Solutions to Tutorial Sheet - 4 IEC103.

(Q1). Find the output voltage V_o of the amplifier circuit shown in Fig. Q1. Assume that all op-amps are ideal and none of the op-amps saturate.

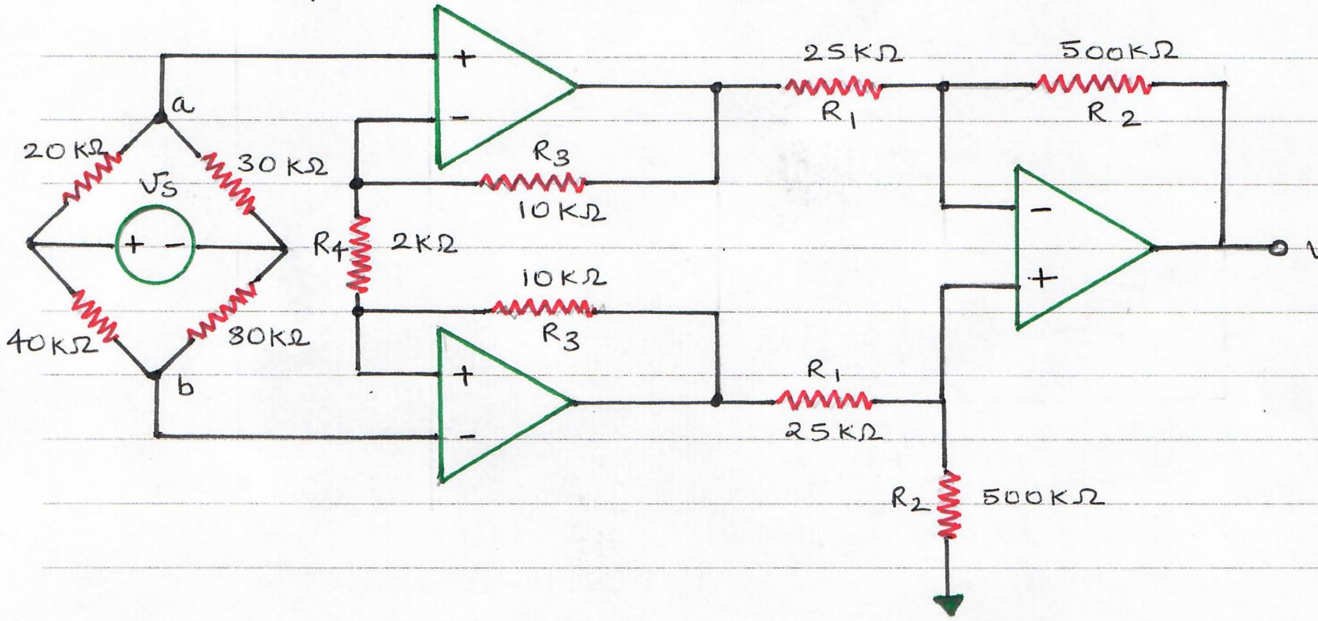
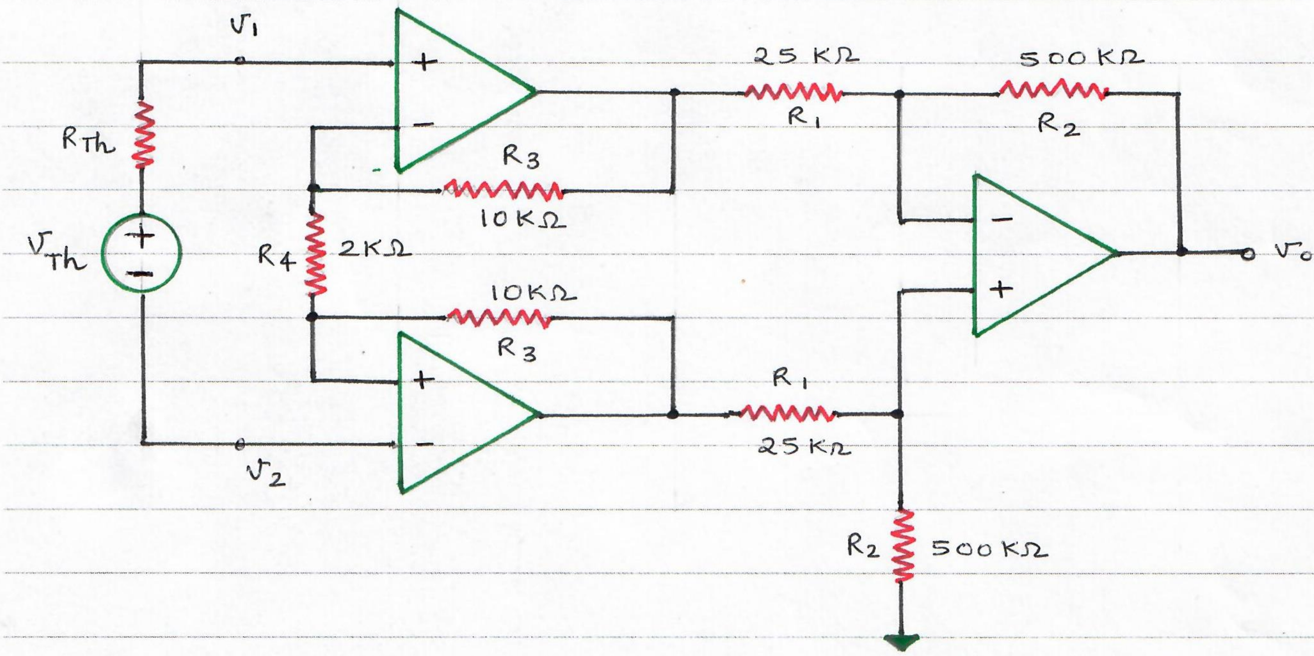


