

Basic Electronic Circuits

(IEC-103)

Lecture-02

Frequency Response Analysis

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- In AC sinusoidal steady state analysis covered in ES-1 the source frequency is assumed constant (50Hz). Here we consider the source frequency as a variable and examine the performance as the source frequency is varied.

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- Frequency response of a circuit/system is the steady state response of the circuit/system when excited by range of sinusoidal signals.

Applications

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- Filters play critical roles in blocking or passing specific frequencies or ranges of frequencies.
- Without filters, it would be impossible to have multiple channels in radio communication.

Frequency Response

- A circuit that is intended to amplify human speech must treat all the sinusoids with frequencies in the range 20 Hz to 20 KHz in the same way or the amplified signal will be distorted.

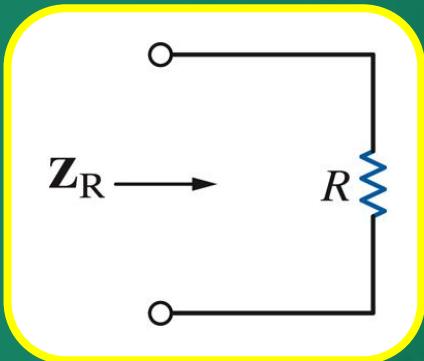
Frequency Response

- A circuit that is intended to amplify human speech must treat all the sinusoids with frequencies in the range 20 Hz to 20 KHz in the same way or the amplified signal will be distorted.
- The amplifier output will be an undistorted copy of input voltage only if the gain of the circuit is constant over the entire frequency range and the phase shift is proportional to the input frequency.

Impedance Functions

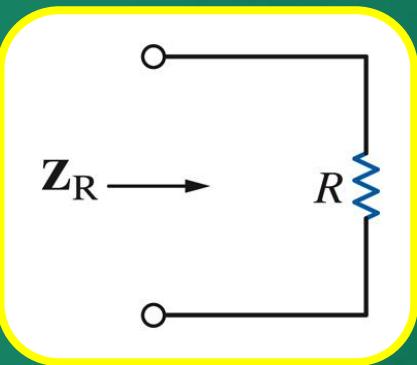
Impedance of a Resistor

Resistor



Impedance of a Resistor

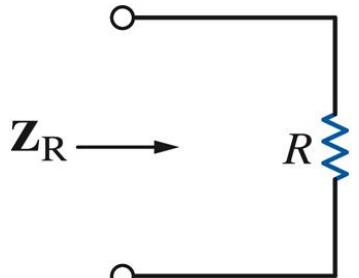
Resistor



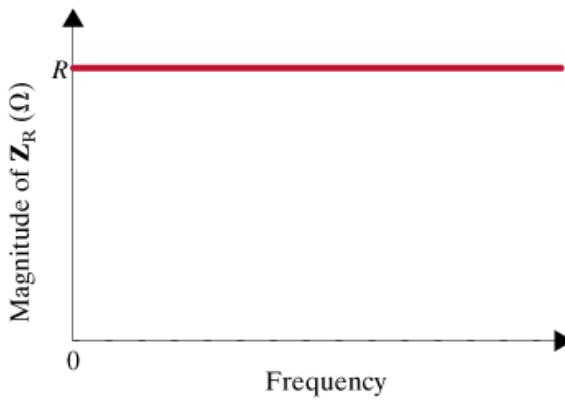
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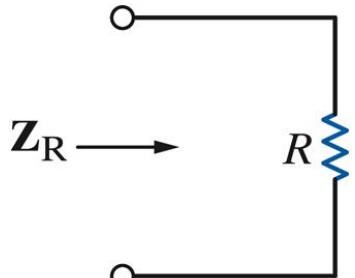


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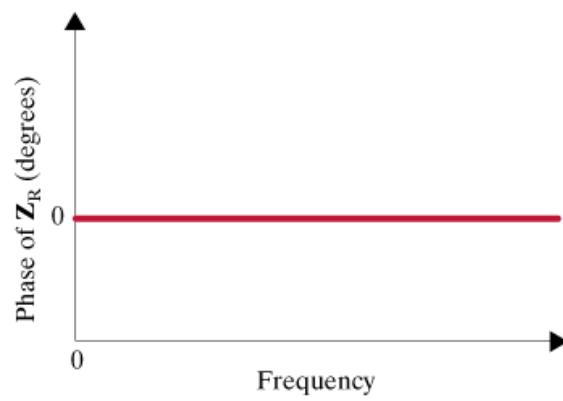
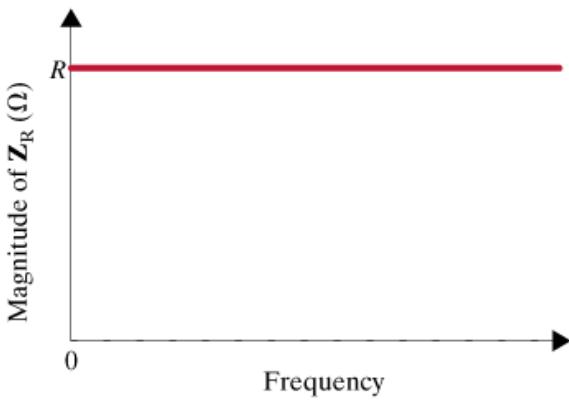


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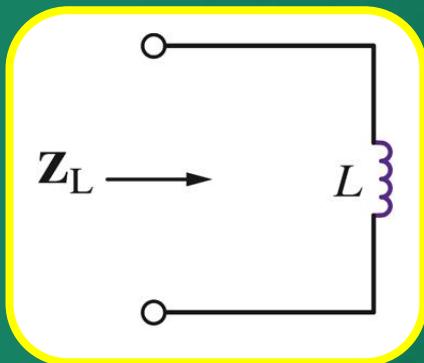


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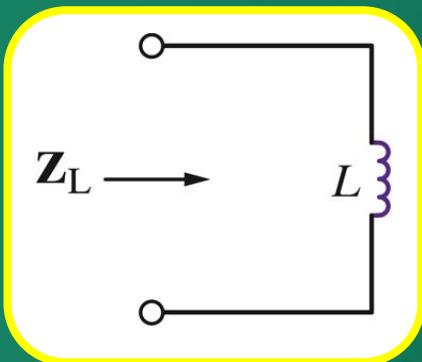
Impedance of an Inductor

Inductor



Impedance of an Inductor

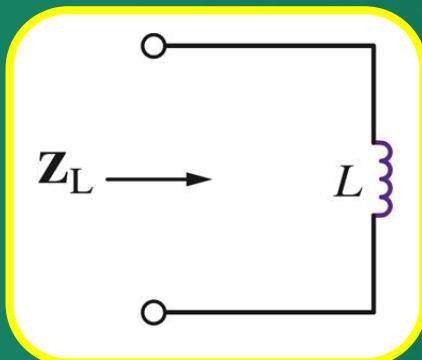
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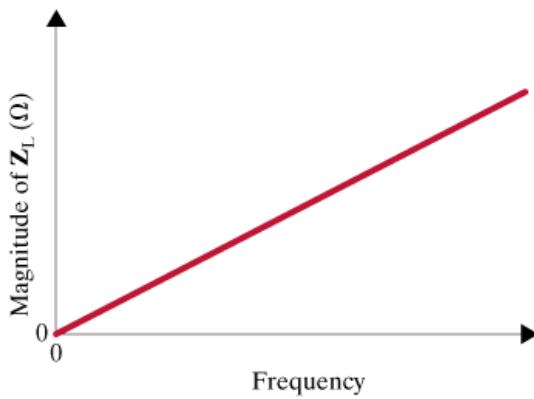
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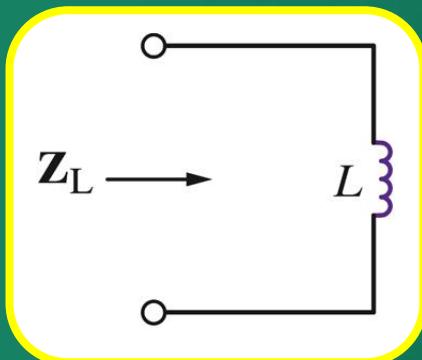


