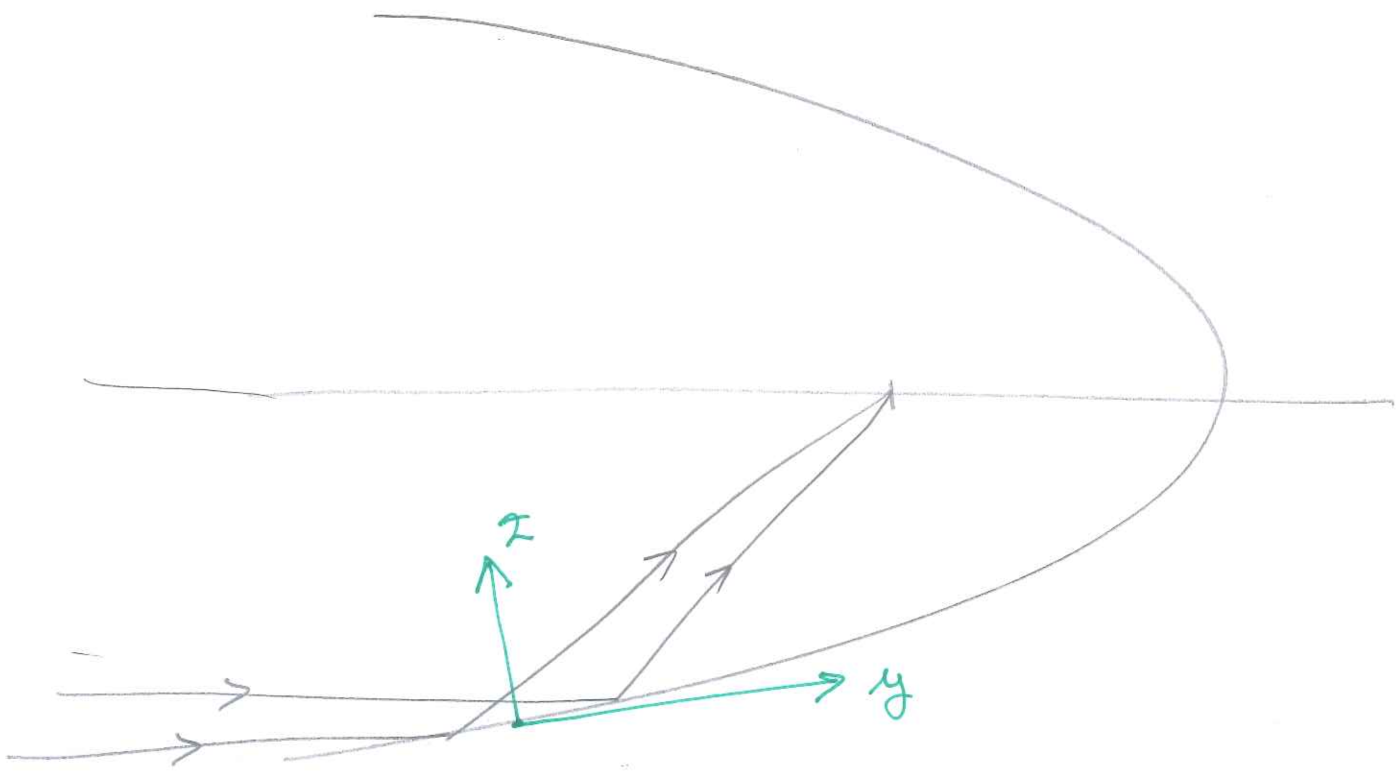
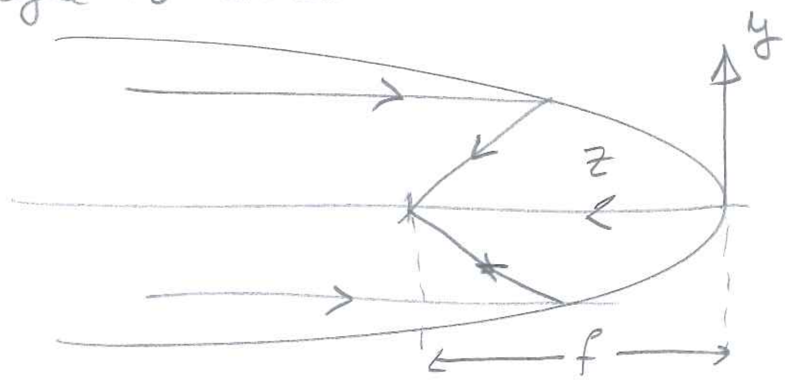
# Parabolic mirror

