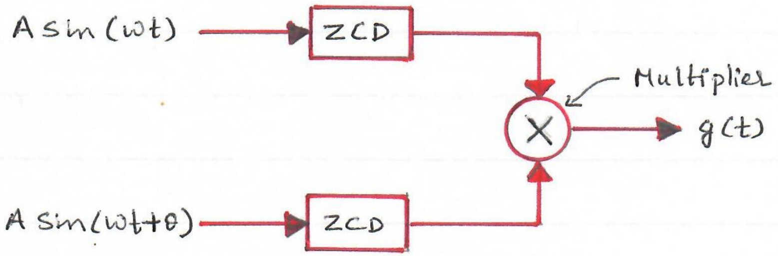
Solutions to Tutorial Sheet-2 IEC103

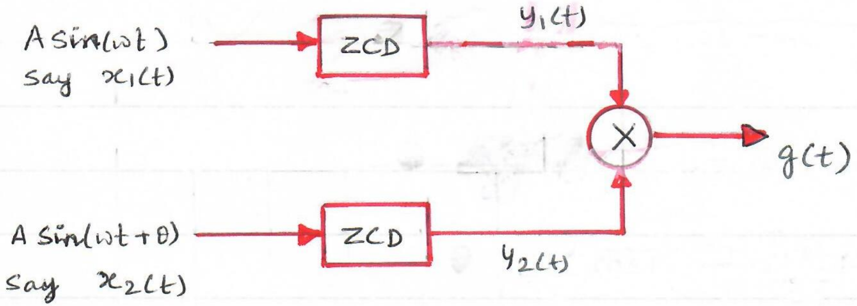
(Q1) A device called zero crossing detector (ZCD) has the following property.

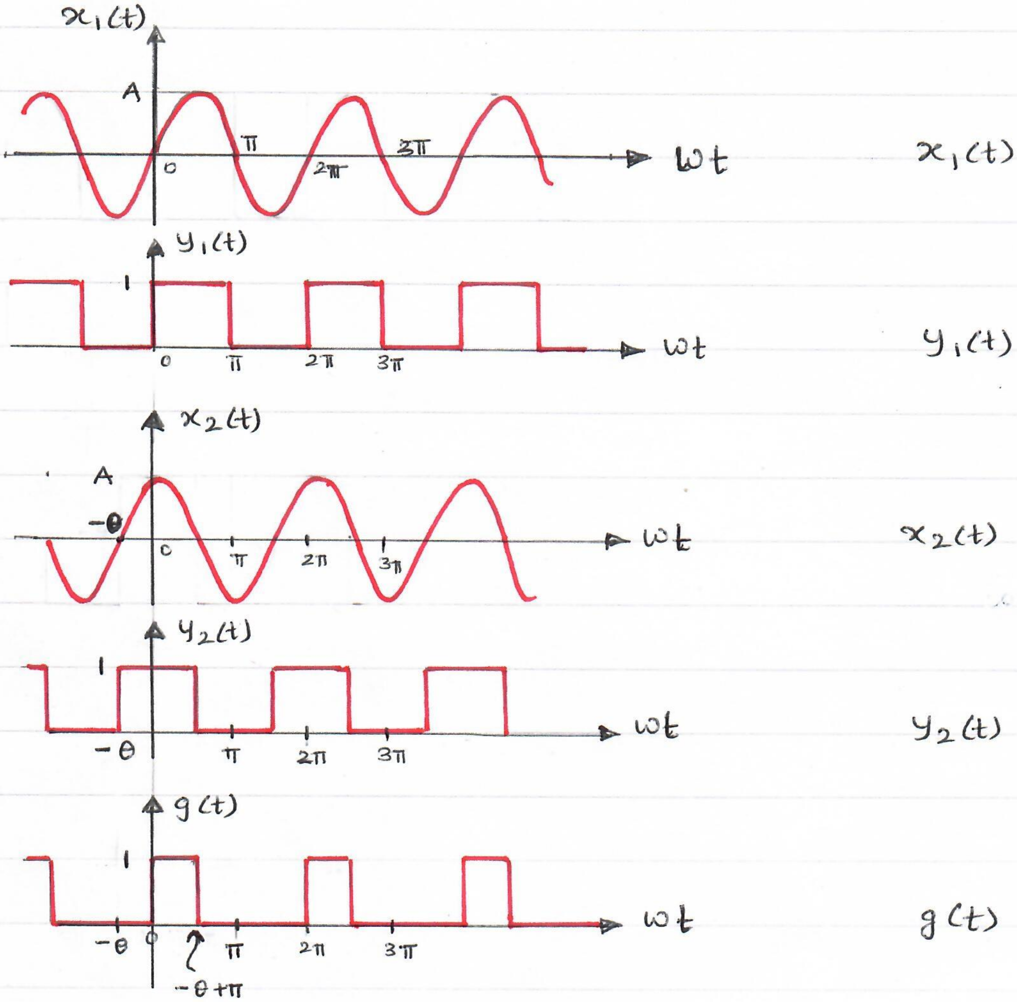
$$\text{output Amplitude} = \begin{cases} 1, & \text{for input amplitude} > 0 \\ 0, & \text{for input amplitude} < 0 \end{cases}$$