Fig. Q1

Sol. Thevenizing bridge circuit across a-b.



$$R_{Th} = 20K\Omega \,||\, 30K\Omega + 40K\Omega \,||\, 80K\Omega$$

$$= \frac{20 \times 30}{50} + \frac{40 \times 80}{120} \; K\Omega$$

$$= \frac{60}{5} + \frac{80}{3} \; K\Omega = \frac{116}{3} \; K\Omega$$

$$v_{th} = v_1 - v_2 = \frac{30}{30+20} v_s - \frac{80}{80+40} v_s$$

(∵ no current flows through the 1st stage op-amps)

$$= \frac{30}{50} v_s - \frac{80}{120} v_s$$

$$= \frac{3}{5} v_s - \frac{8}{12} v_s = \frac{3}{5} v_s - \frac{2}{3} v_s$$

$$= -\frac{1}{15} v_s = v_1 - v_2$$

$$v_o = \left(\frac{R_2}{R_1}\right)\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

$$= \left(\frac{500k}{25k}\right)\left(1 + \frac{2 \times 10k}{2k}\right) \times \frac{v_s}{15} \qquad \because v_2 - v_1 = \frac{v_s}{15}$$

$$= 20 \times 11 \times \frac{v_s}{15} = \frac{220}{15} v_s = \frac{44}{3} v_s = 14.667\, v_s$$

(Q2) The opamp circuit shown in Fig. Q2 is designed to amplify a voltage signal and current signal from two transducers. Applying superposition, find the output voltage $v_o(t)$ interms of inputs $v_{IN}(t)$ and $i_{IN}(t)$. You may find it convenient to first find the values of intermediate variables $v_1(t)$ and $v_2(t)$ shown in the Fig. 2 and then find $v_o(t)$. Assume that op amps are ideal.