10/4/2019




$$c_2 y^2 + c_3 z^2 + c_5 y z + c_9 z = 0$$

if incident angle is zero




$$c_2 y^2 + c_9 z = 0$$

$$y^2 = -\frac{c_9}{c_2} z$$

$$\boxed{p = -\frac{1}{2}\frac{c_9}{c_2}}$$

$$y^2 = 2 p z$$

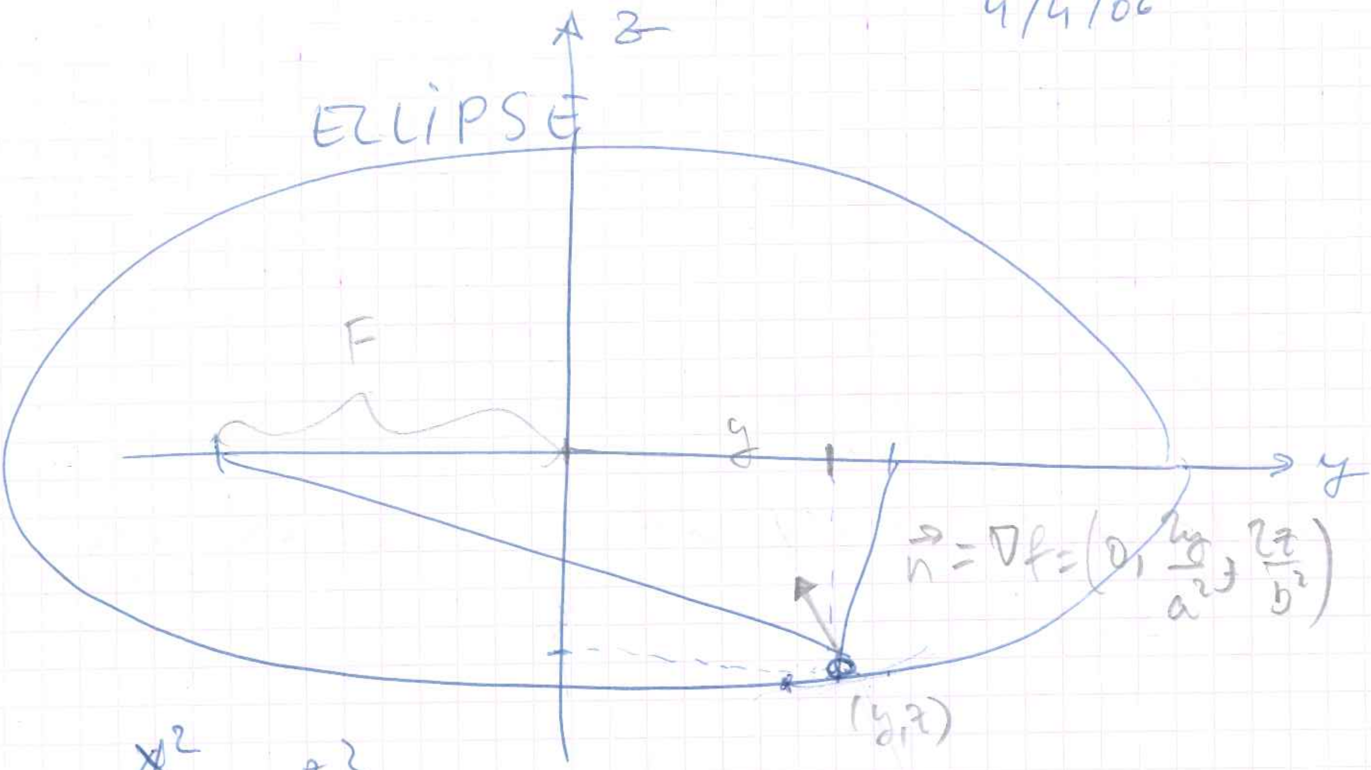
parameter

$$p = 2f$$

focal distance

4/4/06

# ELLIPSE

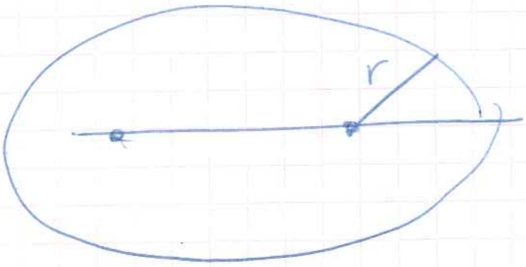


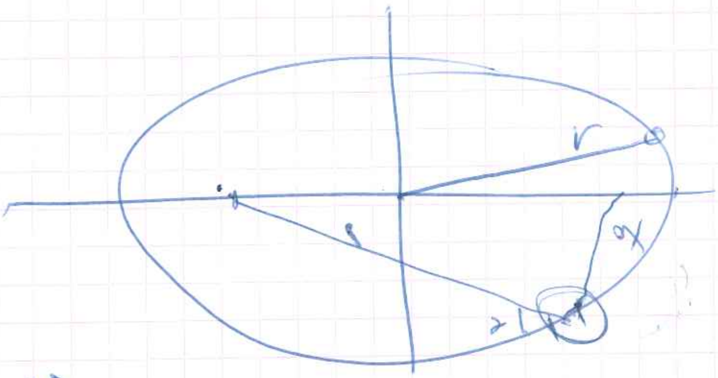

$$\frac{\cancel{x}^2}{a^2} + \frac{z^2}{b^2} = 1$$

focal distance $F^2 = c^2 = a^2 - b^2$ $= AFOCI^2$

eccentricity $\varepsilon = \frac{c}{a} = \frac{\sqrt{a^2-b^2}}{a}$ = ECCEN

Eq. Polar