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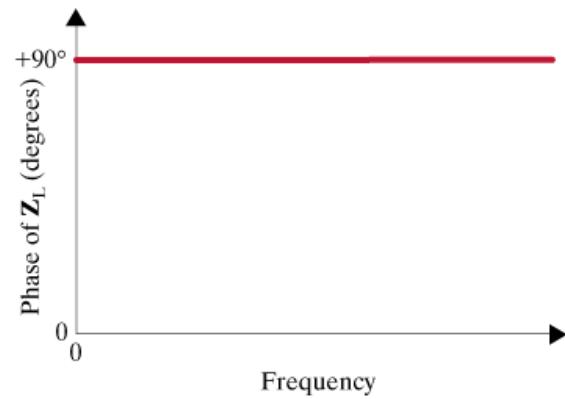
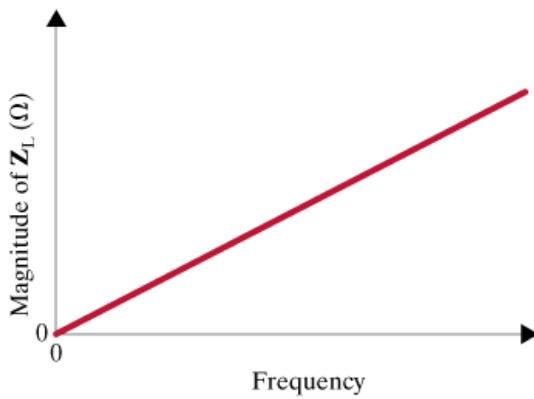


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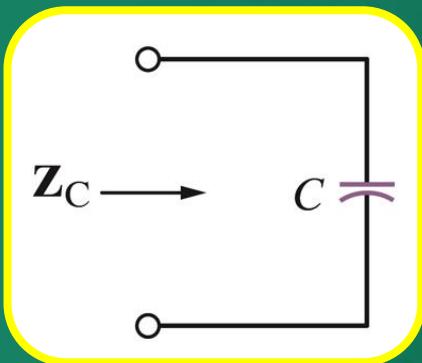


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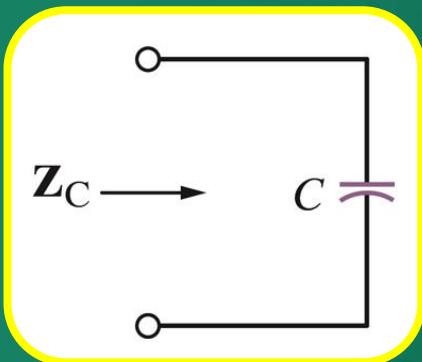
Impedance of a Capacitor

Capacitor



Impedance of a Capacitor

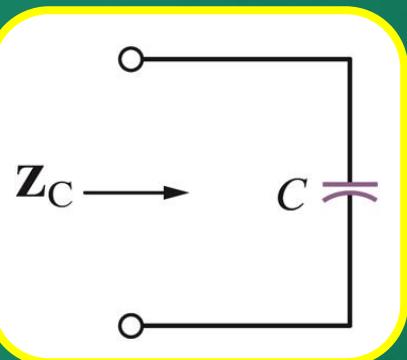
Capacitor



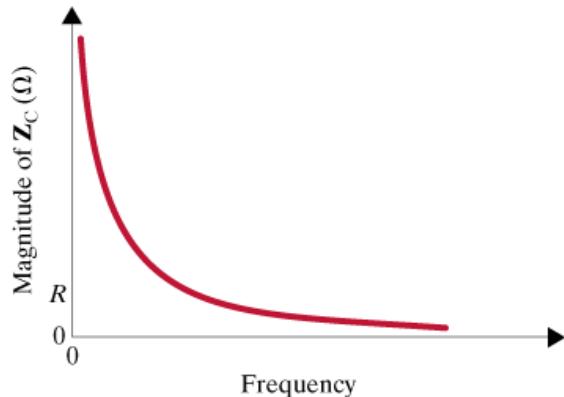
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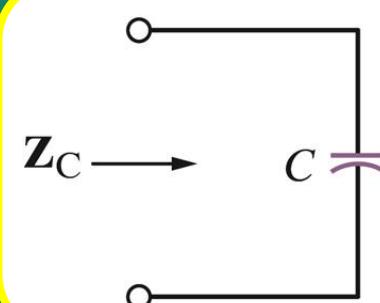


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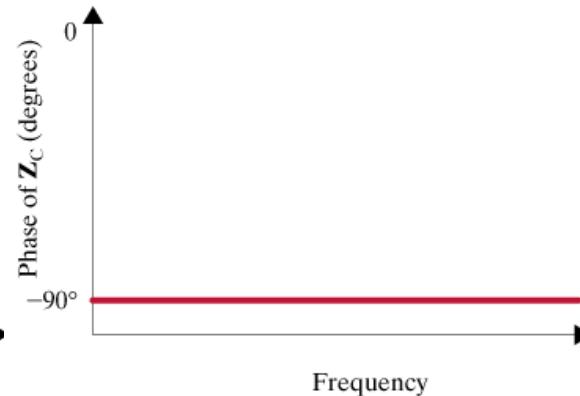
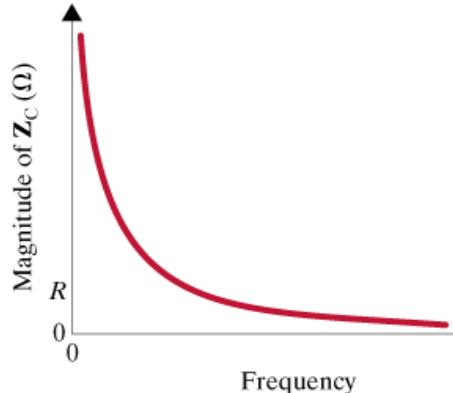


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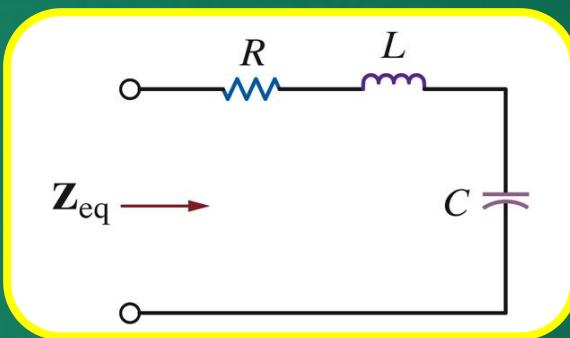
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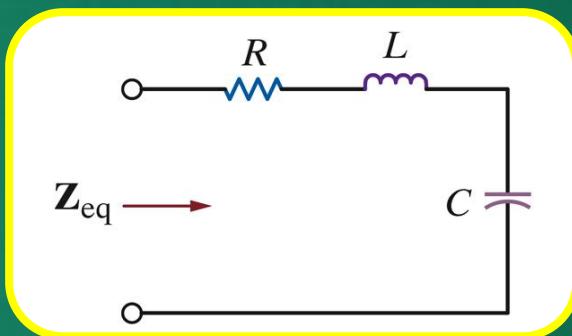
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Series RLC Circuit

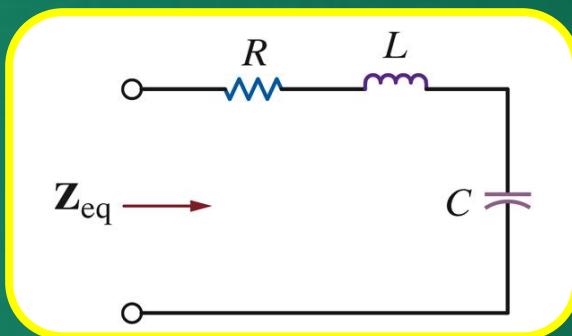


Series RLC Circuit



$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C} = \frac{(j\omega)^2 LC + j\omega RC + 1}{j\omega C} \times \frac{-j}{-j} = \frac{\omega RC + j(\omega^2 LC - 1)}{\omega C}$$

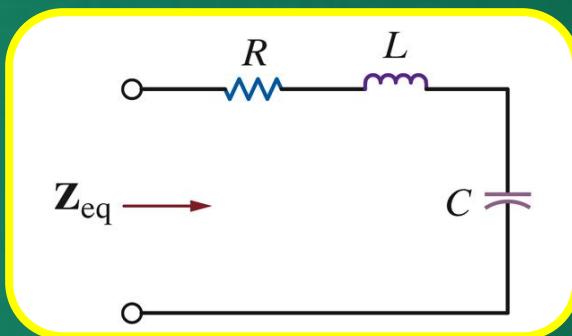
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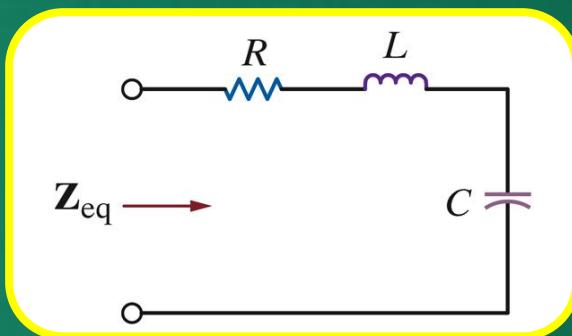


