Attending Over Beliefs

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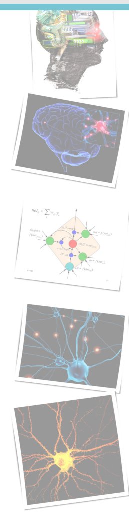
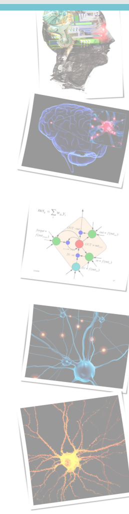


From Reinforcement Learning to Beliefs

From Beliefs to Attention Networks



What is Reinforcement Learning (RL)?

- I will introduce everything in the context of RL
- *Reinforcement learning is the study of how animals and artificial systems can learn to optimize their behavior in the face of rewards and punishments* – Peter Dyan, Encyclopedia of Cognitive Science
- **Not** supervised learning - the animal/agent is not provided with examples of optimal behaviour, it has to be discovered!
- **Not** unsupervised learning either - we have more guidance than just observations

Links to other fields

- It subsumes most artificial intelligence problems
- Forms the basis of most modern "intelligent agent" frameworks
- Ideas drawn from a wide range of contexts, including psychology (e.g., Skinner's "Operant Conditioning"), philosophy, neuroscience, operations research, **Cybernetics**
- This will form the main framework of our talk - no need to understand everything
- Some of the things I am going to say might sound too introductory for someone familiar with RL

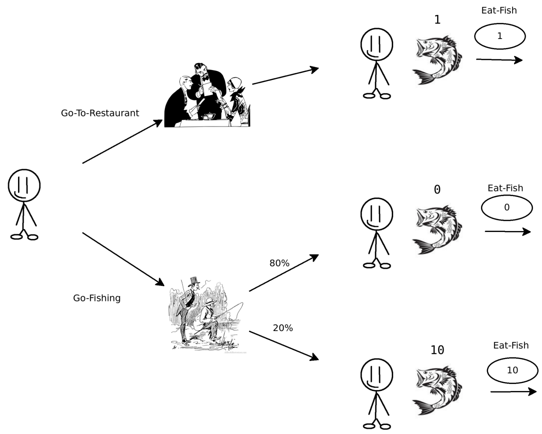
The Markov Decision Process

- The primary abstraction we are going to work with is the Markov Decision Process (MDP).
- MDPs capture the dynamics of a mini-world/universe/environment
- An MDP is defined as a tuple $< S, A, T, R, \gamma >$ where:
  - S, $s \in S$ is a set of states
  - A, $a \in A$ is a set of actions
  - $R : S \times A$, $R(s, a)$ is a function that maps state-actions to rewards
  - $T : S \times S \times A$, with $T(s'|s, a)$ being the probability of an agent moving from state s to state s' after taking a
  - γ is a discount factor - the impact of time on rewards

Policy

- The MDP (the world) is populated by an agent (an actor)
- You can take actions (e.g., move around, move blocks)
- The type of actions you take under a state is called the *policy*
- $\pi : S \times A$, $\pi(s, a) = P(a|s)$, a probabilistic mapping between states and actions
- Finding an optimal policy is *mostly* what the RL problem is all about
- Goal is to maximise long term reward
- $J(\theta) = E_\pi\{\sum_{t=0}^{\infty} R(s, a)\}$

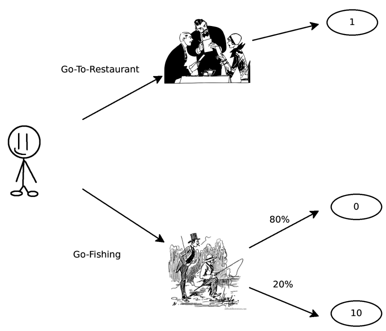
Fishing Toon: Pictorial Depiction



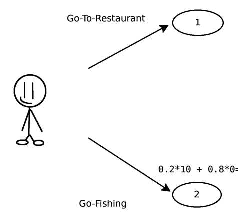
Expected Reward

- Our toon has to choose between two different actions
- `Go-To-Restaurant` or `Go-Fishing`
- We assume that toon is interested in maximising *the expected sum* of happiness/reward
- We can help the toon reason by going through the tree backwards