The waveforms $A\sin(\omega t)$ and $B\sin(\omega t+\theta)$ are passed through a system shown below. Find the amplitude and RMS value of $g(t)$



Sol.





Amplitude of $g(t) = 1$

RMS value of $g(t)$

$$g_{RMS}^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (g(t))^2 \, d\omega t$$

$$= \frac{1}{2\pi}\int_{0}^{\pi-\theta} (1) \, d\theta'$$

$$= \frac{1}{2\pi}(\pi - \theta)$$

$$g_{RMS} = \sqrt{\frac{(\pi-\theta)}{2\pi}}$$

Q2 In the circuit shown in Fig. Q, $R_1 = 1.5\,K\Omega$; $R_2 = 2.0\,K\Omega$, and $C = 0.5\,\mu F$.

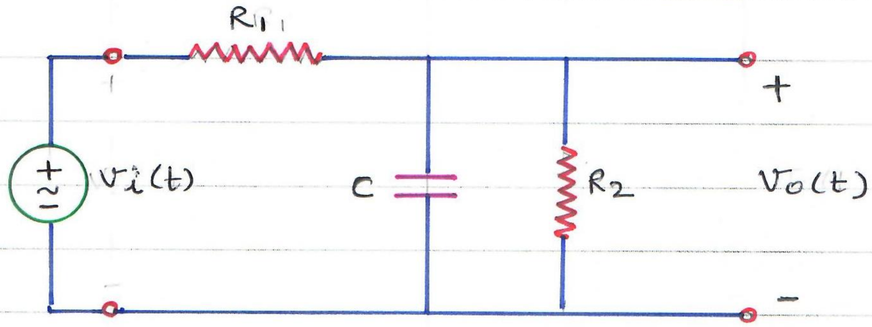


Fig. Q2

Determine

a) The voltage transfer function $H_v(\omega) = \dfrac{V_o(\omega)}{V_i(\omega)}$

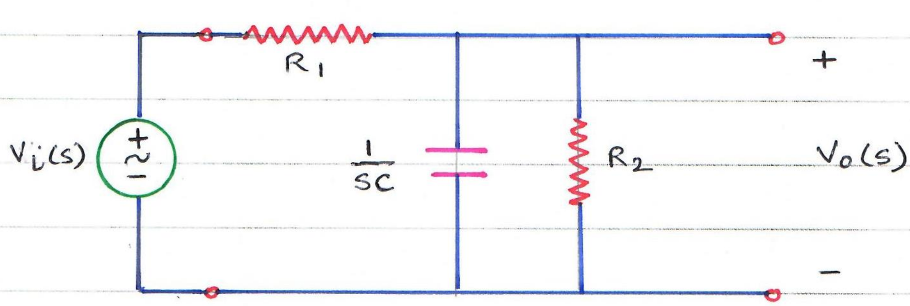
b) How the voltage transfer function $H_v(\omega)$ behaves at extremely high and low frequencies.

c) Show that the transfer function can be manipulated into the form $H_v(\omega) = \dfrac{H_o}{1 + jf(\omega)}$

Find H_o and $f(\omega)$.

d) Find the frequency at which $f(\omega) = 1$ and the value of $H_v(\omega)$ in decibels.

