

# ML Performance Optimization

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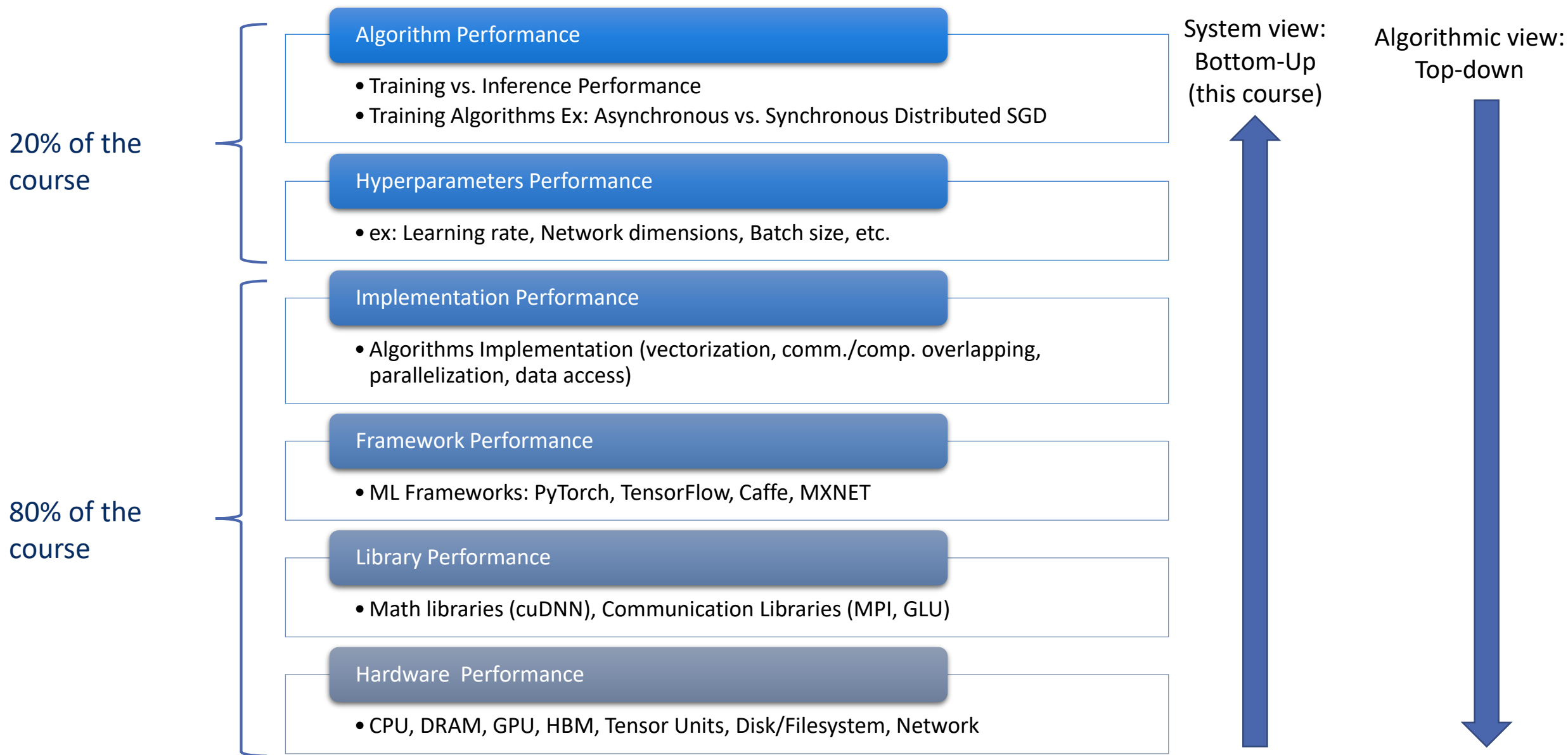
CSCI-GA.3033-020 HPML

# Summary

- Problem definition
- System vs. Algorithmic view
- Performance Optimization Methodology:
  - Measurement
  - Analysis
  - Optimization

# System vs. Algorithmic view

# ML Performance Factors



# A couple of examples

- Implementation Performance:
  - too many mallocs() in C (or *new* in C++): easily 10 – 100x slowdown
- Algorithmic Performance:
  - Search 1 element in 10 billion stored in an array
    - Linear search:  $O(n)$  – average: about 5 billions comparisons expected (\*)
    - Binary search:  $O(\log n)$  – average: about 32 comparisons expected (\*)

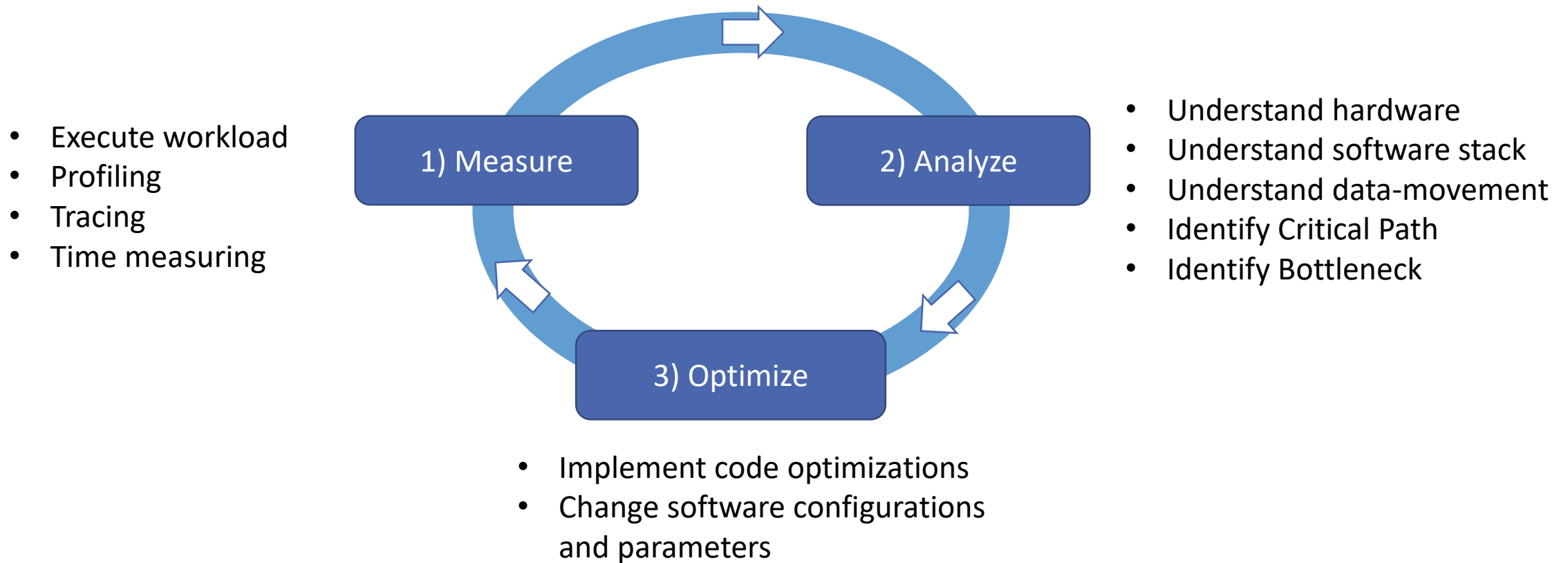
(\*) Assuming exactly one matching element exists and elements are uniformly distributed

# Software Performance Optimization

# ML Performance Optimization Definition

- Software Performance Optimization for ML
  - Given:
    - A **system** (ex: NYU Compute node + PyTorch)
    - An **algorithm** (ex: Distributed SGD training) + **hyperparameters**
    - A **dataset** (ex: CIFAR100)
  - Obtain the **maximum** performance

# Performance Optimization Methodology





# Performance optimization methodology (1): Measurement

# What is performance?

- Basic metrics:

- Execution time:  $t$  (for a single operation is called **latency**)

- Performance:  $\frac{1}{t}$

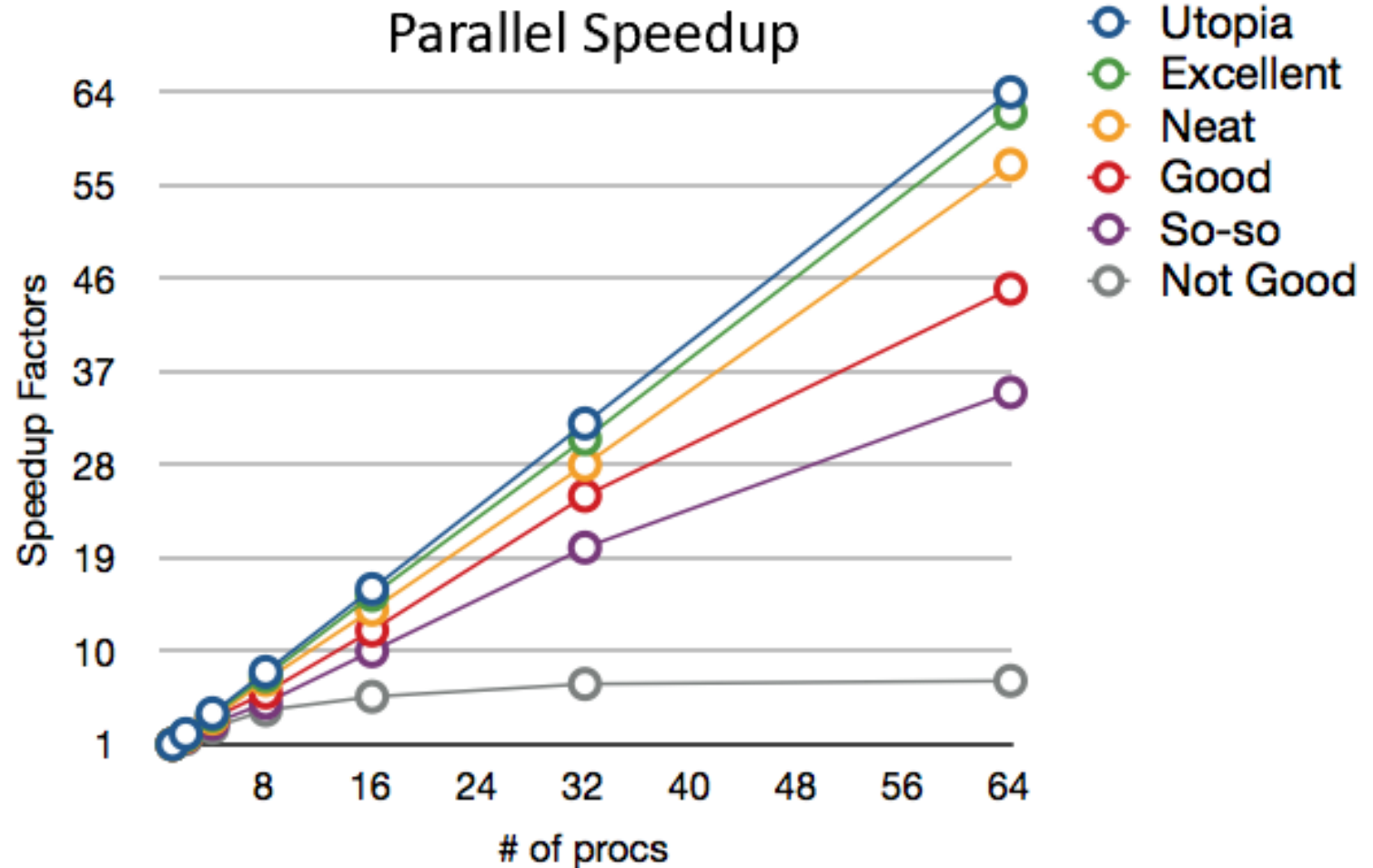
- Derived metrics:

