UNIVERSITY DEPARTMENT OF MATHEMATICS

TMBU, Bhagalpur

Due Date: 30–05-19 | *Linear Algebra* | Session: 2018–20

1. Problems on system of linear equation
   (a) Solve the following system of linear equation using Gaussian elimination method and describe the type of system of linear equations using solutions

   i.
   $$\begin{aligned} 2x+y+3z&=1\\ 2x+6y+8z&=3\\ 6x+8y+18z&=5 \end{aligned}$$

   ii.
   $$\begin{aligned} x+2y-3z+w&=-2\\ 3x-y-2z-4w&=1\\ 2x+3y-5z+w&=-3 \end{aligned}$$

   iii.
   $$\begin{aligned} x+y+2z&=-2\\ 3x-y+14z&=6\\ x+2y&=-5 \end{aligned}$$

   iv.
   $$\begin{aligned} 2x+3y-5z+w&=-3\\ 3x-y-2z-4w&=1 \end{aligned}$$

   (b) For what value of k does the following system of linear equation has infinitely many solutions
   $$\begin{aligned} x+ky+z&=1\\ y+z&=2\\ x+y+2&=3 \end{aligned}$$

2. Problem on finding coordinate in given vector space
   (a) Find the coordinate of $x=(8,-9,6)$ w.r.t the basis $\beta=\{(1,-1,3),(-3,4,9),(2,-2,4)\}$ of $\mathbb{R}^3$.
   (b) Let $\mathcal{P}_3$ be the set of all polynomial of degree less than or equal to 3. Find the coordinate of the vector $f(x)=1+x+x^2+x^3$ and $g(x)=-3+2x^3$ relative to the following basis of $\mathcal{P}_3$
   $$\beta=\{1-x,1+x,x^2-x^3,x^2+x^3\}$$
   (c) Consider the the vector space of all 2×2 matrices of real numbers with basis
   $$\beta=\left\{\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}1&0\\0&-1\end{pmatrix},\begin{pmatrix}0&1\\-1&0\end{pmatrix}\right\}.$$
   Find the coordinate of $A=\begin{pmatrix}5&4\\3&2\end{pmatrix}$ and $B=\begin{pmatrix}8&6\\2&2\end{pmatrix}$, with relative to β.

3. Problem on matrix of linar transformation
   (a) Let T be the linear map from $\mathbb{R}^3\to\mathbb{R}^3$ is defined by
   $$T(x,y,z)=(3x+2y+z,x+3z,y+4z),$$
   then find the matrix of linear transformation given by standard basis.
   (b) Find the matrix of linear transformation that reflects any vector along the the line inclined at an angle θ with $x-$axis.
   (c) Find the matrix of linear transformation that rotates any vector at an angle θ.

 Question 3 continues on the next page. . .

(d) For an integer $n > 0$, let $\mathcal{P}_n$ denote the vector space of polynomials with real coefficients of degree n or less. Define the map $T : \mathcal{P}_2 \to \mathcal{P}_4$ by

$$T(f)(x) = f(x^2).$$

i. Determine if T is a linear transformation.

ii. If it is, find the matrix representation for T relative to the basis $B = \{1, x, x^2\}$ of $\mathcal{P}_2$ and $C = \{1, x, x^2, x^3, x^4\}$ of $\mathcal{P}_4$.

(e) Let $\mathcal{P}_1$ be the vector space of all real polynomials of degree 1 or less. Consider the linear transformation $T : \mathcal{P}_1 \to \mathcal{P}_1$ defined by

$$T(ax + b) = (3a + b)x + a + 3,$$

i. With respect to the basis $B = 1, x$, find the matrix of the linear transformation T.

ii. Find a basis B of the vector space $\mathcal{P}_1$ such that the matrix of T with respect to B is a diagonal matrix.

iii. Express $f(x) = 5x + 3$ as a linear combination of basis vectors of B.