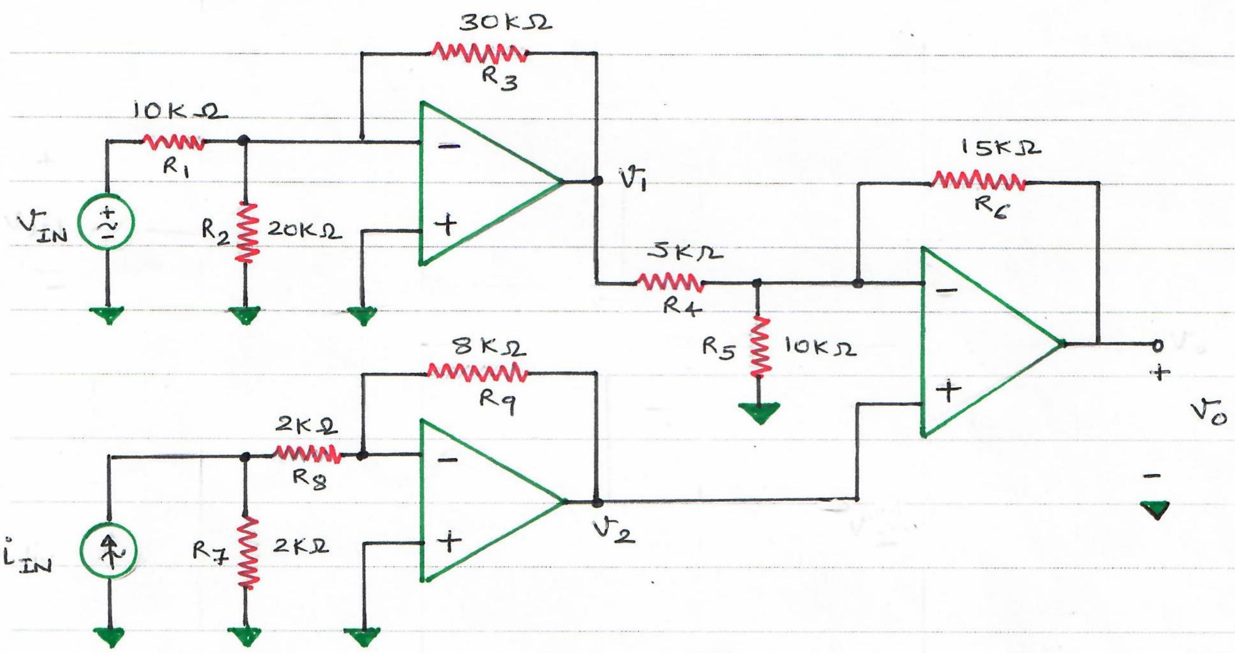
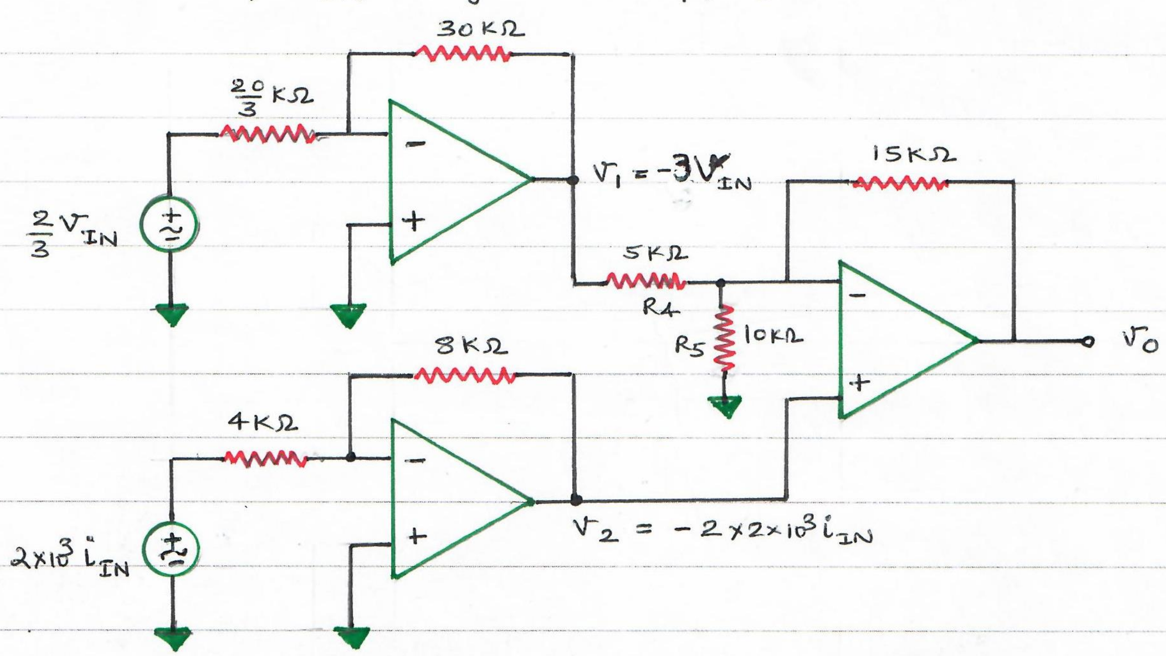


