Solutions to Mid Exam-1 Paper IEC103

(Q1) You are given 2 voltage amplifiers A and B to be connected in cascade between source with $V_S = 10\,mV$ & $R_S = 100\,K\Omega$ and a load of $R_L = 100\Omega$.

The voltage amplifiers have voltage gain (A_V), input resistance (R_{in}), and output resistance (R_o) as follows.

For amplifier A: $A_V = 100\,V/V$; $R_i = 10\,K\Omega$; $R_o = 10\,K\Omega$

For amplifier B: $A_V = 1\,V/V$; $R_i = 100\,K\Omega$; $R_o = 100\Omega$

Your task is to decide how the amplifiers should be connected. To proceed, evaluate the two possible connections between source (S) and load (L), namely SABL and SBAL. Find the overall voltage gain (v_L/v_S) for both cases in decibels. Which amplifier arrangement is best?

Sol.

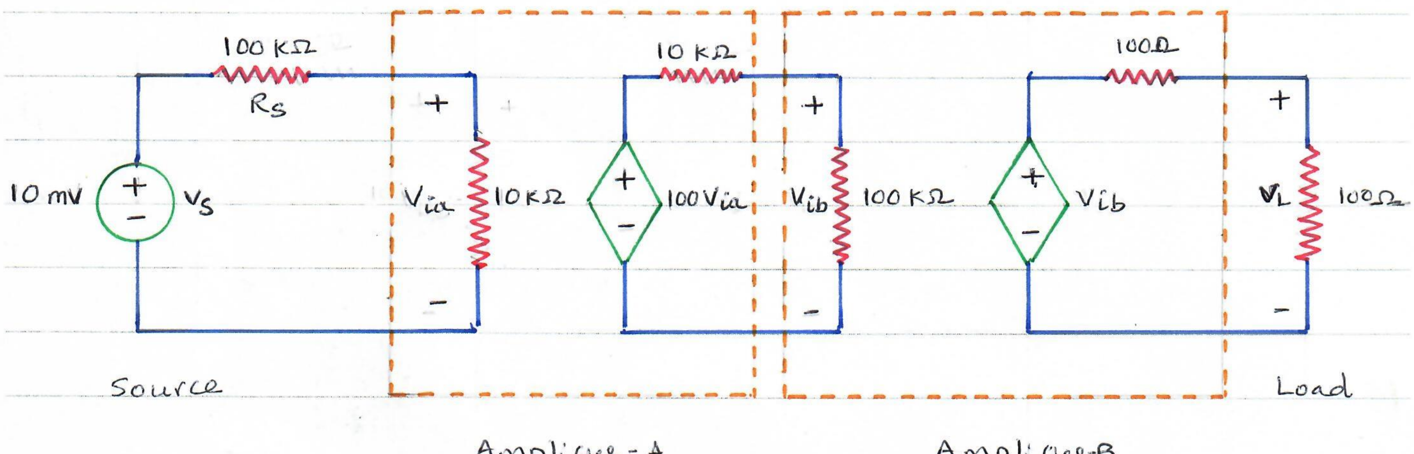
Source: $V_S = 10\,mV$; $R_S = 100\,K\Omega$

Amplifier A: $A_V = 100\,V/V$; $R_i = 10\,K\Omega$; $R_o = 10\,K\Omega$

Amplifier B: $A_V = 1\,V/V$; $R_i = 100\,K\Omega$; $R_o = 100\Omega$

Load: $R_L = 100\Omega$

Case 1) SABL



$$V_{ia} = \frac{10K}{100K + 10K} \times 10\,mV = \frac{10}{110} \times 10\,mV = \frac{10}{11}\,mV$$

$$V_{ib} = \frac{100K}{10K+100K} \times 100 V_{ia}$$

$$= \frac{100}{110} \times 100\, V_{ia} = \frac{10}{11} \times 100 \times \frac{10}{11}\ mV$$

$$= \frac{10000}{121}\ mV$$

$$V_L = \frac{100}{100+100} \times V_{ib} = \frac{1}{2} \times V_{ib} = \frac{5000}{121}\ mV$$

