

# Monte Carlo simulation of radiation emitted from the BM ports of the new ESRF lattice

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## Abstract

This document explains a model to Monte Carlo sample wiggler sources. The model follows the implementation in the SHADOW ray tracing code. The results are applied to the simulation of the emission of the wigglers to be used at the bending magnet beamlines for the new ESRF lattice.

## 1 Introduction

This paper includes the equations needed to simulate the emission from wigglers. The model is based on the fact that there is no interference of the photons emitted from different points along the electron trajectory, usually a good approximation for insertion devices with deviation parameter  $K \gg 1$ . The computation goes through different phases: calculation of the trajectory, velocity and curvature for an electron moving in an arbitrary magnetic field, computation of the spectrum of the emitted radiation, sampling mechanisms to create rays from the radiation distribution, and lately the effect of the electron beam emittance. The effects of the electron beam emittance have been treated in depth. The SHADOW model has been corrected and a new implementation is described.

## 2 Electron in a magnetic field

Let us define our reference system with coordinate  $s$  along the electron orbit,  $y$  tangent to the orbit at a given point,  $x$  in the horizontal plane, and  $z$  in vertical.

In usual wigglers the magnetic field is directed along the  $z$  direction to make the electron moving in the horizontal plane. See Fig. 2 for an example of wiggler magnetic field.

An electron of energy  $E$  will move with velocity:

$$\frac{v_x}{c} \approx (0.3/E) \int B_z(y) dy \quad (1)$$

where  $v_x$  is the velocity along  $x$ ,  $c$  is the speed of the light,  $E$  is the electron energy in GeV,  $B_z$  the magnetic field in T and coordinates and distances are in  $m$ .

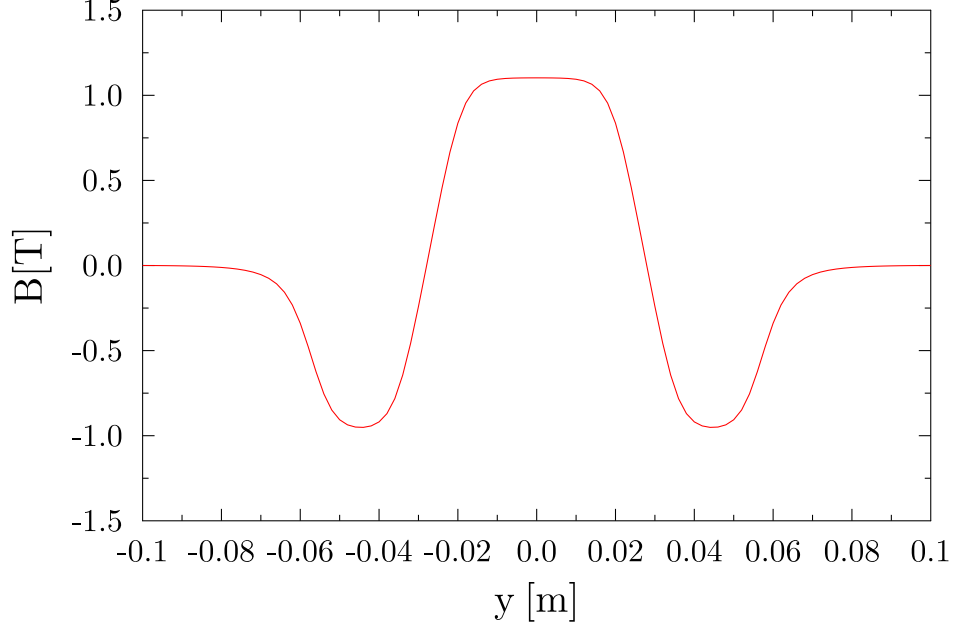


Figure 1: Magnetic field for the wigglers to be placed at the ESRF new lattice.

The trajectory is given by the integral of the velocity:

$$x \approx (0.3/E) \int \int B_z(y) dy^2 \quad (2)$$

To keep the electron in orbit, the electron must have the same position and direction at the entrance and exit of the wiggler. This is guaranteed if:

$$\int B_z(y) dy = 0 \quad (3)$$

In most practical cases we can write the magnetic field along the axis  $y$  far from its ends as a sinusoidal function with period  $\lambda_w$  and  $B_0$  the peak magnetic field. An arbitrary magnetic field  $B_z(y)$  can be defined numerically, but it may be convenient to describe it as a sum of different harmonic components:

$$B_z(y) = \sum_{n=1}^N B_n \sin\left(\frac{2\pi}{\lambda_w} ny + \theta_n\right) \quad (4)$$

A general expression of harmonic decomposition is in Ref. [4]. The interest of the harmonic decomposition is that trajectories and velocities are calculated analytically thus with less errors than using numerical integration. For the standard wiggler:

$$\frac{v_x}{c} = \frac{K}{\gamma} \cos\left(\frac{2\pi}{\lambda_w} y\right) \quad (5)$$

where  $K$  is the deflection parameter:

$$K = 93.4 B_0 [T] \lambda_w [m] \quad (6)$$

( $K \gg 1$  for wiggler, so the emission on different points on the trajectory does not interfere) and  $\gamma$  is the electron energy in units of electron energy at rest ( $m_e c^2 = 0.511 \text{ MeV}$ ).

The trajectory is given by:

$$x \approx \frac{0.3}{E} \int \int B_z(y) dy^2 = \frac{\lambda_w K}{2\pi\gamma} \sin\left(\frac{2\pi}{\lambda_w} y\right) \quad (7)$$

where .

