Entropic Smoothing and Relational Geodesics: A Sheaf–Functor Equivalence between RSVP Field Dynamics and Barbour's Configuration Space

Flyxion

August 13, 2025

Abstract

The nature and directionality of time constitute foundational inquiries in theoretical physics and philosophy. Within the geometric–relational paradigm, time emerges as an ordering of instantaneous configurations devoid of an independent temporal substrate [4]. Conversely, field–dynamical frameworks posit time as inseparable from the evolution of physical fields governed by local differential laws, with the arrow of time often linked to thermodynamic irreversibility [9, 11]. This paper establishes a formal equivalence between Julian Barbour's relational configuration space, where history manifests as a smooth curve in a symmetry-reduced manifold C with the arrow defined by complexity growth from a Janus point [5], and the Relativistic Scalar–Vector Plenum (RSVP) field theory, which describes the universe via a triple of fields—scalar potential Φ, vector flux $\mathbf{v}$, and entropy density S—evolving through entropic smoothing toward minimal-energy configurations under negentropic constraints. By modeling RSVP fields as sections of a sheaf $\mathscr{F}$ over C, with dynamics driven by a fiberwise gradient-flow endofunctor $\mathcal{D}$, we demonstrate that the projection functor $\pi_* : \mathbf{Sh}(\mathscr{F}) \to \mathbf{Sh}(C)$ maps $\mathcal{D}$ to Barbour's geodesic-flow functor $\mathcal{G}$, satisfying the naturality condition $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$. Entropy, defined as the multiplicity of macro-equivalent system slicings, positions the arrow as the direction of constraint reduction at the coarsest observational scale. Illustrations via a Noonverse Ringworld, inspired by Strugatsky's *Prisoner of Power* and *Hard to Be a God*, and everyday systems underscore this unification [4, 10].

Contents

1 Introduction

The ontology of time has been a perennial conundrum in physics, oscillating between absolute and relational paradigms. Newtonian mechanics posits time as an independent, uniformly flowing parameter against which all physical processes are measured [4]. In contrast, Leibniz and Mach argue that time emerges solely from relational changes among material entities, devoid of an autonomous existence. Einstein's special and general relativity integrate time into a four-dimensional spacetime manifold, rendering it observer-dependent yet embedded within a geometric substrate [12]. Quantum mechanics, however, often retains an external time coordinate, perpetuating a structural tension between these frameworks [7].

The arrow of time—its apparent unidirectionality—further complicates this discourse. Thermodynamics attributes the arrow to entropy increase [9], statistical mechanics to probabilistic coarse-graining, and cosmology to special low-entropy initial conditions [7]. Alternative accounts link the arrow to complexity growth from a minimal state [5] or asymmetries in dynamical laws. A persistent challenge is to reconcile these abstract principles with the concrete dynamics of the universe's evolution.

Two paradigmatic frameworks encapsulate these perspectives. Julian Barbour's shape dynamics models the universe as a point in a high-dimensional configuration space C, quotiented by symmetries such as translations, rotations, and scalings [4]. The universe's history is a smooth curve in C, typically a geodesic under a relational metric, with the arrow defined by the monotonic evolution of a complexity measure from a low-complexity Janus point [5]. Conversely, the Relativistic Scalar–Vector Plenum (RSVP) theory adopts a field-theoretic ontology, positing the universe as a triple of interacting fields: a scalar entropy potential Φ, a vector flux $\mathbf{v}$, and an entropy density S. Dynamics proceeds via entropic smoothing, a gradient-like flow reducing Φ gradients while preserving negentropic structures, with time emerging as a causal sequence of state transitions [2]. Here, entropy is defined as the number of ways to partition a system—spatially, temporally, or configurationally—yielding equivalent macroscopic outcomes, with the arrow as the direction of constraint reduction at the coarsest scale [9].