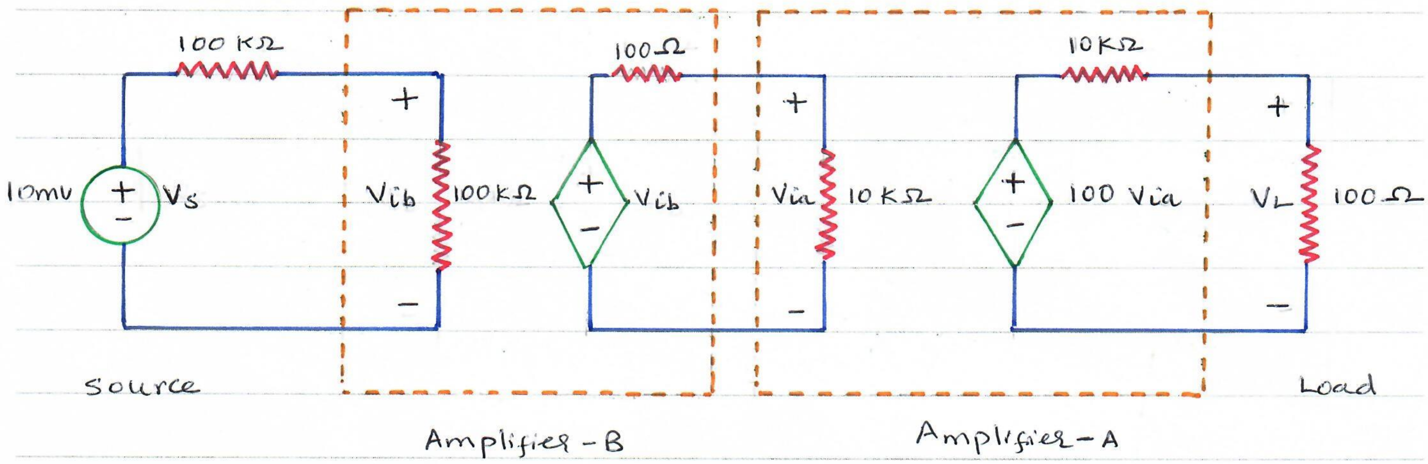
$$= 41.32\ mV$$

$$\text{overall gain} = \frac{V_L}{V_S} = \frac{41.3\ mV}{10\ mV} = 4.132$$

$$= 20\log_{10}(4.132)\ dB$$

$$= 12.323\ dB$$

Case 2) SBAL



$$V_{ib} = \frac{100K}{100K+100K} \times V_S = \frac{1}{2} \times 10mV = 5mV$$

$$V_{ia} = \frac{10K}{100+10K} \times V_{ib} = \frac{10000}{10100} \times 5\ mV = \frac{500}{101}\ mV$$

$$V_L = \frac{100}{10000+100} \times 100\, V_{ia} = \frac{100}{10100} \times 100 \times \frac{500}{101}\, mV = \frac{100}{101} \times \frac{500}{101}\ mV = 4.9\, mV$$

$$\frac{V_L}{V_S} = \frac{4.9\, mV}{10\, mV} = 0.49 = 20\log_{10}(0.49)\ dB = -6.196\, dB$$

Connection SABL is better than SBAL as it provides a better overall voltage gain.

(Q2) Determine V_o of the op-amp circuit shown in Fig-Q2

Assume that the op-amps are ideal.

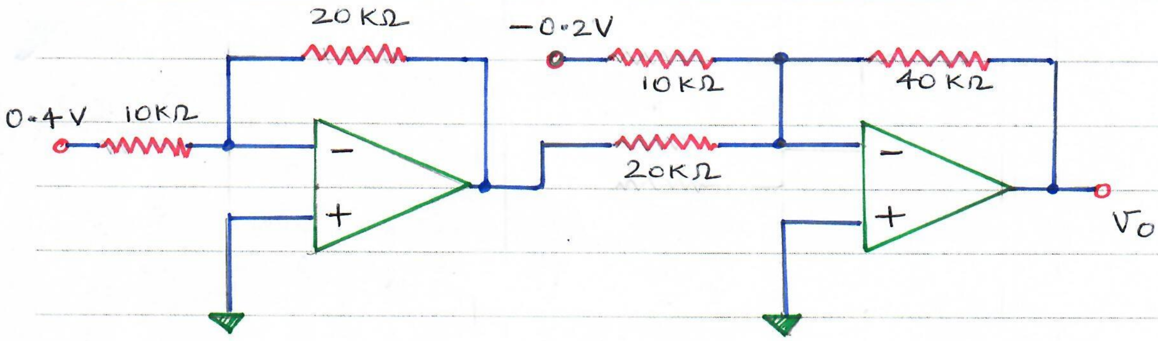


Fig. Q2

Sol.

