Solutions to Tutorial sheet-7 IEC103

Q1. Derive expression for ripple factor of the unregulated power supply using capacitor filter shown in Fig. Q1 below.

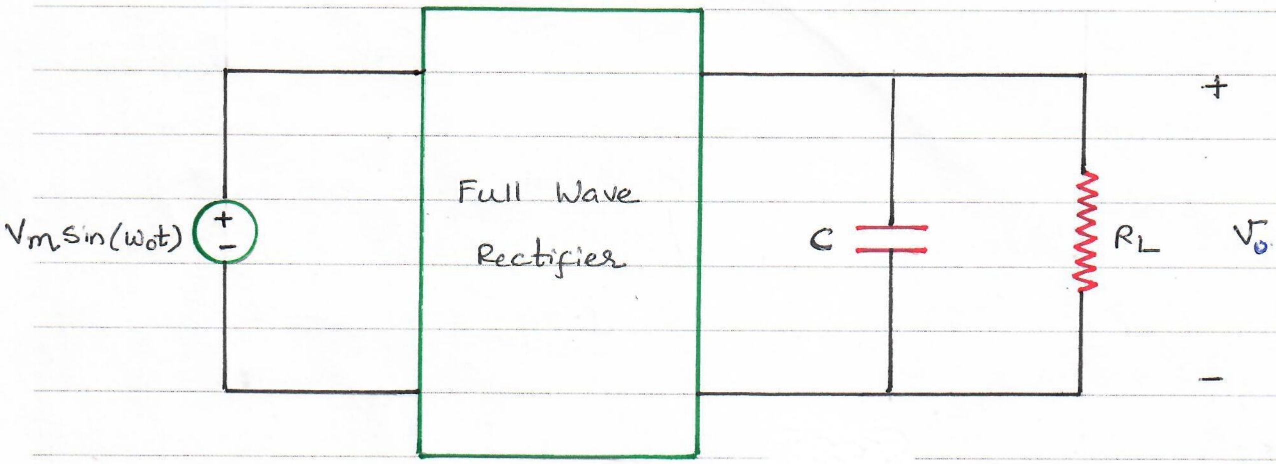


Fig. Q1

Assume the time constant $R_L C$ is much greater than the time period of the source voltage.

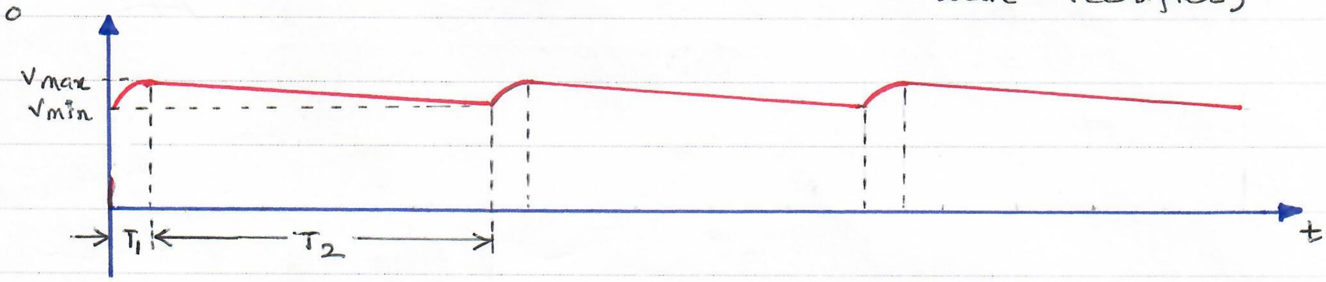
Sol.

$$\omega_o = 2\pi f$$

The output waveform will be as shown below.

$$T = T_1 + T_2 = \frac{1}{2f}$$ (since rectifier is a full wave rectifier)