In SHADOW [2] the computation of the electron trajectories for standard wigglers are done by a preprocessor called "epath". The calculations for an arbitrary magnetic field (numerically defined or using harmonic decomposition) is done by the preprocessor **epath\_b** which defines:

$$\begin{aligned} B_z(y) &= \sum_{n=1}^N B_n \cos\left(\frac{2\pi}{\lambda_w} ny\right) \\ \frac{v_x(y)}{c} &= \frac{-0.3}{E} \frac{\lambda_w}{2\pi n} \sum_{n=1}^N B_n [\sin\left(\frac{2\pi}{\lambda_w} ny\right) - \sin(-\pi n)] \\ x(y) &= \frac{-0.3}{E} \left(\frac{\lambda_w}{2\pi n}\right)^2 \sum_{n=1}^N B_n [\cos\left(\frac{2\pi}{\lambda_w} ny\right) - \cos(-\pi n)] \end{aligned} \quad (8)$$

and:

$$\begin{aligned} \frac{v_y(y)}{c} &= \sqrt{\left(\frac{v_0}{c}\right)^2 - \left(\frac{v_x}{c}\right)^2} \\ \gamma &= \frac{E}{m_e c^2} = \left(1 - \left(\frac{v_x}{c}\right)^2\right)^{-1/2} \\ x(y) &= \frac{-0.3}{E} \left(\frac{\lambda_w}{2\pi n}\right)^2 \sum_{n=1}^N B_n [\cos\left(\frac{2\pi}{\lambda_w} ny\right) - \cos(-\pi n)] \\ \frac{1}{R(y)} &= -\frac{e}{\gamma m_e c} \frac{c}{v_0} B_z(y) \end{aligned} \quad (9)$$

Table 1: Harmonic decomposition  $n, B_n$  for the magnetic field in Fig. 2.

1	0.393193
2	0.735493
3	0.313539
4	-0.163942
5	-0.168566
6	-0.035670
7	-0.020342
8	-0.006085
9	0.034613
10	0.022417
11	-0.009862
12	-0.005189
13	0.007606
14	0.001296
15	-0.004822
16	-0.001565
17	-0.000223
18	-0.001313
19	0.000344
20	0.001928
21	0.000684
22	-0.000531
23	-0.000300
24	-0.000006
25	-0.000133
26	-0.000051
27	0.000076
28	0.000026
29	-0.000046
30	0.000021
31	0.000047
32	0.000020
33	-0.000021
34	-0.000011
35	-0.000014
36	0.000000
37	-0.000002
38	0.000011
39	-0.000000
40	0.000003

### 3 wiggler radiation

The full emission spectrum produced by the 3-pole wiggler with magnetic field in Fig. 2 (BMHard) is compared with a the 0.4  $T$  bending magnet (integrated over 2  $mrad$  horizontal divergence) in Fig. 3.

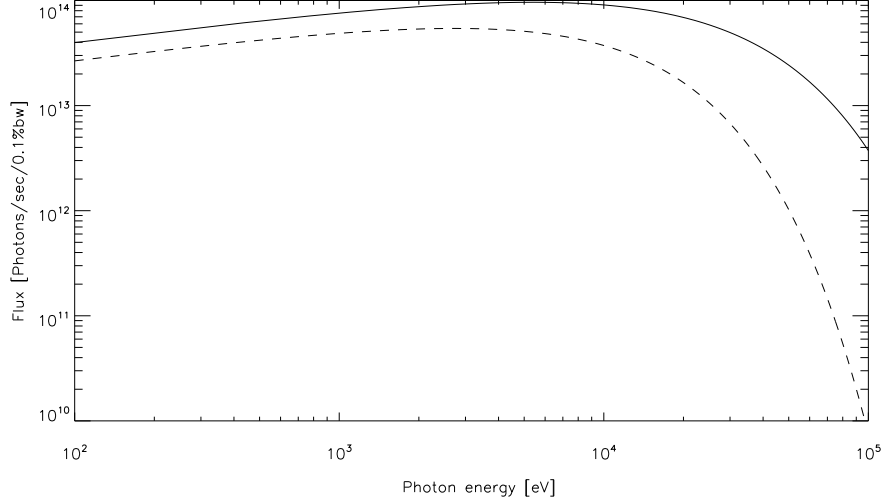


Figure 2: Calculated (XOP) spectra produced by the 3-pole wiggler at BMHard location (solid line) and the 0.4T bending magnet at BMSOft (dashed line).

#### 4 Electron beam size and divergence characterisation. Evolution in a straight section.

Let us suppose we are given the Twiss parameters  $\beta(s_0)$ ,  $\alpha(s_0)$ ,  $\gamma(s_0)$  at position  $s_0$ , and the emittance is  $\epsilon$ . We consider the horizontal phase space here, with coordinates  $(x, x')$ . An electron beam with non-zero emittance  $\epsilon_{x,z}$  is distributed about the orbit with individual electrons executing betatron oscillations. In the case of a Gaussian electron beam (expected in electron storage ring), the emittance is related to the so-called Courant Snyder invariant  $J_{x,z}$  via

$$\epsilon = \langle J \rangle \quad (10)$$

The Courant-Snyder invariant at  $s$  is given by

$$J = \frac{1}{2}(\gamma x^2 + 2\alpha x x' + \beta x'^2) \quad (11)$$

We note that for a constant value of  $J$ , this represents the equation of an ellipse. As the Twiss parameters change, the shape will change, but the area remains the same. Now, we recall that the Twiss parameters must satisfy the relationships [3]:

$$\begin{aligned} \beta\gamma &= 1 + \alpha^2 \\ \alpha &= -\frac{1}{2} \frac{d\beta}{ds} \end{aligned} \quad (12)$$

and one may compute their value at another position ( $s$ ) (assuming no focussing element between  $s$  and  $s_0$ ) via

$$\beta(s) = \beta_0 - 2\alpha_0(s - s_0) + \gamma_0(s - s_0)^2$$

$$\begin{aligned}\alpha(s) &= \alpha_0 - \gamma_0(s - s_0) \\ \gamma(s) &= \gamma_0\end{aligned}\tag{13}$$