Output of the 1st op-amp = $V_{01} = -\dfrac{20K}{10K} \times 0.4 = -0.8V$

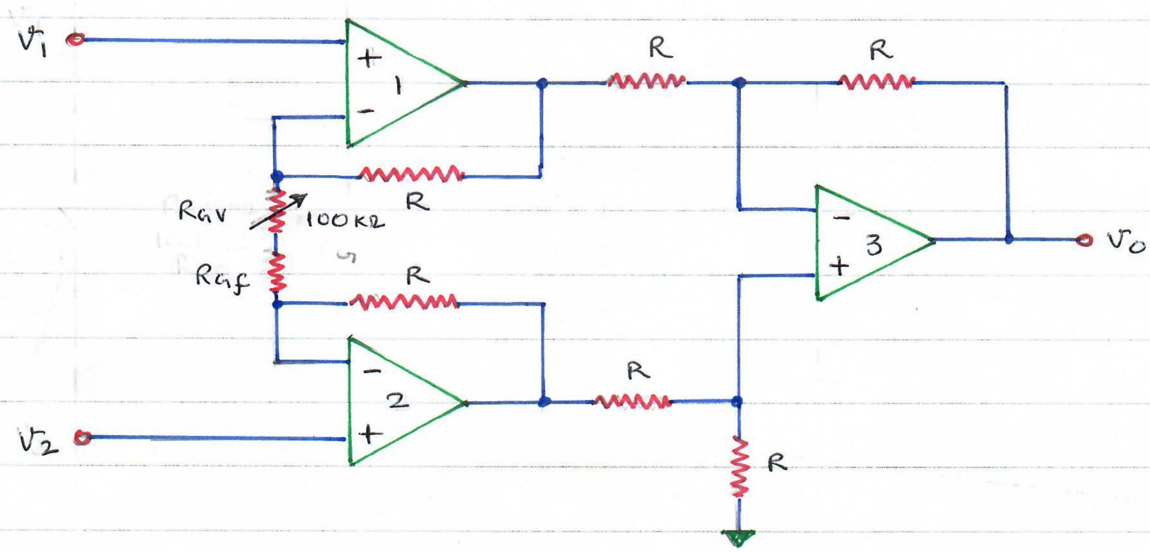
Output of the 2nd op-amp $= V_o = -\dfrac{40K}{10K} \times (-0.2) + \left(-\dfrac{40K}{20K}\right) \times V_{01}$

$= -4 \times (-0.2) + (-2) \times (-0.8)$

$= 0.8 + 1.6 = 2.4V$

Q3 Design an instrumentation amplifier circuit to provide a gain that can be varied over a range of 2 to 1000 utilizing a 100 KΩ variable resistance (a potentiometer). Draw the complete circuit diagram specifying the values of all the resistors. Consider op-amps to be ideal.

Sol.



The circuit diagram of the instrumentation amplifier is as given above. A variable resistance (or potentiometer) along with fixed resistor Rif is used as a gain control resistor R_G to obtain variable gain. The given pot is 100 KΩ.

$$v_o = \left(1 + \frac{2R}{R_G}\right)(v_2 - v_1)$$

$$1 + \frac{2R}{R_G} = 2 \text{ to } 1000$$

$R_G = R_{av} + R_{GF}$ where R_{av} is variable resistance (100K pot) and R_{af} is fixed resistance

$$\therefore \; 1 + \frac{2R}{R_{GF} + R_{aV}} = 2 \text{ to } 1000$$

$$1 + \frac{2R}{R_{af}} = 1000 \quad (R_{av} = 0) \quad \ldots (A)$$

and $$1 + \frac{2R}{R_{af} + 100K} = 2$$

$$\therefore \frac{2R}{R_{af}} = 999 \quad \text{and} \quad \frac{2R}{R_{af} + 100K} = 1$$

$$\Rightarrow \frac{R_{af} + 100K}{2R} = 1$$

$$\Rightarrow \frac{R_{af}}{2R} + \frac{100K}{2R} = 1$$

$$\Rightarrow \frac{1}{999} + \frac{50000}{R} = 1$$

$$\Rightarrow \frac{50000}{R} = 1 - \frac{1}{999} = \frac{998}{999}$$

$$\Rightarrow R = \frac{999}{998} \times 50000$$

$$\Rightarrow \boxed{R = 50.05 K\Omega}$$

$$\frac{2R}{R_{af}} = 999$$

$$\Rightarrow R_{af} = \frac{2R}{999} = \frac{2 \times 50.05}{999} K\Omega = 100.2\,\Omega$$

$$\therefore \boxed{R_{af} = 100.2\,\Omega}$$

Designed values of resistances: $R_{af} = 100.2\,\Omega$

$R_{av} = 100K\Omega$ pot

$R = 50.05K\Omega$

As the resistance of pot is varied from $100K\Omega$ to 0Ω, the gain of the instrumentation amplifier