Fig. Q2

Sol.

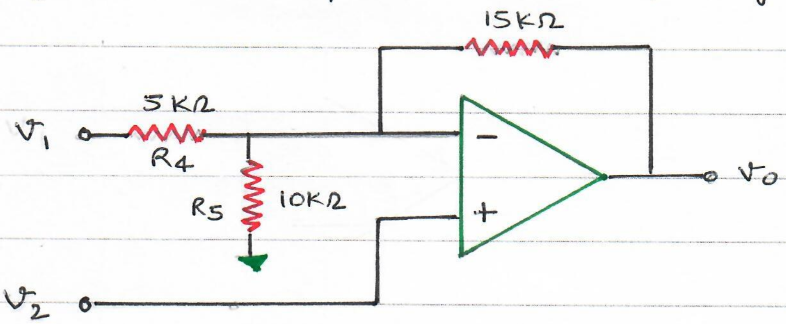
Using the Thevenin equivalent of source-1, v_{IN} along with R_1 & R_2 & source-2, i_{IN} along with R_7, R_8.



Free Lined Graph Paper from http://incompetech.com/graphpaper/lined

PTO

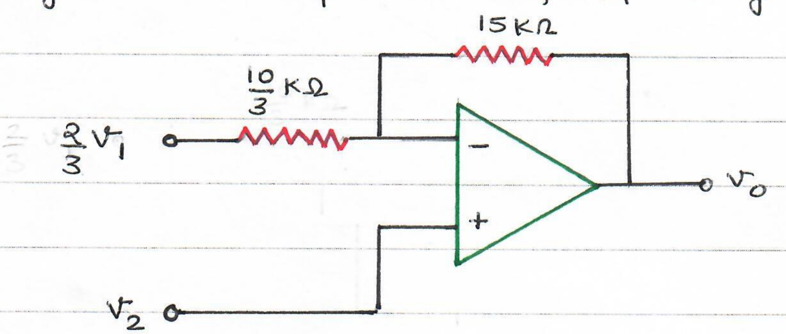
Using Thevenin equivalent of v_i along with R_4, R_5



$v_1 = -3v_{IN}$

$v_2 = -4\times10^3 i_{IN}$

Using Thevenin equivalent of v_1 along with R_4, R_5.



$$v_o = \left(\frac{-15K}{(\frac{10}{3})K}\right)\times\frac{2}{3}v_1 + \left(1+\frac{15}{(10/3)}\right)v_2$$

$$= \frac{-45}{10}\times\frac{2}{3}v_1 + \left(1+\frac{45}{10}\right)v_2$$

$$= -3v_1 + 5.5v_2$$

$$= -3\times-3\,v_{IN} + 5.5\times-4\times10^3 i_{IN}$$

$$\Rightarrow \boxed{v_o = 9v_{IN} - 22\times10^3 i_{IN}}$$

Q3. i) What type of feedback is incorporated in Q.3a, Q.3b, Q.3c, and Q.3d.

ii) Assuming ideal op amp, find β and A_f relevant to each circuit.