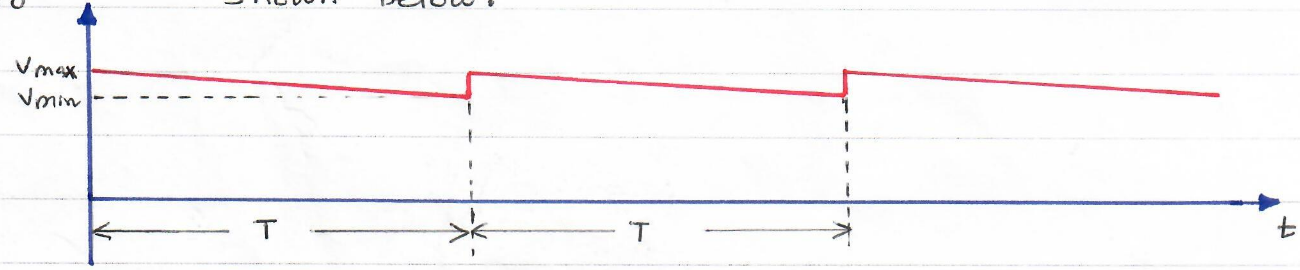
v_o



$T = T_1 + T_2$

The above waveform can be approximated to waveform shown below.

v_o



$$R_L C >> T$$

$$V_{min} = V_{max}\, e^{-T/R_L C}$$

$$= V_{max}\left(1 - \frac{T}{R_L C} + \frac{T^2}{2R_L^2 C^2} - \cdots\right)$$

$$\simeq V_{max}\left(1 - \frac{T}{R_L C}\right) \qquad \because R_L C >> T$$

$$\text{Ripple factor} = \frac{\text{RMS value of ripple voltage}}{\text{DC value of the voltage}}$$

$$V_{dc} = \frac{(V_{max} + V_{min})}{2} = \frac{1}{2}\left[V_{max} + V_{max}\left(1 - \frac{T}{R_L C}\right)\right]$$

$$= \frac{1}{2}\left[V_{max} + V_{max} - V_{max}\frac{T}{R_L C}\right]$$

$$= \frac{1}{2}\left[2V_{max} - V_{max}\frac{T}{R_L C}\right]$$

$$= V_{max}\left(1 - \frac{T}{2R_L C}\right)$$

$$\text{Peak - peak to ripple voltage} = V_{max} - V_{min} \qquad \because = V_{p\text{-}pr}$$

$$= V_{max} - V_{max}\left(1 - \frac{T}{R_L C}\right)$$

$$= V_{max}\frac{T}{R_L C} = V_{p\text{-}pr}$$

$$\text{RMS value of ripple voltage} = \frac{V_{p\text{-}pr}}{2\sqrt{3}} = \frac{V_{max}T}{2\sqrt{3}R_L C}$$

$$\therefore \gamma = \frac{V_{max}T}{2\sqrt{3}R_L C}$$

$$\therefore \gamma = \frac{\frac{V_{max}T}{2\sqrt{3}R_L C}}{V_{max}\left(1 - \frac{T}{R_L C}\right)} = \frac{T}{2\sqrt{3}R_L C\left(1 - \frac{T}{R_L C}\right)} = \frac{T}{2\sqrt{3}(R_L C - T)}$$

$$\simeq \frac{T}{2\sqrt{3}R_L C} = \frac{1}{4\sqrt{3} f R_L C}$$

$$\because T = \frac{1}{2f}$$

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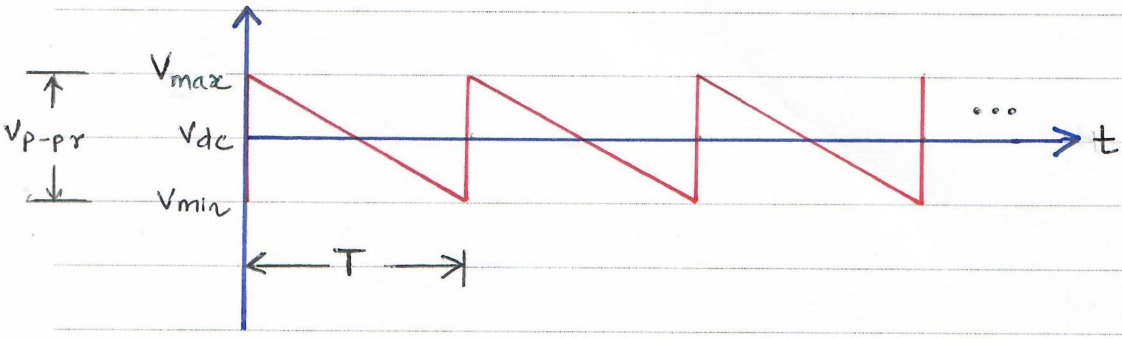
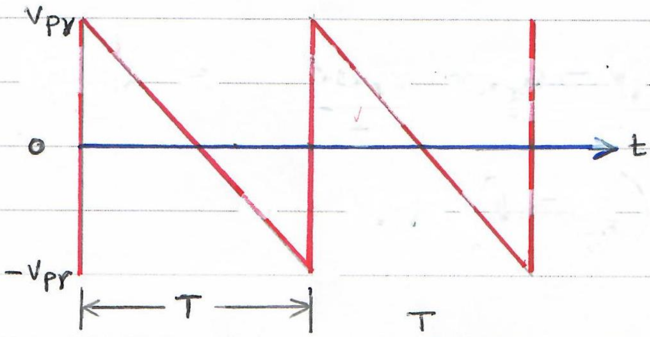
2nd method