Q4. The circuit shown in Fig. Q4 utilizes a 10 KΩ potentiometer to realize an adjustable gain amplifier. Derive the expression for the gain (V_o/V_s) as a function of potentiometer position 'x'. Assume op-amp to be ideal. What is the range of gains obtained. Show how to or where to add a fixed resistor so that the gain can be inbetween 1 and 11. What should be the value of the resistance?

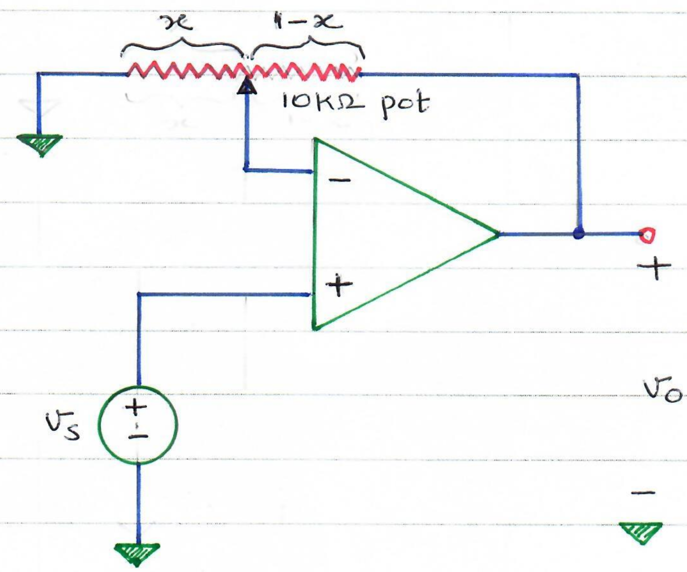


Fig. Q4

Sol.

The above circuit is an non-inverting amplifier with feedback resistor $R_f = (1-x) \times 10000\ \Omega$ and $R_1 = x \times 10000\ \Omega$

Gain of the amplifier circuit $= 1 + \frac{R_f}{R_1} = 1 + \frac{10000(1-x)}{10000x}$

$$= 1 + \frac{1-x}{x} = \frac{1}{x}$$

Gain will vary from 0 to ∞ as x goes from 1 to 0.

A fixed resistor R can be inserted between ground and one end of the pot to ~~varies~~ vary gain from 1 to 11

Amplifier gain with fixed resistance inserted $= 1 + \frac{10000(1-x)}{R + 10000x}$

Maximum gain is obtained when $x = 0$

$$\therefore \; 1 + \frac{10000}{R} = 11$$

$$\text{or} \quad \frac{10000}{R} = 10$$

$$\text{or} \quad R = \frac{10000}{10} = 1000 = 1K\Omega$$

Minimum gain is obtained when $x = 0$ (voltage follower)

$$\text{gain} = 1 + \frac{0}{R+10K} = 1$$

(Q5) For the active band pass filter shown in Fig. Q5, find $H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$ and also the lower and upper cut-off frequencies.