Reasoning Backwards (1)



Reasoning Backwards (2)



Correct Action

- Toon should go `Go-Fishing`
- **Would you do the same?**
- **Would a pessimist toon do the same?**
- We just went through the following equation:

$$Q^*(s,a) = R(s,a) + \gamma \sum_{s' \in S} T(s'|s,a) \max_{a' \in A} Q^*(s',a')$$

- Looks intimidating - but it's really simple

No model?

- The existence of an a-priori model is extremely unrealistic
- Three major classes of methods that do not assume this:
  - **Model-free**
    - Learn a policy directly by roaming around the world
  - **Model based**
    - Learn a policy indirectly
    - Learn a model of the world
    - Learn a policy by “thinking hard”
  - **Some combination of both (e.g., Dyna)**
    - Learn both a model and a policy from interactions
    - Improve the policy based on fictitious replays

Examples of model free methods

- **Q-learning**: $Q(s,a) \leftarrow Q(s,a) + \eta \left[R(s,a) + \gamma \max_{a' \in A} Q(s',a') - Q(s,a)\right]$
- **SARSA(0)**: $Q(s,a) \leftarrow Q(s,a) + \eta \left[R(s,a) + \gamma Q(s',a') - Q(s,a)\right]$
- **SARSA(1)/MC**: $Q(s,a) \leftarrow Q(s,a) + \eta \left[\mathrm{v}_\tau - Q(s,a)\right]$
  $\mathrm{v}_\tau \leftarrow R(s,a) + \gamma R(s',a') + ...\gamma^2 R(s'',a'') + \gamma^{\tau-1} R(s^\tau,a^\tau)$
- η is a small learning rate, e.g., $\eta = 0.001$

Function Approximation

- States often have some kind of feature (e.g., distance, velocity, etc)
- Too many states in any problem
- Learning based on these features
  - Approximate Q-values, policies...
  - Neural Networks, Linear Regressors, n-tuple networks etc...
  - Global vs Local function approximators

Some issues with model learning

- Errors Compound!
- If you try to think far away, minor errors in the model add up
- You end up with a completely broken future reward/transition structure
  - Unpublished
  - See here http://www.iclr.cc/lib/exe/fetch.php?media=iclr2015:silver-iclr2015.pdf
- Takes time to do

Why learn a model?

- I can't find any good reasons to learn a model
- Learning a model is most of the time seems harder than learning to act based on states directly
- Contradicts with our (my?) intuition
  - We make inferences about the future all the time + logical reasoning, can't be that Q-values are enough!
- Doing any large-scale learning requires tons of tricks (see Deep-Q learning by DeepMind)
  - Maybe we haven't found the right tricks for model-based stuff?
- Computationally - depends too much on exploration vs exploitation parameters

Partial Observability

- Not learning a model is unintuitive
  - We tend to think a lot - not everything is habit
- The Markov property does NOT hold!
- An observation is not enough to recover the state
  - Can you think of an example?
- You cannot observe reality...

POMDPs

- The usual MDP stuff:
  - S, $s \in S$ is a set of states
  - A, $a \in A$ is a set of actions
  - $R : S \times A$, $R(s,a)$ is a function that maps state-actions to rewards
  - $T : S \times S \times A$, with $T(s'|s,a)$ being the probability of an agent moving from state s to state s' after taking a
  - γ is a discount factor - the impact of time on rewards
- Plus an observation model
  - O is a finite set of observations, $o \in O$
  - $\Omega : O \times S \to A$ is an observation function, with $\Omega(o|s',a)$ the probability observation o was emitted when the agent landed in state s after taking action a

The belief state

- Reality is unobservable
- We can keep track of observations and act on their histories

$$h(s) = \{o_0 ... o_n\} \tag{1}$$

*...Or we can find worthy stuff to believe in....