RMS value of ripple voltage



$$V_r(t) = V_{pr} + \frac{(-V_{pr} - V_{pr})\,t}{T}$$

$$= V_{pr}\left(1 - \frac{2t}{T}\right)$$

$$V^2_{r(RMS)} = \frac{1}{T}\int_0^T \left[V_{pr}\left(1 - \frac{2t}{T}\right)\right]^2 dt$$

$$= \frac{V_{pr}^2}{T}\int_0^T \left(1 + \frac{4t^2}{T^2} - \frac{4t}{T}\right) dt$$

$$= \frac{V_{pr}^2}{T}\left[t + \frac{4t^3}{3T^2} - \frac{4t^2}{2T}\right]$$

$$= \frac{V_{pr}^2}{3}$$

$$\Rightarrow V_{r(RMS)} = \frac{V_{pr}}{\sqrt{3}} = \frac{V_{pr}}{\sqrt{3}} = \frac{V_{p-pr}}{2\sqrt{3}} \qquad \boxed{V_{p-pr} = 2V_{pr}}$$

$$V_{min} = V_{max}\, e^{-T/R_LC} \simeq V_{max}\left(1 - \frac{T}{R_LC}\right)$$

$$V_{dc} = \frac{V_{max} + V_{min}}{2} = \frac{V_{max}}{2} + \frac{V_{max}}{2}\left(1 - \frac{T}{R_LC}\right) = V_{max}\left(1 - \frac{T}{2R_LC}\right)$$

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$$V_{p\text{-}pr} = V_{max} - V_{min} = V_{max} - V_{max}\left(1 - \frac{T}{R_LC}\right)$$

$$= \frac{V_{max}\,T}{R_LC}$$

$$\therefore \quad V_{r(RMS)} = \frac{V_{max}\,T}{2\sqrt{3}R_LC}$$

$$V_{dc} = V_{max}\left(1 - \frac{T}{2R_LC}\right)$$

$$r = \frac{V_{r(RMS)}}{V_{dc}} = \frac{\dfrac{V_{max}\,T}{2\sqrt{3}R_LC}}{V_{max}\left(1 - \dfrac{T}{2R_LC}\right)}$$

$$= \frac{T}{2\sqrt{3}R_LC\left(1 - \dfrac{T}{2R_LC}\right)}$$

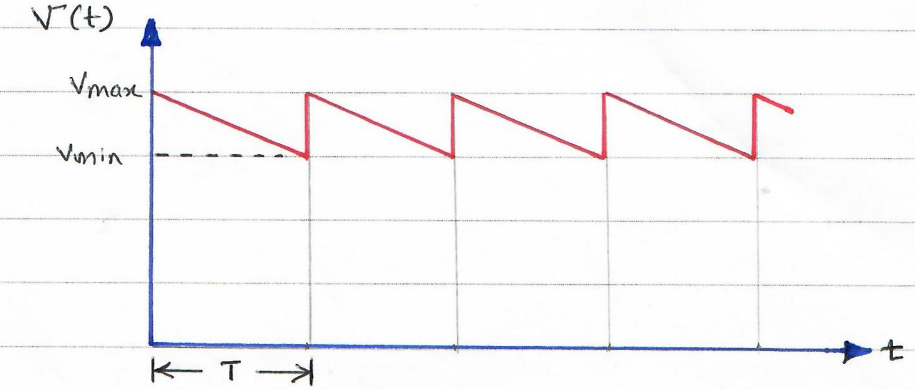
$$= \frac{T}{2\sqrt{3}\left(R_LC - \frac{T}{2}\right)R_LC}$$