Sol. a) converting the circuit to frequency domain



where $s = j\omega$

$$V_o(s) = \frac{\frac{1}{sC} \parallel R_2}{R_1 + \frac{1}{sC} \parallel R_2} \times V_i(s)$$

$$= \frac{\dfrac{\frac{R_2}{sC}}{R_2 + \frac{1}{sC}}}{R_1 + \dfrac{R_2/sC}{R_2 + 1/sC}} \times V_i(s)$$

$$= \frac{\dfrac{R_2}{1 + sR_2C}}{R_1 + \dfrac{R_2}{1 + sR_2C}} \times V_i(s)$$

$$= \frac{R_2}{R_1(1 + sR_2C) + R_2} \times V_i(s)$$

$$= \frac{R_2}{R_1 + R_2 + sR_1R_2C} \times V_i(s)$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = H_v(s) = \frac{R_2}{(R_1 + R_2) + sR_1R_2C}$$

$$\Rightarrow H_v(\omega) = \frac{R_2}{(R_1 + R_2) + j\omega R_1R_2C}$$

b) $$H_v(\omega) = \frac{R_2}{(R_1 + R_2) + j\omega R_1R_2C}$$

when $\omega << 1$ ie, $\omega \to 0$ $H_v(\omega) \simeq \dfrac{R_2}{R_1 + R_2} = \dfrac{2K}{1.5K + 2K} = \dfrac{2}{3.5} = \dfrac{20}{35} = \dfrac{4}{7}$

when $\omega >> 1$ i.e, $H_v(\omega) \simeq \dfrac{R_2}{j\omega R_1R_2C} = \dfrac{2}{j\omega \times 1.5 \times 10^3 \times 2 \times 10^3 \times 0.5 \times 10^{-6}}$

as $\omega \to \infty$ $= \dfrac{2}{j\omega \times 1.5} \simeq 0$

since ω is very large

c) $$H_V(\omega) = \frac{R_2}{(R_1+R_2) + j\omega R_1R_2C}$$

$$= \frac{R_2/(R_1+R_2)}{1 + \dfrac{j\omega R_1R_2C}{(R_1+R_2)}} = \frac{H_o}{1 + jf(\omega)}$$

where $H_o = \dfrac{R_1}{R_1+R_2}$ is the gain at zero frequency

$$f(\omega) = \frac{\omega R_1R_2C}{(R_1+R_2)}$$

d) $$f(\omega) = \frac{\omega R_1R_2C}{(R_1+R_2)} = 1$$

$$\Rightarrow \omega = \frac{(R_1+R_2)}{R_1R_2C} = \frac{(1.5+2)\times10^{3}}{1.5\times10^{3}\times2\times10^{3}\times0.5\times10^{-6}}$$

$$= \frac{3.5\times10^{3}}{1.5} \text{ rad/sec}$$

$$= 2.33 \text{ k rad/sec}$$

$$H_V(\omega) = \frac{H_o}{1+jf(\omega)}$$

$$H_V(2.33 \text{ k rad/sec}) = \frac{4/7}{1+j}$$

$$H_o = 4/7 = 0.57 = 20\log_{10}(0.57)\text{ dB}$$

$$= -4.86 \text{ dB}$$

(Q3) One application of narrowband filters is seen in rejecting interference due to AC line power. Any undesired 50 Hz signal originating in the AC line power can cause serious interference in sensitive instruments. In medical instruments such as the electrocardiograph, 50 Hz notch filters are often provided to reduce the effect of this interference on cardiac measurements. Fig. Q3 depicts a circuit in which the effect of 50 Hz noise is represented by way of 50Hz sinusoidal generator connected in series with a signal source (V_S), representing the desired signal. In this example we design a 50Hz narrowband (or notch) filter to remove the unwanted 50 Hz noise.