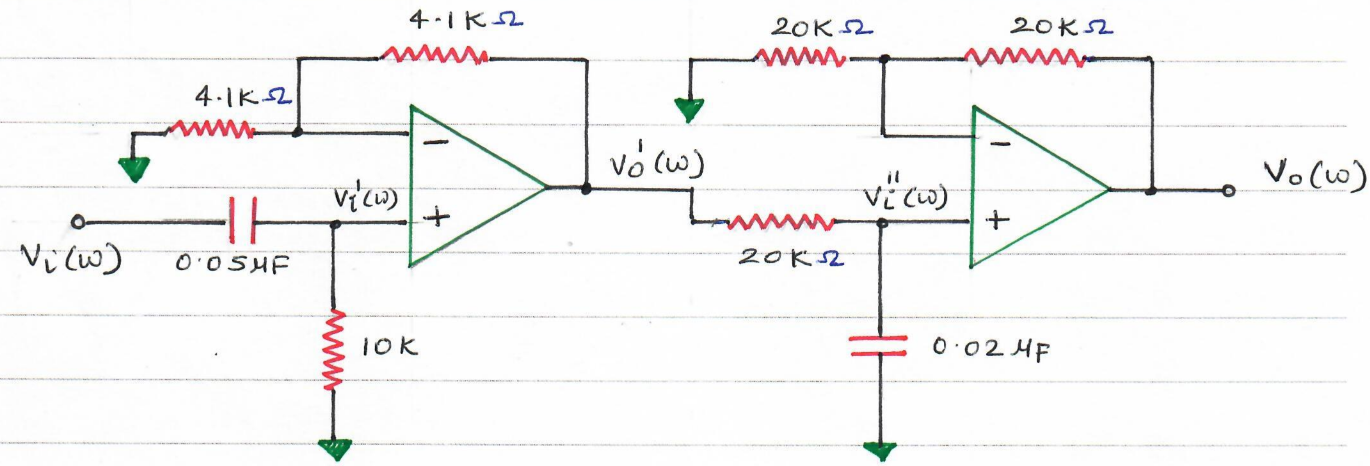


Fig. Q5

Sol. Let $R_1 = 10\,K\Omega,\ C_1 = 0.05\,\mu F$

$R_2 = 20K\Omega,\ C_2 = 0.02\,\mu F$

$$V_o'(\omega) = V_i'(\omega)\left(1 + \frac{4.1K}{4.1K}\right) = 2V_i'(\omega)$$

$$V_i'(\omega) = \frac{R_1\ V_i(\omega)}{R_1 + (1/j\omega C_1)} = \frac{j\omega R_1 C_1 . V_i(\omega)}{(j\omega R_1 C_1 + 1)}$$

$$V_o'(\omega) = 2V_i'(\omega) = \frac{j2\omega R_1 C_1\ V_i(\omega)}{(j\omega R_1 C_1 + 1)} \quad \ldots \quad (A)$$

$$V_o(\omega) = \left(1 + \frac{20K}{20K}\right) V_i''(\omega) = 2V_i''(\omega)$$

$$V_i''(\omega) = \frac{(1/j\omega C_2)}{\left[R_2 + (1/j\omega C_2)\right]} V_o'(\omega) \quad \ldots \quad (B)$$

$$\therefore V_o(\omega) = 2 \times \frac{(1/j\omega C_2)}{\left[R_2 + (1/j\omega C_2)\right]} V_o'(\omega)$$

$$\Rightarrow V_o(\omega) = \frac{\left(\frac{2}{j\omega C_2}\right)}{\left[R_2 + \left(\frac{1}{j\omega C_2}\right)\right]} \times 2V_i'(\omega)$$

$$\Rightarrow V_o(\omega) = \frac{\left(2/j\omega C_2\right)}{\left[R_2 + \left(1/j\omega C_2\right)\right]} \times 2 \times \frac{j\omega R_1C_1}{(j\omega R_1C_1 + 1)} V_i(\omega)$$

$$= \frac{2}{(j\omega R_2C_2 + 1)} \times 2 \times \frac{j\omega R_1C_1}{(j\omega R_1C_1 + 1)} V_i(\omega)$$

$$V_o(\omega) = \frac{j4\omega R_1C_1}{(j\omega R_1C_1 + 1)(j\omega R_2C_2 + 1)} V_i(\omega)$$

$$\frac{V_o(\omega)}{V_i(\omega)} = H(\omega) = \frac{j4\omega R_1C_1}{(j\omega R_1C_1 + 1)(j\omega R_2C_2 + 1)}$$

$$= \frac{j4\omega R_1C_1}{R_1C_1\left(j\omega + 1/R_1C_1\right) R_2C_2\left(j\omega + 1/R_2C_2\right)}$$

$$= \frac{j4\omega/R_2C_2}{(j\omega + 1/R_1C_1)(j\omega + 1/R_2C_2)}$$

$$\frac{1}{R_1C_1} = \frac{1}{10\times10^3 \times 0.05\times10^{-6}} = 2000 \text{ rad/s}$$

$$\frac{1}{R_2C_2} = \frac{1}{20\times10^3 \times 0.02\times10^{-6}} = 2500 \text{ rad/sec}$$