- Throughput:  $\frac{\# \text{ operations}}{t}$  or  $\frac{\# \text{ programs}}{t}$

- FLOPS:  $\frac{\# \text{ floating\_point\_operations}}{t}$  (<https://en.wikipedia.org/wiki/FLOPS>)

# Speedup

- Speedup of B w.r.t. A:  $\frac{t_A}{t_B}$
- Parallel Speedup:  $\frac{t_{serial}}{t_{parallel}}$
- Slowdown is inverse of Speedup



from: [http://web.eecs.utk.edu/~huangj/hpc/hpc\\_intro.php](http://web.eecs.utk.edu/~huangj/hpc/hpc_intro.php)

# Scalability

- **Scaling Efficiency**

- $E = \frac{t_{serial}}{t_{parallel} * p} \leq 1$

$p$  is the number of processes/threads/...

- **Strong Scaling:** Constant problem size while increasing  $p$

- Increasing synchronization cost, but fixed amount of work

- **Weak Scaling:** Increasing problem size proportional to  $p$

- Work per process is constant
  - Increasing synchronization cost, increasing work

# Computing Averages

- Average Execution Time

- Arithmetic mean:  $\frac{1}{n} \sum_{i=1}^n t_i$

- Average Performance or Throughput

- If  $t$  is held constant  $\Rightarrow$  Arithmetic mean
- If  $\#operations$  is held constant  $\Rightarrow$  Harmonic mean:

$$\frac{n}{\sum_{i=1}^n \frac{t_i}{\#operations}}$$

- Average Speedup, Slowdown or any Ratio

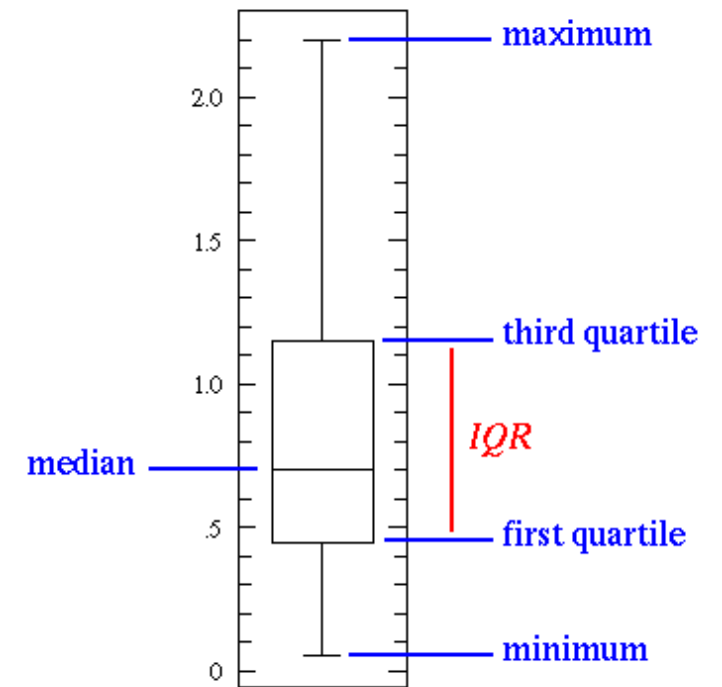
- Geometric mean:  $\sqrt[n]{\prod_{i=1}^n speedup_i}$

# Benchmarking Workloads

- Benchmarks in ascending order of complexity:
  1. Micro-kernels: test a specific processor feature  
Examples: Floating point, L1 Cache, L2 Cache,
  2. Micro-benchmark: small program from a programming assignment  
Examples: Merge sort in isolation
  3. Kernels: a specific algorithm in a real program  
Examples: Quicksort, Binary Search, DGEMM, DAXPY with context
  4. Synthetic Benchmarks: try to reproduce the workload of a class of applications  
Examples: Dhrystone, Linpack
  5. Real Applications: a real application used for a specific purpose  
Examples: Word, MySQL, NAMD (Molecular Dynamics)
  6. Real Workflows: a set of applications working together  
Example: CANDLE workflow

# Measuring and Reporting Performance

- Reproducibility
  - Always include absolute execution time
  - Report relevant hardware and software info:
    - CPU, Memory, Network, Disk, etc.
    - Experiment configuration
    - Code, Pseudo code
    - Compiler ver., Compilation Flags, Libraries ver., OS ver.
- Accuracy
  - Repetitions: 5, 10, 100, ... (depends on variability)
  - If high-variability results:
    - Try to understand why and reduce it
    - Include stddev, variance, max-min, inter-quartile range
    - Use box-plot for chart representation as shown in figure



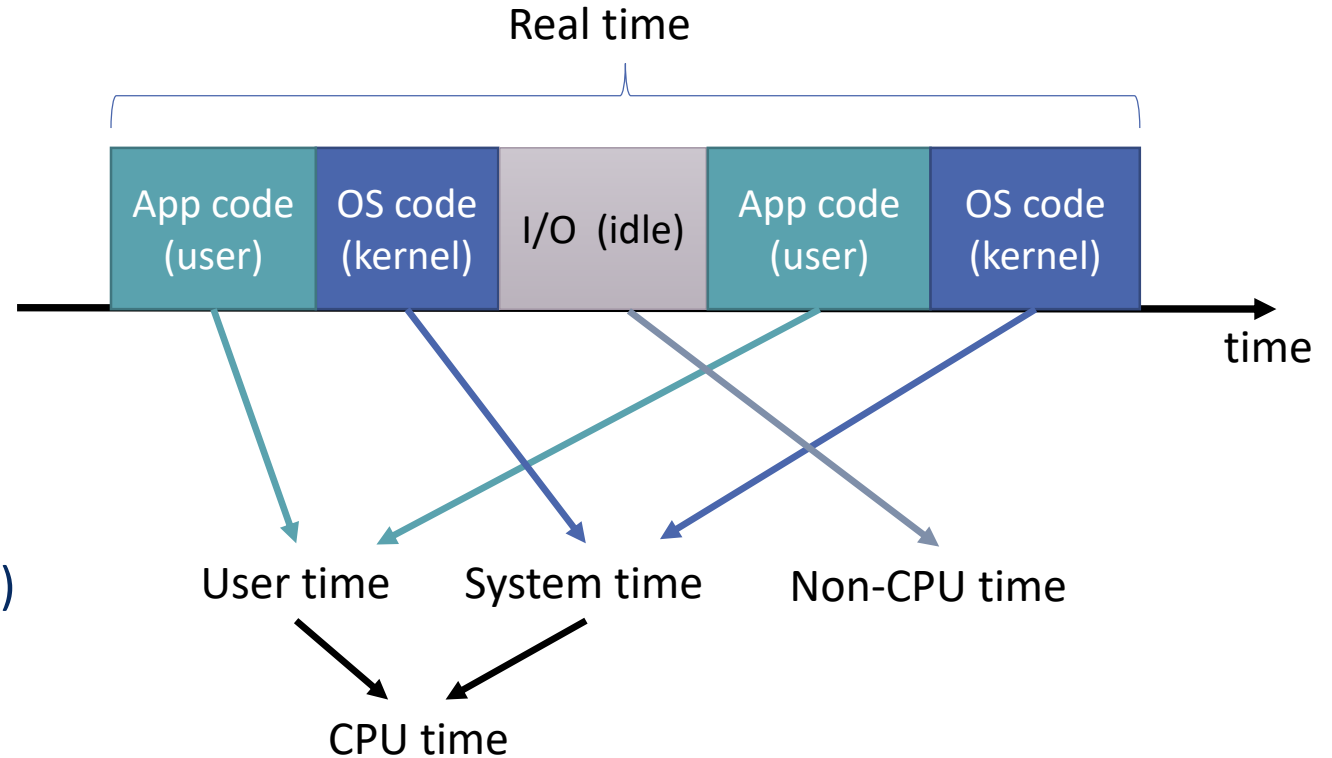
# Basic and advanced measurement techniques

- Basic:
  - Time measurement
  - Application Throughput
  - Breakdown phases or iterations
- Advanced:
  - Profiling
  - Tracing



# Time definitions

- **Real (or Wall Clock or Elapsed) Time :** actual elapsed time from a point in the past
- **CPU (or Process) Time:** time spent executing CPU instructions
  - **User Time :** time spent in user space
  - **System Time :** time spent in kernel space (OS)
- **Non-CPU Time:** time spent waiting (idle CPU) for: I/O, Virtualization, etc.



