Space-time varying Poisson auto-regressive model

An application to Covid-19 cases at the England local authority level

P. Alaimo Di Loro S. Sahu D. Bohning

StanConnect through Space and Time

October 31, 2022

Outline

Data-driven models for the COVID-19 pandemic

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Standard Poisson auto-regressive specification

Let $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T$ be series of **detected cases** over $s_1, \ldots, s_I$ regions
Poisson auto-regression:

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\begin{aligned}
Y_{st} \,|\, \boldsymbol{y}_{s(1:t-1)} &\sim Poi\left(\lambda_{st}\right) \\
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- $\widetilde{r}_s$ are region-specific **auto-regressive coefficients**
  Determine the **epidemic growth rate** r_s
- b_s are location specific **baseline rates**
  It is a long-term **endemic rate** and accounts for **hidden infections**

Standard Poisson auto-regressive specification

In the **standard framework**

- $\widetilde{r}_s$ is **constant** in time
- $\widetilde{r}_s < 1$ to ensure **stationarity**
- $\widetilde{r}_s$'s **independently estimated** across regions

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- When **regions** are small we want to **borrow information**

Extending Poisson auto-regression for epidemics

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- Include **infection latency** up to some lag τ

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$$

- Time-varying location-specific rates $\widetilde{r}_{st},\ b_{st}$
- Include infection latency up to some lag τ
- Non-exploding condition through a **non-reinfection window:**

$$
d_{st} = 1 - \frac{\sum_{i=1}^{\widetilde{t}} y_{s(t-i)}}{\text{pop}_s}
$$

Modeling the coefficients

Model **observed (covariates $\boldsymbol{X}$, $\boldsymbol{V}$)** and **unobserved (random effects)** heterogeneity

- **Generalized linear mixed model**

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Auto-regressive coefficient

$$\log\left(\tilde{r}_{st}\right) = lr_0 + \boldsymbol{X\beta} + \phi_{st}$$

Baseline rate

$$\log\left(b_{st}\right) = lOff_s + \boldsymbol{V\eta} + \psi_{st}$$

CAR-AR on $\mathbf{\Phi}$, $\mathbf{\Psi}$ (Rushworth et al., 2014)

Independence across random effects $\mathbf{\Phi} \perp \mathbf{\Psi}$

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Spatial dependence $\leftrightarrow$ **Leroux model**

$$\phi_t^*, \psi_t^* \sim \mathcal{N}\left(\mathbf{0}\,,\, \sigma^2 \boldsymbol{Q}(\alpha, \boldsymbol{W})^{-1}\right)$$

with $\boldsymbol{Q}(\alpha, \boldsymbol{W}) = (\alpha(\boldsymbol{D} - \boldsymbol{W}) + (1 - \alpha)\boldsymbol{I}_I)$

Temporal dependence $\leftrightarrow$ **AR(1) structure**

$$\begin{aligned} \phi_1, \psi_1 &\sim \mathcal{N}\left(\mathbf{0}\,,\, \sigma^2 \boldsymbol{Q}(\alpha, \boldsymbol{W})^{-1}\right) \\ \phi_t, \psi_t &\sim \mathcal{N}\left(\rho \cdot \phi_{t-1}\,,\, \sigma^2 \boldsymbol{Q}(\alpha, \boldsymbol{W})^{-1}\right), \quad t = 2, \ldots, T \end{aligned}$$

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  2. **Prior** choice is driven only by **prior information or regularization**

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- Doesn't ask much **in exchange**
  1. Implements the **NUTS** that adaptively sets sampling paths
  2. **Prior** choice is driven only by **prior information or regularization**
- It is **honest** and **sincere**
  1. It **efficiently explore** the full posterior (even when complex)
  2. Provides **incredibly useful diagnostic tools**

Why STAN?

How can we facilitate `STAN` job for the proposed model?