$\therefore$ Lower cut off frequency $= \omega_1 = 2000$ rad/s

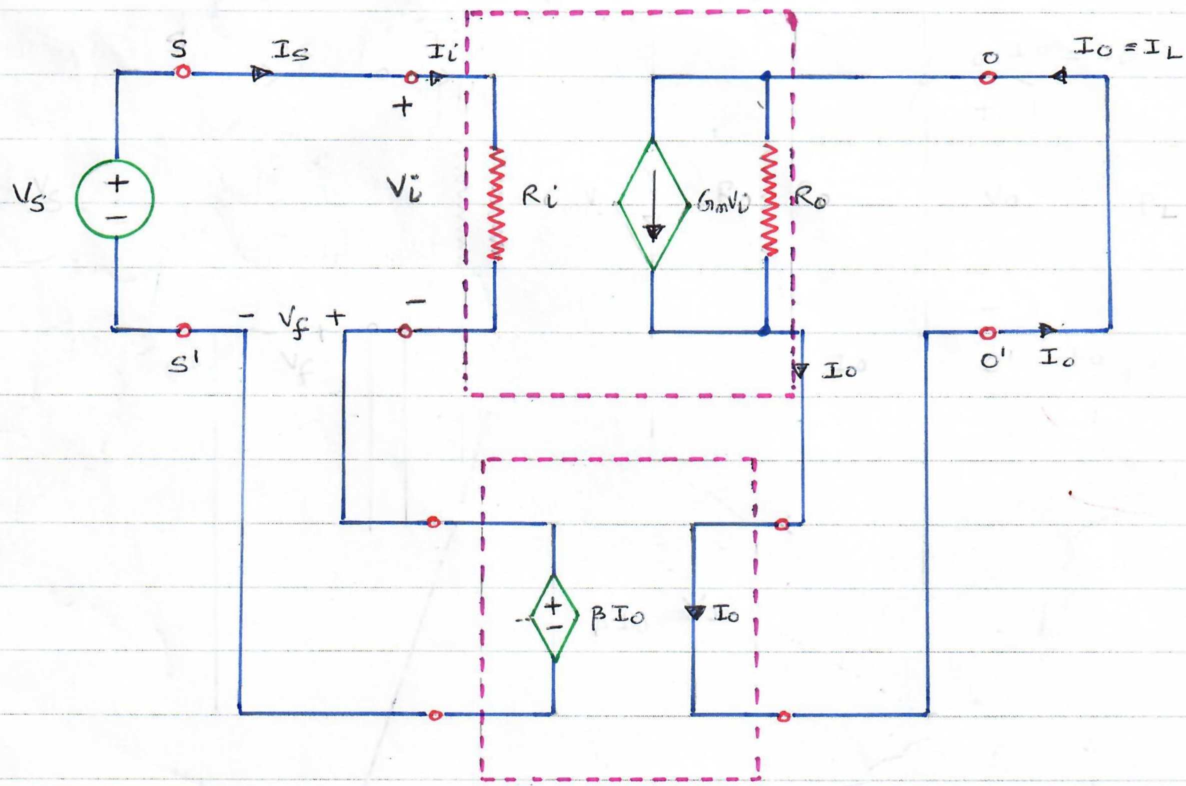
Upper cut off frequency $= \omega_2 = 2500$ rad/s

Q6 Derive the expression for the gain (GMf), input resistance (Rif), and output resistance (Rof) of a transconductance amplifier with -ve feedback (feedback factor is β). The transconductance amplifier is a practical amplifier with gain $\frac{V_o}{I_i} = G_M$, Input resistance R_i and output resistance R_o.

Draw the complete circuit diagram and derive the expressions mentioned above.

Assume that the feedback network is ideal.

Sol. The circuit diagram is as shown below.



$$G_{Mf} = \frac{I_o}{V_s} = \frac{I_o}{V_i + V_f}$$

but $V_f = \beta I_o$; $I_o = G_M V_i \Rightarrow V_i = I_o/G_M$

$$\therefore G_{Mf} = \frac{I_o}{I_o/G_M + \beta I_o} = \frac{1}{\frac{1}{G_M} + \beta} = \frac{G_M}{1 + G_M\beta}$$