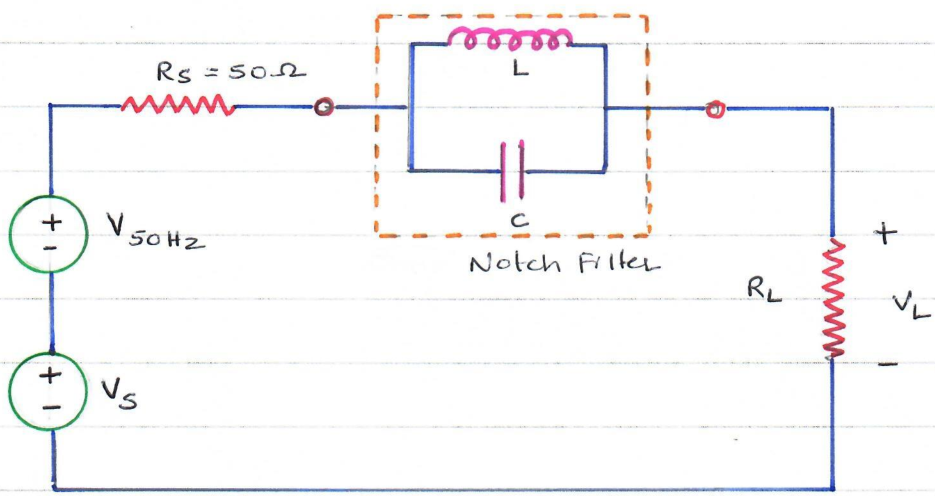


Fig. Q3 50Hz Notch Filter

Find the appropriate value of L and C for the notch filter.

Sol.

Sol.

Impedance of the notch filter $= j\omega L \,||\, \frac{1}{j\omega C}$

$$= \frac{j\omega L \times \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{j\omega L}{1 - \omega^2 LC}$$

Note that when $\omega^2 LC = 1$, the impedance of the circuit is infinite ! The frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ is the resonant frequency of the LC circuit. If the resonant frequency were selected to be equal to 50 Hz, the series circuit would show an infinite impedance to 50 Hz currents, and would therefore block the interference signal, while passing most of the other frequency components. We thus select values of L and C that results in $\omega_0 = 2\pi \times 50$.

Let $L = 100\,mH$, then $C = \frac{1}{\omega_0^2 L} = 101\,\mu F$

The frequency response of the complete circuit is given below

$$H(j\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{R_L}{R_S + Z_{notch} + R_L}$$

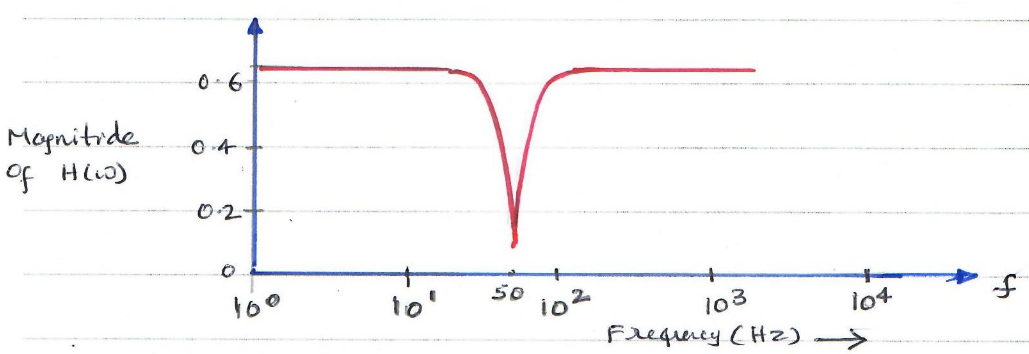
$$= \frac{R_L}{R_S + \frac{j\omega L}{1-\omega^2 LC} + R_L}$$

$$= \frac{R_L}{R_S + R_L + j\omega L/(1-\omega^2 LC)}$$

$$|H(j\omega)| = \frac{R_L(1-\omega^2 LC)}{\left[(R_S+R_L)^2(1-\omega^2 LC)^2 + (\omega L)^2\right]^{1/2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega L}{(R_S+R_L)(1-\omega^2 LC)}\right)$$

Magnitude response (qualitave) as given below.