$R_LC >> T$

$$\simeq \frac{T}{2\sqrt{3}R_LC} = \frac{1}{4\sqrt{3}fR_LC}$$

$\because \quad T = \frac{1}{2f}$

3rd Method



$R_LC >> T$

$$V_{min} = V_{max}\, e^{-T/RC} \simeq V_{max}\left(1 - \frac{T}{R_LC}\right)$$

$$\text{Ripple factor} = r = \frac{\sqrt{V_{rms}^2 - V_{dc}^2}}{V_{dc}}$$

$$V_{dc} = \frac{V_{max} + V_{min}}{2} = \frac{1}{2}\left[V_{max} + V_{max}\left(1 - \frac{T}{R_LC}\right)\right]$$

$$= \frac{1}{2} V_{max}\left(1 + 1 - \frac{T}{R_LC}\right)$$

$$\Rightarrow \boxed{V_{dc} = V_{max}\left(1 - \frac{T}{2R_LC}\right)}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\, dt$$

where $v(t) = V_{max} + \frac{(V_{min} - V_{max})\, t}{T}$ $\quad 0 < t < T$

$$= V_{max}\left(1 - \frac{t}{T}\right) + \frac{V_{min}\, t}{T}$$

$$= V_{max}\left(1 - \frac{t}{T}\right) + V_{max}\left(1 - \frac{T}{R_LC}\right)\frac{t}{T}$$

$$= V_{max}\left[\left(1 - \frac{t}{T}\right) + \left(1 - \frac{T}{R_LC}\right)\frac{t}{T}\right]$$

$$\Rightarrow \boxed{v(t) = V_{max}\left(1 - \frac{t}{R_LC}\right)}$$

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$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{T}\int_0^T V_{max}^2\left(1-\frac{t}{R_LC}\right)^2 dt$$

$$= \frac{V_{max}^2}{T}\int_0^T\left(1+\frac{t^2}{R_L^2C^2}-\frac{2t}{R_LC}\right)dt$$

$$= \frac{V_{max}^2}{T}\left[\left. t+\frac{t^3}{3R_L^2C^2}-\frac{2t^2}{2R_LC}\right|_0^T\right]$$

$$= \frac{V_{max}^2}{T}\left[T+\frac{T^3}{3R_L^2C^2}-\frac{T^2}{R_LC}\right]$$

$$= V_{max}^2\left[1+\frac{T^2}{3R_L^2C^2}-\frac{T}{R_LC}\right]$$

V_r = Ripple voltage (RMS value)

$$V_r^2 = V_{rms}^2 - V_{dc}^2$$

$$\therefore V_r^2 = V_{max}^2\left(1+\frac{T^2}{3R_L^2C^2}-\frac{T}{R_LC}\right) - \underbrace{V_{max}^2\left(1-\frac{T}{2R_LC}\right)^2}_{V_{dc}^2}$$

$$= V_{max}^2\left(1+\frac{T^2}{3R_L^2C^2}-\frac{T}{R_LC}\right) - V_{max}^2\left(1+\frac{T^2}{4R_L^2C^2}-\frac{T}{R_LC}\right)$$

$$= V_{max}^2\left(\cancel{1}+\frac{T^2}{3R_L^2C^2}-\cancel{\frac{T}{R_LC}}-\cancel{1}-\frac{T^2}{4R_L^2C^2}+\cancel{\frac{T}{R_LC}}\right)$$

$$= \frac{V_{max}^2T^2}{12R_L^2C^2} = V_r^2$$

$$\therefore V_r = \frac{V_{max}T}{2\sqrt{3}R_LC}$$

$$\text{Ripple factor} = \frac{V_r}{V_{dc}} = \frac{\dfrac{V_{max}T}{2\sqrt{3}R_LC}}{V_{max}\left(1-\dfrac{T}{2R_LC}\right)} = \frac{\cancel{V_{max}}T}{\cancel{2}\sqrt{3}\cancel{R_LC}} \times \frac{\cancel{2}\cancel{R_LC}}{\cancel{V_{max}}(2R_LC-T)}$$

$$\Rightarrow \quad \gamma = \frac{T}{\sqrt{3}\,(2R_LC - T)} \simeq \frac{T}{2\sqrt{3}\,R_LC} \qquad \because \; R_LC >> T$$

$$\therefore \; \gamma = \frac{T}{2\sqrt{3}\,R_LC} = \frac{1}{4\sqrt{3}\,R_LCf} \qquad \because \; T = \frac{1}{2f}$$

$$\therefore \; \boxed{\gamma = \frac{1}{4\sqrt{3}f\,R_LC}}$$

Q2 For the circuit shown in Fig. Q2, find the turns ratio of the transformer, the current rating of the diodes, and the value of C, so that the maximum value of voltage across the load should not exceed 15V and minimum value should not drop below 13V.
Take the charging time of capacitor is $\frac{1}{10}$ th of the time period of the output waveform.