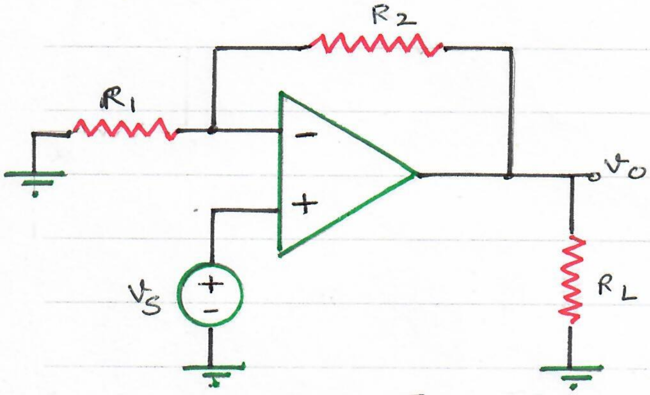


Fig. Q3a

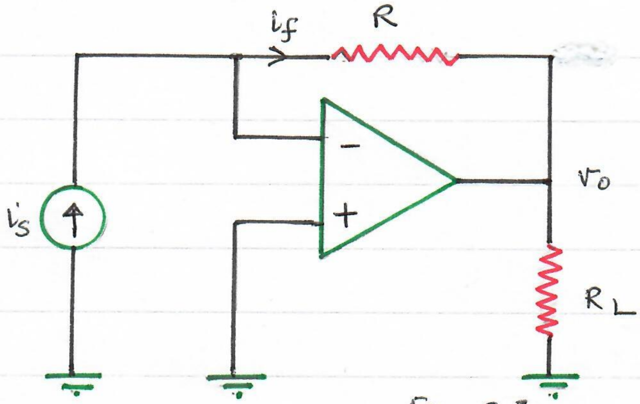


Fig. Q.3b

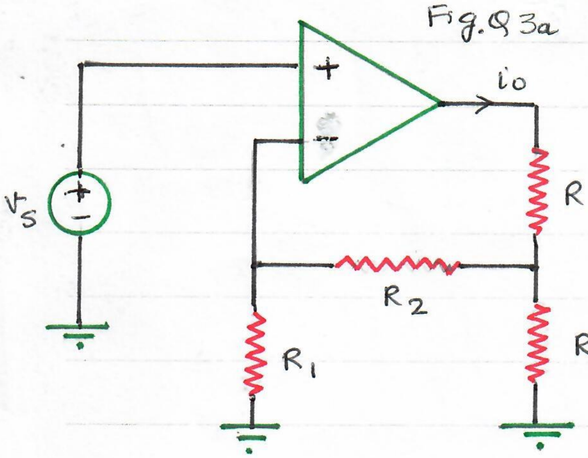


Fig. Q3c

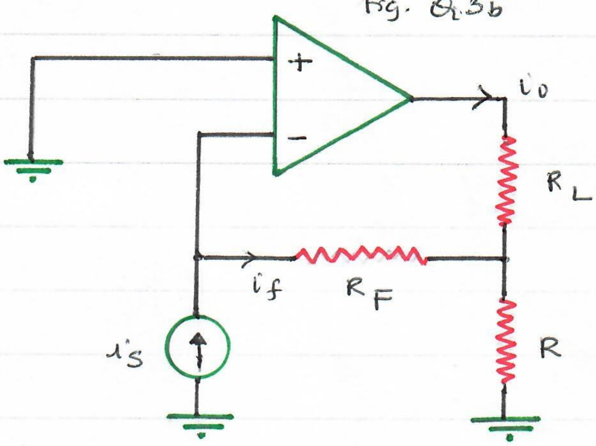


Fig. Q.3d

Sol.

a) series - shunt configuration

(voltage mix) - (voltage sense)

$$\beta = \frac{v_f}{v_o} = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{1}{\beta} \qquad (\because A = \infty \text{ for ideal op-amp})$$

$$= \frac{1}{\left(\frac{R_1}{R_1+R_2}\right)} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

$$\boxed{A_f = 1 + \frac{R_2}{R_1} = \frac{v_o}{v_s}}$$

b) shunt – shunt configuration
(current mixing) (voltage sense)

$$\beta = \frac{i_f}{v_o} = \frac{-(v_o/R)}{v_o} = -\frac{1}{R}$$

$$A_f = \frac{v_o}{i_s} = \frac{A}{1+A\beta} = \frac{1}{\beta} \quad (\because A=\infty)$$

$$= \frac{1}{(-1/R)} = -R$$

$$\therefore \boxed{A_f = \frac{v_o}{i_s} = -R}$$

c) series – series configuration
(voltage mix) (current sense)

$$\beta = \frac{v_f}{i_o} = \frac{i_o R' \times \frac{R_1}{(R_1+R_2)}}{i_o} \simeq \frac{R R_1}{(R_1+R_2)}$$

