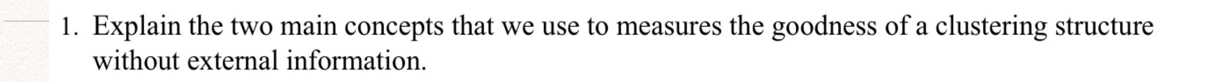


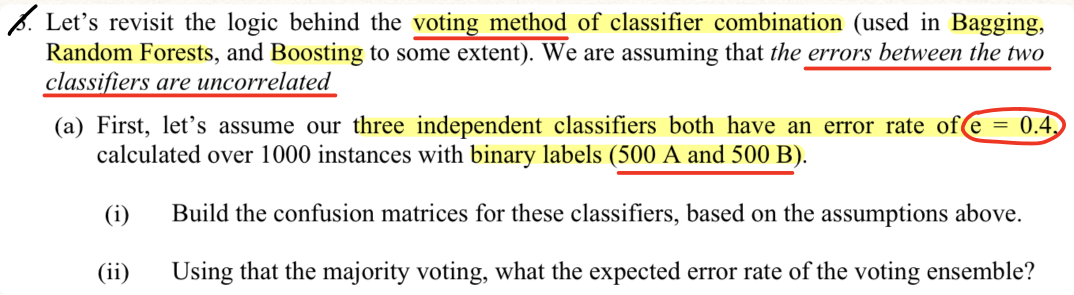
1. Explain the two main concepts that we use to measures the goodness of a clustering structure without external information.

① Cohesiveness

Members of each cluster to be integrated and close to each other as possible.

② Separability

Clusters to be separate & independent as possible from the other clusters.

2. 

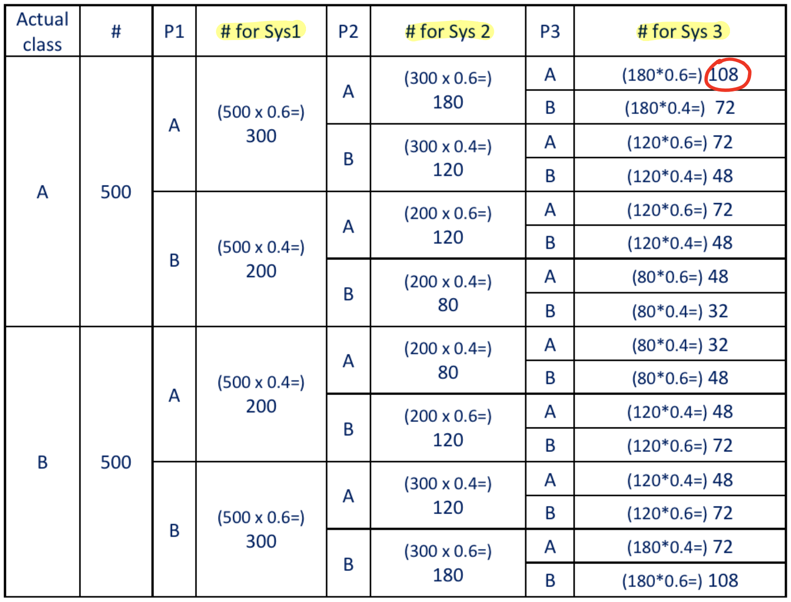
~~5~~. Let's revisit the logic behind the voting method of classifier combination (used in Bagging, Random Forests, and Boosting to some extent). We are assuming that *the errors between the two classifiers are uncorrelated*

(a) First, let's assume our three independent classifiers both have an error rate of e = 0.4, calculated over 1000 instances with binary labels (500 A and 500 B).

(i) Build the confusion matrices for these classifiers, based on the assumptions above.

(ii) Using that the majority voting, what the expected error rate of the voting ensemble?