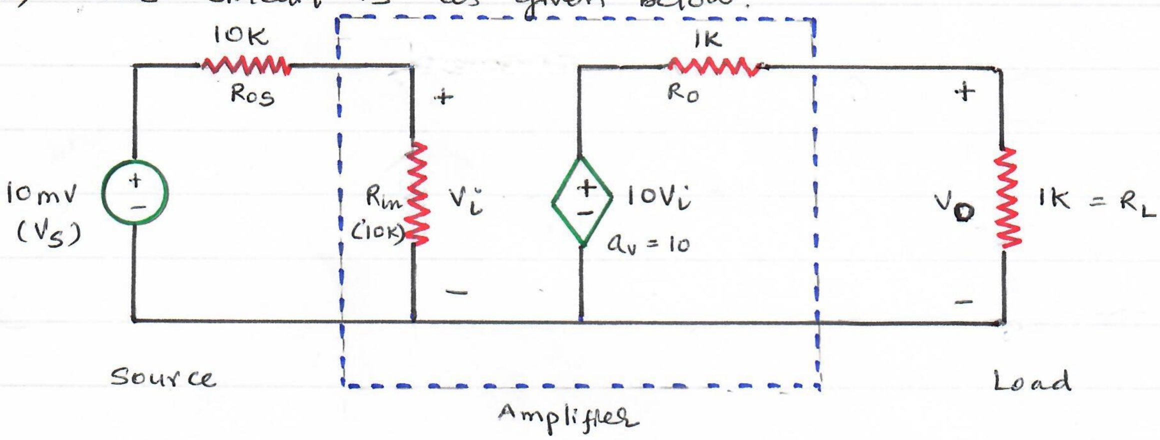
(Q7) A source with output resistance (a source) of 10K puts out a signal of 10 mV amplitude. It is intended to amplify the signal to feed into a load of 1K. An amplifier is available with open-loop voltage gain of 10 and output resistance 1K and input resistance of 10K.

i) What would be the output amplitude if the available amplifier is used.

ii) If N such amplifiers are connected in cascade and used between source and load, what is the signal amplitude at the output ?

Sol.

i) The circuit is as given below.



$$V_i = \frac{V_S \times R_{in}}{R_{in} + R_{os}} = \frac{10m \times 10K}{10K + 10K} = \frac{10\,mV}{2} = 5\,mV$$

$$I_L = \frac{10V_i}{R_o + R_L} = \frac{10 \times 5 \times 10^{-3}}{1K + 1K} = \frac{10 \times 5 \times 10^{-3}}{2 \times 10^3}$$