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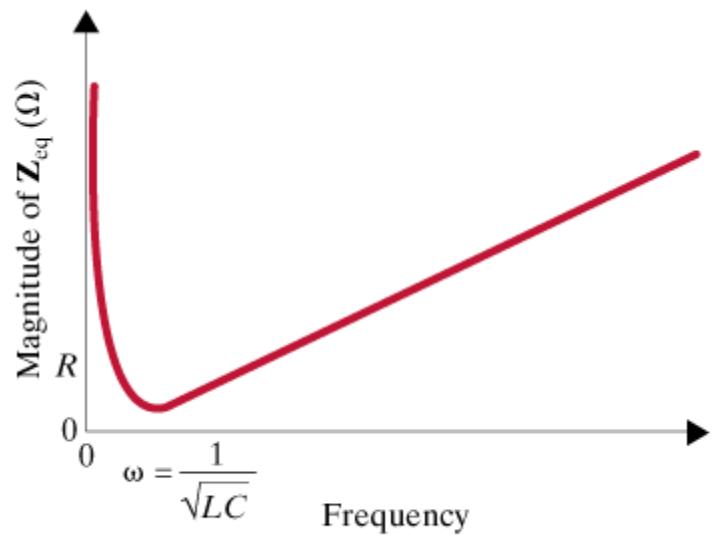
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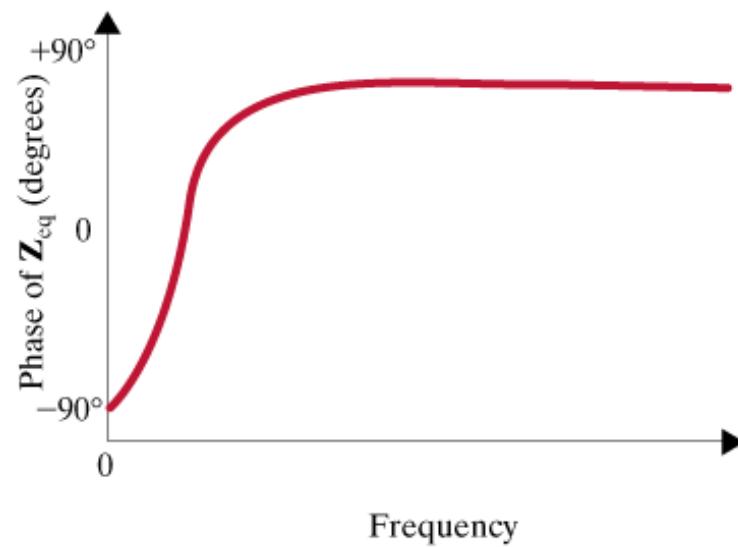
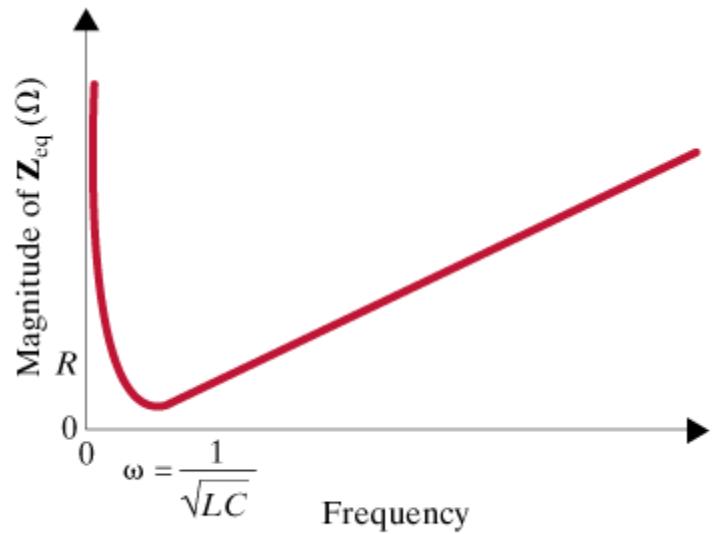
"Simplification in notation" $j\omega \approx s$

$$Z_{eq}(s) = \frac{s^2 LC + sRC + 1}{sC}$$

Series RLC Circuit



Series RLC Circuit



Impedance Function

Simplified notation for basic components

$$Z_R(s) = R, \quad Z_L(s) = sL, \quad Z_C = \frac{1}{sC}$$

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For all cases seen, and all cases to be studied, the impedance is of the form

$$Z(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

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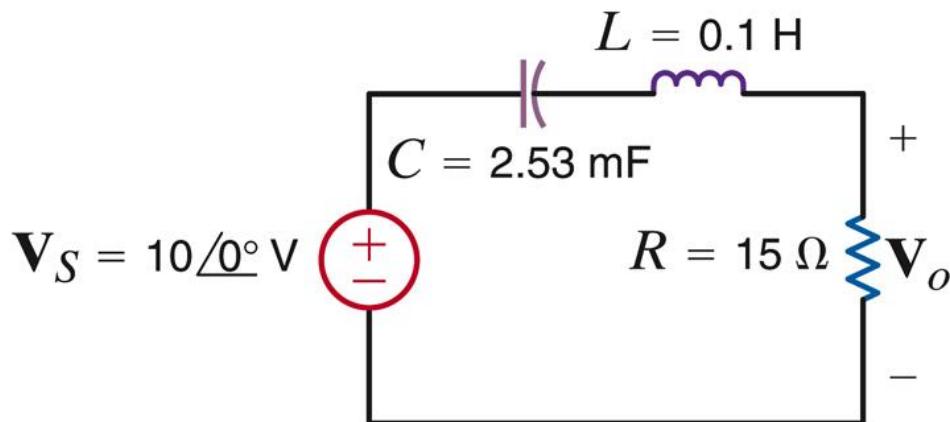
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Moreover, if the circuit elements (L, R, C, dependent sources) are real then the expression for any voltage or current will also be a rational function in S.

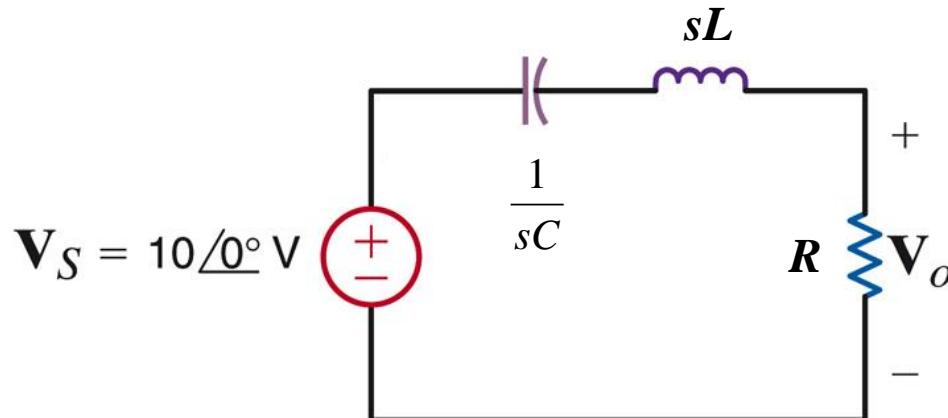
Example

Find the expression for magnitude and Phase of V_o as a function of frequency ω and plot the response as ω is varied from 0.1 Hz to 1 KHz.



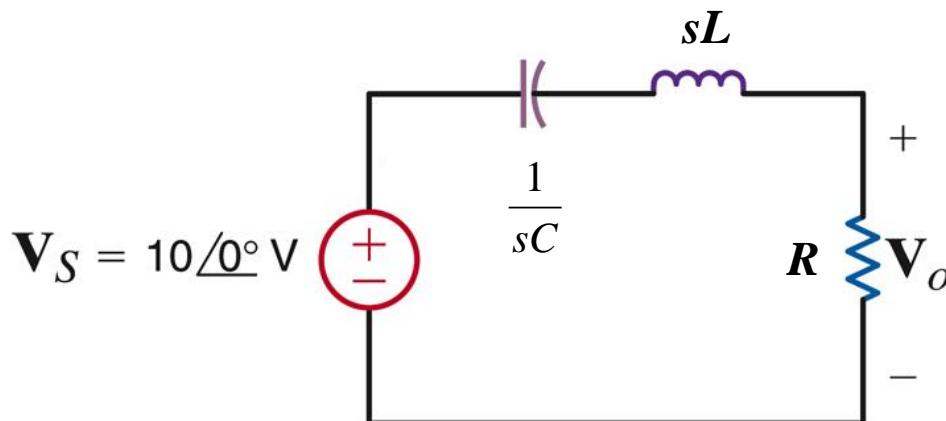
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$$V_o = \frac{j\omega(15 \times 2.53 \times 10^{-3})}{(j\omega)^2(0.1 \times 2.53 \times 10^{-3}) + j\omega(15 \times 2.53 \times 10^{-3}) + 1} 10 \angle 0^\circ$$

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$$V_o = \frac{0.3795\omega j}{(1 - 2.53 \times 10^{-4})\omega^2 + 0.03795\omega j}$$