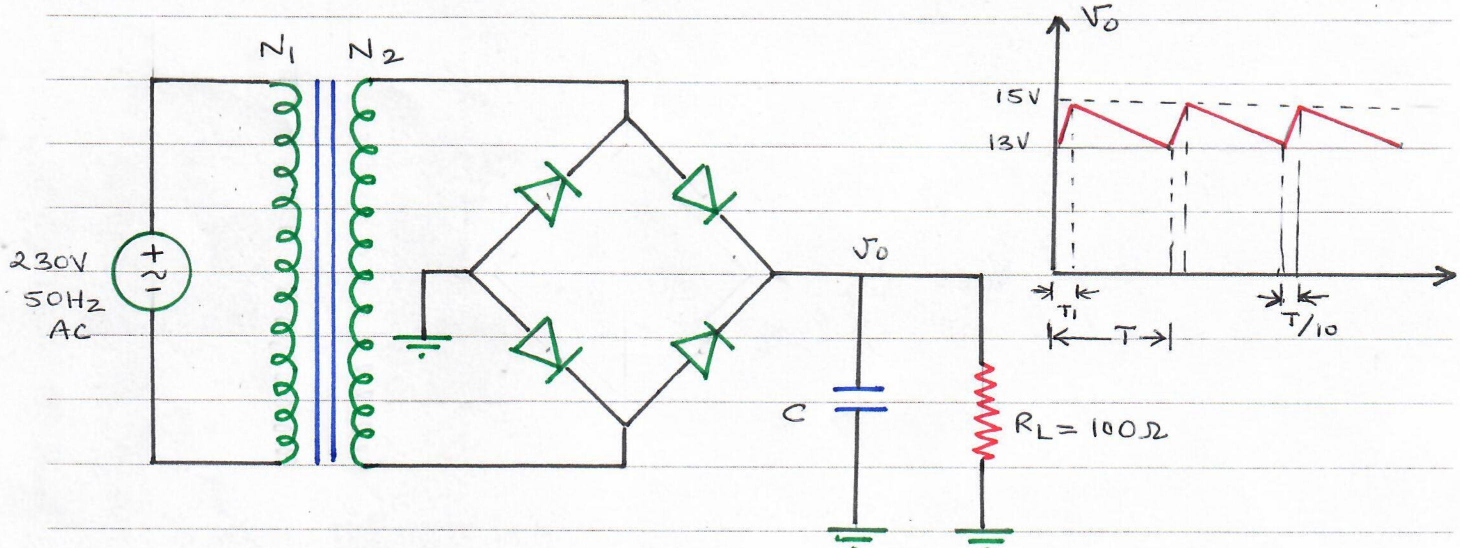


Fig. Q2

What will be the maximum current through diodes.

Sol.

Peak value of voltage across load = 15V = peak value of secondary voltage of transformer

Peak value of secondary voltage of the transformer = 15 V

Peak value of primary voltage of the transformer = $230\sqrt{2} = 325.27$ V

$$\frac{N_1}{N_2} = \frac{325.27}{15} = 21.685 \simeq 22$$

$V_o(max) = V_p = 15$ V

$V_o(min) = 13$V

Time period of the output waveform $= \frac{1}{100} = 10\ ms$

T_1 = charging time ; T_2 = discharging time

$T_1 + T_2 = T$

$T_1 = \frac{T}{10} = 1ms$; $T_2 = 9\ ms$

$T_2 \simeq T = 10\ ms$

During the discharging of capacitor

$$V_o = V_{o(max)}\, e^{-t/R_L C}$$

$$V_o \simeq V_{o(max)}\left(1 - \frac{t}{R_L C}\right)$$

$$V_{o(min)} \simeq V_{o(max)}\left(1 - \frac{T_2}{R_L C}\right)$$

$$\Rightarrow C = \frac{V_{o(max)}\, T_2}{R_L\left[V_{o(max)} - V_{o(min)}\right]}$$

$$= \frac{15 \times 9 \times 10^{-3}}{100 \times (15-13)} = \frac{15 \times 9 \times 10^{-3}}{200}$$