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- The computational complexity of the **posterior evaluation**
- The **geometry** of the posterior

Sparse Leroux

Determinant and a **quadratic form on**:

$$\boldsymbol{Q}(\alpha, \boldsymbol{W}) = (\alpha(\boldsymbol{D} - \boldsymbol{W}) + (1 - \alpha)\boldsymbol{I}_I)$$

- These two operations scale with I^3 and I^2
- When I increases they become **rapidly costly**

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We can exploit the **sparsity pattern** in $\boldsymbol{Q}(\alpha, \boldsymbol{W})$

- Sparse-representation $\rightarrow \phi_t^\top \boldsymbol{Q}(\alpha, \boldsymbol{W})\phi_t$ only non-zero entries

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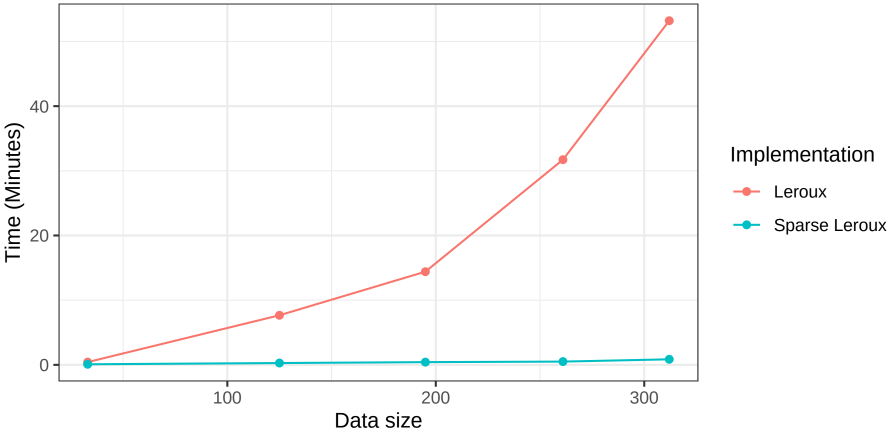
- Sparse-representation $\rightarrow \phi_t^\top \boldsymbol{Q}(\alpha, \boldsymbol{W})\phi_t$ only non-zero entries
- Drawing from Jin et al. (2005) in the **proper CAR**:

$$\boldsymbol{Q}(\alpha, \boldsymbol{W}) = (\boldsymbol{I}_I - \alpha\,(\boldsymbol{I}_I - \boldsymbol{D} + \boldsymbol{W})) \rightarrow$$

$$\rightarrow |\boldsymbol{Q}(\alpha, \boldsymbol{W})| \propto \prod_{i=1}^{I} (1 - \alpha\lambda_i)$$

where λ_i are the eigen-values of $(\boldsymbol{I}_I - \boldsymbol{D} + \boldsymbol{W})$

Sparse Leroux - 1000 iterations



Non-centered parametrization

- The **NUTS** samples and adapts its tuning parameters in the **unconstrained parameters space**
- Sampling is the more efficient the more the posterior in the unconstrained space is **uncorrelated/approximately Gaussian**

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The **non-centered** parametrization helps (often a lot)

`Model block`

$$\phi_t^*, \psi_t^* \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{Q}(\alpha, \boldsymbol{W})^{-1}\right), \quad t = 1, \ldots, T$$

`Transformed parameters block`

$$\phi_1, \psi_1 = \phi_1^*$$

$$\phi_t, \psi_t = \rho \cdot \phi_{t-1} + \phi_t^*, \quad t = 2, \ldots, T$$

Application on the real data

Full dataset

- $I = 313$ districts of England
- $8th$ of May 2020 to the $8th$ of March 2021, $T = 43$
- $N = 13'459$ records