(a) (i)



Actual class	#	P1	# for Sys1	P2	# for Sys 2	P3	# for Sys 3	
A	500	A	(500 x 0.6=) 300	A	(300 x 0.6=) 180	A	(180*0.6=) 108	A
						B	(180*0.4=) 72	A
				B	(300 x 0.4=) 120	A	(120*0.6=) 72	A
						B	(120*0.4=) 48	B
		B	(500 x 0.4=) 200	A	(200 x 0.6=) 120	A	(120*0.6=) 72	A
						B	(120*0.4=) 48	B
				B	(200 x 0.4=) 80	A	(80*0.6=) 48	B
						B	(80*0.4=) 32	B
B	500	A	(500 x 0.4=) 200	A	(200 x 0.4=) 80	A	(80*0.4=) 32	A
						B	(80*0.6=) 48	A
				B	(200 x 0.6=) 120	A	(120*0.4=) 48	A
						B	(120*0.6=) 72	B
		B	(500 x 0.6=) 300	A	(300 x 0.4=) 120	A	(120*0.4=) 48	A
						B	(120*0.6=) 72	B
				B	(300 x 0.6=) 180	A	(180*0.4=) 72	B
						B	(180*0.6=) 108	B

(Same pred as above)

(ii) For Actual A: 48 + 48 + 48 + 32 = 176

For Actual B: 32 + 48 + 48 + 48 = 176

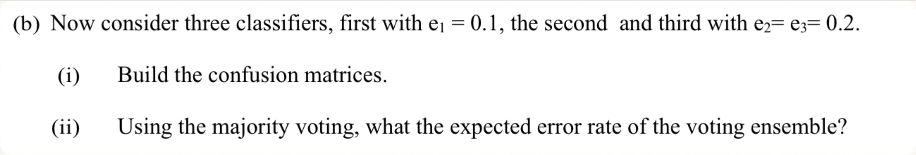
Total error count = 176 + 176 = 352

Expected $ER = \frac{352}{1000} = 0.352 < 0.4$ (individual ER)

learners correct each others' mistake (assume errors uncorrelated)

Alternative:

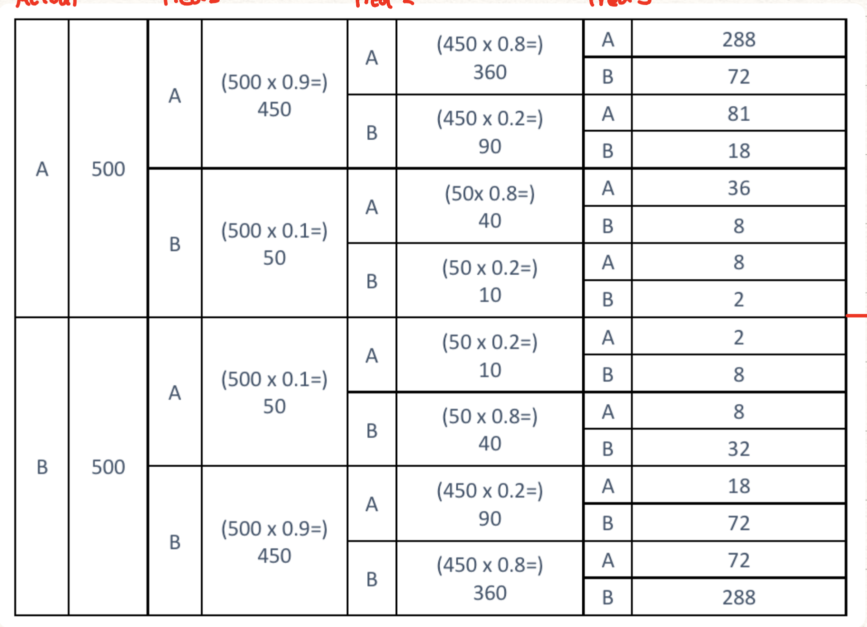
$$\underbrace{\binom{3}{2}(0.4)^2(0.6)}_{\text{two making mistakes}} + \underbrace{\binom{3}{3}(0.4)^3}_{\text{all wrong}} = 0.352$$



(b) Now consider three classifiers, first with $e_1 = 0.1$, the second and third with $e_2 = e_3 = 0.2$.

(i) Build the confusion matrices.

(ii) Using the majority voting, what the expected error rate of the voting ensemble?