$$b'(s') = (1/\eta)\Omega(o|s', a) \sum_{s' \in S} T(s'|s, a)b(s) \tag{2}$$

$$\eta = \sum_{s' \in S} \Omega(o|s', a) \sum_{s \in S} T(s'|s, a)b(s) \tag{3}$$

$$\tag{4}$$

Acting on beliefs

- Now we can switch states, transitions, rewards with belief states, transitions, rewards
- We can try learning $Q(b(s), a)$, since s cannot be observed
- We have no clue what $b(s)$ is unless we have a model!
- But there is no way we can get this directly because we don't even have samples of $T(s'|s, a)$
- This is still possible (free energy learning schemes make this transparent)

Scaling up

- We need to encode the belief model is some kind of distributed form
- Obviously, Recurrent Neural Networks (more later...)
- Notice that the internal model has a relationship with the real world, but can have any form
- Game of Poker:
  - External Model: The game + opponent model
  - Internal Model: Could be a simple state machine
    - "Believe in AA if you see two raises"

Attending over beliefs

- Most of the things you believe in are unrelated to your current situation
- You must choose to believe in what is relevant
  - Your beliefs about Poker are irrelevant when driving
  - You need to focus only on relevant beliefs!
  - But how?
- Let's call this attention mechanism $\mathcal{T}$
- Possibly learning $Q(\mathcal{T}(b(s)), a)$ (attending over beliefs)??
- Or $Q(\mathcal{T}(o_0 ... o_n), a)$ (attending over observations)??

Research Questions

- What beliefs are worth having?
  - Not all beliefs are created the same
  - Some beliefs are counter-productive
  - What's the impact of reward on broken beliefs?
- **What beliefs are worth attending?**
  - **How do we find out which beliefs to focus on?**
  - **What does it mean to focus on different beliefs?**
  - **How should we learn when we have attention mechanisms in place?**
- How are beliefs transmitted?
  - You can't possibly learn everything from experience
  - Someone has to transmit beliefs directly to you

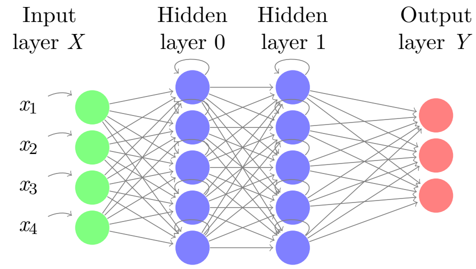
Neural Networks

- Non-linear global function approximators
- Can be learned iteratively through SGD
- Hard to get proofs of convergence for RL, work really well in practice
- Some issues:
  - Catastrophic forgetting
  - Global approximators
  - A new sample will affect all knowledge stored
  - Compare to updating a table (local approximator)

Modern tricks of the trade

- Better initialisation methods (e.g., unsupervisded pre-training, Glorot initialisation)
- Better training methods (e.g., ADAM, RMSPROP)
- Better activation functions/units (e.g., Rectifiers, Maxout)
- Better regularisation methods (e.g., Dropout)
- Better weight sharing/convolutional layers (e.g., Fractional Max-pooling)
- Better hardware (GPUs)

Recurrent Neural Networks



- NNs are ***NOT*** Turing Complete - no internal state
- Cannot handle inputs of arbitrary length not form belief
- Sometimes you can substitute *time* with *space*
- Thus can only act on histories of observations at best
- ... but we can add recurrences...