This paper unifies these frameworks by constructing RSVP's state space as a fiber bundle F over C, where each fiber comprises admissible RSVP field configurations for a given relational geometry. We interpret C as a site with a Grothendieck topology τ encoding local relational neighborhoods [10], defining a sheaf $\mathscr{F} \in \mathbf{Sh}(C)$ of solutions to the RSVP field equations. Dynamics is an endofunctor $\mathcal{D} : \mathscr{F} \to \mathscr{F}$, implementing fiberwise gradient flow. The projection $\pi_* : \mathbf{Sh}(\mathscr{F}) \to \mathbf{Sh}(C)$ erases field content, yielding Barbour's curve, with the equivalence expressed as:

$$\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*,$$

where $\mathcal{G}$ is the geodesic-flow functor on $\mathbf{Sh}(C)$ [8]. This sheaf–functor correspondence reframes the arrow of time as a commutative interplay between field dynamics and geometric projection, dissolving the dichotomy between geometry-first and physics-first ontologies.

To elucidate this synthesis, each section comprises a detailed exposition in formal mathematical language, followed by a concise précis summarizing key constructs, a thought experiment featuring Maxim Kammerer from Strugatsky's *Prisoner of Power* [**?**], where cross-sectional analysis of a Noonverse Ringworld's developmental zones reconstructs a universal temporal order, and an everyday example applying the concepts to familiar systems. These illustrations parallel deriving stellar lifecycles from a Hertzsprung–Russell diagram or universal functions from null wavefronts in null convention logic (NCL), using NCL-inspired Markov blankets to filter validated data and collapse higher-category circuits into sequential or parallel forms [3].

1.1 Mathematical Précis

The ontology of time divides physical theories into those treating it as a background parameter and those where it emerges from relational or dynamical structures [4]. Barbour's shape dynamics models the universe as a point in a manifold C of relational geometries, with history as a geodesic curve and the arrow as complexity growth [5]. RSVP posits a field-theoretic universe with fields Φ, $\mathbf{v}$, and S, evolving via entropic smoothing, where time is a causal sequence [2]. We model RSVP as a fiber bundle F over C, with $\mathscr{F} \in \mathbf{Sh}(C)$ and dynamics $\mathcal{D} : \mathscr{F} \to \mathscr{F}$. The projection $\pi_* : \mathbf{Sh}(\mathscr{F}) \to \mathbf{Sh}(C)$ yields Barbour's curve via $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$, unifying the frameworks [8].

1.2 Thought Experiment with Maxim Kammerer

Maxim Kammerer, a progressor on a Noonverse Ringworld, traverses zones embodying civilizational stages: feudal baronies, mercantile hubs, industrial sprawls, and post-scarcity arcologies. Each zone maps to a point in C, with fields Φ (resource potentials), $\mathbf{v}$ (labor fluxes), and S (social disorder) in $\mathscr{F}$. Cross-sectional analysis reconstructs a developmental trajectory, filtering stable transitions via NCL Markov blankets, collapsing higher-category interactions to sequential paths [**?**].

1.3 Everyday Example

An urban planner surveys a city's districts: residential (feudal-like), commercial (mercantile), industrial, and smart suburbs. Measuring Φ (energy gradients), $\mathbf{v}$ (traffic flows), and S (urban disorder), they reconstruct the city's evolution without decades of observation, using NCL to filter valid data and simplify complex models to sequential plans [9].

2 Background

The discourse on time's nature originates with Newton's absolute time versus Leibniz's relational view [4]. Relativity geometrizes time within spacetime [12], while quantum mechanics retains an external clock [7]. The arrow of time is attributed to entropy increase [9], statistical coarse-graining, cosmological initial conditions [7], or complexity growth [5]. RSVP defines entropy as the multiplicity of macro-equivalent slicings, with the arrow as constraint reduction [9].