$$= \frac{15 \times 9}{2} \times 10^{-5}$$

$$= 67.5 \times 10^{-5}\ F$$

$$\Rightarrow \boxed{C = 675\ \mu F}$$

$R_L C = 100 \times 675 \times 10^{-6} = 67.5\ ms \gg T$

Maximum current through diodes $= \frac{C\,(V_{o(max)} - V_{o(min)})}{T_1}$

$i = C\frac{dv}{dt} = C\frac{\Delta V}{\Delta T} = \frac{C\,(V_{o(max)} - V_{o(min)})}{(T_1 - 0)}$

$$= \frac{67.5 \times 10^{-5} \times (15-13)}{10^{-3}}$$

$$= 2 \times 67.5 \times 10^{-2}$$

$$= 1350\ mA$$

$$= 1.35\ A$$

The current rating of each diode should be more than 1.35 A

$I_{D(max)} = 1.35\ A$

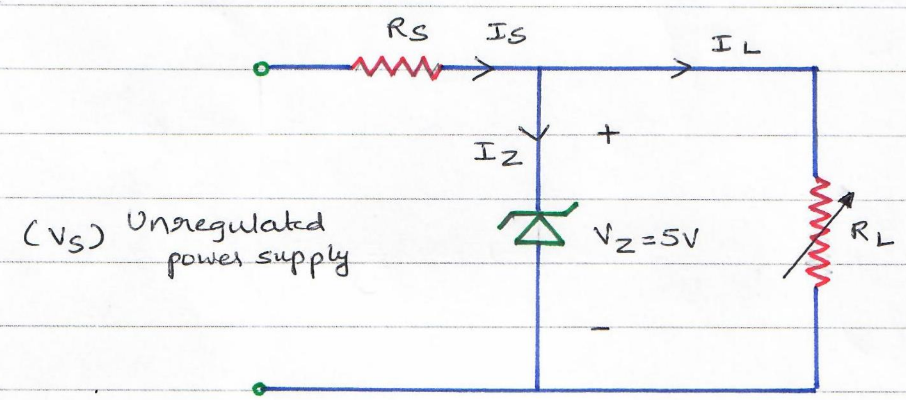
Q3) Find the maximum and minimum value of R_S for a zener voltage regulator so as to maintain a constant DC voltage of 5 V across load.

The zener diode ratings are

$V_Z = 5V$, I_{ZK} (knee current) $= 0\,A$, $P_{Z(max)} = 0.5\,W$

The input voltage can vary from 7 to 10V and maximum load current is 20 mA.

sol.



$$I_Z = I_S - I_L$$

$$\Rightarrow \quad I_Z = \frac{V_S - V_Z}{R_S} - I_L$$

$$I_{Z(max)} = \frac{V_{S(max)} - V_Z}{R_{S(min)}} - I_{L(min)}$$

$$I_{Z(min)} = \frac{V_{S(min)} - V_Z}{R_{S(max)}} - I_{L(max)}$$

$$I_{Z(min)} < I_Z < I_{Z(max)}$$

$$I_{Z(knee)} < I_Z \quad I. \frac{P_{Z(max)}}{V_Z}$$

$V_{S(max)} = 10V$; $V_{S(min)} = 7V$; $I_{Z(knee)} = 0\,A$; $I_{Z(max)} = \frac{P_{Z(max)}}{V_Z}$

$I_{L(min)} = 0\,A$; $I_{L(max)} = 20\,mA$ $\quad = \frac{0.5}{5}$

$$I_{Z(max)} = \frac{10-5}{R_{S(min)}} - 0 < 0.1 \qquad = 0.1\,A$$

$$\Rightarrow \frac{5}{R_{S(min)}} - 0 < 0.1$$

$$\Rightarrow \frac{5}{R_{S(min)}} < 0.1 \Rightarrow R_{S(min)} > \frac{5}{0.1} = 50$$

$$\Rightarrow R_{S(min)} > 50\Omega$$

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$$I_{Z(min)} = \frac{V_{S(min)} - V_Z}{R_{S(max)}} - I_{L(max)} > I_{Z(knee)}$$

$$\Rightarrow \frac{7-5}{R_{S(max)}} - 0.02 > 0$$

$$\Rightarrow \frac{2}{R_{S(max)}} > 0.02$$

$$\Rightarrow R_{S(max)} < \frac{2}{0.02} = 100\Omega$$

$\therefore$ R_S should lie between 50Ω to 100Ω for proper operation of zener regulator.