[https://en.wikipedia.org/wiki/CPU\\_time](https://en.wikipedia.org/wiki/CPU_time)

# Time Measurement - Linux

- *time* command - Real, User and System times

```
$ time ./executable
```

```
real    0m1.057s
user    0m1.015s
sys     0m0.000s
```

- millisecond granularity, accuracy may vary between systems!
- $\text{real} \geq \text{user} + \text{sys} + \text{non-CPU time}$

# Time measurement in C

- `clock_gettime(CLOCK_MONOTONIC,..)` - Real time
  - Nanosecond granularity (neither precision nor accuracy!) - measuring in usec:

```
#include <time.h>
struct timespec start, end;

clock_gettime(CLOCK_MONOTONIC, &start);
<CODE TO MEASURE>
clock_gettime(CLOCK_MONOTONIC, &end);

double time_usec = (((double)end.tv_sec * 1000000 + (double)end.tv_nsec / 1000)
    - ((double)start.tv_sec * 1000000 + (double)start.tv_nsec / 1000));
printf("a=%d time: %.03lf\n", a, time_usec);
```

- <http://btorpey.github.io/blog/2014/02/18/clock-sources-in-linux/>

# Execution Time measurement in Python

- Real Time:

- granularity fractions of seconds – printing in seconds (Python 3.3)

```
import time
```

```
start=time.monotonic()
```

```
<CODE TO MEASURE>
```

```
end=time.monotonic()
```

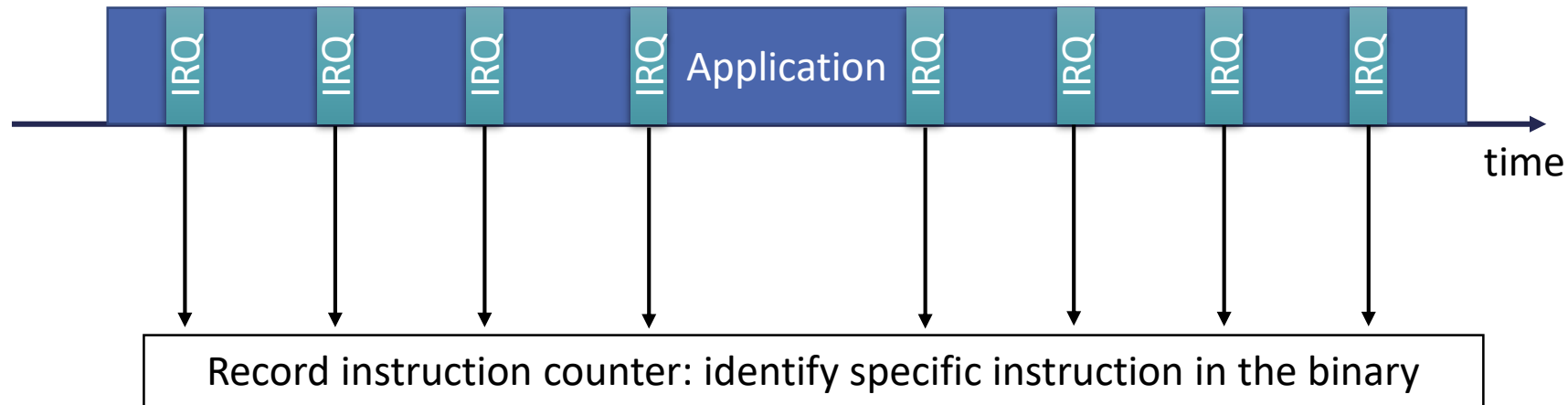
```
print("time: " + str(end-start))
```

- From Python 3.7: *time.monotonic\_ns()* (granularity in nanoseconds)
- <https://docs.python.org/3.7/library/time.html>

# Profiling

- Sampling:
  - Sample applications during execution to infer a statistical distribution:  
Example: approximate time spent in each instruction of the code
- Counting:
  - Count exact events
  - Software counters: count specific events
    - Example: count number of memory allocations (malloc())
  - **Hardware performance counters (aka Performance Counters)**
    - Examples: count number of L2 misses, Floating-point ops, Integer ops.

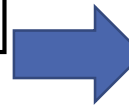
# Profiling - Sampling



- IRQ: interruption of the application to execute a different routine
- Profiling uses IRQs to register instruction counter and other metrics at regular intervals
- Relatively low-overhead, depending on IRQ frequency

# Profiling - Sampling

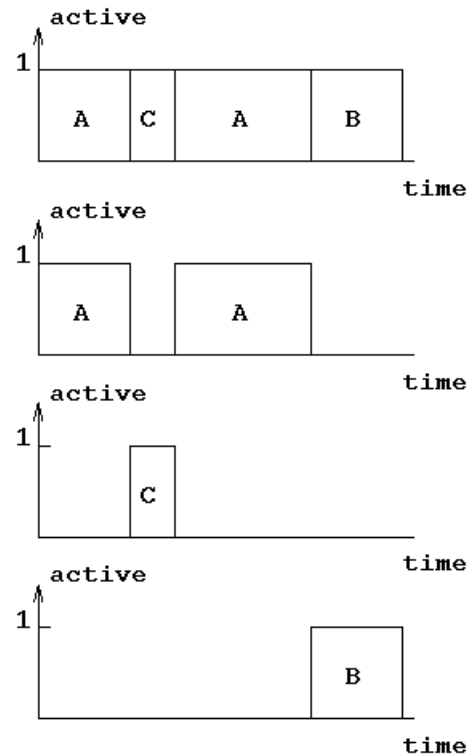
```
int
main() {
    long i, a=1;
    for ( i=0; i<1000000; i++)
        a += a*i;
    return a;
}
```



main /mnt/nfs/nfsshare/user_homes/ufsecond/HPML/dummy	
Percent	
Disassembly of section .text:	
00000000004004cd <main>:	
main():	
	push %rbp
	mov %rsp,%rbp
	movq \$0x1,-0x10(%rbp)
	movq \$0x0,-0x8(%rbp)
	↓ jmp 2f
16:	mov -0x8(%rbp),%rax
	lea 0x1(%rax),%rdx
20.00	mov -0x10(%rbp),%rax
80.00	imul %rdx,%rax
	mov %rax,-0x10(%rbp)
	addq \$0x1,-0x8(%rbp)
2f:	cmpq \$0xf423f,-0x8(%rbp)
	↑ jle 16
	mov -0x10(%rbp),%rax
	pop %rbp
	← retq

- Example of Linux *perf* *annotate*
  - Annotated code showing time percentage
  - All the time associated with only 2 instructions ?
  - <https://perf.wiki.kernel.org/index.php/Tutorial>

# Sampling Analysis



Operation (A,B,C) activity during a single threaded program run

$$T = \sum_{\psi} T_{\psi}$$

$$T_{\psi} = \int_0^T p_{\psi}(t) dt$$

Monte Carlo numerical integration

$$T_{\psi} \approx T \frac{1}{N} \sum_{i=1}^N p_{\psi}(i\tau) \pm T \sqrt{\frac{\langle p_{\psi}^2 \rangle - \langle p_{\psi} \rangle^2}{N}}$$

Via the Koksma-Hlawka inequality the error estimate is bound by the product of the variance  $V$  of the integrated function and the discrepancy  $D_N$  of the sequence.

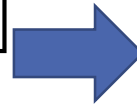
$$\left| \frac{1}{N} \sum_{i=1}^N f(x_i) - \int f(u) du \right| \leq V(f) D_N(x_1, \dots, x_N)$$

Periodic sampling  $\Rightarrow$  error declines with  $1/N$



# Profiling – Sampling 2

```
int
main() {
    long i,a=1;
    for ( i=0; i<1000000000UL; i++)
        a += a*i;
    return a;
}
```



main /mnt/nfs/nfsshare/user_howes/ufsecond/HPML/dummy	
Percent	
Disassembly of section .text:	
00000000004004cd <main>:	
main():	
	push %rbp
	mov %rsp,%rbp
	movq \$0x1,-0x10(%rbp)
	movq \$0x0,-0x8(%rbp)
	↓ jmp 2f
0.04	16: mov -0x8(%rbp),%rax
0.04	lea 0x1(%rax),%rdx
75.86	mov -0x10(%rbp),%rax
11.99	imul %rdx,%rax
0.16	mov %rax,-0x10(%rbp)
0.04	addq \$0x1,-0x8(%rbp)
0.56	2f: mov -0x8(%rbp),%rax
	cmp \$0x3b9ac9ff,%rax
11.31	↑ jbe 16
	mov -0x10(%rbp),%rax
	pop %rbp
	← retq

- *Linux perf annotate*
  - Annotated code showing time percentage
  - More samples => more realistic time association
  - <https://perf.wiki.kernel.org/index.php/Tutorial>

# Profiling Call Trees

```
extern int fa(unsigned size) {
    unsigned j,tmp=0;
    for (j=0;j<size;j++) {
        tmp+=j; tmp = tmp%5555555;
    }
}
extern int fsmall(unsigned size) {
    return fa(size);
}
extern int flarge(unsigned size) {
    return fa(size);
}
int main(void) {
    unsigned j, tmp;
    for (j=0;j<1000;j++) {
        tmp += fsmall(10);
        tmp += flarge(1000000);
    }
    return tmp;
}
```

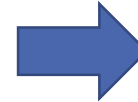
## Gprof, RHEL7.6

index	%time	self	children	called	name
[1]	100.0	0.00	65.56		main[1]
	0.00	32.78	1000/1000		fsmall[4]
	0.00	32.78	1000/1000		flarge[3]

- Gprof only samples the last stack entry
- Assembles call chains incrementally
- Assumes all calls to the same function F take the same time to derive call tree annotation!

# Tracing

```
int main(int argc, char **argv) {  
    RECORD_TRACE_EVENT("after_main");  
    struct timespec start, end;  
    int i,a=1;  
    clock_gettime(CLOCK_MONOTONIC,&start);  
    RECORD_TRACE_EVENT("before_loop");  
    for ( i=0; i < 1000000000; i++) {  
        RECORD_TRACE_EVENT("in_loop");  
        a += a*i;  
    }  
    RECORD_TRACE_EVENT("after_loop");  
    clock_gettime(CLOCK_MONOTONIC,&end);  
    RECORD_TRACE_EVENT("before_return");  
    return 0;  
}
```



## TRACE Example

usec	event
[000012]	after_main
[000013]	before_loop
[000021]	in_loop
[000024]	in_loop
...	...
[012122]	in_loop
[012132]	after_loop
[012223]	before_return

- Explicit code instrumentation with tracing primitives
- Higher overhead than profiling
- Linux perf tracing can be applied to any code: Applications, Runtime, Kernel, Etc.
- <https://perf.wiki.kernel.org/index.php/Tutorial>

# Performance optimization methodology (2): Analysis

# Amdahl's Law

- $S(p, s) = \frac{1}{(1-p) + p/s}$ 
  - $S$ : speedup of the entire application (or runtime, OS, etc.)
  - $p$ : portion of the execution time that is spent in the code section before improvement (if time for  $p$  is high the section is called **critical section**)
  - $s$ : speedup of the improved code section
- Overall speedup is limited by how much time the improved code takes compared to the rest