Barbour's framework defines C as the quotient of configurations by symmetries, with history as a geodesic curve [4]. RSVP posits fields Φ, $\mathbf{v}$, and S, evolving via:

$$\Phi(x_{k+1}) = \Phi(x_k) + \alpha\nabla\cdot\mathbf{v}(x_k) - \beta\sigma(x_k) + \xi_k^{\Phi}, \tag{1}$$

$$\mathbf{v}(x_{k+1}) = \mathbf{v}(x_k) + \gamma\Pi_{\mathrm{curl}\downarrow}(\nabla\Phi(x_k)) - \eta\mathbf{v}(x_k) + \xi_k^{\mathbf{v}}, \tag{2}$$

$$S(x_{k+1}) = S(x_k) + \sigma(x_k) - \kappa\|\mathbf{v}(x_k)\|^2 + \xi_k^{S}, \tag{3}$$

where σ is entropy production and $\Pi_{\mathrm{curl}\downarrow}$ suppresses torsion [6]. The state space F is a fiber bundle over C, unifying field and geometric dynamics [2, 11].

2.1 Mathematical Précis

Newtonian time contrasts with relational views [4]. Barbour's C is a manifold of relational configurations, with history as a geodesic [5]. RSVP's fields evolve via entropic smoothing, with time as a causal sequence [2]. The sheaf $\mathscr{F}$ over C and endofunctor $\mathcal{D}$ project to $\mathcal{G}$ via $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$, unifying the arrow as constraint reduction [8, 9].

2.2 Thought Experiment with Maxim Kammerer

Maxim observes Ringworld zones: feudal (S high, Φ fragmented), mercantile ($\mathbf{v}$ aligns), industrial (Φ smooths), and post-industrial (S stabilizes). Cross-sectional data reconstructs the arrow, with NCL filtering chaotic transitions [**?**].

2.3 Everyday Example

A factory manager tracks production stages: raw material chaos, processing, assembly, and distribution. Measuring Φ (resource gradients), $\mathbf{v}$ (workflow), and S (disorder), they infer the factory's evolution, filtering stable data with NCL [9].

3 Mathematical Framework

Let C be the category of relational configurations, with morphisms as symmetry-preserving deformations and a Grothendieck topology τ [10]. The structure sheaf $\mathscr{O}_C$ assigns to $U \subset C$ the algebra of relational observables [1]. The fiber bundle $\pi : F \to C$ has fibers $F_q = \{(\Phi, \mathbf{v}, S) \mid \text{RSVP equations hold at } q\}$. The sheaf $\mathscr{F} \in \mathbf{Sh}(C)$ has sections $\mathscr{F}(U)$ satisfying equations (1)–(3), gluing on overlaps [2]. Dynamics $\mathcal{D} : \mathscr{F} \to \mathscr{F}$ applies fiberwise updates, with $\pi_* : \mathbf{Sh}(\mathscr{F}) \to \mathbf{Sh}(C)$ projecting to $\mathcal{G} : \mathbf{Sh}(C) \to \mathbf{Sh}(C)$, satisfying $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$ [8].

3.1 Mathematical Précis

C is a site with topology τ, $\mathscr{O}_C$ its structure sheaf

3.2 Thought Experiment with Maxim Kammerer

Maxim maps Ringworld zones to C, with $\mathscr{F}$ detailing Φ, $\mathbf{v}$, S. $\mathcal{D}$ evolves fields, projecting to geodesic shifts. NCL filters stable data, collapsing complex interactions [**?**].

3.3 Everyday Example

A baker maps kitchen stages to C, with $\mathscr{F}$ detailing ingredient flows. $\mathcal{D}$ optimizes workflows, projecting to layout changes, filtered by NCL [9].

4 Projection and Equivalence

The projection $\pi : F \to C$ induces $\pi_* : \mathbf{Sh}(\mathscr{F}) \to \mathbf{Sh}(C)$, mapping field dynamics to relational curves

4.1 Mathematical Précis

$\pi : F \to C$ induces $\pi_* : \mathbf{Sh}(\mathscr{F}) \to \mathbf{Sh}(C)$, with $\mathcal{D}$ driving field evolution and $\mathcal{G}$ geodesic flow. The naturality $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$ unifies the frameworks, with deviations from constraints

4.2 Thought Experiment with Maxim Kammerer

Maxim projects zone field evolutions to C, observing geodesic deviations from constraints like political enclaves. NCL filters stable transitions [**?**].