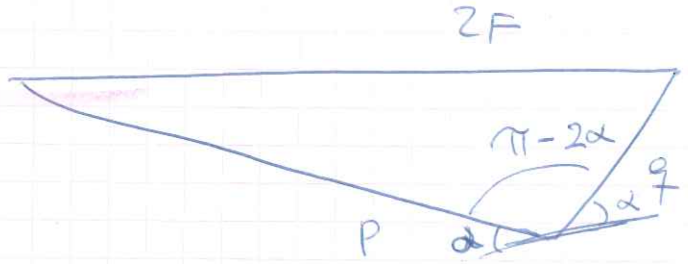




$$r = \frac{p}{1+\varepsilon\cos\theta} \quad \theta \in [0, 2\pi)$$

$$r = \frac{b^2}{1-\varepsilon^2\cos^2\theta} \quad \theta \in [0, 2\pi)$$

$$p = \frac{b^2}{a}$$

Mirror




cosine theorem

$$(2F)^2 = p^2 + q^2 - 2pq \cos(\pi - 2\alpha)$$

$$p + q = 2a \longrightarrow \boxed{a = \frac{p+q}{2}} = \text{AXMAJ}$$

$$4(a^2 - b^2) = p^2 + q^2 + 2pq \cos 2\alpha$$

$$4\left(\frac{(p+q)^2}{4} - b^2\right) = p^2 + q^2 + 2pq \cos 2\alpha$$

$$\cancel{p^2} + \cancel{q^2} + 2pq - 4b^2 = \cancel{p^2} + \cancel{q^2} + 2pq \cos 2\alpha$$