Example

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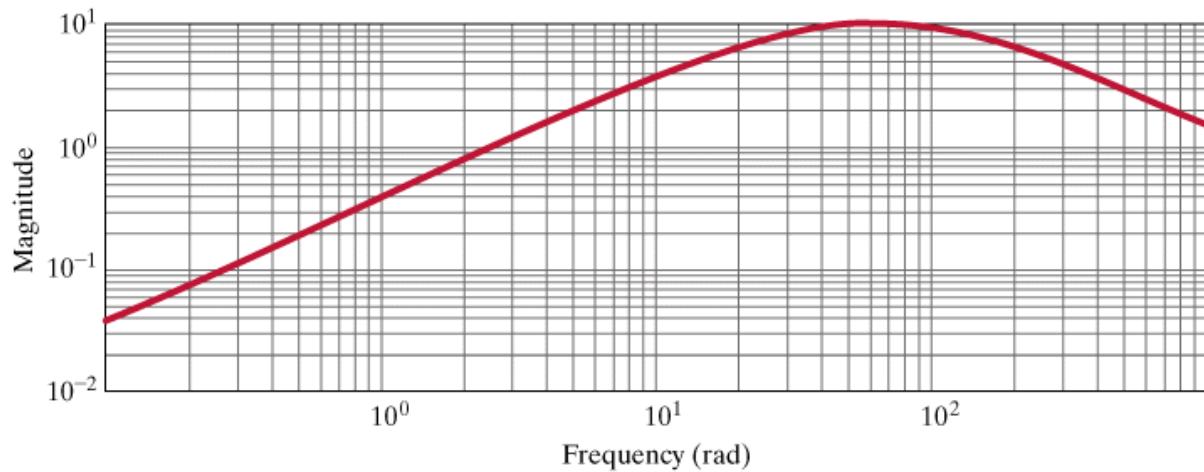
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$$\angle V_o = \frac{\pi}{2} - \tan^{-1} \left(\frac{0.03795\omega}{(1 - 2.53 \times 10^{-4})\omega^2} \right)$$

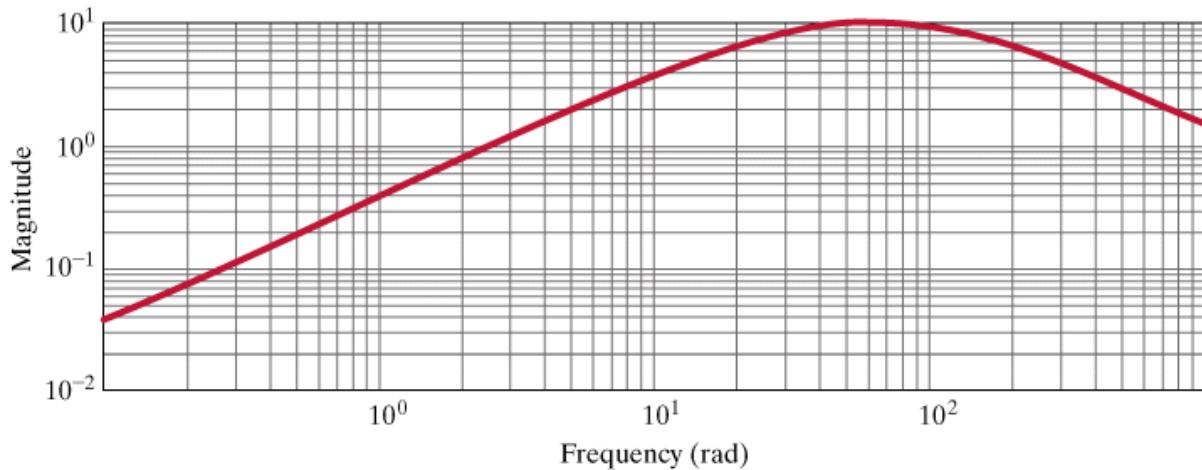
Frequency Response

Log-log plot

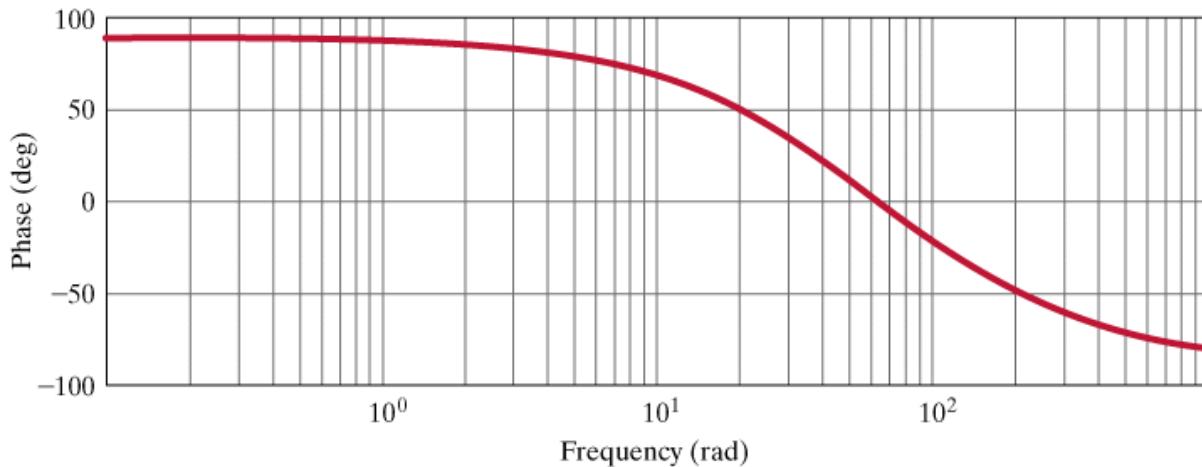


Frequency Response

Log-log plot



Semi-log plot



(b)

Frequency in log scale

Use of logarithms expands the range of frequencies portrayed on the horizontal axis.

Gain in bels

In communication systems, gain is measured in bels. The bel is used to measure the ration of two levels of power or power gain G, that is

$$G = \text{Number of bels} = \log_{10} \frac{P_2}{P_1}$$

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The ear, responds in a logarithmic fashion to changes in audio power levels.

Bels & Decibels

The decibel (dB) provides us with a unit of less magnitude. It is 1/10th of a bel and is given by

$$G_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$$

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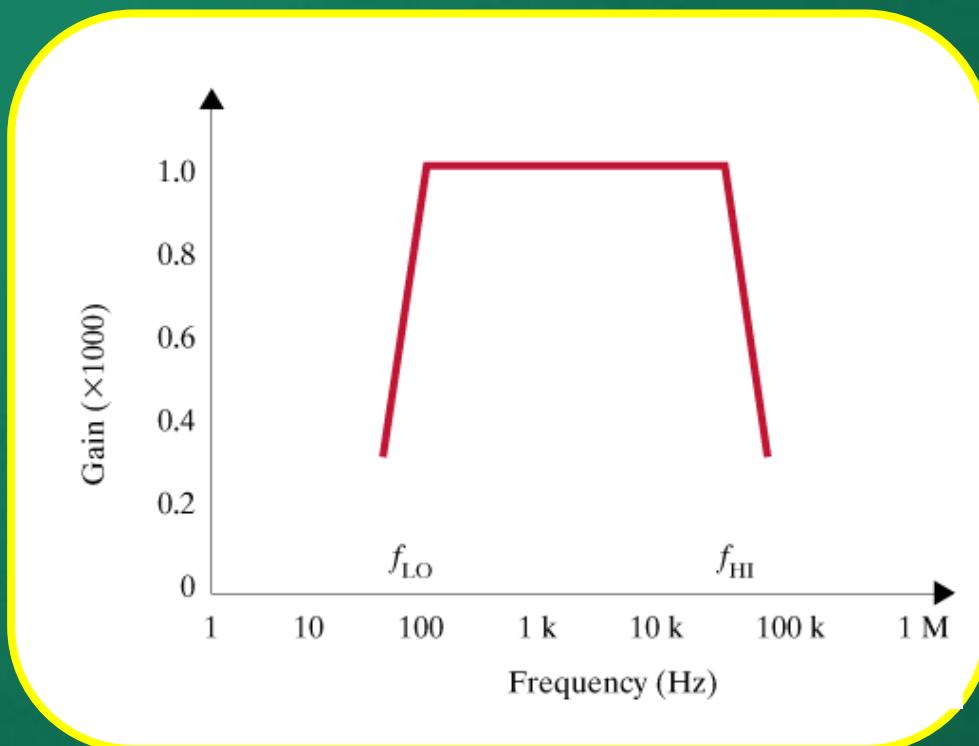
$$G_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$$

For comparing voltages or current levels, we use

$$G_{\text{db}} = 20 \log_{10} \frac{V_2}{V_1} \text{ or } G_{\text{db}} = 20 \log_{10} \frac{I_2}{I_1}$$