$\therefore$ We can choose $R_S = 75\Omega$

Q4 For the zener regulator shown in Fig. Q4, the input voltage can vary from 20 to 25V and the load resistance vary from 500Ω to 2kΩ.
The zener ratings are

$V_Z = 3.3\text{ V}$; $I_{Z(knee)} = I_{Z(min)} = 3\text{ mA}$; $P_{Z(max)} = 330\text{ mW}$

Determine the acceptable range of R_S for proper operation of the regulated power supply.

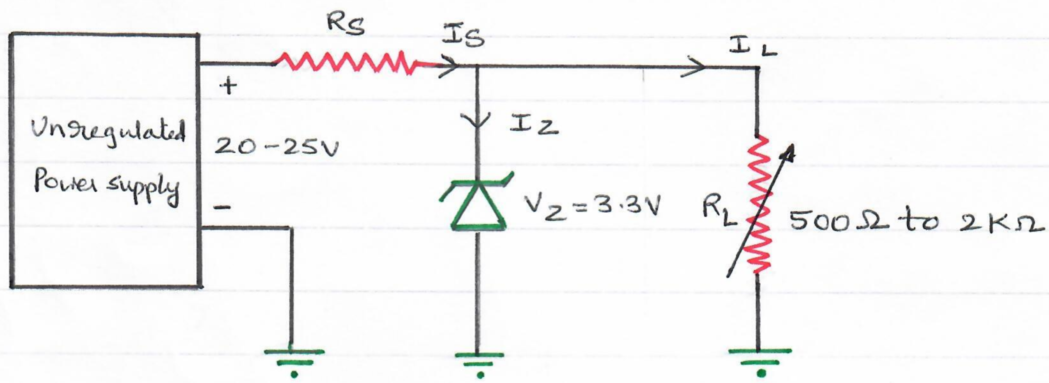

Fig. Q4

Sol. The current through the zener diode should be between $I_{Z(knee)}$ and $I_{Z(max)} = \frac{P_{Z(max)}}{V_Z}$ for the diode to be operated in the zener breakdown regim to maintain constant voltage V_Z across it (thus across R_L)

$I_{Z(min)} = I_{Z(knee)} = 3\text{mA}$ (Minimum current needed through zener)

$I_{Z(max)} = \frac{P_{Z(max)}}{V_Z} = \frac{330\text{ mW}}{3.3\text{ V}} = 100\text{mA}$ (maximum allowable current through zener)

Maximum current passes through zener when I_S is maximum (i.e., when V_{in} is maximum) and I_L is minimum.
& also R_S is minimum)

∴ Maximum current through zener diode $= I_{S(max)} - I_{L(min)}$

$$= \frac{V_{in(max)} - V_Z}{R_{S(min)}} - \frac{V_Z}{R_{L(max)}}$$

This maximum current through the zener diode should be less than the maximum allowable current ($I_Z(max)$)

$$\therefore \frac{V_{in}(max) - V_Z}{R_S(min)} - \frac{V_Z}{R_L(max)} < I_Z(max)$$

$$\Rightarrow \frac{25 - 3.3}{R_S(min)} - \frac{3.3}{2K} < 100\,m$$

$$\Rightarrow \frac{21.7}{R_S(min)} < 101.65\,m$$

$$\Rightarrow R_S(min) > \frac{21.7}{101.65}\,K\Omega \simeq 215\Omega$$

$$\boxed{R_S(min) > 215\Omega}$$

Minimum current passes through zener diode when I_S in minimum (i.e., V_{in} is minum and R_S is maximum) and I_L is maximum

$\therefore$ Minimum current through zener diode $= I_S(min) - I_L(max)$

$$= \frac{V_{in}(min) - V_Z}{R_S(max)} - \frac{V_Z}{R_L(min)}$$

This minimum current should be more than the knee current of the zener i.e., $I_Z(knee) = I_Z(min)$

$$\therefore \frac{V_{in}(min) - V_Z}{R_S(max)} - \frac{V_Z}{R_L(min)} > I_Z(min)$$

$$\Rightarrow \frac{20 - 3.3}{R_S(max)} - \frac{3.3}{500} > 3\,m$$

$$\Rightarrow \frac{16.7}{R_S(max)} > 9.6\,m \Rightarrow R_S(max) < \frac{16.7}{9.6}\,K\Omega \simeq 1.74K\Omega$$

$$\boxed{R_S(max) < 1.74K\Omega}$$

$\therefore$ Acceptable value of R_S should lie between 215Ω and $1.74K\Omega$

So we can choose $R_S = 1K\Omega$ for proper operation of the regulator

$$\boxed{215\Omega < R_S < 1.74K\Omega}$$

Q5 The circuit shown in Fig. Q5 is an adjustable voltage regulator. Assume that the op-amp is ideal. The zener ratings are as given below.