Actual		($e_1 = 0.1$) Pred 1		($e_2 = 0.2$) Pred 2		($e_3 = 0.2$) Pred 3		
A	500	A	(500 x 0.9=) 450	A	(450 x 0.8=) 360	A	288	A
						B	72	A
				B	(450 x 0.2=) 90	A	81	A
						B	18	B
		B	(500 x 0.1=) 50	A	(50x 0.8=) 40	A	36	A
						B	8	B
				B	(50 x 0.2=) 10	A	8	B
						B	2	B
B	500	A	(500 x 0.1=) 50	A	(50 x 0.2=) 10	A	2	A
						B	8	A
				B	(50 x 0.8=) 40	A	8	A
						B	32	B
		B	(500 x 0.9=) 450	A	(450 x 0.2=) 90	A	18	A
						B	72	B
				B	(450 x 0.8=) 360	A	72	B
						B	288	B

Error count = (18 + 8 + 8 + 2) + (2 + 8 + 8 + 18)

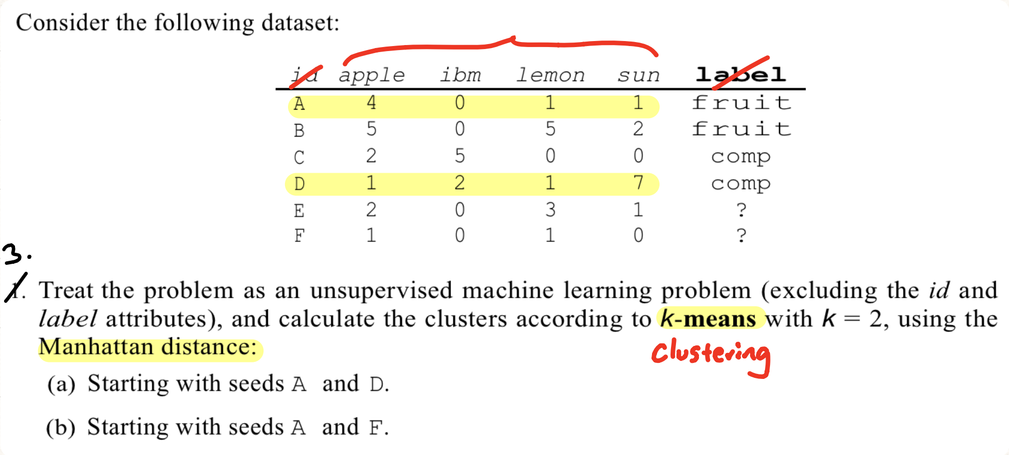
= 72

$ER = \frac{72}{1000} = 0.072 < 0.1$ (sys 1)

Alternative:

$$ER = \underbrace{(0.1)(0.2)^2}_{\text{all wrong}} + \underbrace{(0.9)(0.2)^2}_{S_2 \,\&\, S_3 \text{ wrong}} + \underbrace{2 \times (0.1)(0.2)(0.8)}_{S_1 \,\&\, S_2/S_3 \text{ wrong}}$$

= 0.072



Consider the following dataset:

~~id~~	apple	ibm	lemon	sun	~~label~~
A	4	0	1	1	fruit
B	5	0	5	2	fruit
C	2	5	0	0	comp
D	1	2	1	7	comp
E	2	0	3	1	?
F	1	0	1	0	?

3. ~~1.~~ Treat the problem as an unsupervised machine learning problem (excluding the *id* and *label* attributes), and calculate the clusters according to ***k*-means** with $k = 2$, using the Manhattan distance:

Clustering

(a) Starting with seeds A and D.

(b) Starting with seeds A and F.

(a) Seed A, D (K=2)

⇒ two initial centroids:

C_1 : < 4, 0, 1, 1 >

C_2 : < 1, 2, 1, 7 >

① Calculate distances from each instance to the (of clusters) centroids

$d_M(A, C_1) = 0$

$d_M(A, C_2) = |4-1| + |0-2| + |1-1| + |1-7| = 11$

$d_M(B, C_1) = |5-4| + |0-0| + |5-1| + |2-1| = \underline{6}$

$d_M(B, C_2) = \underline{15}$

$\left.\begin{array}{l} d_M(C, C_1) = 9 \\ d_M(C, C_2) = 12 \end{array}\right\}$ $\left.\begin{array}{l} d_M(D, C_1) = 11 \\ d_M(D, C_2) = 0 \end{array}\right\}$ $\left.\begin{array}{l} d_M(E, C_1) = 4 \\ d_M(E, C_2) = 11 \end{array}\right\}$