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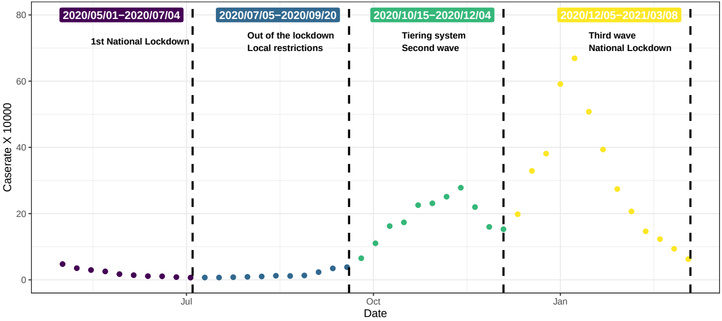

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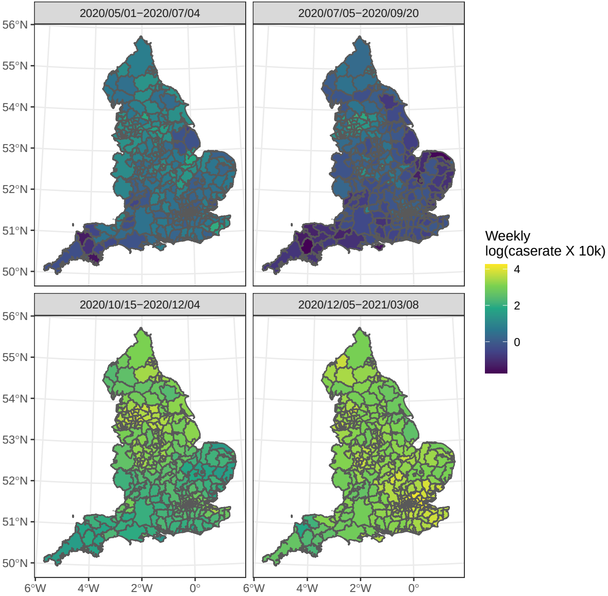

Modeling UK covid-19 cases

$$\lambda_{st} = \left(\left(\sum_{j=1}^{\tau} w_j \cdot y_{s(t-j)} \right) \cdot \widetilde{r}_{st} + b_{st} \right) \cdot d_{st}$$

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Fixed components

- No reinfections
- Latency up to $\tau = 3$

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Covariates

$\boldsymbol{X}$ Population density, jsa, median house price
Tier level (II, III, IV)

$\boldsymbol{V}$ Only intercept

Off_s Population size divided by 10000

Application - STAN parameters

Estimation

- 4 parallel chains
- 8000 iterations each
- Thinning of 2 to reduce memory burden
- $80\% - 20\%$ train-test split

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Runtime of ≈ 40 hours

Application - Parameter estimates

Weights	$\hat{\theta}$	CI_{95}	*Rhat*
w_1	0.70	$(0.67, 0.69)$	1.00
w_2	0.19	$(0.16, 0.18)$	1.00
w_3	0.11	$(0.08, 0.10)$	1.00