Recurrent Neural Networks and Beliefs

- Recurrent networks have internal state
- Hence they form some kind of distributed belief
- But have no notion of attention/focus
- Everything gets updated when a new observation comes in
- In effect, spurious correlations (i.e. overfitting) come up all the time
- When acting one has to focus ONLY on the relevant task

Catastrophic forgetting (related)

- Effectively this means you can teach a network how to ride a bike
- Once you start teaching how to ride a car, it will practically forget everything
  - Unless you mix, replay experiences
- But humans don't seem to need that
- You can choose which beliefs to attend over AND what to update
  - Don't think there has been any work on this

Fixing Catastrophic forgetting

- Some work here and there
- We need to discover good mechanisms for doing this in artificial systems
- Nothing really good IMHO
- Rapid context switching/focusing on relevant beliefs is key for general intelligence

BABI Dataset

- Language is strongly non-markov
- Words don't mean anything by themselves
- Context extremely important
- Not RL, beliefs are not necessary, but attention almost certainly is
- 20 different types of questions

Babi Example - the importance of attention (Question Type 3)

Mary moved to the bathroom. Sandra journeyed to the bedroom. Mary got the football there. John went back to the bedroom. Mary journeyed to the office. John journeyed to the office. John took the milk. Daniel went back to the kitchen. John moved to the bedroom. Daniel went back to the hallway. Daniel took the apple. John left the milk there. John travelled to the kitchen. Sandra went back to the bathroom. Daniel journeyed to the bathroom. John journeyed to the bathroom. Mary journeyed to the bathroom. Sandra went back to the garden. Sandra went to the office. Daniel went to the garden. Sandra went back to the hallway. Daniel journeyed to the office. Mary dropped the football. John moved to the bedroom.

Where was the football before the bathroom? office

Babi Example - the importance of attention (Question Type 3)

Mary moved to the bathroom. Sandra journeyed to the bedroom. Mary got the football there. John went back to the bedroom. Mary journeyed to the office. John journeyed to the office. John took the milk. Daniel went back to the kitchen. John moved to the bedroom. Daniel went back to the hallway. Daniel took the apple. John left the milk there. John travelled to the kitchen. Sandra went back to the bathroom. Daniel journeyed to the bathroom. John journeyed to the bathroom. Mary journeyed to the bathroom. Sandra went back to the garden. Sandra went to the office. Daniel went to the garden. Sandra went back to the hallway. Daniel journeyed to the office. Mary dropped the football. John moved to the bedroom.

Where was the football before the bathroom? office

Distance Networks (my stuff)

- Encoding Paragraph
  - Embed Paragraph word by word
  - (Use RNN to get beliefs)
  - MaxPooling (get only most important features of each sentence)
- Encoding Question
  - RNN (get only last state)
- Attend:
  - Subtract a the question vector from each sentence
  - Use a recurrent softmax mechanism to get the most relevant sentences
- Predict:
  - Usual softmax stuff
  - Cross-entropy error

Word embeddings

- Each word in a sentence is given a numerical representation (i.e., “Bob” becomes 1)
- Each numerical representation is linked to a position within a matrix/vector, which points to weights
- Weights (after training) will provide us with a vectorial representation of words
- You tend to get nice properties like: $King - Queen = Man - Woman$

MaxPooling

- Get only the most important features from the words of each sentence
- Pool on sentence level
  - For every sentence, find the each feature maximum (per word) and keep only this
- Let’s see an example - two features, single example, poolsize = 3
  - $[3, 4, 2, 6, 7, 8]$
  - $[2, 1, 4, 5, 1, 2]$

RNN encoder

- For the question sentence:
  - Get the final vector representation as provided by an RNN
- RNNs exactly as described previously
- Rectifers
- GRU/LSTM cells might be better

Attention

- Subtract each sentence vector from each question vector
- Find the absolute, i.e a distance measurement
- Let's call this absolute X
- Some code which I have to convert to math at some point:
- Weight matrix size $[X.shape[0], 2]$
- for each timestep:

$$att = softmax(X \cdot W + b) \tag{5}$$
$$z_0 = att_0 \tag{6}$$
$$z_1 = att_1 \tag{7}$$
$$h_t = z_0 x_{t-1} + z_1 x_t \tag{8}$$

Training

- Goal is to find the most relevant sentence
- Focus on it - get results
- Same mechanism that would work for finding $Q(\mathcal{T}(b(s)), a)$
- But this is just a mechanism I came up with
- Training involves using all 20 Question - not one by one
- Learning one question at a time would require 20 networks
- Or a new *learning* attention mechanism

The End - Rest of the JC