Specs of an Amplifier

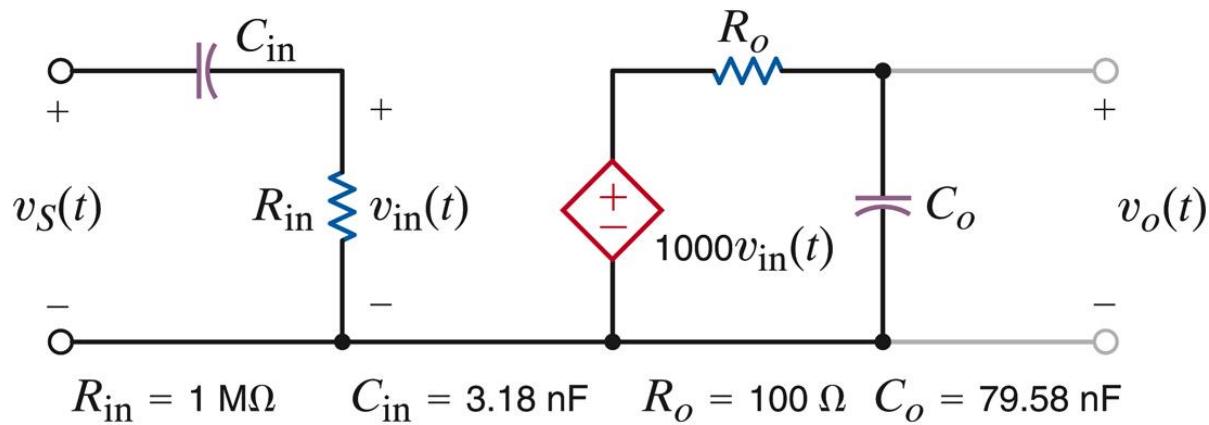
**Desired frequency characteristic
(flat between 50 Hz and 15 KHz)**



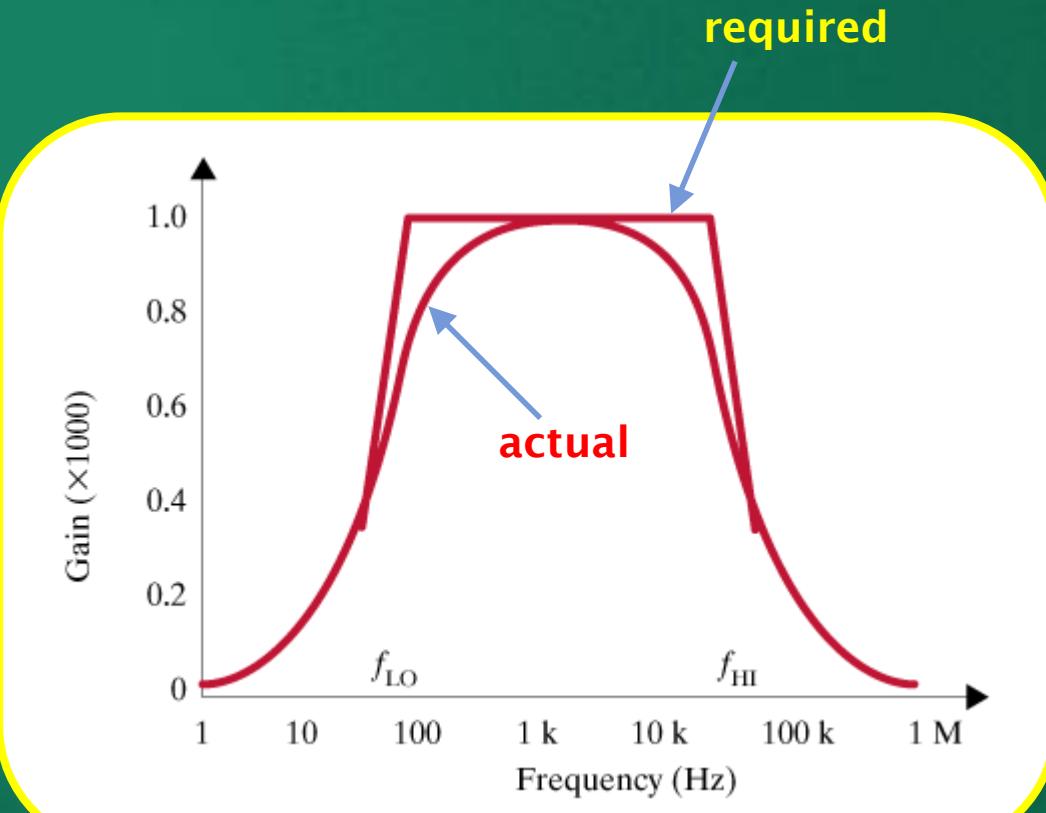
Log scale (frequency)

Synthesis of an Amplifier

Postulated amplifier



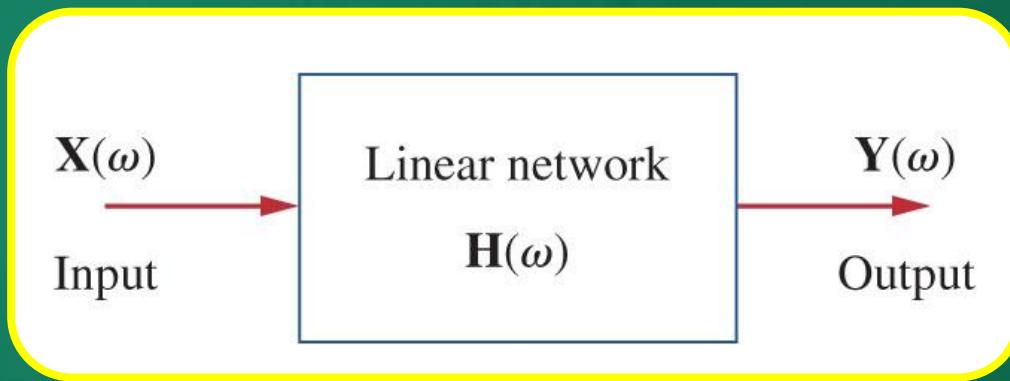
Specs and actual response



Transfer Function

Definition

One useful way to analyze the frequency response of a circuit is the concept of the transfer function $H(\omega)$.



$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$H(\omega)$ is the frequency dependent ratio of a forced function or response $Y(\omega)$ to the forcing function $X(\omega)$.

Terminology

One useful way to analyze the frequency response of a circuit is the concept of the transfer function $H(\omega)$.

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

Zeros & Poles

To obtain $H(\omega)$, first convert to frequency domain equivalent components in the circuit.

$H(\omega)$ can be expressed as the ratio of numerator $N(\omega)$ and denominator $D(\omega)$ polynomials.

$$H(\omega) = \frac{N(\omega)}{D(\omega)}$$

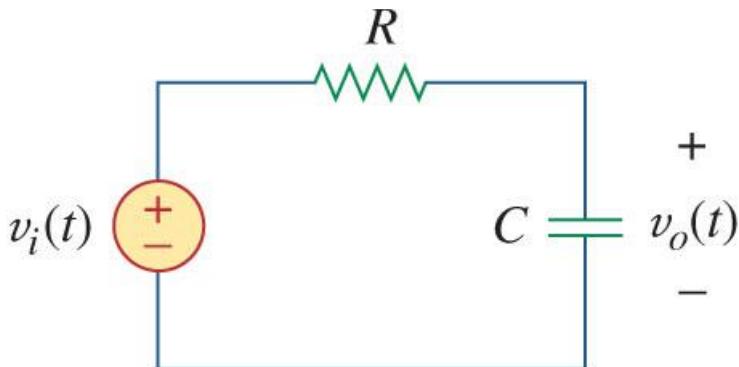
Zeros are where the transfer function goes to zero.

Poles are where it goes to infinity.

They can be related to the roots of $N(\omega)$ and $D(\omega)$.

