of $m^2(t) \sin(4\pi f_c t) = 0$



You have assumed $m(t)$ to BB with BW W then what about $m^2(t)$

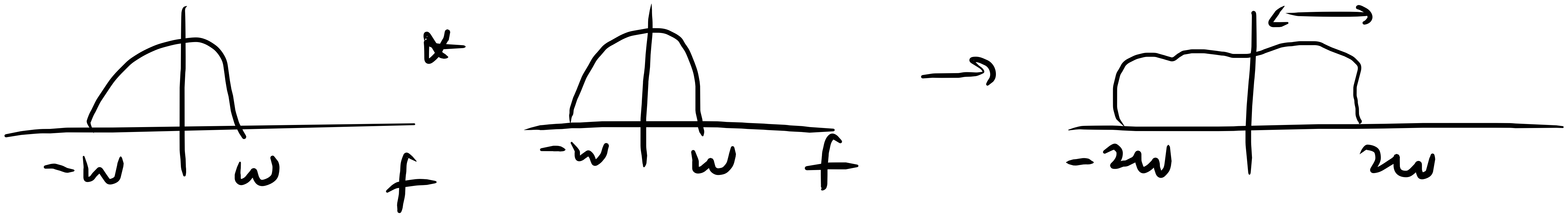
FS coefficient

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} m(t)\, e^{-jk\omega t}\, dt$$

we know that $f_c > W \Rightarrow 2f_c > 2W$

$a_0 =$

$m(t) \times m(t) \longrightarrow M(f) * M(f)$



$m^2(t) \sin(2\pi\, 2f_c t) \rightarrow$ so this is a PB signal. $\Rightarrow$ that DC value is zero.

hence $\int m^2(t) \sin(4\pi f_c t)\, dt = 0$. so $v_p(t)$ & $u_p(t)$ are orthogonal.

What about two different BB signals using the same carrier frequency?

$$u_p(t) = m_1(t)\cos(2\pi f_c t)$$

$$v_p(t) = m_2(t)\sin(2\pi f_c t)$$

where $m_1(t) \to W_1$

$m_2(t) \to W_2$

$W_1 < f_c$ & $W_2 < f_c$

$\Downarrow$

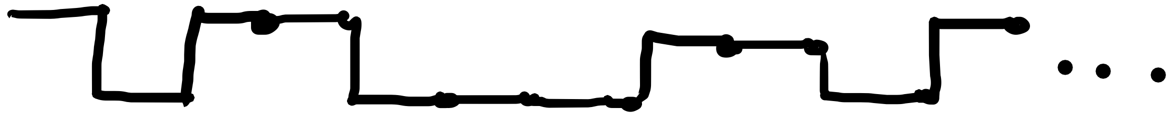
$W_1 + W_2 < 2f_c$

Will the orthogonality hold?

$$\underbrace{\underset{W_1}{m_1(t)}\;\underset{W_2}{m_2(t)}}_{W_1+W_2} \sin(2\pi\, 2f_c\, t)$$

10110001101011 $\rightarrow$ convert to a CT signal using a suitable pulse

Use rect( ) pulse



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So, using both cosine & sine carriers, we can construct a PB signal of the form

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$

where $u_c(t)$ & $u_s(t)$ are real BB signals of BW at most W, with $f_c > W$.