A waist (minimum in beam size) is given when  $\alpha = 0$ . Thus, for a given  $(\beta_0, \alpha_0, \gamma_0)$ , we find that the waist occurs at a distance

$$s_w = \frac{\alpha_0}{\gamma_0}\tag{14}$$

with the value of  $\beta$  at the waist,

$$\beta_w = \frac{1}{\gamma_0}\tag{15}$$

Now, the normalized electron beam distribution is given by

$$f(x, x') = \frac{1}{2\pi\epsilon} e^{-\frac{J(x, x')}{\epsilon}}\tag{16}$$

This Gaussian distribution may be characterized by the matrix of second moments

$$\Sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}\tag{17}$$

with, for example

$$\langle x^2 \rangle = \int dx dx' x^2 f(x, x')\tag{18}$$

and likewise for the other second moments. Notationally, we may also write

$$\begin{aligned}\sigma_x &= \sqrt{\langle x^2 \rangle} \\ \sigma'_x &= \sqrt{\langle x'^2 \rangle}\end{aligned}\tag{19}$$

Computing these quantities, using eqns. (11) and (16), we find that

$$\begin{aligned}\langle x^2 \rangle &= \beta\epsilon \\ \langle xx' \rangle &= -\alpha\epsilon \\ \langle x'^2 \rangle &= \gamma\epsilon\end{aligned}\tag{20}$$

One may also confirm that

$$\langle J \rangle = \epsilon\tag{21}$$

It is sometimes useful to introduce a set of normalized coordinates  $(u, u')$  in which the motion in phase space is simply circular. In the uncoupled linear context here, we may represent this transformation as

$$\begin{pmatrix} u \\ u' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}\tag{22}$$

and the inverse transformation is given by

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} u \\ u' \end{pmatrix}\tag{23}$$

In terms of the normalized coordinates, the Courant-Snyder invariant may be represented simply as

$$J = \frac{1}{2}(u^2 + u'^2)\tag{24}$$

## 5 Inclusion of dispersion

There is an additional effect that increases the horizontal beam size, even in the ideal ring with no errors. This is the effect of orbit changes with energy, and is captured in the concept of the dispersion function  $\eta(s)$  and its derivative  $\eta' = d\eta/ds$ . For a particle with a given energy deviation  $\delta = \Delta E/E$ , the particle will oscillate about an orbit displaced by in the horizontal by  $\eta\delta$ . Each particle in the bunch will have a slightly different energy with a Gaussian distribution given by

$$\rho(\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta} e^{-\frac{\delta^2}{2\sigma_\delta^2}} \quad (25)$$

Now, one applies the analysis of the previous section, but to the coordinate

$$x_\beta = x - \eta\delta \quad (26)$$

One then sees that the horizontal beam size and divergence are increased via

$$\langle x^2 \rangle = \langle x_\beta^2 \rangle + \eta^2 \sigma_\delta^2 \quad (27)$$

$$\langle x'^2 \rangle = \langle x'_\beta{}^2 \rangle + \eta'^2 \sigma_\delta^2 \quad (28)$$

In case one is given  $\eta$  and  $\eta'$  at one position, one can compute them at another position, assuming there is not dipole or quadrupole field in between:

$$\begin{aligned} \eta_2 &= \eta_1 + (s_2 - s_1)\eta'_1 \\ \eta'_2 &= \eta'_1 \end{aligned} \quad (29)$$

## 6 Electron beam sampling with emittance

The electron coordinates (position and divergence or angle) in both horizontal  $x$  and vertical  $z$  directions follow a bivariate normal distribution with moments:  $\langle x^2 \rangle, \langle x'^2 \rangle, \langle xx' \rangle$ . At the beam waist  $\langle xx' \rangle = 0$ ,  $\langle x^2 \rangle = \sigma_x^2$ ,  $\langle x'^2 \rangle = \sigma_{x'}^2$ , and the emittance is:  $\epsilon_x = \sigma_x \sigma_{x'}$ . The same equations hold for the vertical plane by replacing  $x$  by  $z$ .

We are interested in computing these moments at any point  $y$  of the wiggler knowing their values at the waist. The optical functions at a  $y$  position in the wiggler ( $s = y - s_w$  from waist) we can calculate using Eqs. 13. The  $s_w$  is the position of the waist from the center of the wiggler ( $y = 0$ ) calculated using Eq. 14. The moments can be expressed from the electron orbit functions using Eq. 20.