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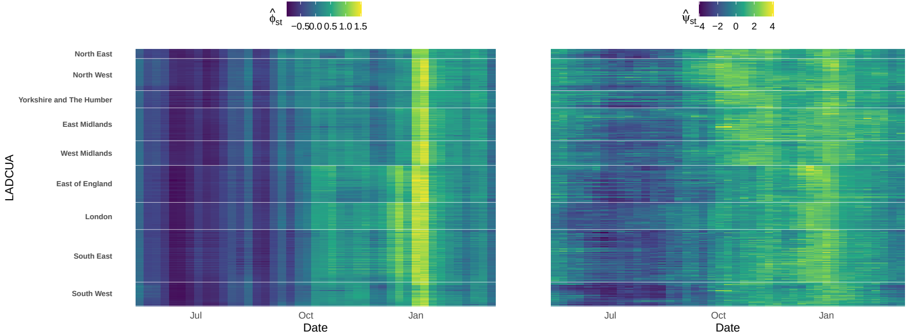
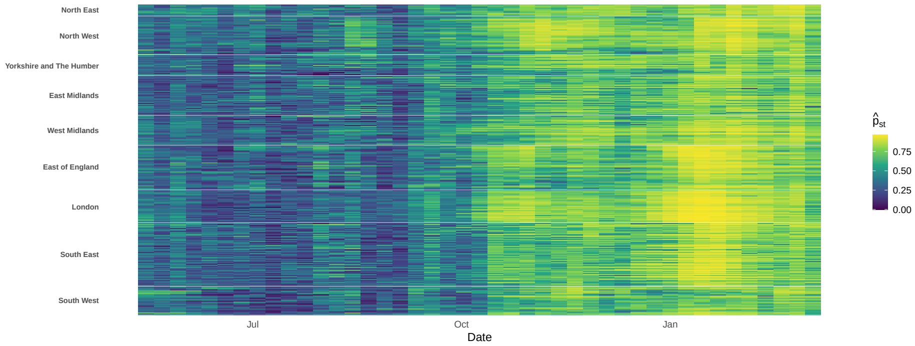
Coefficients	$\hat{\beta}$	CI_{95}	*Rhat*
lr_0	-0.55	$(-0.64, -0.45)$	1.01
Log-pop density	0.06	$(0.05, 0.08)$	1.00
Log-jsa	0.18	$(0.01, 0.03)$	1.00
Log-hprice	-0.06	$(-0.08, -0.04)$	1.00
Tier II	-0.08	$(-0.14, -0.03)$	1.00
Tier III	-0.23	$(-0.30, -0.16)$	1.00
Tier IV	-0.24	$(-0.32, -0.16)$	1.01

Application - Random effects

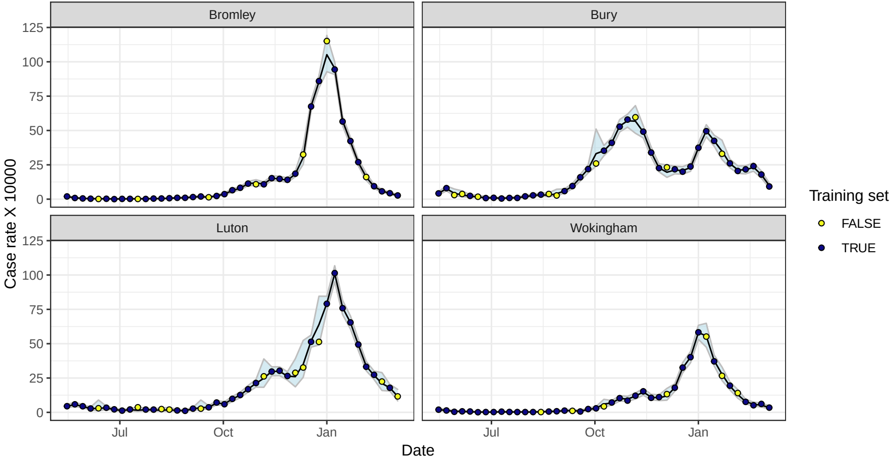


Application - Autoregressive and baseline components

$$\hat{p}_{st} = \frac{\sum_{i=1}^{\tau} \left(w_i \cdot y_{s,t-i}\right) \cdot \hat{r}_{s,t}}{\sum_{i=1}^{\tau} \left(w_i \cdot y_{s,t-i}\right) \cdot \hat{r}_{s,t} + \hat{b}_{st}}$$



Application - Predictive performances



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Conclusion

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- ✓ Efficient implementation of the model in `STAN`
- ✓ Estimation of the efficacy of the tiering policies and description of the space-time heterogeneity

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- ✓ Efficient implementation of the model in `STAN`
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Work in progress

- × Estimate (rather than fix) the latency τ
- × Include dependence across the two sets of random effects $\mathbf{\Phi}$ and $\mathbf{\Psi}$
- × Implement some variable-selection scheme

Main references

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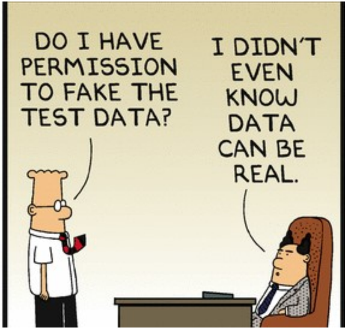
THANK YOU FOR YOUR ATTENTION!