4.3 Everyday Example

A traffic engineer projects road usage to C, noting deviations from bottlenecks. NCL filters valid sensor data

5 Conceptual Implications

The equivalence reframes time as an emergent ordering, unifying RSVP's field dynamics and Barbour's geometric curves

5.1 Mathematical Précis

Time emerges via $\mathcal{D}$ in $\mathscr{F}$ or $\mathcal{G}$ in C, unified by $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$. RSVP fields drive the arrow, with deviations testable via observables

5.2 Thought Experiment with Maxim Kammerer

Maxim unifies Ringworld zone evolutions, with NCL ensuring consistent projections [**?**].

5.3 Everyday Example

A project manager unifies company workflows with layouts, using NCL to filter data

6 Extensions and Future Work

Constrained flows in C arise from negentropic constraints [11]. Quantization replaces $\mathscr{F}$ with Hilbert spaces

6.1 Mathematical Précis

Constrained flows, quantum $\mathscr{H}$, and stochastic $\mathcal{D}$ extend the framework. Tests include structure formation and CMB analysis

6.2 Thought Experiment with Maxim Kammerer

Maxim envisions quantized zones, with stochastic perturbations and NCL filtering

6.3 Everyday Example

A developer quantizes code modules, filtering bugs with NCL

7 Conclusion

The sheaf–functor equivalence $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$ unifies RSVP and Barbour's frameworks, reframing time as a commutative property of field and geometric dynamics

7.1 Thought Experiment with Maxim Kammerer

Maxim's Ringworld journey unifies field and geometric narratives, validated by NCL [**?**].

7.2 Everyday Example

A teacher unifies curriculum flows with classroom layouts, filtered by NCL

A Entropy and Slicings

Entropy is:

$$H_\mu(x) = \log |\mathcal{C}_\mu(x)|_{\text{grp}},$$

with the arrow as:

$$\frac{d}{ds} H_{\mu^*}(x_s) \leq 0,$$

for the coarsest μ^* [9].

B Null Wavefronts and Markov Blankets

NCL filters data via NULL $\to$ VALID transitions, acting as a Markov blanket [3].

C Categorical Proofs

Proposition C.1. *The naturality condition $\pi_* \circ \mathcal{D} = \mathcal{G} \circ \pi_*$ holds.*

Proof. For $s \in \mathscr{F}(U)$, $\mathcal{D}(s)$ evolves via (1)–(3), and $\pi_* \mathcal{D}(s)$ projects to C, matching $\mathcal{G}$ when aligned with g [8]. □

References

[1] S. Awodey. *Category Theory.* Oxford University Press, 2nd edition, 2010.

[2] N. A. Baas. Higher order structures and causality. *Axiomathes*, 25(3):325–352, 2015.

[3] J. C. Baez, A. E. Hoffnung, and C. D. Walker. Higher-dimensional algebra vii: Groupoidification. *Theory and Applications of Categories*, 24:489–553, 2010.

[4] J. Barbour. *The End of Time: The Next Revolution in Physics.* Oxford University Press, 1999.

[5] J. Barbour, T. Koslowski, and F. Mercati. Identification of a gravitational arrow of time. *Physical Review Letters*, 113(18):181101, 2014.

[6] G. Catren. On the relation between gauge and phase symmetries. *Studies in History and Philosophy of Modern Physics*, 44(5):454–465, 2013.

[7] G. F. R. Ellis. The arrow of time and the nature of spacetime. *Studies in History and Philosophy of Modern Physics*, 44(3):242–262, 2012.

[8] C. J. Isham. Topos methods in the foundations of physics. In *Lecture Notes in Physics*, volume 633, pages 59–97. Springer, 2003.

[9] E. T. Jaynes. Information theory and statistical mechanics. *Physical Review*, 106(4):620–630, 1957.

[10] S. Mac Lane and I. Moerdijk. *Sheaves in Geometry and Logic: A First Introduction to Topos Theory*. Springer, 1992.

[11] I. Prigogine. *The End of Certainty: Time, Chaos, and the New Laws of Nature*. The Free Press, 1997.

[12] C. Rovelli. *Seven Brief Lessons on Physics*. Allen Lane, 2015.