The moments at any point of the wiggler axis distant  $s$  from the horizontal waist are<sup>1</sup>:

$$\langle x^2 \rangle = s^2 \sigma_{x'}^2 + \sigma_x^2$$

---

<sup>1</sup>

they ones implemented in SHADOW (routine `gauss`) were different (bug fixed, using new routine `binormal`):

$$\begin{pmatrix} \sqrt{d^2 \sigma_{x'}^2 + \sigma_x^2} & \frac{d\sigma_x}{\sqrt{d^2 \sigma_{x'}^2 + \sigma_x^2}} \\ \frac{d\sigma_{x'}}{\sqrt{d^2 \sigma_{x'}^2 + \sigma_x^2}} & \frac{\sigma_x \sigma_{x'}}{\sqrt{d^2 \sigma_{x'}^2 + \sigma_x^2}} \end{pmatrix} \quad (30)$$

$$\begin{aligned}\langle xx' \rangle &= s\sigma_x'^2 \\ \langle x'^2 \rangle &= \sigma_x'^2\end{aligned}\tag{31}$$

Again, the same equations hold for the vertical plane replacing subindices  $x$  by  $z$ .

To do the sampling from a general covariance matrix, we assume that one can find a matrix  $L$  such that

$$\Sigma = LL^T\tag{32}$$

For any 2x2 covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix}\tag{33}$$

The matrix  $L$  is found by the Cholesky decomposition method to be

$$L = \begin{pmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}\tag{34}$$

which, when the  $\Sigma$  matrix is expressed in terms of the Twiss parameters reproduces equation (23). If you get two random numbers  $(x_1^u, x_2^u)$  from the computer generator in  $U(0, 1)$  we can create two numbers following a Normal distribution in  $N(0, 1)$  using the Box and Muller decomposition:

$$\begin{aligned}x_1^n &= \sqrt{-2\ln(x_1^u)} \cos 2\pi x_2^u \\ x_2^n &= \sqrt{-2\ln(x_1^u)} \sin 2\pi x_2^u\end{aligned}\tag{35}$$

the result of applying  $L$  to these two numbers gives two new numbers following a binormal correlated distribution with covariance  $\Sigma$ .

## 7 Hard and Soft bend locations for upgrade lattice

The center of the soft bend source location (BMSoft, in the lattice) is located at a distance  $s = 13.33$  meters from the start of the cell, and the center of the hard bend source location (BMHard) is located at  $s = 13.717$  meters. A 3-pole wiggler (3PW) is only used for the current Hard bend dipoles ( $s=13.717$  m). The soft bend dipoles, located at  $s=13.33$  m will come from the dipole itself which has a field of  $0.41$  T (radius  $R = 50.02$  m). The full BM length is  $s_{end} - s_{start} = 13.565 - 12.807 = 0.76$  m covering an emission angle of  $15.7$  mrad, separated in the Hard and Soft BM (at  $s = 13.133$ ) thus in  $10.7 + 4.8$  mrad. The 3PW will be between the dipole (DQ2C) and the quadrupole QF8D. The Twiss parameters for these positions on the S28 lattice are in Table 7.

Table 2: Twiss parameters at the wiggler center for the Hard and Soft locations.

location	$s$ [m]	$\alpha_x$	$\beta_x$ [m]	$\gamma_x$ [ $m^{-1}$ ]	$\alpha_z$	$\beta_z$ [m]	$\gamma_z$ [ $m^{-1}$ ]
BMSoft	13.33	-0.49	0.46	2.72	1.38	5.10	0.57
BMHard	13.717	-2.03	1.44	3.56	2.68	3.16	2.59



Table 3: Second moments at the wiggler center for the Hard and Soft locations.

location	$\sqrt{\langle x^2 \rangle}$ [ $\mu m$ ]	$\langle xx' \rangle$ [ $pm$ ]	$\sqrt{\langle x'^2 \rangle}$ [ $\mu rad$ ]	$\sqrt{\langle z^2 \rangle}$ [ $\mu m$ ]	$\langle zz' \rangle$ [ $pm$ ]	$\sqrt{\langle z'^2 \rangle}$ [ $\mu m$ ]
BMSoft	8.306624	73.5	20.199	5.049752	-6.9	1.688194
BMHard	14.697	304.50	23.108	3.975	-13.400	3.599

Table 4: Moments at three positions (center and edges) of the wiggler [1].

moment	unit	$y = -0.1 \text{ m}$	$y = 0$	$y = 0.1 \text{ m}$
$\langle x^2 \rangle$	$\mu m^2$	12.67	15.70	16.8
$\langle xx' \rangle$	$pm$	251.10	304.5	357.9
$\langle x'^2 \rangle$	$\mu rad^2$	23.11	23.11	23.11
$\langle z^2 \rangle$	$\mu m^2$	4.31	3.97	3.64
$\langle zz' \rangle$	$pm$	-14.69	-13.40	-12.11
$\langle z'^2 \rangle$	$\mu rad^2$	3.60	3.60	3.60

The equilibrium horizontal emittance is 150 pm. The vertical emittance may vary depending on the conditions. Let us suppose it is 5 pm. We then find for the second moments at BMSoft and BMHard (see Table 7)).

The evolution of the Twiss parameters along the 3PW axis is determined by Eqs. 13. In terms of the moments, the values at a given point  $y$  are calculated from the values at the given location in Table 7 using:

$$\begin{aligned}
 \langle x^2 \rangle_y &= \langle x^2 \rangle + 2\langle xx' \rangle y + \langle x'^2 \rangle y^2 \\
 \langle xx' \rangle_y &= \langle xx' \rangle + \langle x'^2 \rangle y \\
 \langle x'^2 \rangle_y &= \langle x'^2 \rangle
 \end{aligned} \tag{36}$$

The change of the electron beam moments change along the wiggler is summarized in Table 7 and a plot of the X space phase at the widget edges is in Fig. 7.