## Amdahl's law effect example

Two independent parts **A** **B**

Original process



Make **B** 5x faster



Make **A** 2x faster



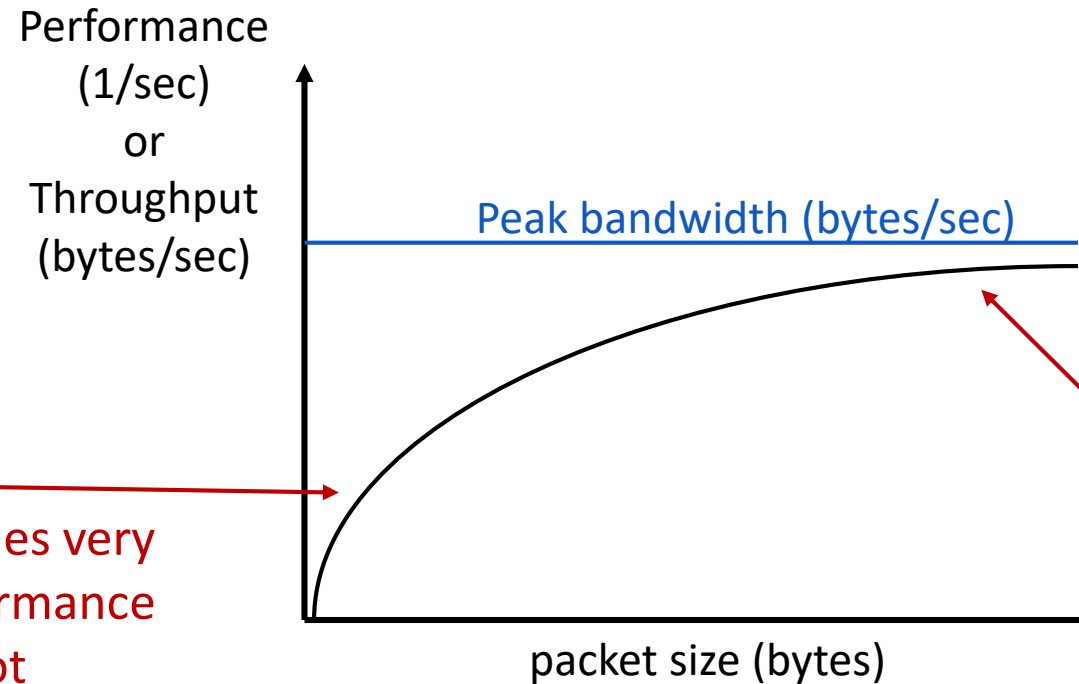
From: [https://en.wikipedia.org/wiki/Amdahl%27s\\_law](https://en.wikipedia.org/wiki/Amdahl%27s_law)

# Performance Analysis Step

1. Identify **Critical Path**: the section of the program that accounts for most of the time (high value of Amdahl's  $p$ )
  - Critical Path characteristics: very slow to execute (high latency) and/or executed many times
  - Use output of performance measurement step (profiling, tracing, etc.)
  - Verify hypothesis of critical path: comment code and run again
2. Identify the **Bottleneck**: the **system resource** that affects the execution time of the critical path
  - Need to Understand Software/Hardware architecture
  - Bottleneck type: **Data Movement** vs. **Computation**

# Data Movement and Packet Size

- True for any **Data Movement**: Network, PCIe, DRAM, etc.



- **SMALL Packets:**

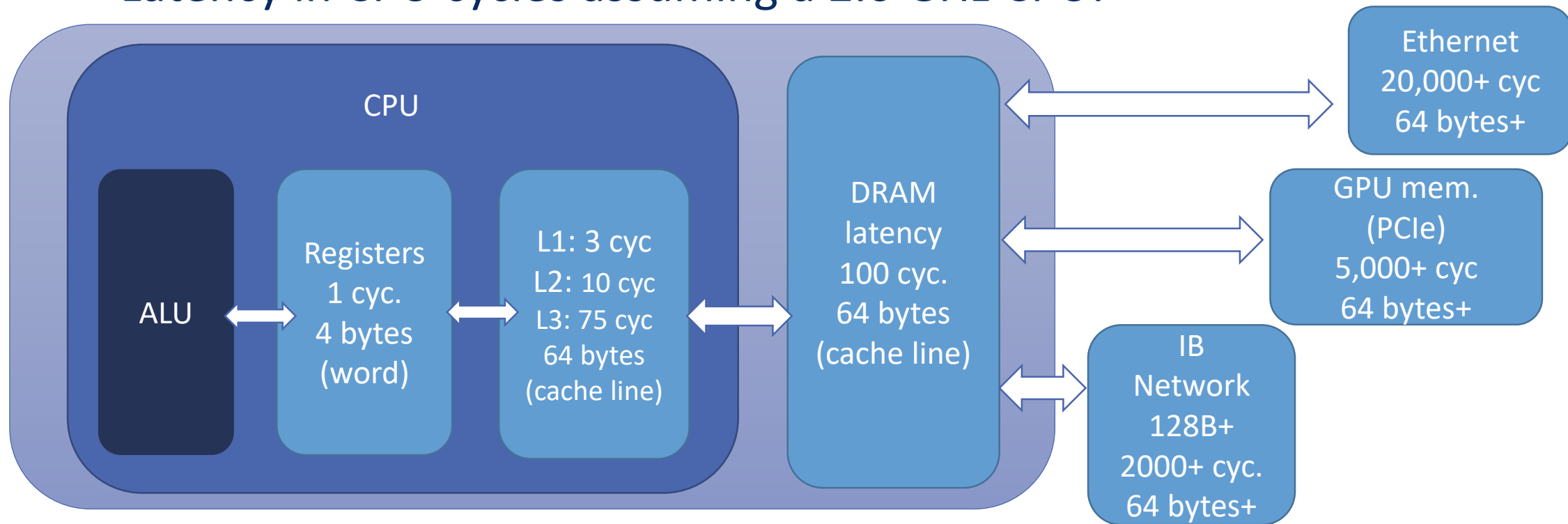
- **Low Latency** becomes very important for performance
- Full Bandwidth is not reached

- **LARGE Packets:**

- **High Bandwidth** becomes very important for performance
- Latency is not relevant

# Data movement Locality Principle - Latency

- Latency in CPU cycles assuming a 2.0 GHz CPU:



- small granularity
- low latency
- high bandwidth

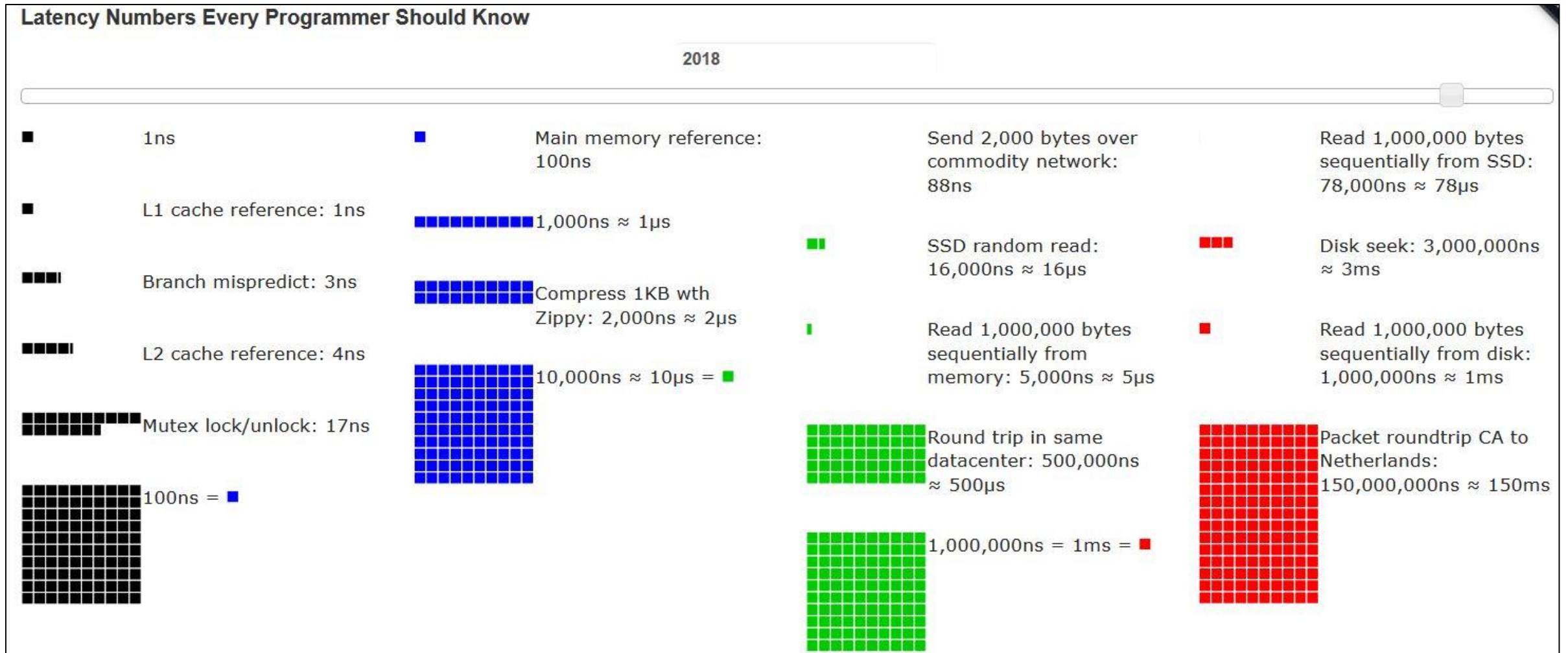


- large granularity
- high latency
- low bandwidth

**Latencies every programmer should know: [link](#)**



# Latency values over the years – very cool tool!

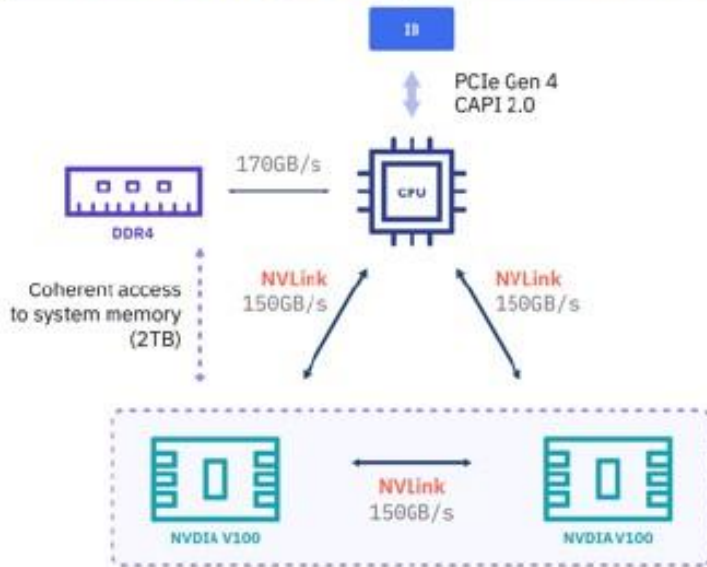


- [https://people.eecs.berkeley.edu/~rcs/research/interactive\\_latency.html](https://people.eecs.berkeley.edu/~rcs/research/interactive_latency.html)