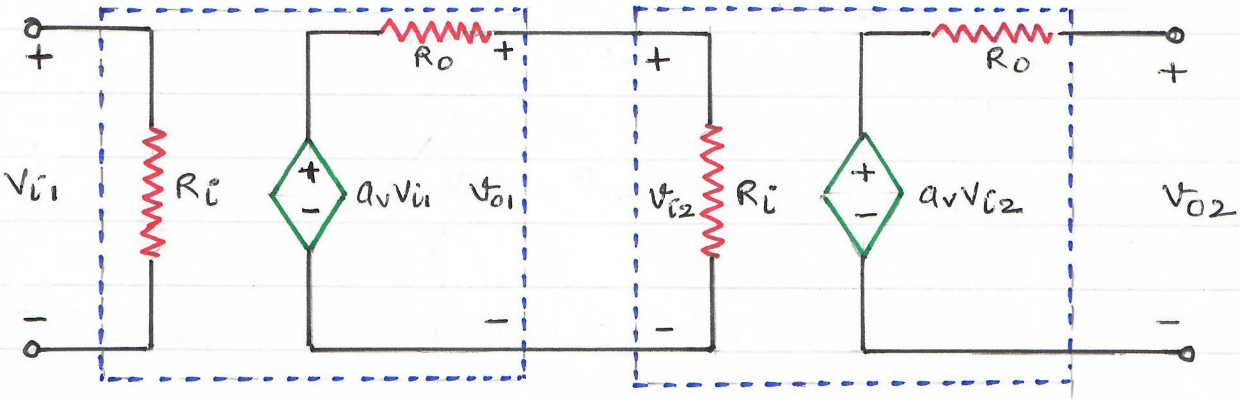
$$= 25 \times 10^{-6}\,A$$

$$= 25\,\mu A$$

$$V_o = I_L R_L = 25 \times 10^{-6} \times 10^3 = 25 \times 10^{-3} = 25\,mV$$

$$\therefore \boxed{V_o = 25\,mV}$$

ii) If 2-amplifiers are connected in cascade, the overall voltage gain can be found out as given below.



Amplifier -1 Amplifier - 2

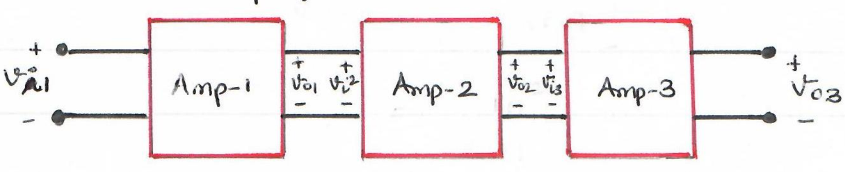
Since the amplifiers are connected in cascade, the output (voltage) of amplifier-1 is input voltage to the amplifier -2 i.e., $V_{o1} = V_{i2}$

$$V_{o1} = \left(\frac{a_v V_{i1}}{R_o + R_i}\right) \times R_i = V_{i2}$$

$$V_{o2} = a_v V_{i2} = a_v \times \left(\frac{a_v V_{i1}}{R_o + R_i}\right) \times R_i$$

$\therefore$ overall gain $= \dfrac{V_{o2}}{V_{i1}} = \dfrac{a_v^2}{\left(\frac{R_o + R_i}{R_i}\right)} V_{i1}$

If 3-such amplifiers are connected in cascade



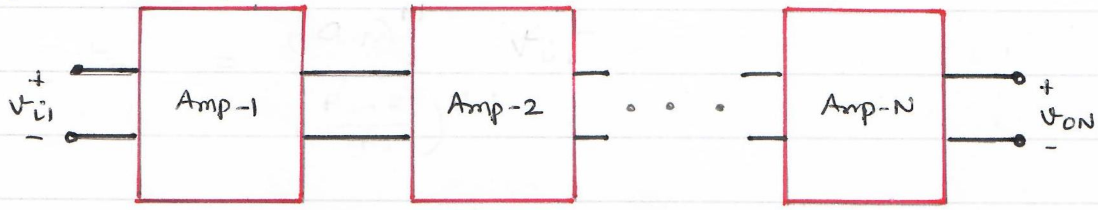
V_{o2} will be the output voltage of 2nd amplifier and is input to the 3rd amplifier

$$V_{o1} = \frac{a_v}{(R_o + R_i)} R_i V_{i1} = V_{i2}$$

$$V_{o2} = \frac{a_v V_{i2}}{(R_o + R_i)} \times R_i = \frac{a_v}{(R_o + R_i)} \times R_i \times \frac{a_v R_i}{(R_o + R_i)} V_{i1} = \frac{a_v^2}{\left(\frac{R_o + R_i}{R_i}\right)^2} V_{i1}$$

$$V_{o3} = a_v V_{o2} = \frac{a_v^3}{\left(\frac{R_i + R_o}{R_i}\right)^2} V_{i1}$$