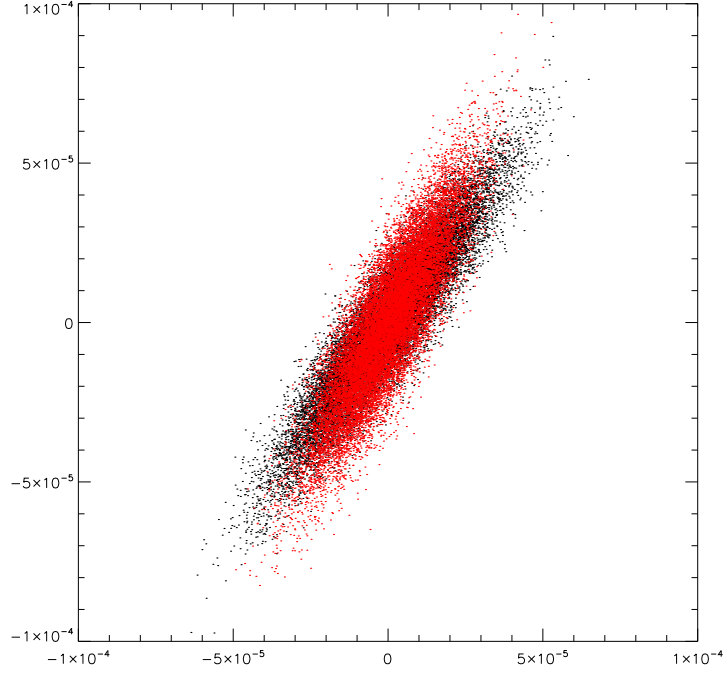


Figure 3: Monte Carlo sampling of the electron phase space  $x'[\text{rad}]$  vs.  $x[\text{m}]$  at the wiggler edges:  $y = -0.1$  (red) and  $y = 0.1$  (black).

It is also possible to calculate the moments at a given point knowing the distance to the waist. From Eq. 14 we calculate the positions of the horizontal ( $x$ ) and vertical ( $z$ ) with respect to the BMSOft and BMHard positions (Tables 7 and 7). These tables also contain the moments at these waist positions (parameters used by SHADOW). Note that at the waist position the relation  $\epsilon = \sigma\sigma'$  holds, because correlations are zero.

Note that due to the quadrupole, the waist is actually not where we calculate supposing no focusing. But for the purpose of beam size evolution just within the wiggler it is correct, as parameters are calculated back and forth from the wiggler position always supposing free space. The correct values and the calculated ones are shown in Fig. 7.

The plots of phase spaces  $(x, x')$ ,  $(z, z')$  and real  $(x, z)$  and divergence  $(x', z')$  space is given in Fig. 8 for the photons emitted by the 3-pole wiggler neglecting electron emittances. Fig. 8 contains the plots of the electron phase space with the same plot limits. Last, Fig. 8 contains the plots for the photon beam including all effects (wiggler emission and electron dispersion) which is approximately the convolution of the two previous models.

Table 5: Horizontal waist parameters and positions. Relative positions means from ID to waist. Note that SHADOW input requites the distance from the waist to the ID center, thus the opposite of the values given here.

location	X waist [m] (relative)	X waist [m] (absolute)	$\sigma_x$ [ $\mu m$ ]	$\sigma'_x$ [ $\mu rad$ ]	$\epsilon_x$ [ $pm$ ]
BMSoft	-0.180	13.150	7.46	20.20	150
BMHard	-0.570	13.147	6.51	23.11	150

Table 6: Vertical waist parameters and positions. Relative positions means from ID to waist. Note that SHADOW input requites the distance from the waist to the ID center, thus the opposite of the values given here.

location	Z waist [m] (relative)	Z waist [m] (absolute)	$\sigma_z$ [ $\mu m$ ]	$\sigma'_z$ [ $\mu rad$ ]	$\epsilon_z$ [ $pm$ ]
BMSoft	2.421	15.751	2.97	1.69	5
BMHard	1.035	14.752	1.39	3.60	5

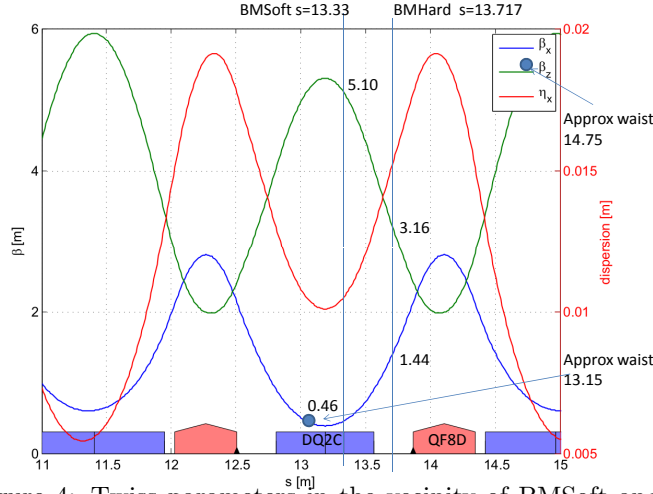


Figure 4: Twiss parameters in the vicinity of BMSoft and BMHard positions [1].