# Data Movement Locality Principle - Bandwidth

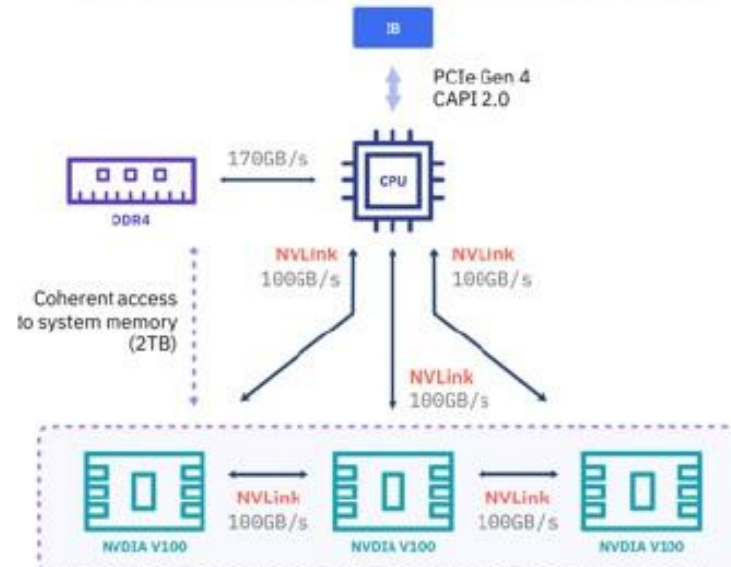
- IBM POWER9 + NVIDIA Volta GPU

## 4 GPUs - Air (4Q'17)/Water Cooled (2Q'18)



- Up to 4 GPUs, air/water cooled options
- 150GB/s of bandwidth from CPU-GPU

## 6 GPUs - Water Cooled (2Q'18)



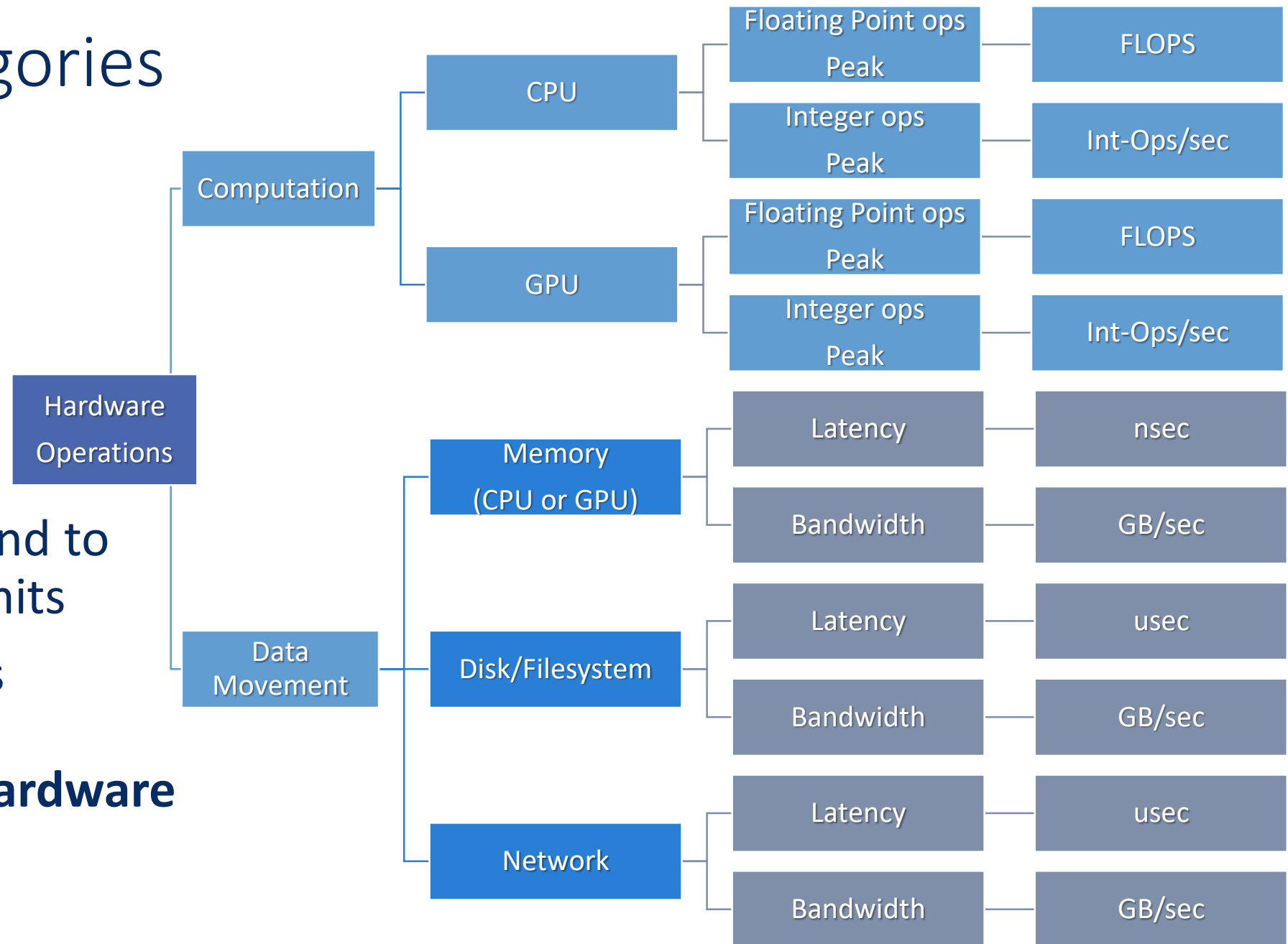
- Up to 6 GPUs, water cooled only
- 100 GB/s of bandwidth from CPU-GPU

- Coherent access to system memory
- PCIe Gen 4 and CAPI 2.0 to InfiniBand
- Water cooled options available in 2Q'18

- (Bi-directional bandwidth)

# Bottleneck categories

- Bottlenecks correspond to specific Hardware Limits
- Performance Analysis Problem: **How far is performance from Hardware Limits?**



# Performance Models Objectives

- Identify performance bottlenecks
- Determines Hardware Limits to Optimization
  - Determines how far we are from hardware limits
  - Motivate algorithmic changes
- Project performance on future hardware or applications

# Peak FLOPS

- Peak FLOPS depend on:
  - Compute unit architecture: CPU, GPU, TPU, FPGA etc.
  - #cores and #threads
  - Clock Frequency
  - Precision: DP (64 bit), SP (32 bit), HP (16 bit)
  - SIMD instructions in the cores: Intel AVX, IBM Altivec
- CPU formula for Peak FLOPS:  $\#tot\_cores \cdot \frac{cycles}{seconds} \frac{FLOPs}{cycles}$
- see <https://en.wikipedia.org/wiki/FLOPS>

# Performance Model – Constants (HW specs)

- Examples of HW Specs for the performance model
  - **CPU peak DP/SP FLOPS:** GFLOPS/s
  - **DRAM peak Bandwidth:** GB/s
  - **GPU peak DP FLOPS:** TFLOPS/s
  - **HBM peak Bandwidth:** TB/s
- How to obtain:
  - Vendor hardware specifications
  - Alternative: run very micro-benchmarks for compute and memory (lower bounds than specs)

# Performance Model - Variables

- Actual Experimental Measurements :
  - Computation (CPU/GPU) Performance: FLOPS
  - Memory throughput: GB/s or TB/s
- How to measure:
  - FLOPS: **hw performance counters** for FLOP divided by time
  - GB/s: **hw performance counters** for memory-ops divided by time

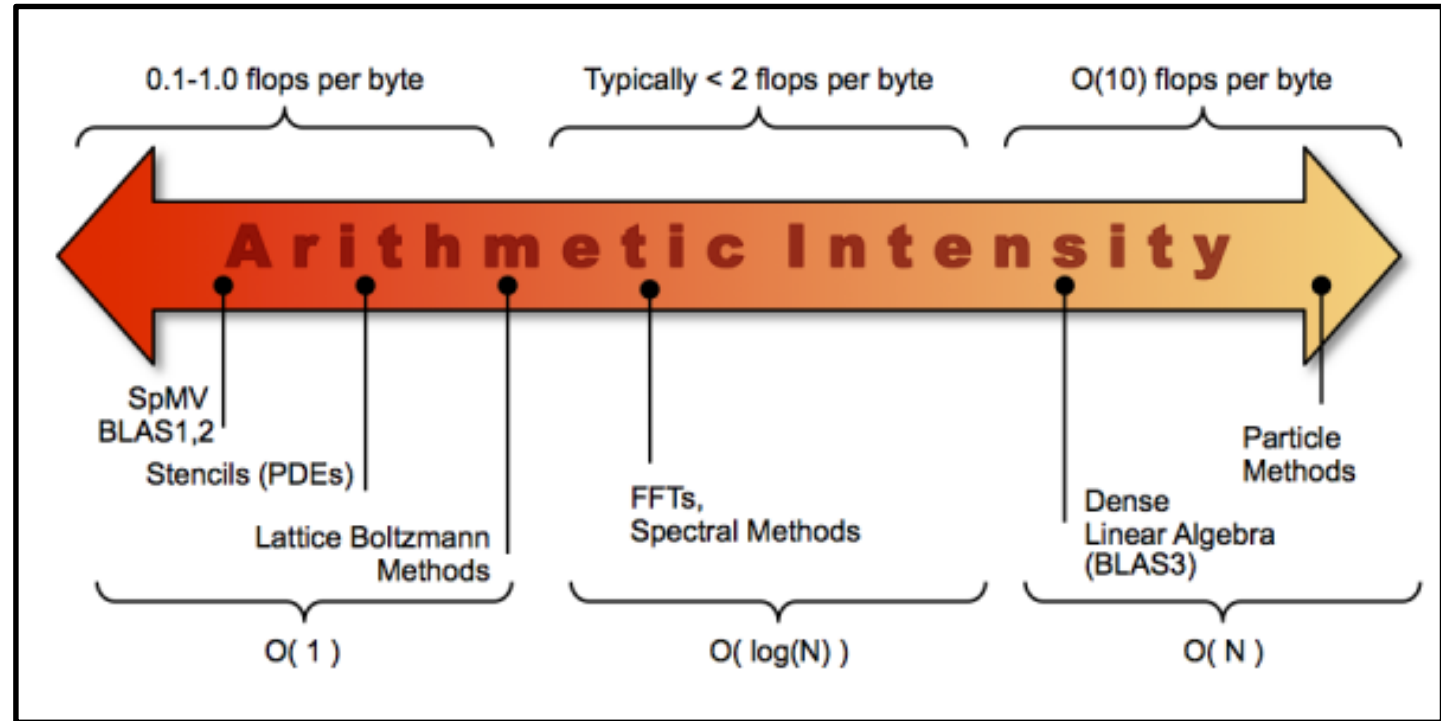
# Roofline Performance Model (1)

- Throughput-based model
- Developed at DOE Lawrence Berkeley Labs
- Metrics:
  - Peak FLOPS
  - Memory Bandwidth:  $\frac{\text{data}}{\text{time}} \left[ \frac{\text{bytes}}{\text{sec}} \right]$
  - Arithmetic Intensity (program property):

$$\frac{\# \text{arithmetic ops}}{\text{DRAM data}} \left[ \frac{\text{FLOP}}{\text{bytes}} \right]$$