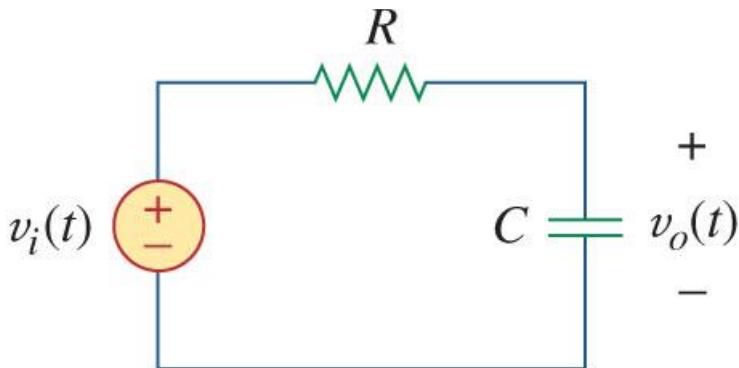
Example

Obtain the transfer function V_o/V_s of RC network shown below and plot the frequency response



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Answer:

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$$\angle H(\omega) = -\tan^{-1}(\omega RC)$$

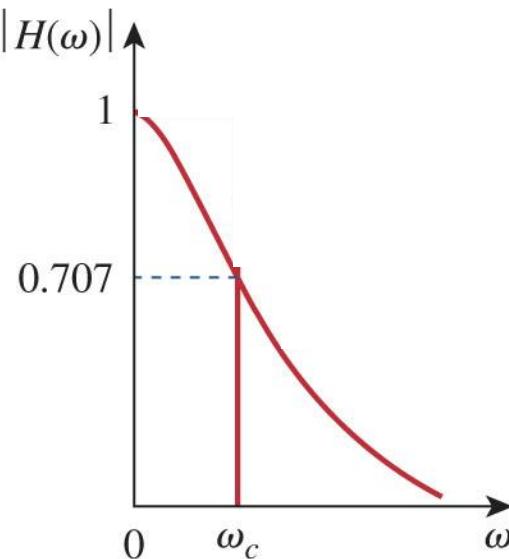
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Frequency Response



Bode Plots

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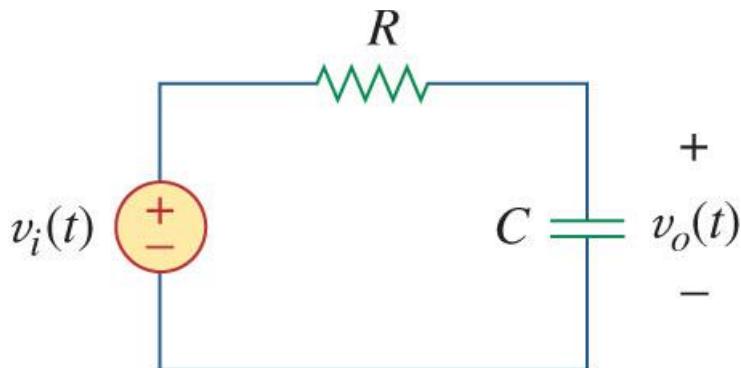
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These plots are referred to as Bode plots.

Bode plots either show magnitude (in decibels) or phase (in degrees) as a function of frequency.

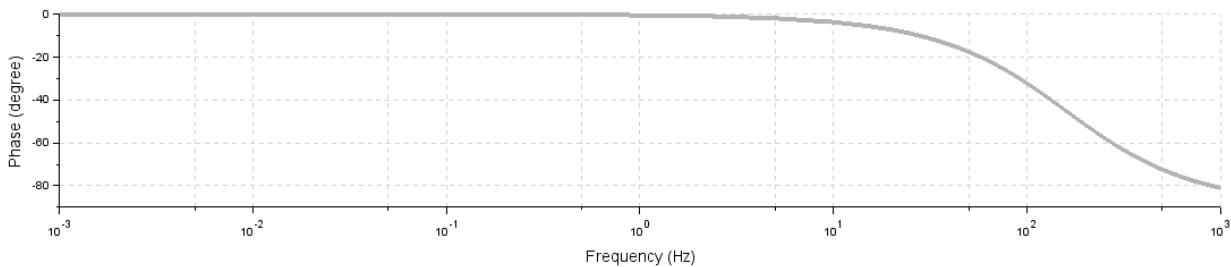
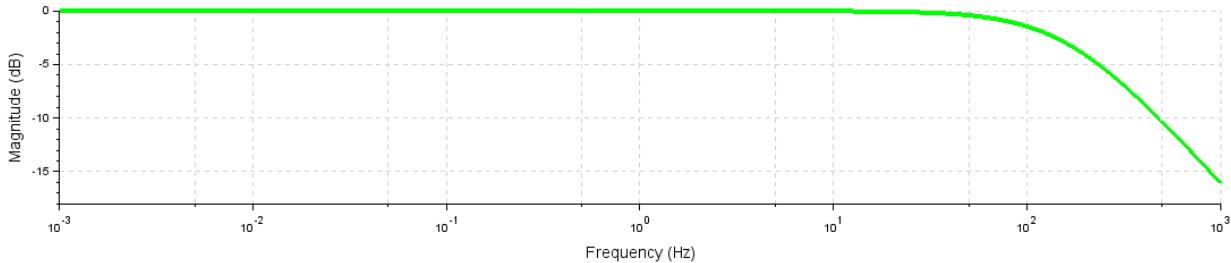
Example

Draw the Bode plot of V_o/V_s of the RC network shown below .



Bode Plot

Bode plot of $H(\omega) = V_o(\omega)/V_s(\omega)$ **with** $R=1\text{k}\Omega$ & $C=1\mu\text{F}$.



Passive Filters

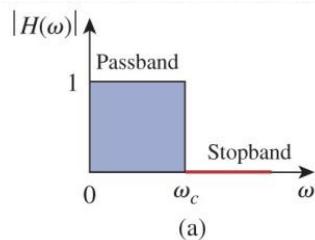
Types of Filters

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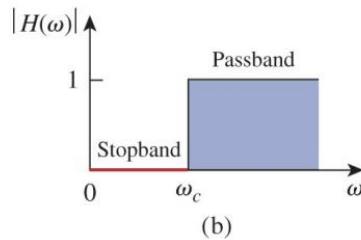
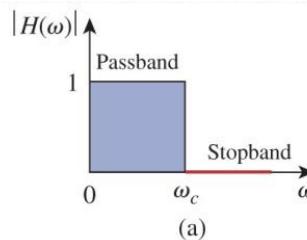
- **Lowpass filter passes only low frequencies and blocks high frequencies.**



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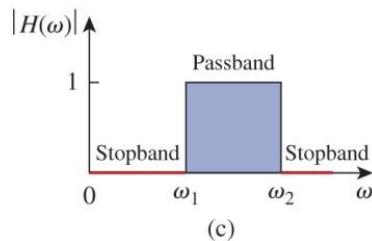
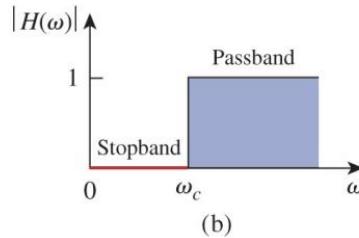
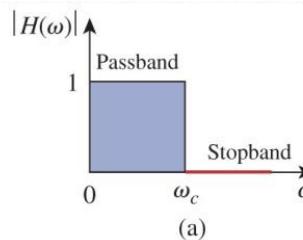
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Types of Filters

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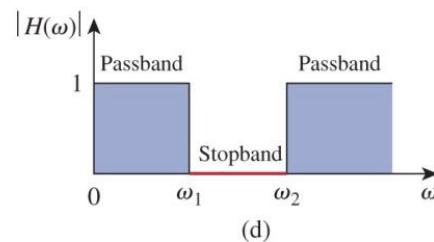
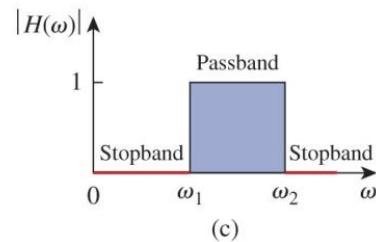
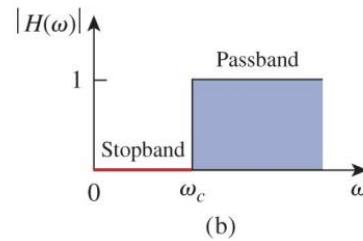
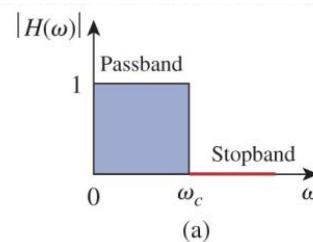
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Types of Filters

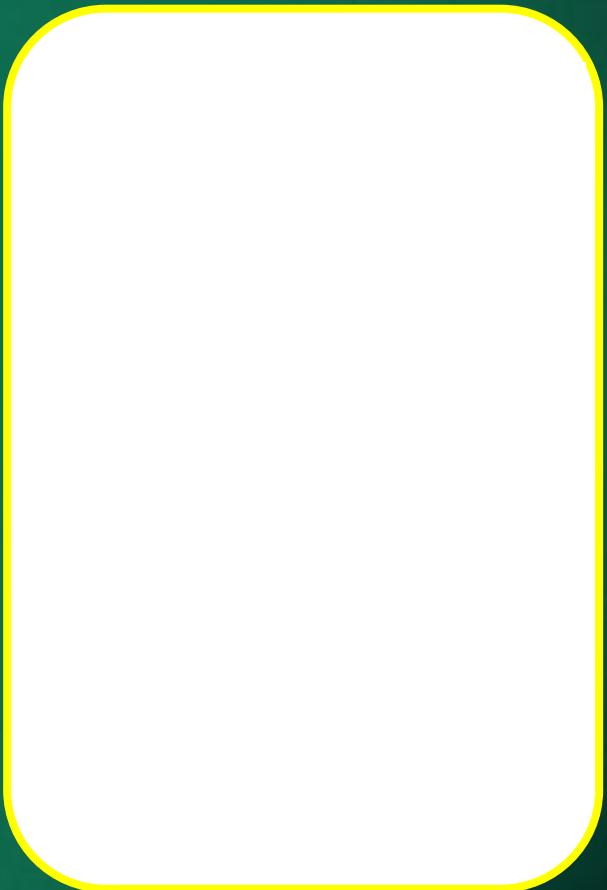
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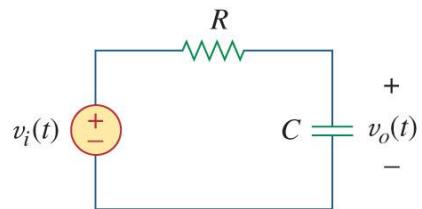
Lowpass Filter

- A typical lowpass filter is formed when the output of a RC circuit is taken across the capacitor.