## 8 Future work

1. inclusion of orbit offset errors and recompute velocities and curvatures from them
2. unify shadow definitions with standard one in literature
3. 3D case: helicoidal wiggler: unify models
4. include dispersion in SHADOW models

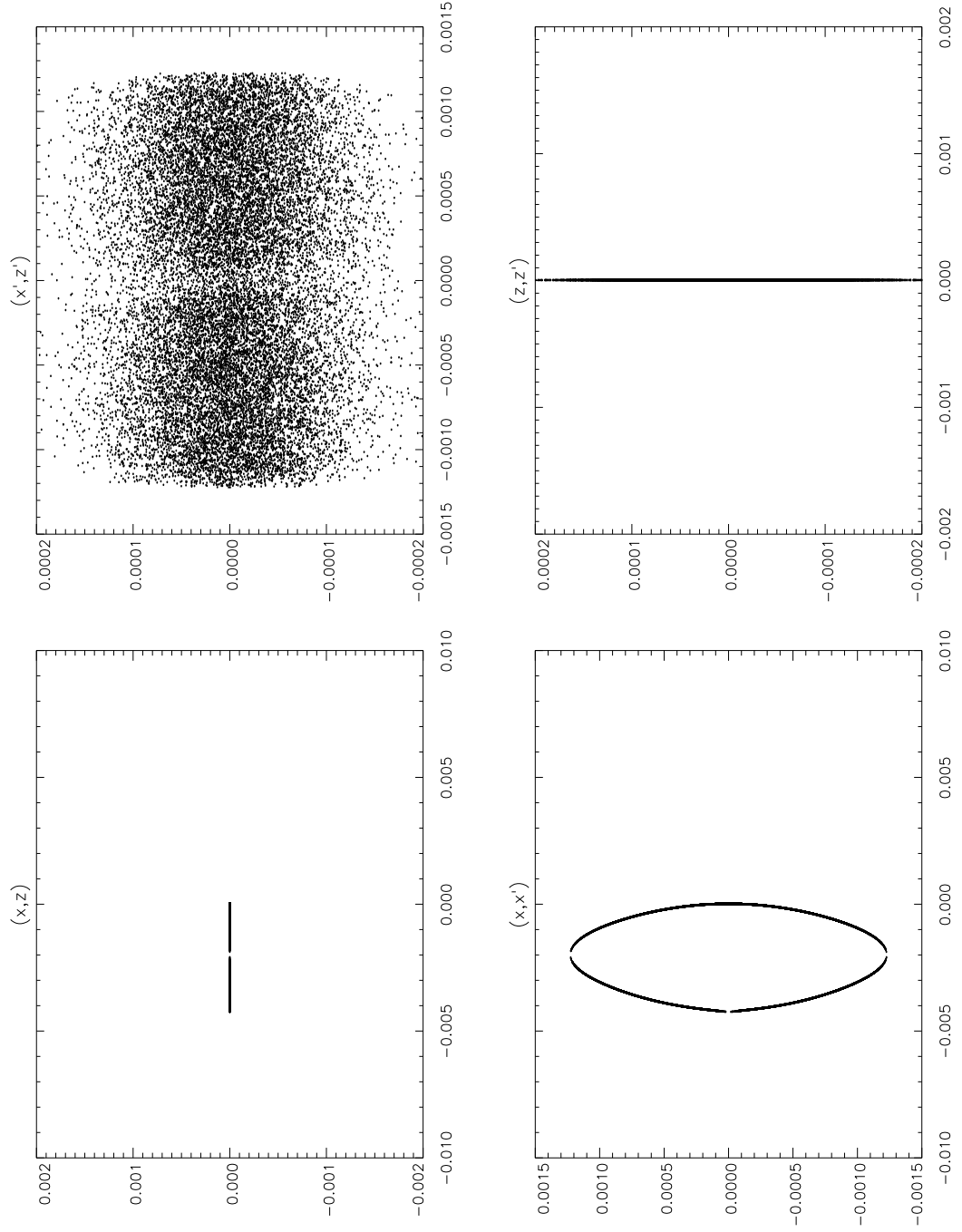


Figure 5: Photon phase space without emittances (length in  $cm$ , angles in  $rad$ ).