(bytes as seen from DRAM)

**note: FLOP  $\neq$  FLOPS**

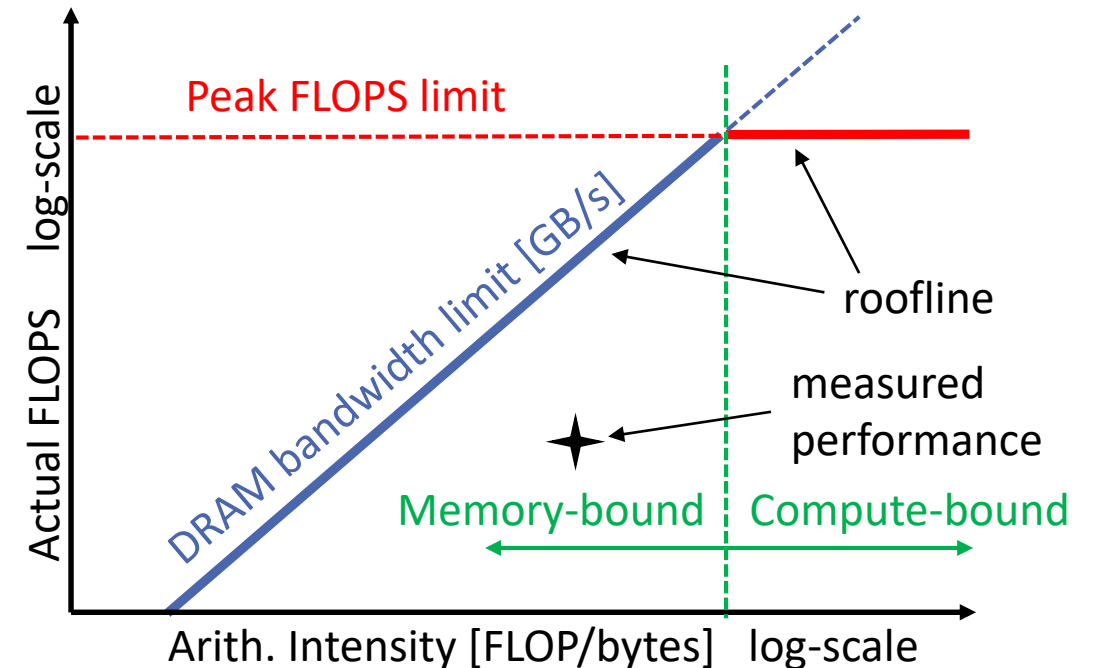


<https://crd.lbl.gov/departments/computer-science/PAR/research/roofline>



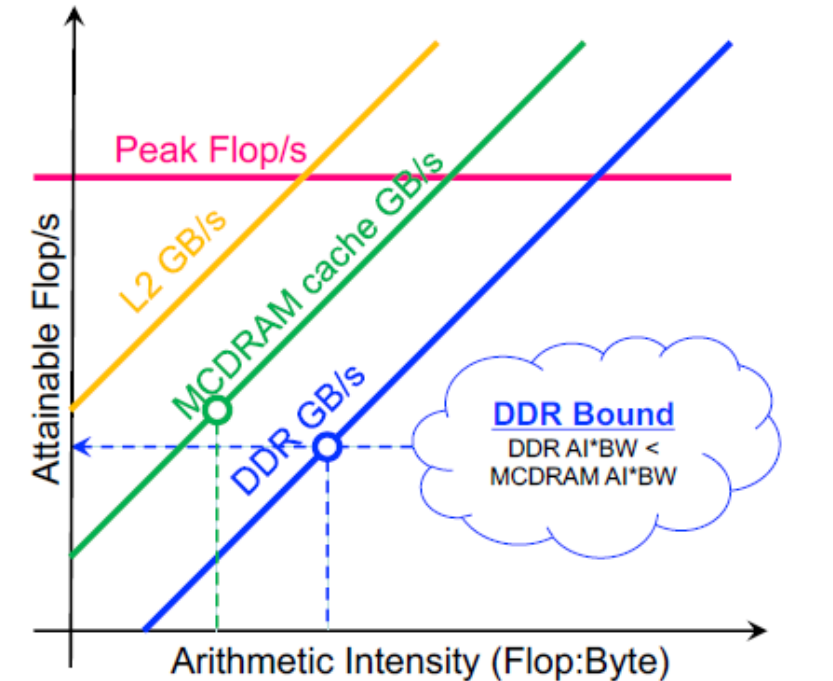
# Roofline Performance Model

- Actual FLOPS are limited first by DRAM bandwidth (memory-bound) and then by CPU (or GPU) Peak FLOPS (compute-bound)
- Actual measured performance is below the roofline
- Depending on Arithmetic Intensity:
  - **memory-bound** code
  - **compute-bound** code
- **Log-log scale is used for clarity**



# More complex Roofline Models

- We considered a basic **DRAM-only Roofline Model**:
  - Bytes as seen from DRAM access
- Not covered: Hierarchical Roof-line
  - for problems that fit in the cache we can add:
    - L1, L2, L3 bandwidth
  - Each Cache Level has its own A.I. (different bytes going through that level of the mem. Hierarchy)
- Also not covered: Cache-aware Roof-line
  - FLOP/bytes as seen from CORE
  - Different roof but same A.I.
  - Need to know from which level data is coming
  - <http://www.inesc-id.pt/ficheiros/publicacoes/9068.pdf>



Hierarchical Roofline

from: LBNL (SC17 Roofline Model Workshop slides)

# crackle1.cims.nyu.edu compute node @NYU

- Intel Xeon E5630@2.53GHz performance:
  1. #cores: 4
  2. LLC (L3) size: 12MB
  3. Clock frequency: 2.53GHz
  4. **DRAM peak bandwidth:** 25.6 GB/s
  5. **CPU Peak FLOPS:** 81.3 DP GFLOPS – 162.56 SP GFLOPS
- DRAM peak bandwidth:
  - [https://ark.intel.com/products/47924/Intel-Xeon-Processor-E5630-12M-Cache-2\\_53-GHz-5\\_86-GTs-Intel-QPI](https://ark.intel.com/products/47924/Intel-Xeon-Processor-E5630-12M-Cache-2_53-GHz-5_86-GTs-Intel-QPI)
- CPU peak FLOPS :
  - $\text{FLOPS} = \text{frequency} * \text{total\_cores} * \text{FLOPS/cyc}$
  - <https://en.wikipedia.org/wiki/FLOPS> (architectures list - this is a *Sandy Bridge*)

```
$ ssh username@access.cims.nyu.edu
$ ssh crackle1.cims.nyu.edu
$ cat /proc/cpuinfo
```

```
processor      : 0
vendor_id     : GenuineIntel
cpu family    : 6
model         : 44
model name    : Intel(R) Xeon(R) CPU E5630 @ 2.53GHz
stepping      : 2
microcode     : 0x15
cpu MHz       : 2527.014
cache size    : 12288 KB
physical id   : 0
siblings      : 8
core id       : 10
cpu cores     : 4
```

# Roofline Model Example – crackle1

- *crackle1*:

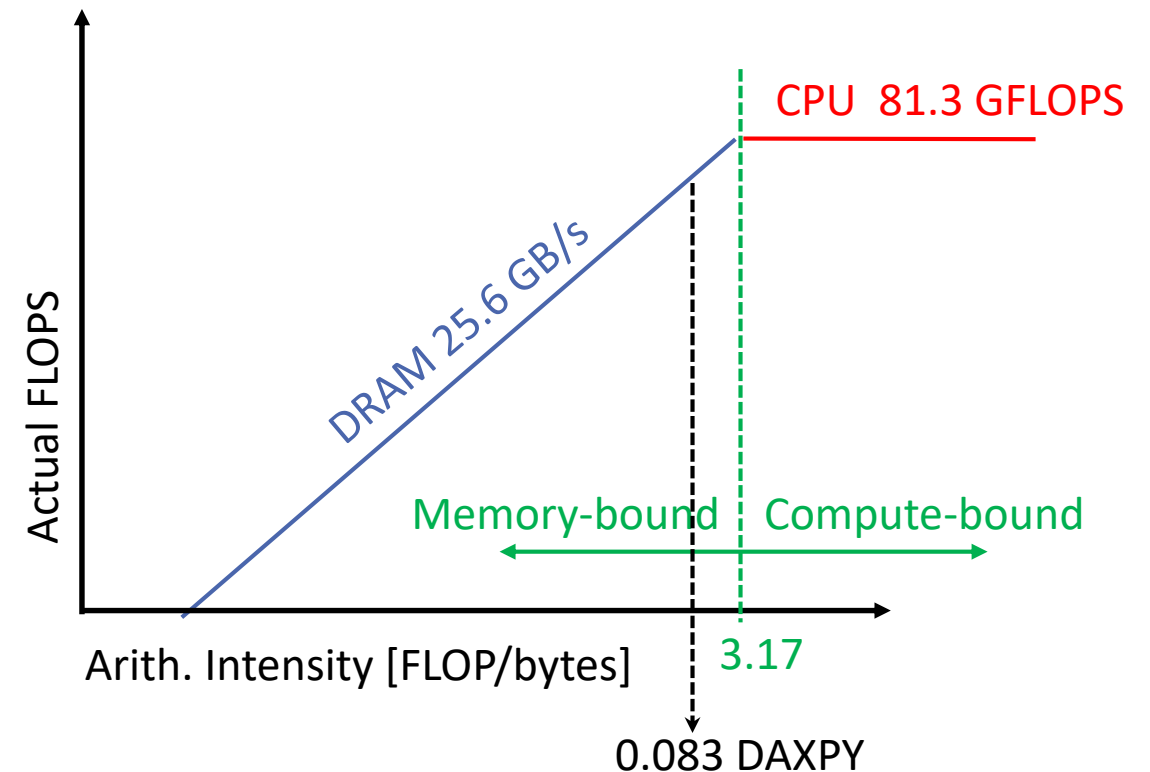
- CPU peak: 81.3 DP GFLOPS
- DRAM peak BW: 25.6 GB/s

- DAXPY code:

```
for (i=0;i<N;i++) {  
  Z[i]= A * (X[i] + Y[i])  
}
```

- Y,A,X are 64 bit float (DP)
- DRAM and CPU cross at:
  - $81.3 \text{ GFLOPS} / 25.6 \text{ GB/s} = 3.17 \text{ FLOP/byte}$
  - CPU\_peak/DRAM\_BW
    - Where DP-bytes=8, SP-bytes=4, HP-bytes=2

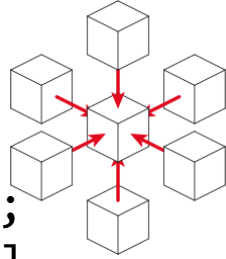
- A.I. =  $2 \text{ FLOP} / (3 \cdot 8) \text{ bytes} = 0.083 \text{ FLOP/byte}$
- Result:  $0.083 < 3.17 \Rightarrow$  **Memory-bound**  $\Rightarrow$  how far for DRAM BW?