$R' = R \parallel (R_1+R_2)$
$\simeq R \quad \because R_1, R_2 \gg R$

$$A_f = \frac{i_o}{v_s} = \frac{A}{1+A\beta} = \frac{1}{\beta} \quad (\because A=\infty)$$

(assuming the R_1, R_2 network does not load o/p network $(R_L - R)$

$$\Rightarrow \boxed{A_f = \frac{R_1+R_2}{R R_1} = \frac{1}{R}\left(1+\frac{R_2}{R_1}\right)}$$

d) shunt – series configuration
(current mix) current sense

$$\beta = \frac{i_f}{i_o} = \frac{-i_o \times \left(\frac{R}{R_F+R}\right)}{i_o} = \frac{-R}{R_F+R}$$

$$A_f = \frac{i_o}{i_s} = \frac{A}{1+A\beta} = \frac{1}{\beta} = \frac{1}{-\left(\frac{R}{R_F+R}\right)} = -\frac{R_F+R}{R} = -\left(1+\frac{R_F}{R}\right)$$

$$\therefore \boxed{A_f = \frac{i_o}{i_s} = -\left(1+\frac{R_F}{R}\right)}$$

(Q4) For the amplifier circuit shown in Fig. Q4, find the expression for i_L in terms of v_S, if $R_2 = R_4$ and $R_1 = R_3$. What type of amplifier is it?
Assume that the op-amp is ideal and operate in linear region.

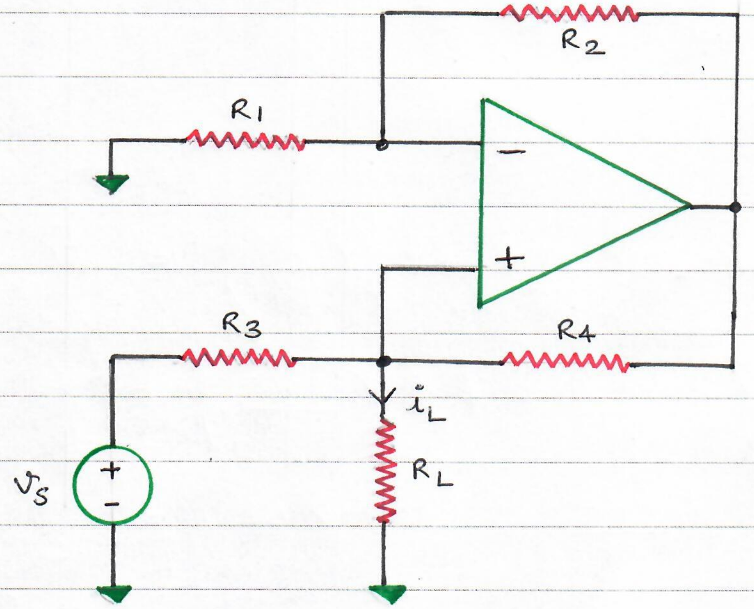
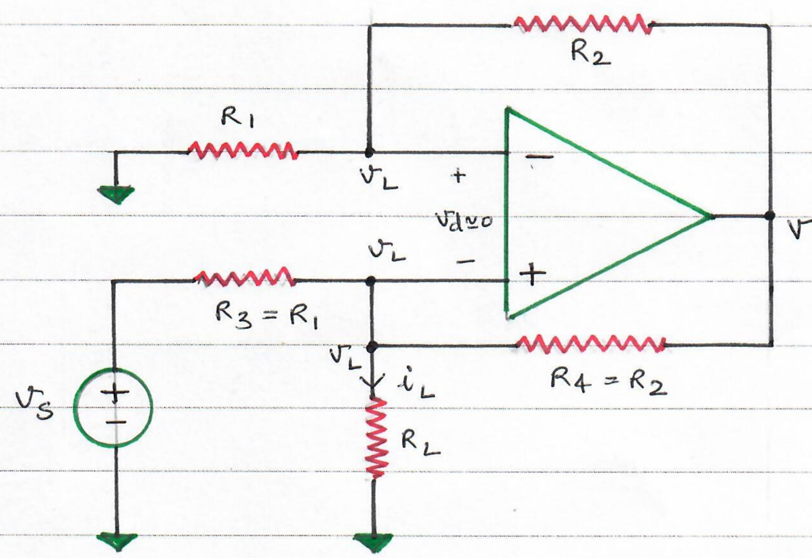


Fig. Q4

Sol.