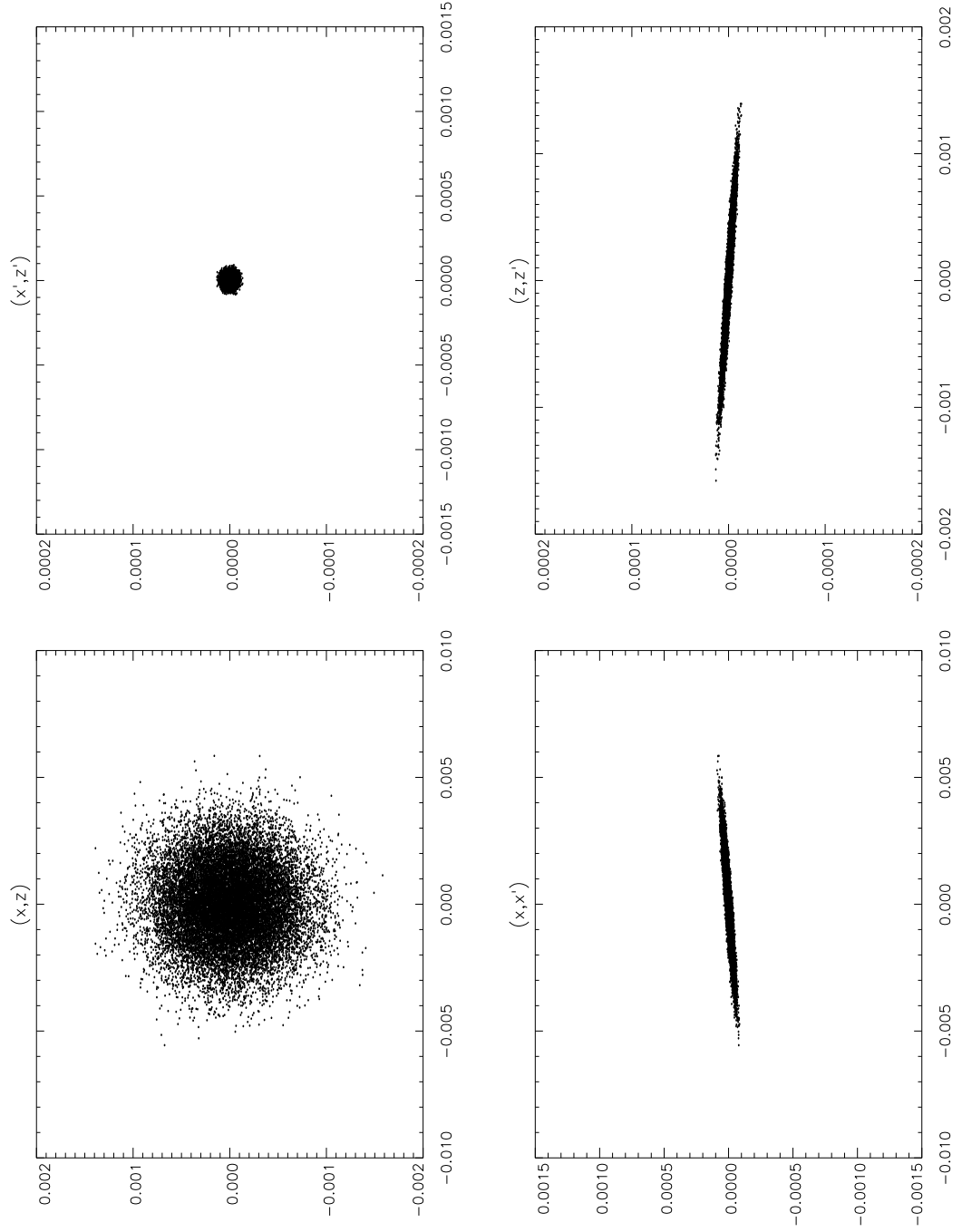


Figure 6: Electron phase space (length in  $cm$ , angles in  $rad$ ).

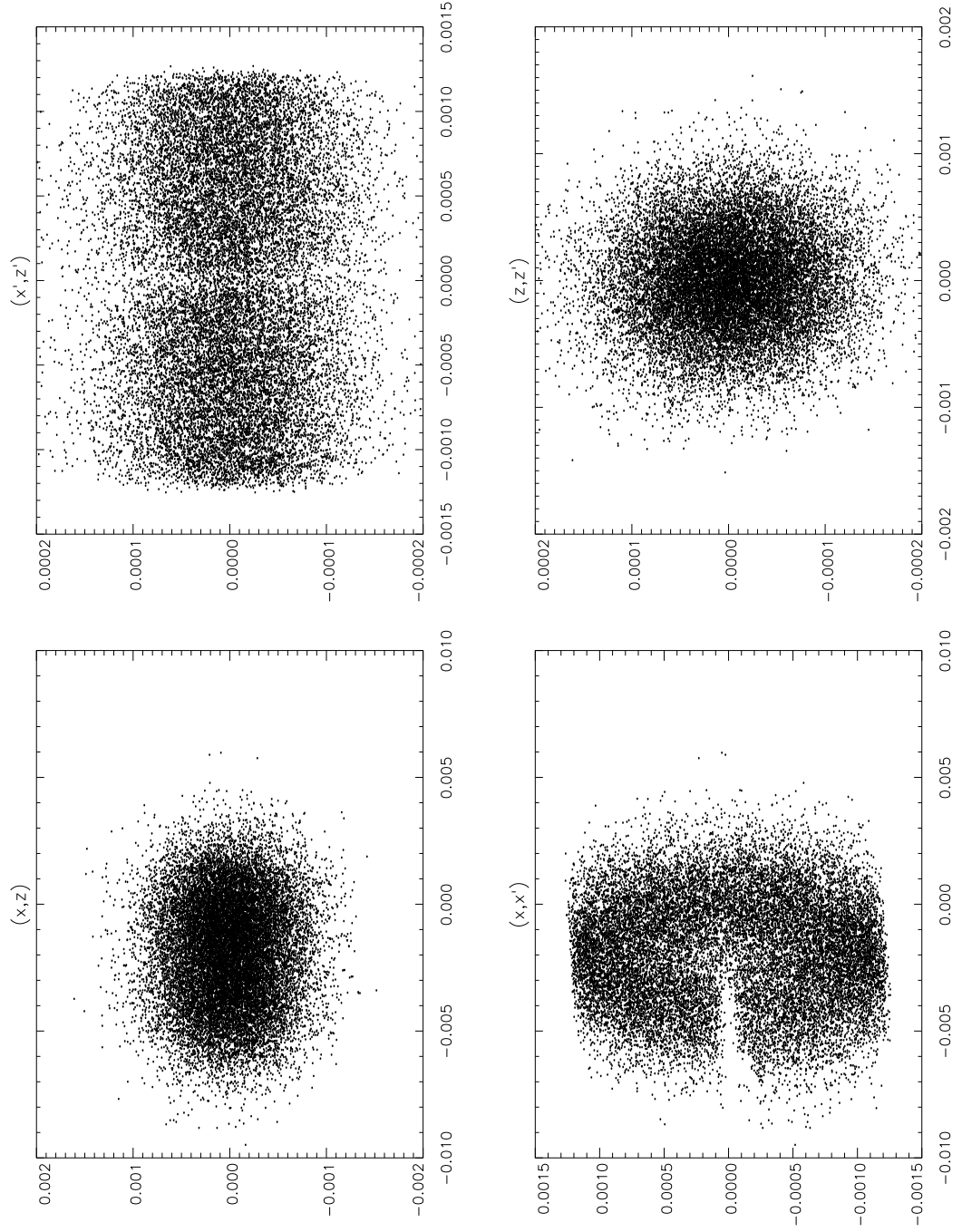


Figure 7: Photon phase space (length in *cm*, angles in *rad*. Full simulation including electron emittance effects.