Lowpass Filter

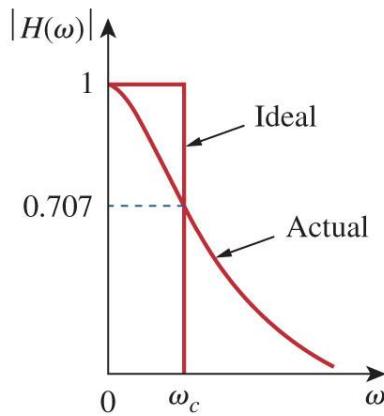
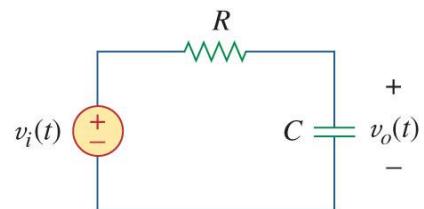
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$$\omega_c = \frac{1}{RC}$$

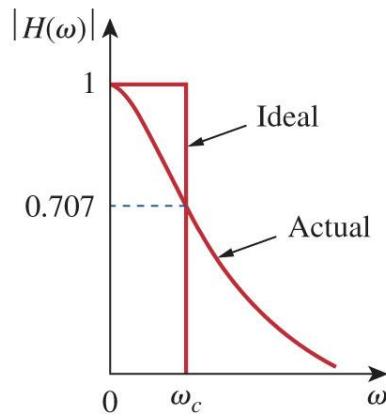
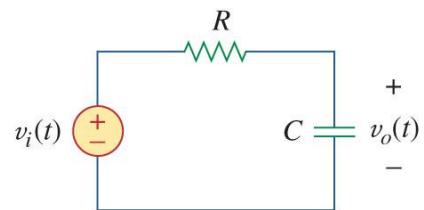


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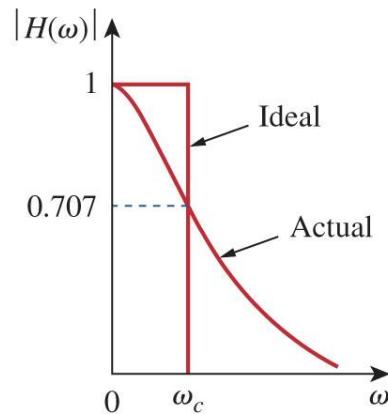
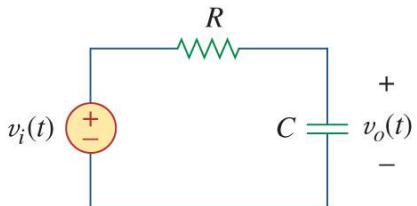


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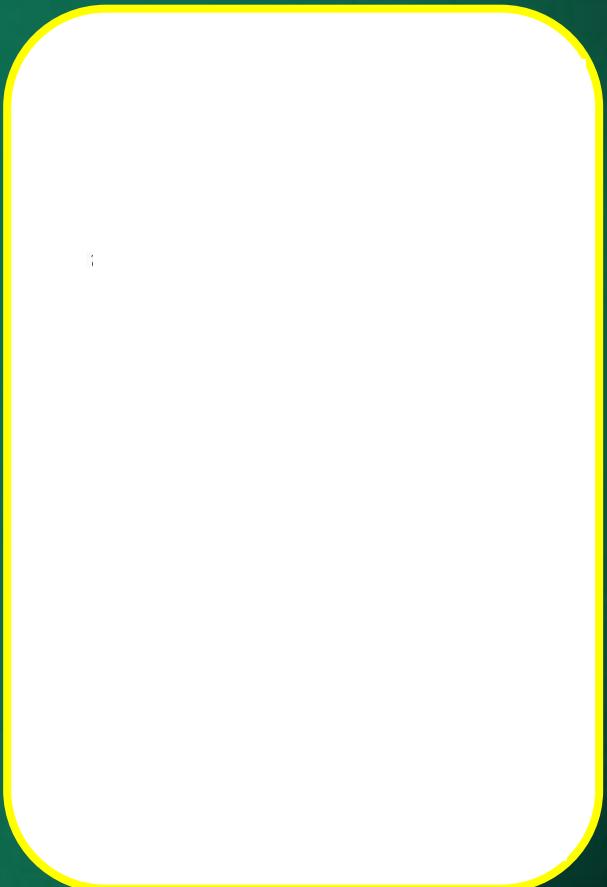
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- The filter is designed to pass from DC up to ω_c



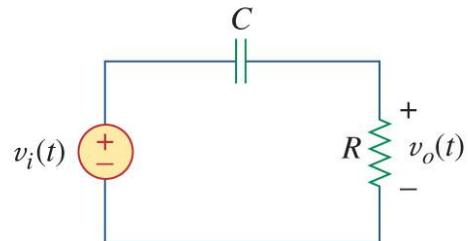
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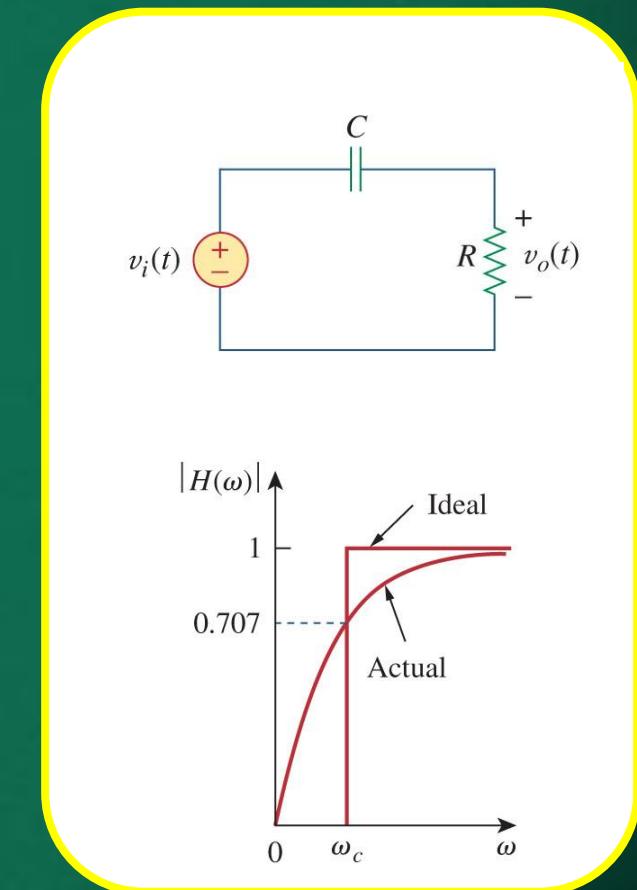
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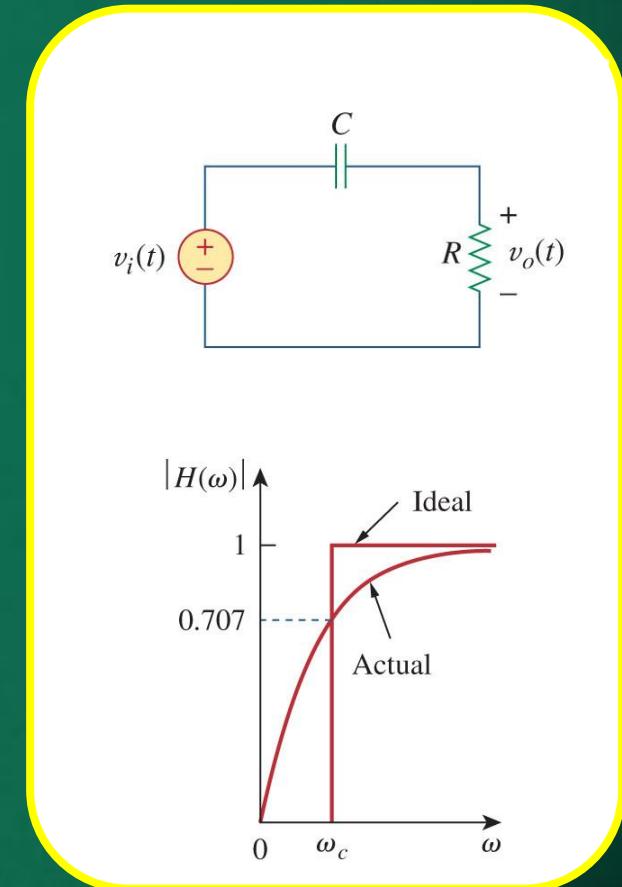
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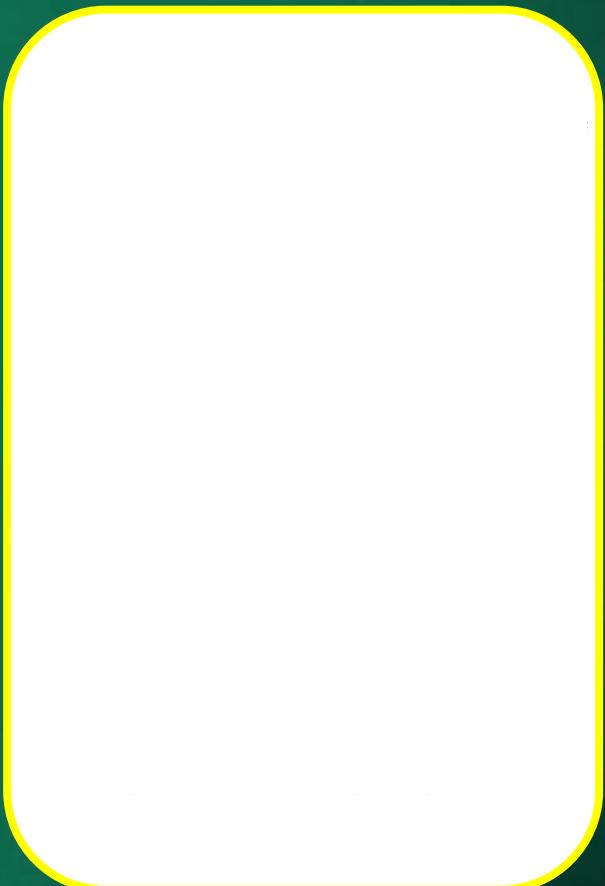
Highpass Filter