$$2pq(1 - \cos 2\alpha) = 4b^2 \Rightarrow$$

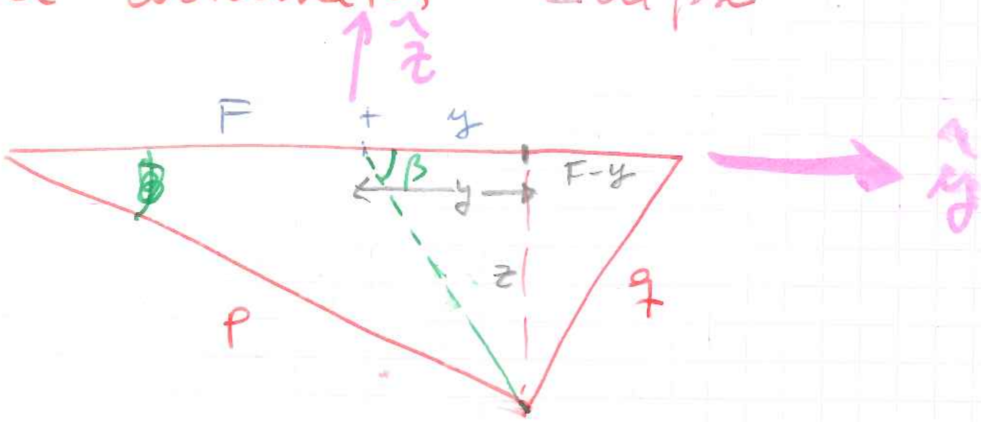
$$b = \sqrt{\frac{pq(1-\cos 2\alpha)}{2}} =$$

$$= \sqrt{\frac{pq}{2}} \sqrt{1 - (\cos^2\alpha - \sin^2\alpha)} =$$

$$= \sqrt{\frac{pq}{2}} \sqrt{1 - (1 - \sin^2\alpha - \sin^2\alpha)} =$$

$$\sqrt{\frac{pq}{2}} \sqrt{2\sin^2\alpha} \Rightarrow \boxed{b = \sqrt{pq} \cdot \sin\alpha} = \text{AXMIN}$$

# Pole coordinates - Ellipse




$$\left.\begin{aligned}(F+y)^2+z^2&=p^2\\(F-y)^2+z^2&=q^2\end{aligned}\right\}\qquad \left.\begin{aligned}F^2+y^2+2Fy+z^2&=p^2\\F^2+y^2-2Fy+z^2&=q^2\end{aligned}\right\}$$

$$4yF=p^2-q^2 \Rightarrow \boxed{y=\frac{p^2-q^2}{4F}}$$

(in shadow $y=\dfrac{p-q}{2E}$ (verified numerically)

YCEN

$$\frac{y^2}{a^2}+\frac{z^2}{b^2}=1 \Rightarrow \boxed{z=b\sqrt{1-\frac{y^2}{a^2}}}$$ - ZCEN

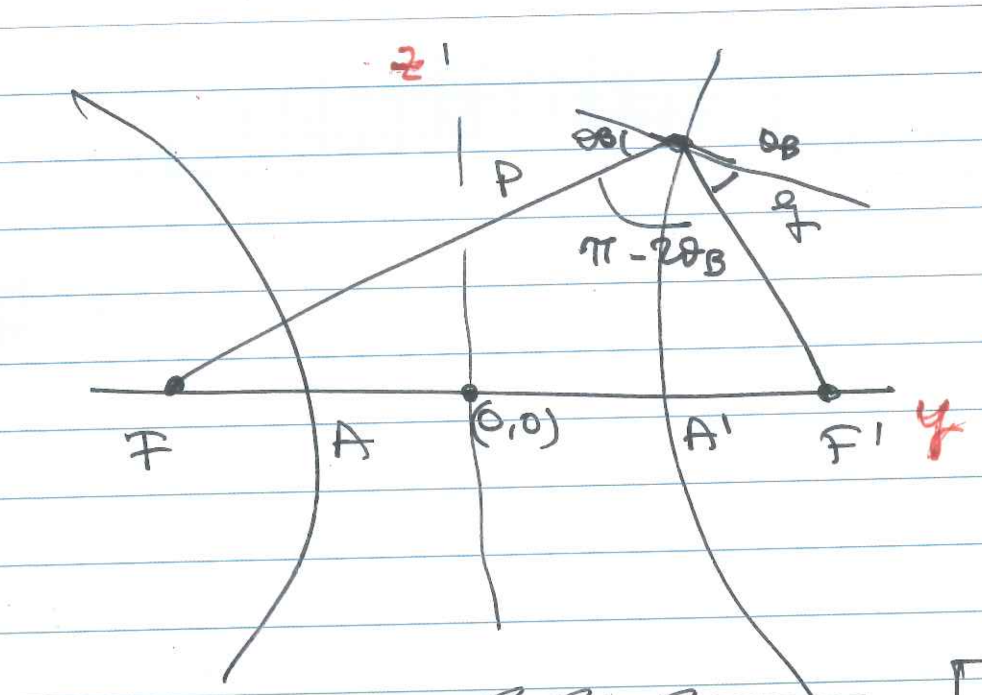
"Angle of MajAx and pole [CCW] [deg]" ← in ShadowVVI