Applying KCL at non-inverting input of op-amp

$$\frac{v_L - v_S}{R_1} + \frac{v_L}{R_L} + \frac{v_L - v'}{R_2} = 0 \quad \cdots (A)$$

Applying KCL at inverting input of op-amp

$$\frac{v_L}{R_1} + \frac{v_L - v'}{R_2} = 0$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v'}{R_2}$$

$$\Rightarrow v' = R_2 \times \frac{v_L(R_1+R_2)}{R_1 R_2}$$

$$\Rightarrow v' = \frac{(R_1+R_2)\,v_L}{R_1} \quad \ldots \quad (B)$$

Substituting the expression for v' in (B) in eq. (A)

Eq (A)

$$\frac{v_L - v_S}{R_1} + \frac{v_L}{R_L} + \frac{v_L - v'}{R_2} = 0$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{v'}{R_2} = \frac{v_S}{R_1}$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{(R_1+R_2)\,v_L}{R_1 R_2} = \frac{v_S}{R_1}$$

$$\Rightarrow v_L\left(\frac{1}{R_1} + \frac{1}{R_L} + \frac{1}{R_2}\right) - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_L = \frac{v_S}{R_1}$$

$$\Rightarrow \frac{v_L}{R_L} = \frac{v_S}{R_1}$$

$$\Rightarrow \frac{i_L R_L}{R_L} = \frac{v_S}{R_1}$$

$$\Rightarrow \boxed{i_L = \frac{v_S}{R_1}}$$

Load current depends on source (input) voltage irrespective of load resistance (R_L). It is a transconductance amplifier.

(Q5) Fig. Q5 shows a feedback transconductance amplifier implemented using an op amp with open-loop gain μ, a very large input resistance (∞), and an output resistance r_o. The output current I_o that is delivered to the load resistance R_L is sensed by the feedback network composed of three resistances R_M, R_1, and R_2, and a proportional voltage V_f is fed back to the negative-input terminal of the op amp. Find the expressions for $A = I_o/V$, $\beta = V_f/I_o$, and $A_f = I_o/V_s$. If the loop gain is large, find the approximate expression for A_f (closed-loop gain)