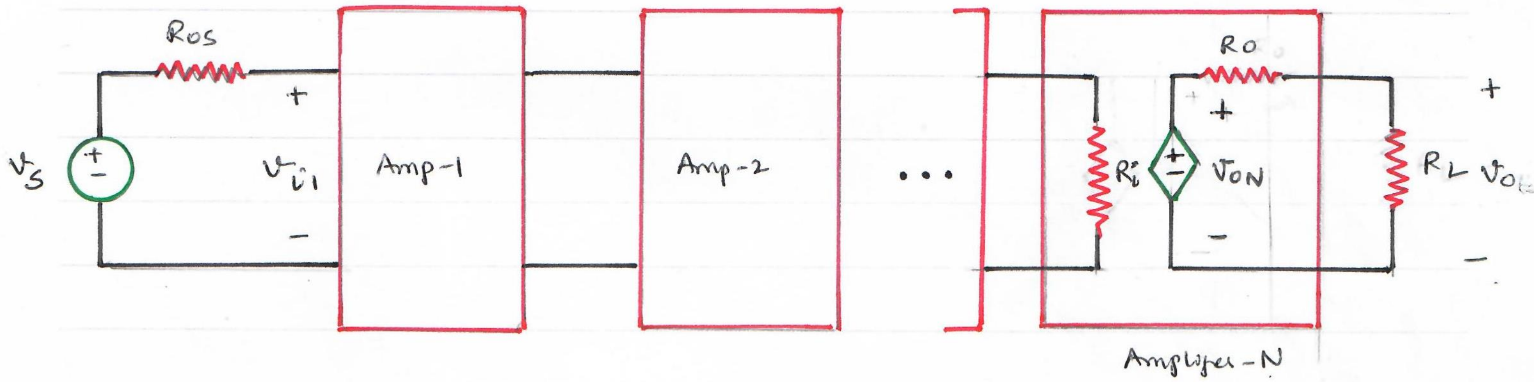
If N such amplifiers are connected in cascade



N such amplifiers are connected in cascade

$$v_{ON} = \frac{(a_v)^N}{\left(\frac{R_i+R_o}{R_i}\right)^{N-1}} v_{i1}$$

Now if we connect the source with resistance R_{os} to i/p of amplifier-1 and connect load of resistance R_L to o/p of amplifier-N.



$$v_{i1} = v_s \times \frac{R_i}{(R_i+R_{os})}$$

$$v_o = v_{ON} \times \frac{R_L}{(R_L+R_o)}$$

where $$v_{ON} = \frac{(a_v)^N}{\left(\frac{R_i+R_o}{R_i}\right)^{N-1}} v_{i1}$$

Substituting the values of v_s, R_{os}, R_i, R_o, & R_L, the output voltage across the load can be calculated.

$$v_{i1} = v_s \times \frac{R_i}{R_i+R_{os}} = 10\times10^{-3} \times \frac{10K}{10K+10K} = 5\times10^{-3} = 5mV$$

$$a_v = 10; \quad \frac{R_i+R_o}{R_i} = \frac{10K+1K}{10K} = \frac{11}{10}$$

$$\therefore v_{ON} = \frac{(10)^N}{\left(\frac{11}{10}\right)^{N-1}} v_{i1} = \frac{(10)^N}{(11/10)^{N-1}} \times 5\times10^{-3}$$

$$\therefore v_o = v_{ON} \times \frac{R_L}{R_L+R_o} = v_{ON} \times \frac{1K}{1K+1K} = \frac{v_{ON}}{2} = \frac{(10)^N}{(11/10)^{N-1}} \times 2.5\times10^{-3} \text{ V}$$

(Q5) Derive the exact formula for the overall gain of an inverting op-amp circuit (v_o/v_s) shown in Fig. Q5.

a) If $R_1 = 1K\Omega$, $R_f = 10K\Omega$, $R_i = 100K\Omega$, $R_o = 100\Omega$, and open-loop gain $A_{OL} = 10^5$, evaluate the gain of this inverting amplifier.