- A highpass filter is also made of a RC circuit, with the output taken off the resistor.
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- The difference being that the frequencies passed go from ω_c to infinity.



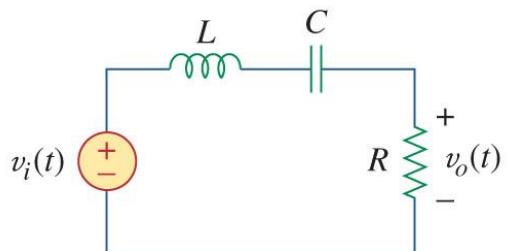
Bandpass Filter

- The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor.



Bandpass Filter

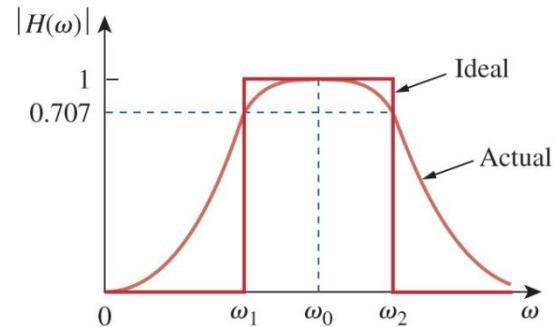
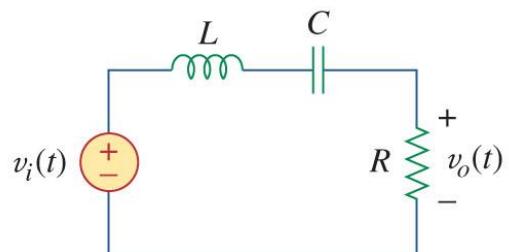
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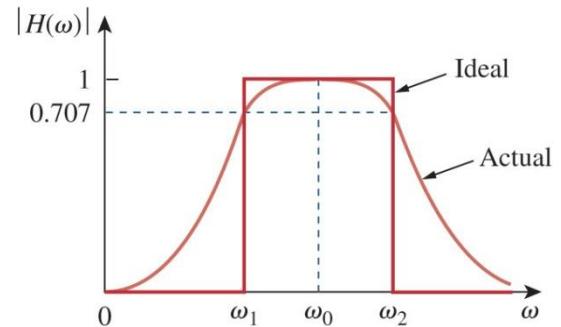
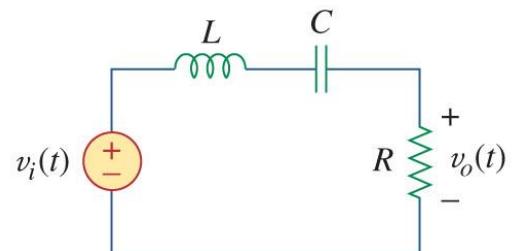
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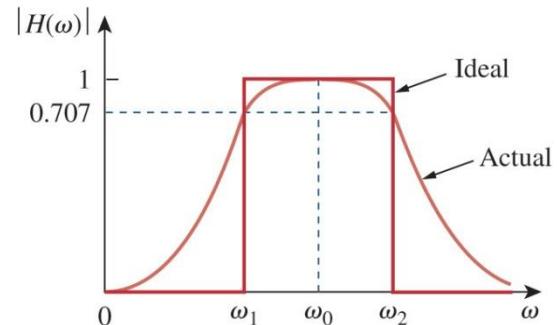
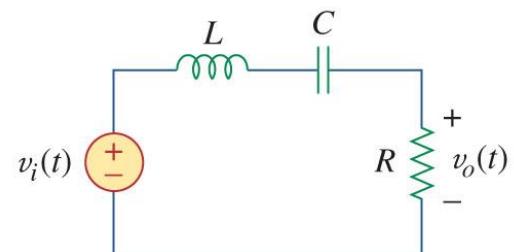
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- The center frequency is $\omega_c = \frac{1}{\sqrt{LC}}$
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- It can also be made by feeding the output from a lowpass to a highpass filter.

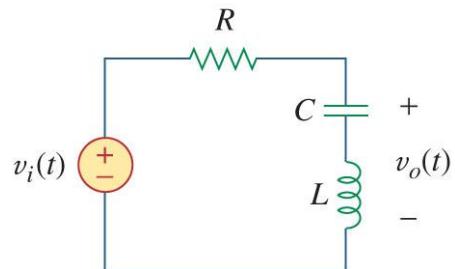


Bandstop Filter

- A bandstop filter can be created from a RLC circuit by taking the output from the LC series combination.

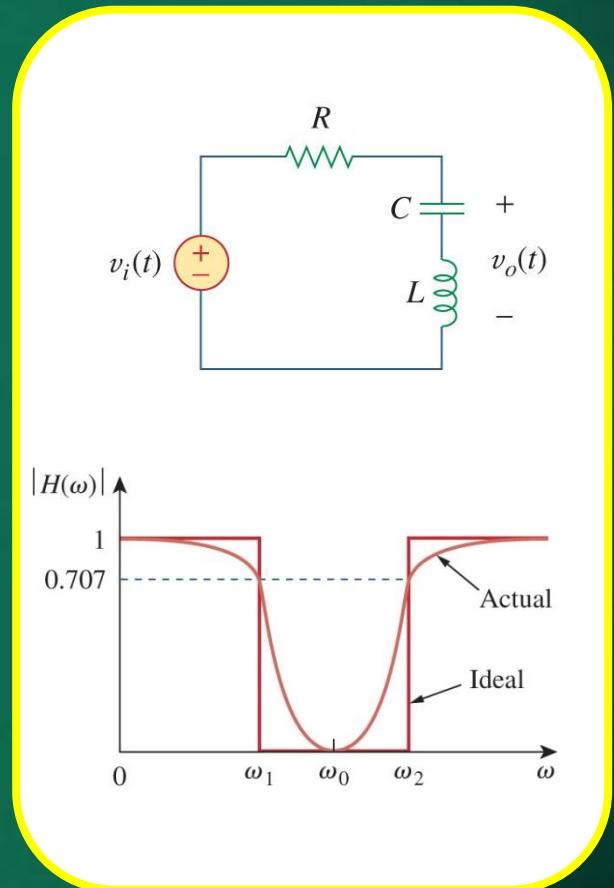
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Bandstop Filter

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- The range of frequencies (from ω_1 and ω_2) are blocked.



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- It is possible, using op-amps, to create all the common filters (active filters).
- Their ability to isolate input and output also makes them very desirable.