$I_{Z(Knee)} = 0.01\,mA$ and $P_{Z(max)} = 0.1\,W$, and $V_Z = 5V$.

Design the value of R_S for proper operation of the zener diode. Also calculate the output voltage as R_2 is varied from 0 to $1K\Omega$. V_S varies from 6V to 10V.

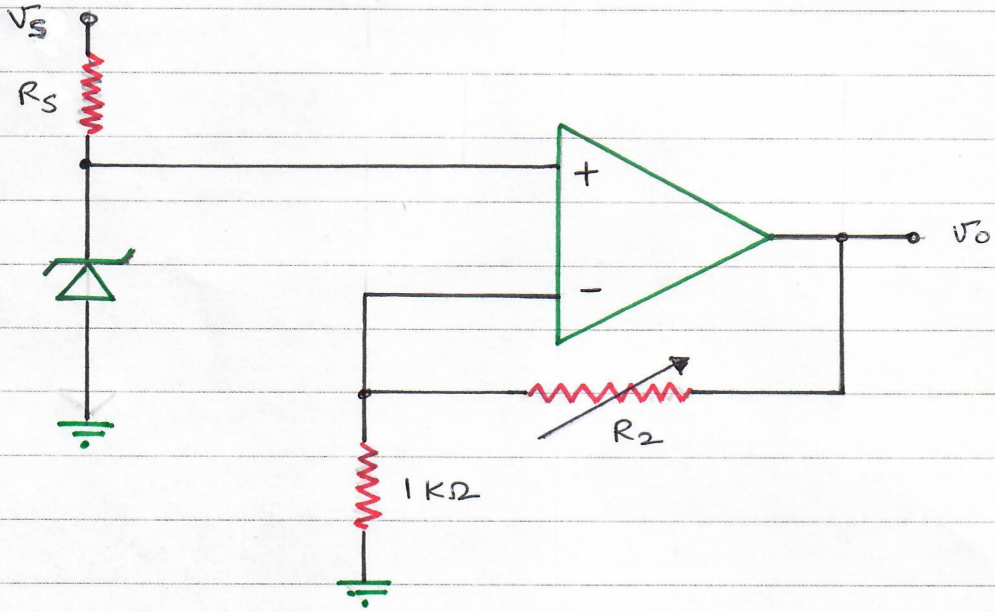

Fig. Q5

Sol.

$$I_Z = \frac{V_i - V_Z}{R_S}$$

$$I_{Z(max)} = \frac{V_{i(max)} - V_Z}{R_{S(min)}} < \frac{P_{Z(max)}}{V_Z}$$

$$\Rightarrow \frac{10-5}{R_{S(min)}} < \frac{0.1}{5}$$

$$\Rightarrow \frac{5}{R_{S(min)}} < 0.02$$

$$\Rightarrow R_{S(min)} > \frac{5}{0.02} = 250\Omega$$

$$I_{Z(min)} = \frac{V_{i(min)} - V_Z}{R_{S(max)}} > 0.01 \times 10^{-3}$$

$$\Rightarrow \frac{6-5}{R_{S(max)}} > 0.01 \times 10^{-3}$$

$$\Rightarrow R_{S(max)} < \frac{1}{10^{-5}} = 10^5 = 100\,K\Omega$$

$$\therefore \quad 250\Omega < R_S < 100\,K\Omega$$

So, I can choose an R_S of $1K\Omega$ for proper operation of zener in the break down region.

$$V_O = V_Z \times \left(1 + \frac{R_2}{1K}\right)$$

$$= \left(1 + \frac{R_2}{1K}\right) \times 5$$

$R_2 \rightarrow 0$ to $1K\Omega$

$\therefore$ V_o can vary between 5 to 10V.

So, we can have an adjustable regulated voltage of 5 to 10V at the output pin of op-amp.