b) Compare the result in part a) with ideal op-amp approximation

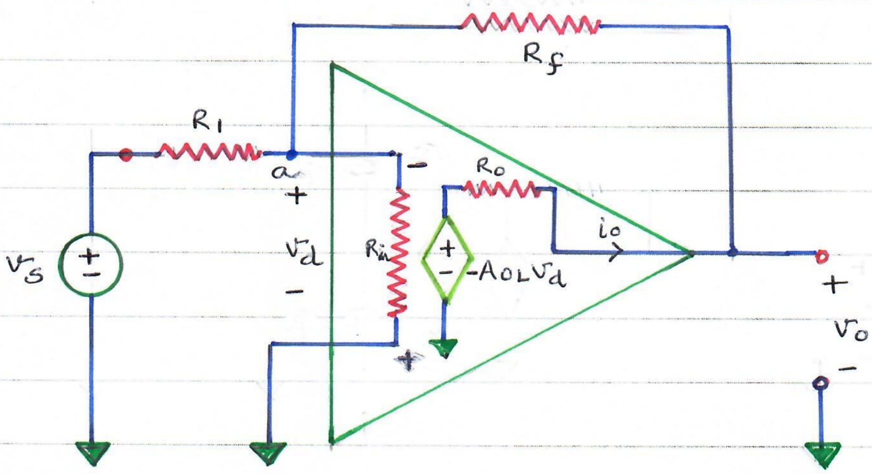


Fig. Q5

Sol. $v_o = -A_{OL}v_d - i_o R_o$ where $i_o = (v_o - v_d)/R_f$

Applying KCL at node 'a'

$$\frac{v_d - v_s}{R_1} + \frac{v_d}{R_i} + \frac{v_d - v_o}{R_f} = 0$$

$$\Rightarrow v_d\left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_f}\right) - \frac{v_o}{R_f} = \frac{v_s}{R_1} \quad \cdots \text{(A)}$$

$$v_o = -A_{OL}v_d - i_o R_o$$

$$= -A_{OL}v_d - R_o\left(\frac{v_o - v_d}{R_f}\right)$$

$$\Rightarrow v_o\left(1 + \frac{R_o}{R_f}\right) = \left(-A_{OL} + \frac{R_o}{R_f}\right)v_d$$

$$\Rightarrow v_d = \frac{(R_f + R_o)\,v_o}{(R_o - R_f A_{OL})} \quad \cdots \text{(B)}$$

Substituting the expression for V_d from (B) in eq. (A), we have

$$V_d\left(\frac{1}{R_1}+\frac{1}{R_i}+\frac{1}{R_f}\right)-\frac{V_o}{R_f}=\frac{V_s}{R_1}$$

$$\Rightarrow \frac{(R_f+R_o)\ V_o}{(R_o-R_f A_{OL})}\left(\frac{1}{R_1}+\frac{1}{R_i}+\frac{1}{R_f}\right)-\frac{V_o}{R_f}=\frac{V_s}{R_1}$$

$$\Rightarrow \left[\frac{(R_f+R_o)}{(R_o-A_{OL}R_f)}\left(\frac{1}{R_1}+\frac{1}{R_i}+\frac{1}{R_f}\right)-\frac{1}{R_f}\right]V_o=\frac{V_s}{R_1}$$

$$\Rightarrow \frac{V_o}{V_s}=\frac{1}{R_1\left[\frac{(R_f+R_o)}{(R_o-A_{OL}R_f)}\left(\frac{1}{R_1}+\frac{1}{R_i}+\frac{1}{R_f}\right)-\frac{1}{R_f}\right]}$$

a) $R_1=1K\Omega$; $R_f=10K\Omega$; $R_i=100\,K\Omega$; $R_o=100\Omega$; $A_{OL}=10^5$

$$\frac{V_o}{V_s}=\frac{1}{10^3\left[\frac{(10*10^3+100)}{(100-10^5\times10\times10^3)}\left(\frac{1}{10^3}+\frac{1}{100\times10^3}+\frac{1}{10\times10^3}\right)-\frac{1}{10\times10^3}\right]}$$

$$=\frac{1}{10^3\left[\frac{10100}{-10^9}\times\frac{1}{10^3}\left(1+\frac{1}{100}+\frac{1}{10}\right)-\frac{1}{10^4}\right]}$$

$$\simeq \frac{-10^4}{10^3}=-10$$

The exact value of V_o/V_s = -9.99888

b) For an ideal op-amp $R_i=\infty$; $R_o=0$; $A_{OL}=0$

$$\frac{V_o}{V_s}=-\frac{R_f}{R_1}=-\frac{10\times10^3}{10^3}=-10$$

$$\therefore \text{error}=\frac{-9.99888-(-10)}{-9.99888}=-0.0112\%$$