ELLiTHE = $\beta$ $\qquad tg\,\beta=\dfrac{z}{y} \Rightarrow z=y\cdot tg\,\beta$

Shadow center

$$\boxed{\begin{aligned}y&=\frac{ab}{\sqrt{b^2+a^2\,tg^2\beta}}\\ z&=y\cdot\tan\beta\end{aligned}}$$

15/10/2008 HYPERBOLA ID24




$p = |p|$

$q = |q|$

$$\frac{y^2}{a^2} - \frac{z^2}{b^2} = 1$$

$AA' = 2a$ $\quad$ $\cancel{2F = 2c = 2\sqrt{a^2+b^2}}$ $\quad$ $\boxed{F^2 = c^2 = a^2 + b^2}$

$$\boxed{\varepsilon = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a}}$$

$$F = \frac{1}{2}\sqrt{p^2 + q^2 - 2pq\cos(\pi - 2\vartheta_B)}$$

$$b = \sqrt{F^2 - a^2}$$

$$(2F)^2 = p^2 + q^2 - 2pq\cos(\pi - 2\vartheta_B)$$

$$\varepsilon = \frac{c}{a} = \frac{F}{a}$$

$p - q = \pm 2a$ $\quad$ $\boxed{a = \pm\frac{p-q}{2}}$

$$4(a^2+b^2) = p^2 + q^2 + 2pq\cos 2\vartheta_B$$

$$4\frac{(p-q)^2}{4} + 4b^2 = \quad "$$

$$\cancel{p^2} + \cancel{q^2} - 2pq + 4b^2 = \cancel{p^2} + \cancel{q^2} + 2pq\cos\theta$$

$$\boxed{b = \sqrt{\frac{2pq\cos 2\theta + 2pq}{4}} = \sqrt{\frac{pq(1+\cos 2\theta)}{2}}}$$

$$= \sqrt{\frac{pq}{2}} \cdot \sqrt{\cancel{1} + (2\cos^2\theta - \cancel{1})} = \sqrt{pq}\cos\vartheta_B$$

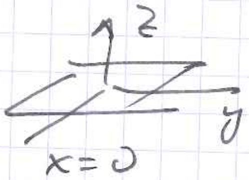
NOR   no   no ~ 1

concave   + ~ 1

ellipse convex   <0   0   +1

hyp convex   ~~+1~~ $\equiv$ !! ??

$\cancel{C_1 x^2 + C_2 y^2 + C_3 z^2 + C_4}$




~~Co~~ x

$$C_1 y^2 + C_2 z^2 + C_4 yz + C_7 y + C_8 z + C_9 = 0$$

$$\cancel{z_1}\quad C_2 z^2 + \underbrace{(C_4 y + C_8)}_{B} z + \underbrace{(C_1 y^2 + C_7 y + C_9)}_{C} = 0$$

$$z = \frac{-B \pm \sqrt{B^2 - 4 C_2 C}}{2 C_2}$$

HYP → focalize

Plane + Sph + Ell no focalize   OK

Problems:

- conicset.pro   $F = \frac{1}{2}\sqrt{p^2 + q^2 \pm 2pq \cos 2\theta_B}$ WHY?
- ~~plane~~ Sph + Ell → shows a best focus $\neq 0$
- Reflectivity does not work?