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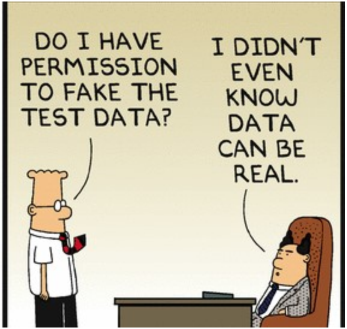
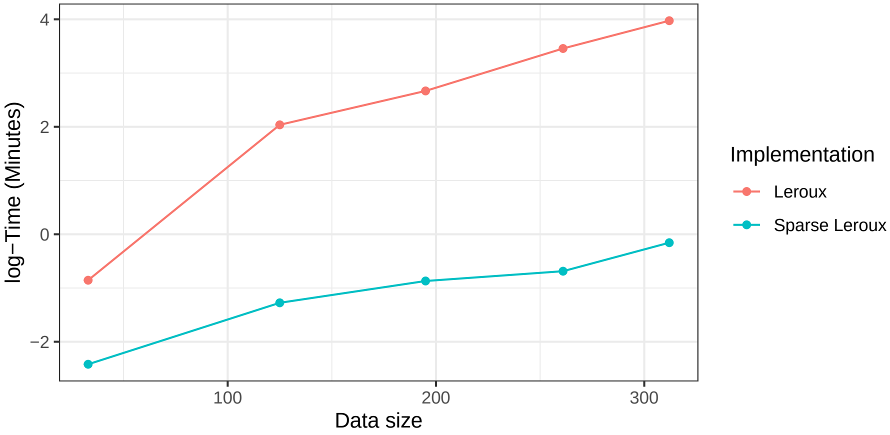


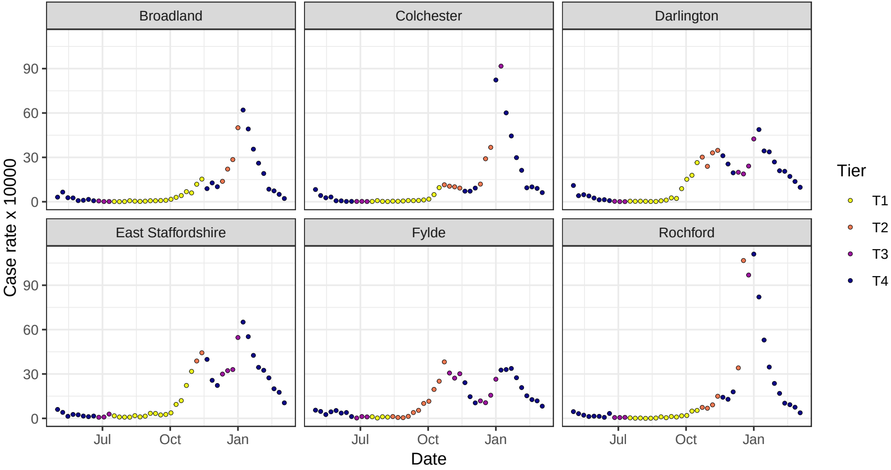
Questions?