$$R_{if} = \frac{V_s}{I_s} = \frac{V_s}{I_i} = \frac{V_i + V_f}{(V_i/R_i)}$$

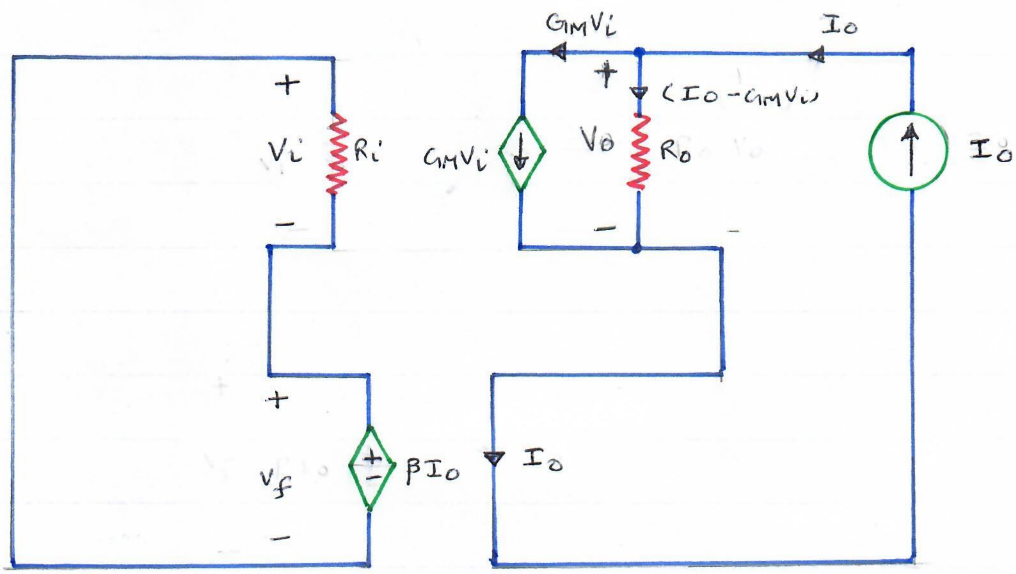
$$= \frac{V_i + \beta I_o}{(V_i/R_i)}$$

$$= \frac{V_i + \beta G_M V_i}{(V_i/R_i)} \qquad \because I_o = G_M V_i$$

$$= \frac{(V_i + \beta G_M V_i) R_i}{V_i}$$

$$\Rightarrow R_{if} = (1 + G_M \beta) R_i$$

$$R_{of} = \left. \frac{V_o}{I_o} \right|_{V_s = 0}$$



$$V_o = (I_o - G_M V_i) R_o$$

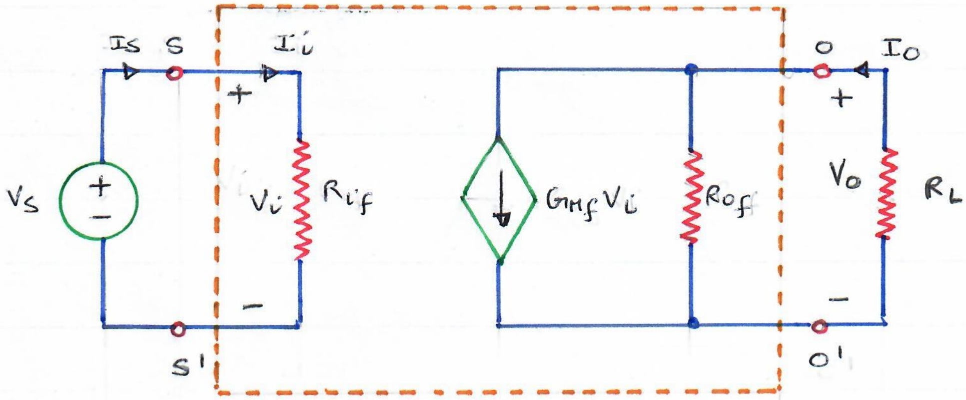
$$V_i = -V_f = -\beta I_o$$

$$\therefore V_o = (I_o - G_M V_i) R_o = (I_o + G_M \beta I_o) R_o$$

$$\Rightarrow V_o = (1 + G_M \beta) R_o I_o$$

$$\Rightarrow \frac{V_o}{I_o} = R_{of} = (1 + G_M \beta) R_o$$

Equivalent circuit diagram is shown in next page



Equivalent circuit

$$R_{if} = (1 + G_M \beta) R_i$$

$$R_{of} = (1 + G_M \beta) R_o$$

$$G_{Mf} = \frac{G_M}{1 + G_M \beta}$$