## A Python code for calculating Twiss parameters and moments at BMHard and BMSoft locations

```

from numpy import sqrt

#S28 ESRF lattice parameters for wigglers to be used at the BM
beamnlines

location = 0 # 0=soft bend, 1=hard bend
# emittances
ex = 150e-12 # m.rad
ez = 5e-12 # m.rad

if location == 0:
    #soft
    s = 13.33 # m

    ax = -0.49
    bx = 0.46 # m
    gx = 2.72 # m^-1

    az = 1.38
    bz = 5.10
    gz = 0.57
else:
    #hard
    s = 13.717 # m

    ax = -2.03
    bx = 1.44 # m
    gx = 3.56 # m^-1

    az = 2.68
    bz = 3.16
    gz = 2.59

print("----- inputs -----\n")
print("emittances [m.rad]: ex, ez: %e, %e \n"%(ex, ez))
if location == 0:
    print("Soft bend location\n")
    print("at lattice location s: %f m"%(s))
else:
    print("Hard bend location\n")
print("X alpha, beta, gamma: %f %f %f\n"%(ax, bx, gx))
print("Z alpha, beta, gamma: %f %f %f\n"%(az, bz, gz))
print("-----\n")

xx2 = ex*bx
xyp = -ex*ax
xp2 = ex*gx

zz2 = ez*bz
zyp = -ez*az
zp2 = ez*gz

print("moments (at the wiggler position): \n")
print(" sqrt(<x2>) : %e"%(sqrt(xx2)))
print(" <x xyp> : %e"%(xyp))
print(" sqrt(<xp2>): %e"%(sqrt(xp2)))
print(" ")
print(" sqrt(<z2>) : %e"%(sqrt(zz2)))
print(" <z zyp> : %e"%(zyp))
print(" sqrt(<zp2>): %e"%(sqrt(zp2)))

# waist positions
dx = ax/gx
dz = az/gz
print(" ")

```

```

print("X waist at (relative) %f, Z waist at (relative) %f \n" %(dx,dz))
print("X waist at (absolute) s=%f, Z waist at (absolute) s=%f \n" %(s+dx
    ,s+dz))

# optical functions at waist
bx_w = bx - 2*ax*dx + gx*dx*dx
ax_w = ax - gx*dx
gx_w = gx

bz_w = bz - 2*az*dz + gz*dz*dz
az_w = az - gz*dz
gz_w = gz

print(" ")
print("optical functions at X waist:")
print("alpha, beta, gamma: %f,%f,%f" %(ax_w, bx_w, gx_w ))
print(" ")
print("optical functions at Z waist:")
print("alpha, beta, gamma: %f,%f,%f" %(az_w, bz_w, gz_w ))

#moments at waist
xx2_w = bx_w*ex
xxp_w = -ax_w*ex
xp2_w = gx_w*ex

zz2_w = bz_w*ez
zzp_w = -az_w*ez
zp2_w = gz_w*ez

print(" ")
print("moments at X waist:")
print("sqrt(<x2>) : %e" %(sqrt(xx2_w)))
print("<x xp> : %e" %(xxp_w))
print("sqrt(<xp2>): %e" %(sqrt(xp2_w)))
print(" ")
print("moments at Z waist:")
print("sqrt(<z2>) : %e" %(sqrt(zz2_w)))
print("<z zp> : %e" %(zzp_w))
print("sqrt(<zp2>): %e" %(sqrt(zp2_w)))

print(" ")
print("emittances at waist (double check):")
print("ex, ez: %e, %e" %(sqrt(xx2_w*xp2_w), sqrt(zz2_w*zp2_w)))

```

## B ShadowVUI workspace for BMHard

A ShadowVUI workspace containing the inputs for creating the 3-pole wiggler source is available in the SHADOW interface. To run it in the ESRF NICE cluster, start the SHADOW interface using XOP 2.4: `/scisoft/xop2.4/xop shadowvui`, then load the workspace with the name: `ESRFnewlattice-BMHard.ws` from the examples directory (ShadowVUI → Load Example Workspace → From Example Dir...).

## References

- [1] ESRF. ASD Upgrade: TDS Ch 2: Accelerator and source upgrade. *TDS Draft version 1.3*, 2014.
- [2] Manuel Sanchez del Rio, Niccolo Canestrari, Fan Jiang, and Franco Cerina. *SHADOW3: a new version of the synchrotron X-ray optics modelling package*. *Journal of Synchrotron Radiation*, 18(5):708–716, Sep 2011.
- [3] H. Wiedermann. *Particle Accelerator Physics*. Springer, 2007.



- [4] Y. K. Wu, E. Forest, and D. S. Robin. Explicit symplectic integrator for s-dependent static magnetic field. *Phys. Rev. E*, 68:046502, Oct 2003.