$\left.\begin{array}{l} d_M(F, C_1) = 4 \\ d_M(F, C_2) = 9 \end{array}\right\}$

② Assign each instance to the nearest cluster

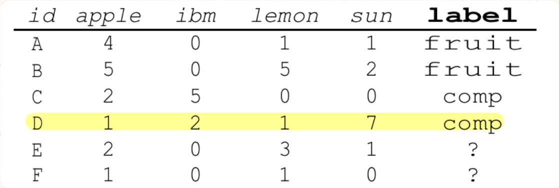
A → C_1 B → C_1 C → C_1 D → C_2

E → C_1 F → C_1

C_1 : A, B, C, E, F C_2 : D

③ Update centroids (average the instances in that cluster)

$C_1 = \left\langle \frac{4+5+2+2+1}{5}, \frac{5}{5}, \frac{1+5+3+1}{5}, \frac{1+2+1}{5} \right\rangle$



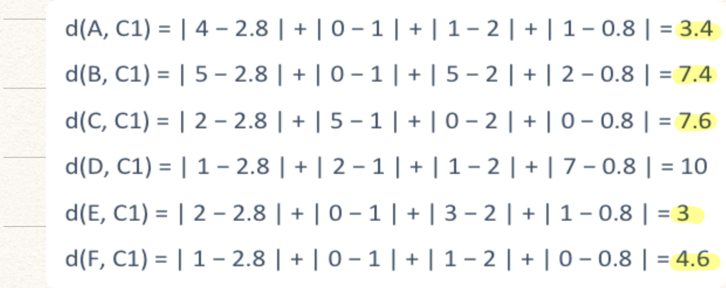
id	apple	ibm	lemon	sun	label
A	4	0	1	1	fruit
B	5	0	5	2	fruit
C	2	5	0	0	comp
D	1	2	1	7	comp
E	2	0	3	1	?
F	1	0	1	0	?

$= \langle 2.8, 1, 2, 0.8 \rangle$

$C_2 = \langle 1, 2, 1, 7 \rangle$ (Just "D")

⇒ repeat! (with new C_1 & C_2)

C_2 hasn't change : reuse dist from last iter.



$d(A, C1) = | 4 - 2.8 | + | 0 - 1 | + | 1 - 2 | + | 1 - 0.8 | = 3.4$

$d(B, C1) = | 5 - 2.8 | + | 0 - 1 | + | 5 - 2 | + | 2 - 0.8 | = 7.4$

$d(C, C1) = | 2 - 2.8 | + | 5 - 1 | + | 0 - 2 | + | 0 - 0.8 | = 7.6$

$d(D, C1) = | 1 - 2.8 | + | 2 - 1 | + | 1 - 2 | + | 7 - 0.8 | = 10$

$d(E, C1) = | 2 - 2.8 | + | 0 - 1 | + | 3 - 2 | + | 1 - 0.8 | = 3$

$d(F, C1) = | 1 - 2.8 | + | 0 - 1 | + | 1 - 2 | + | 0 - 0.8 | = 4.6$

$d_M(A, C_2) = 11$

$d_M(B, C_2) = 15$

$d_M(C, C_2) = 12$

$d_M(D, C_2) = 0$

$d_M(E, C_2) = 11$

$d_M(F, C_2) = 9$

C_1: A, B, C, E, F

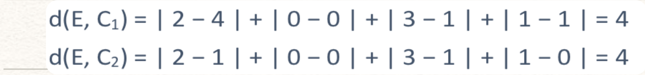
C_2: D

} final assignment

Same as last iter! => Converged! => Stop!

(b) Skip.

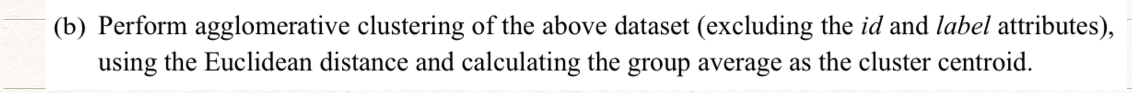
tie:



$d(E, C_1) = | 2 - 4 | + | 0 - 0 | + | 3 - 1 | + | 1 - 1 | = 4$

$d(E, C_2) = | 2 - 1 | + | 0 - 0 | + | 3 - 1 | + | 1 - 0 | = 4$

} randomly pick one



(b) Perform agglomerative clustering of the above dataset (excluding the *id* and *label* attributes), using the Euclidean distance and calculating the group average as the cluster centroid.

In workshop slides.