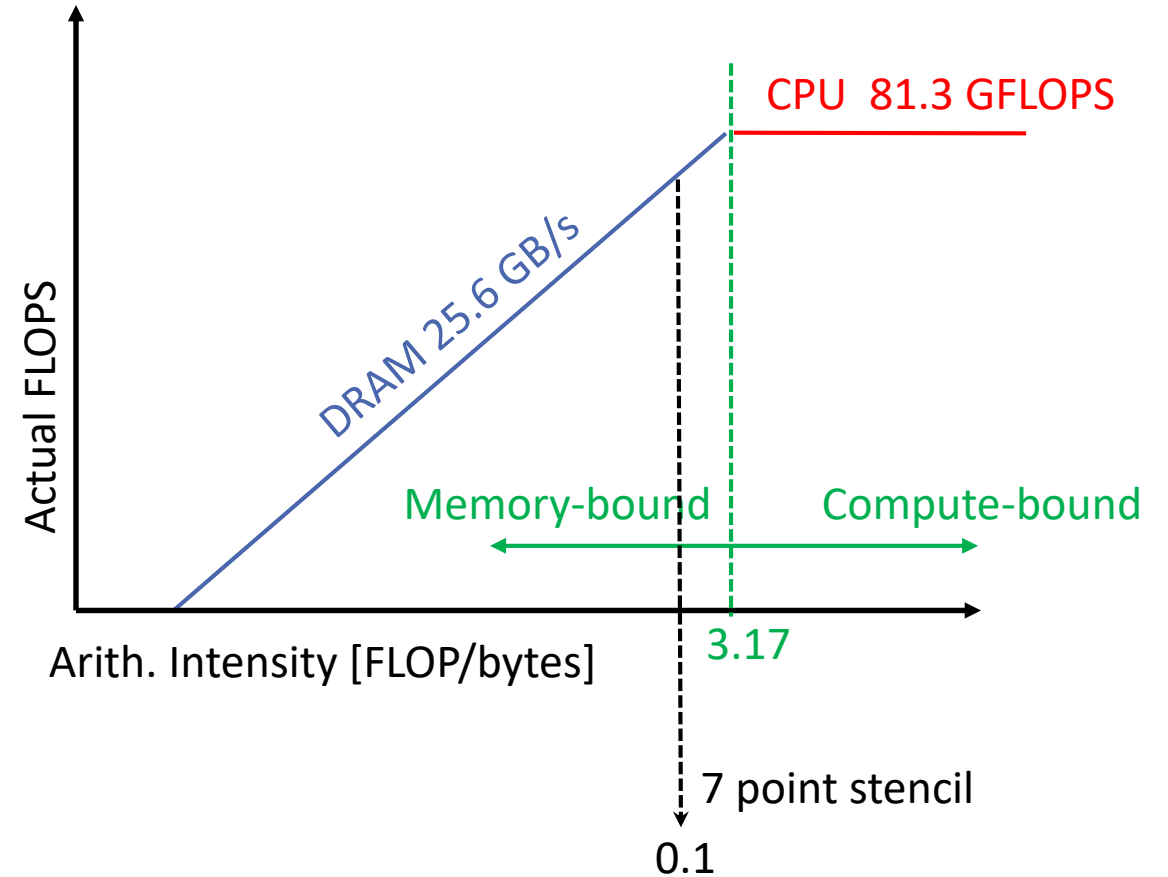
# Roofline Model Example (2) – crackle1

- 7-Points Stencil code:

```
for(k=1;k<N;k++){  
  for(j=1;j<N;j++){  
    for(i=1;i<N;i++){  
      int ijk = i+j*jStride+k*kStride;  
      new[ijk] = -6.0*old[ijk  
        +old[ijk-1  
        +old[ijk+1  
        +old[ijk-jStride  
        +old[ijk+jStride  
        +old[ijk-kStride  
        +old[ijk+kStride ];}}}
```

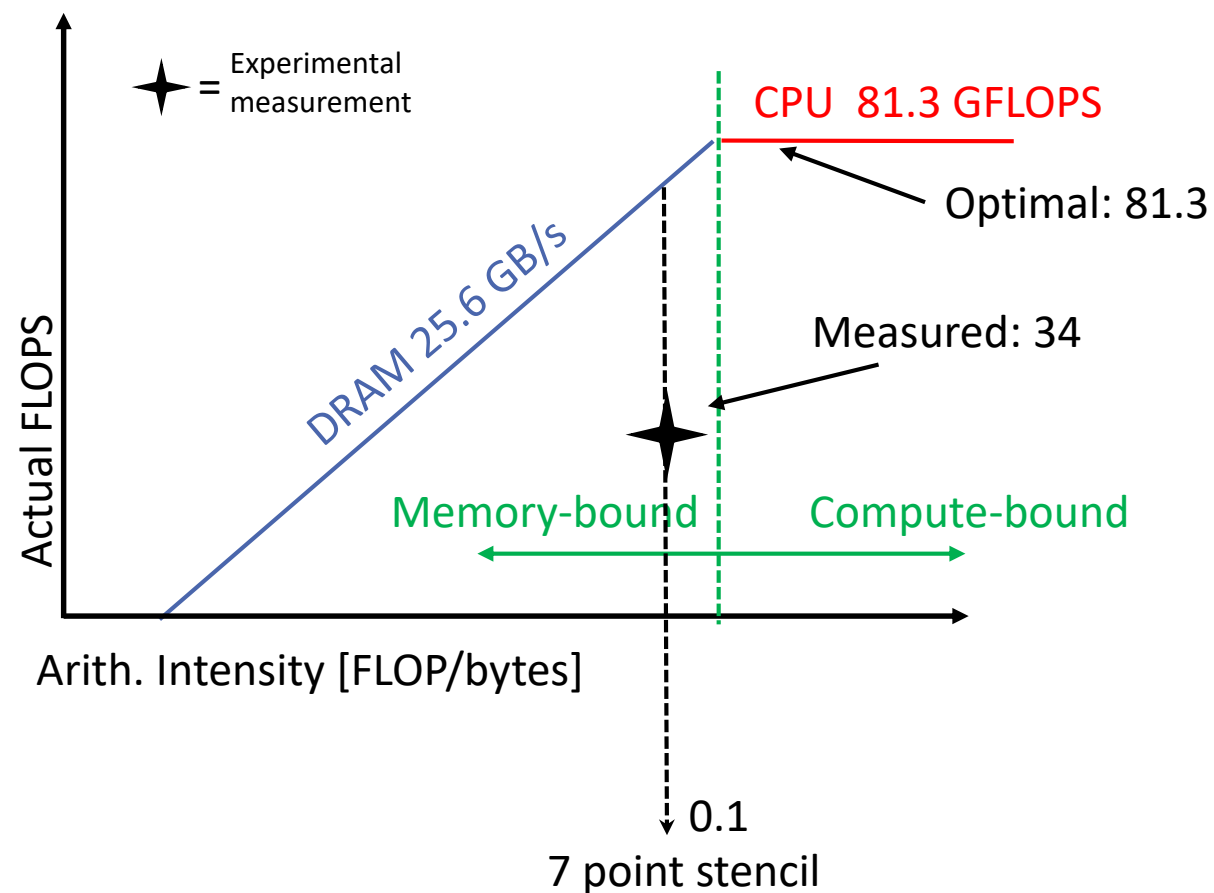


- 7 DP flops (new[] is 64bits)
- 8 memory references
- DRAM/CPU cross = 3.17 FLOP/byte
- AI = 7 FLOP/(8\*8) bytes = 0.109 FLOP/byte
- Result:  $0.109 < 3.17 \Rightarrow$  still **Memory Bound**  $\Rightarrow$  how to optimize?



# Next steps - Optimization

1. Know the limitation from the model: CPU vs. DRAM
2. Measure actual performance:  
Example: 34 GFLOPS
3. Optimize to get close to max FLOPS! (81.3 GFLOPS)



# Performance optimization methodology (3): Optimization

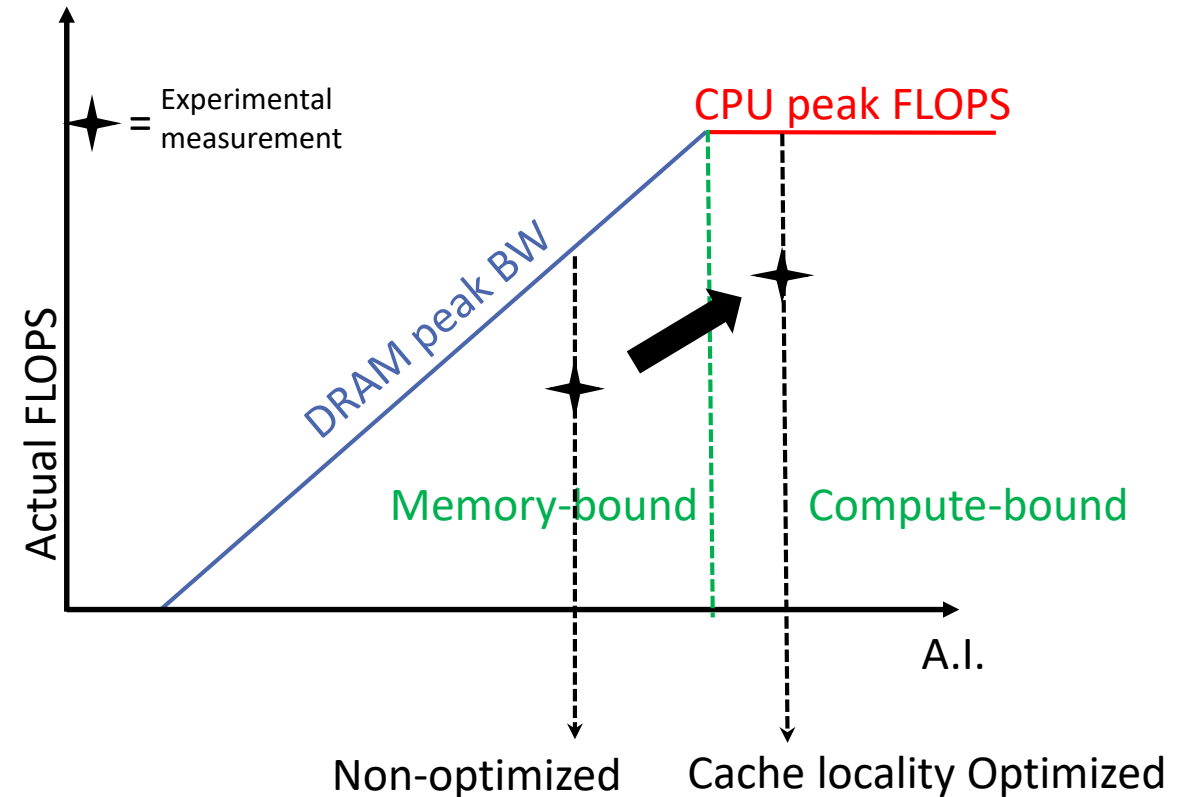
# Two ways to Performance

- Reduce latency (do one operation faster):
  - Dennard Scaling (performance per watt) => Frequency scaling
  - Data access latency reduction
- Increase Parallelism (do more operations at the same time):
  - Vectorization
  - Instruction Level Parallelism
  - Thread Level Parallelism
  - Multi-core design
  - Computer Clusters



