1. $f(x) = \det(xA + I)$

$f'(0) = tr(A)$?

$$h(\lambda) = \det(A - \lambda I) = 0$$
$$= (\lambda - \lambda_1)(\lambda - \lambda_2) \cdot \ldots \cdot (\lambda - \lambda_n) = 0$$
$$= \lambda^n + k_1 \lambda^{n-1} + k_2 \cdot \lambda^{n-2} + \ldots + k_{n-1}\lambda + k_n$$

$$f(x) = x^n \cdot h(-1/x) = x^n\left((-1/x)^n + k_1 \cdot (-1/x)^{n-1} + \ldots + k_{n-1}(-1/x) + \ldots\right)$$
$$= (-1)^n + k_1 \cdot (-1)^{n-1} \cdot x + \ldots + k_{n-1} \cdot (-1) \cdot x^{n-1} + k_n x^n$$

$$f'(x) = (-1)^{n-1} k_1 + 2(-1)^{n-2} k_2 x + \ldots + (n-1)(-1) k_{n-1} x^{n-2} + n k_n x^{n-1}$$

$$f'(0) = (-1)^{n-1} k_1$$

На основу Виетових формула

$$k_1 = (-1)^{n-1}(\lambda_1 + \lambda_2 + \ldots + \lambda_n) = (-1)^{n-1} \cdot \sum_{i=1}^{n} \lambda_i = (-1)^{n-1} tr(A)$$

$$\Rightarrow f'(0) = (-1)^{n-1} \cdot (-1)^{n-1} \cdot tr(A) = (-1)^{2n-2} tr(A) = tr(A)$$

$$\Rightarrow \boxed{f'(0) = tr(A)}$$