APPENDIX

Appendix - Sparse Leroux 1000 iterations



Data example



Prior and hyper-prior settings

$$\lambda_{st} = \left(\left(\sum_{j=1}^{\tau} w_j \cdot y_{s(t-j)} \right) \cdot \widetilde{r}_{st} + b_{st} \right) \cdot d_{st}$$

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Priors

$$lr_0 \sim \mathcal{N}\left(-0.5, 1\right), \quad \boldsymbol{\beta} \sim \mathcal{N}\left(\boldsymbol{0}, \boldsymbol{I}_k\right), \quad \boldsymbol{\eta} \sim \mathcal{N}_\nu\left(\boldsymbol{0}, \boldsymbol{I}_\nu\right),$$
$$\alpha_\phi, \alpha_\psi \sim Beta(1,1), \quad \rho_\phi, \rho_\psi \sim Unif(0,1),$$
$$\sigma_\phi \sim \mathcal{N}^+(0, 0.1), \quad \sigma_\psi \sim \mathcal{N}^+(0, 0.5),$$
$$\boldsymbol{w} \sim Dir(\boldsymbol{1}_\tau)$$

Simulations - Prior predictive check

Restricted dataset

- 33 districts within the region of *London*
- 3rd of July 2020 to the 21st of February 2021
- $B = 300$ simulated sets

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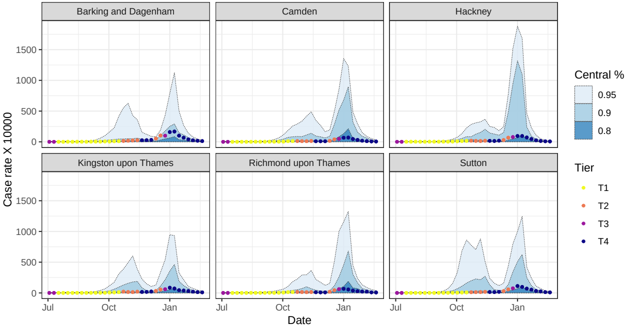
- 3rd of July 2020 to the 21st of February 2021
- $B = 300$ simulated sets



Simulations - Parameter recovery

Estimation

- 2 parallel chains with 2000 iterations each
- 80% – 20% train-test split

Simulations - Parameter recovery

Estimation

- 2 parallel chains with 2000 iterations each
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Metric	lr_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
$\text{r}\bar{\text{M}}\text{SE}$	0.09	0.02	0.02	0.07	0.08	0.08	0.08	0.16	0.12
$\bar{\text{C}}\text{ov}_{95}$	0.95	0.94	0.96	0.95	0.94	0.95	0.92	0.92	0.94

Metric	α_ϕ	α_ψ	ρ_ϕ	ρ_ψ	σ_ϕ	σ_ψ
$\text{r}\bar{\text{M}}\text{SE}$	0.21	0.18	0.21	0.17	0.05	0.08
$\bar{\text{C}}\text{ov}_{95}$	0.93	0.94	0.93	0.93	0.91	0.94

Simulations - Predictive performances

Metric	**In-Sample**	**Out-of-Sample**
$\overline{\text{RMSE}}$	0.03	0.09
$\overline{\text{Cov}}_{95}$	0.99	0.96

Application - Parameter estimates

Baselines	$\hat{\theta}$	CI_{95}	*Rhat*
lr_0	-0.55	$(-0.64, -0.45)$	1.01
η_0	0.43	$(0.35, 0.53)$	1.00

CAR-AR	ϕ			ψ		
	$\hat{\theta}$	CI_{95}	*Rhat*	$\hat{\theta}$	CI_{95}	*Rhat*
α	0.998	$(0.997, 0.999)$	1.01	0.98	$(0.97, 0.99)$	1.00
ρ	0.64	$(0.58, 0.70)$	1.01	0.57	$(0.54, 0.61)$	1.01
σ	0.18	$(0.17, 0.20)$	1.01	0.98	$(0.92, 1.05)$	1.00

Application - Predictive performances

Metric	In-sample	Out-Of-sample
RMSE	≈ 0	0.02
$\bar{\text{Cov}}_{95}$	0.99	0.94
$\bar{\text{IW}}_{95}$	64.3	159