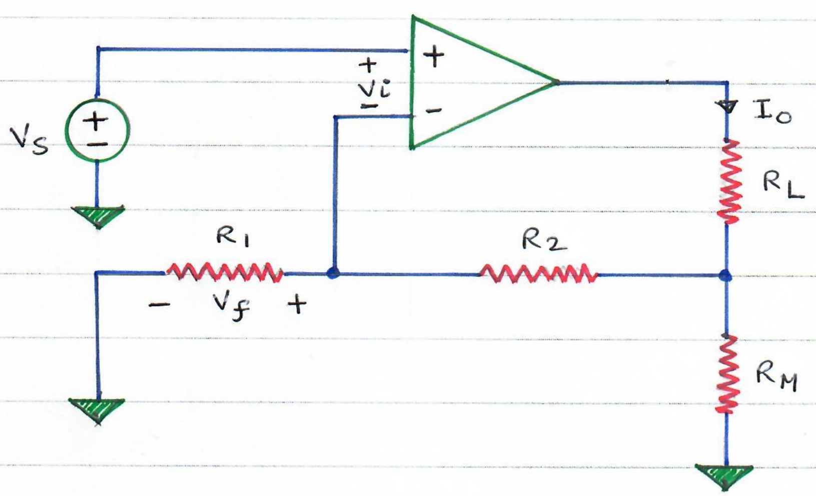
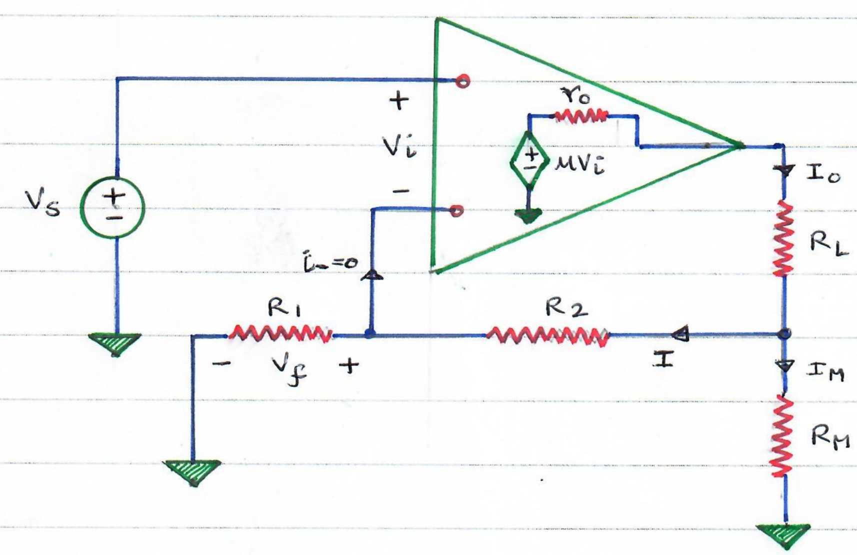


Fig. Q5

Sol.

Replacing op-amp by its equivalent circuit



a) $A = G_M = I_o / V_i$

$$\mu V_i = r_o I_o + R_L I_o + R_M I_M$$

$$\mu V_i = r_o I_o + I_o R_L + \frac{I_o \times (R_1 + R_2)}{(R_1+R_2) + R_M} \times R_M$$

$$\Rightarrow \mu V_i = \left[r_o + R_L + \frac{R_M (R_1 + R_2)}{(R_1 + R_2) + R_M} \right] I_o$$

$$\Rightarrow A = \frac{I_o}{V_i} = \frac{\mu}{r_o + R_L + \dfrac{R_M (R_1 + R_2)}{(R_1 + R_2) + R_M}}$$

b) $\beta = V_f / I_o$

$$V_f = R_1 I$$

$$= R_1 \times \frac{R_M}{(R_1+R_2)+R_M} \times I_o$$

$$\Rightarrow V_f = \frac{R_1 R_M}{(R_1+R_2)+R_M} I_o$$

$$\Rightarrow \beta = \frac{V_f}{I_o} = \frac{R_1 R_M}{(R_1+R_2)+R_M}$$

c) $A_f = I_o / V_s$

$$V_s = V_f + V_i$$

$$= \frac{R_1 R_M}{(R_1+R_2)+R_M} \times I_o + \frac{I_o}{A}$$

$$\Rightarrow V_s = \left[\frac{R_1 R_M}{(R_1+R_2)+R_M} + \frac{1}{A} \right] I_o$$

$$\Rightarrow \frac{I_o}{V_s} = \frac{1}{\left[\dfrac{R_1 R_M}{(R_1+R_2)+R_M} + \dfrac{1}{A} \right]}$$

$$\Rightarrow A_f = \frac{1}{\left[\dfrac{R_1 R_M}{(R_1+R_2)+R_M} + \dfrac{1}{\mu} \left(r_o + R_L + \dfrac{R_M (R_1+R_2)}{(R_1+R_2)+R_M} \right) \right]}$$

Using feedback theory $A_f = \dfrac{A}{1 + A\beta}$ where $A\beta$ = loop gain

If $A\beta$ is very large, $1 + A\beta \simeq A\beta$

$$\therefore A_f \simeq \frac{A}{A\beta} = \frac{1}{\beta}$$

$$\therefore A_f \text{ (with very high loop gain) } = \frac{1}{\beta} = \frac{(R_1 + R_2) + R_M}{R_1 R_M}$$