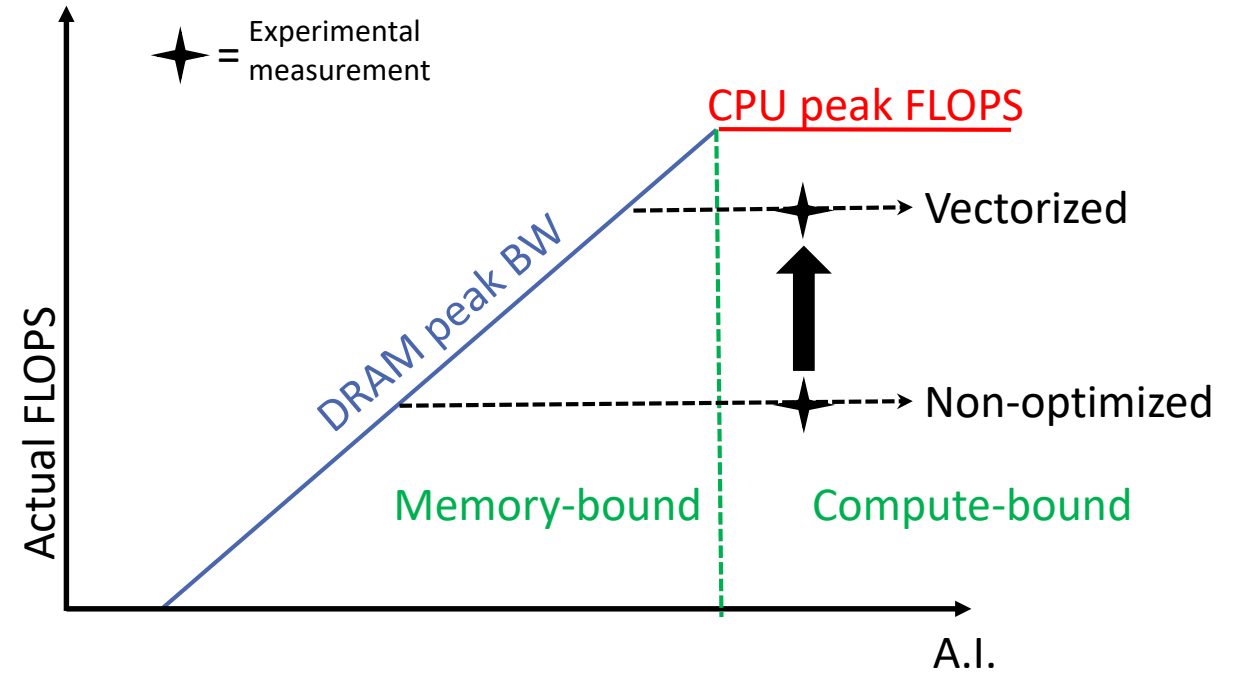
# Optimization Example: Cache Blocking

- Observation: Cache access latency is about 10x lower than DRAM and BW is much higher
- Optimization (cache blocking):
  - Divide program data structures in blocks of the **cache size**
  - Work on each block before switching to the next
  - Less DRAM bytes: **cache is filtering DRAM accesses**
  - A.I. [FLOPS/(DRAM bytes)] is higher
- Result:
  - Bottleneck moves: Code (may) become compute-bound with higher FLOPS!



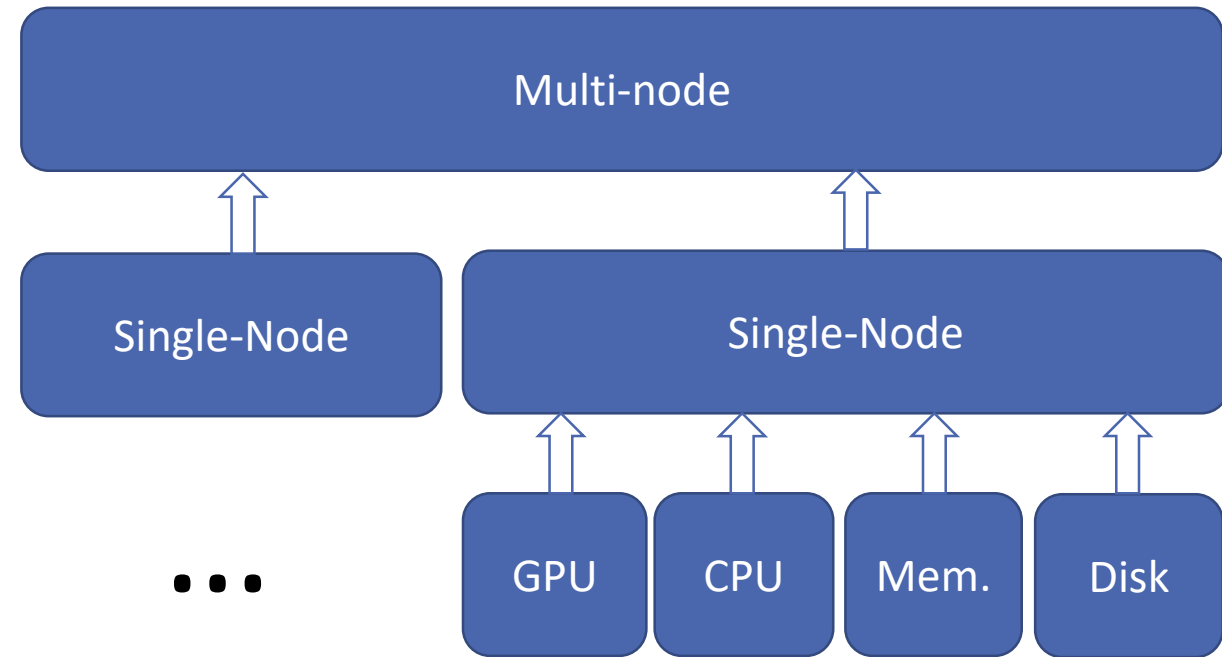
# Optimization Example: Vectorization

- Observation: SIMD instructions execute multiple FLOP (2,4,8,16..) with 1 instruction => higher FLOPS
- Optimization (Vectorization):
  - Replace normal code with SIMD instructions
  - **Hint:** use math libraries like BLAS (CPU) or cuDNN (GPU) and they will do it for you!
- Result:
  - Code reaches higher FLOPS!

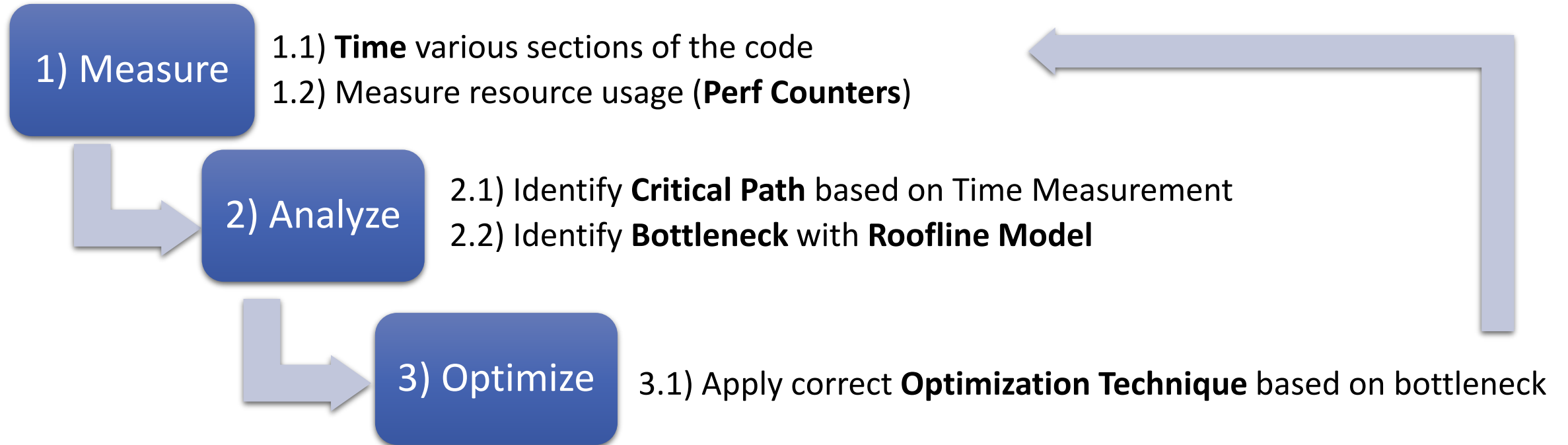


# Hierarchical Perf. Optimization – (next lessons)

- Single Node optimizations:
  - CPU:
    - Vectorize/SIMD optimizations
    - CPU Cache/Memory optimizations
    - Multi-core scalability/parallelism
  - GPU:
    - SM Optimizations
    - SM Cache/Memory optimization
  - Disk and IO
- Multi-node optimizations:
  - Parallelism exposure
  - Domain decomposition
  - Load-balancing
  - Reduce synchronizations
  - Reduce collectives



# Performance Optimization Methodology Recap



# Floating Point Errors

- Error:  $E = |f(x) - F(x)|$ 
  - $F(x)$  is the correct result,  $f(x)$  is the numerically computed result
- Relative error:  $R = E / |F(x)|$ 
  - Floating point 'roundoff' relative error depends on number of bits in the mantissa!
- Cancellation
  - $C = A + B \rightarrow$  may result in  $C = A$  for  $B \ll A$
- Catastrophic cancellation
  - $C = B + A - A \rightarrow$  may result in 0 for  $B \ll A$ , relative error is 1
  - $C = 1 / (B + A - A)!$

# Lesson Key Points

- ML Performance Factors
- Performance Optimization Methodology:
  1. Measurement: Metrics, Time/Resources and Techniques
  2. Analysis: Amdahl's Law, Bandwidth/Latency, Roofline Model
  3. Optimization (in relationship to Roofline model)