

# Planetary Autocatalysis: Hierarchical Selection, Entropy Fields, and the Emergence of Complexity

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## Abstract

The origin of life is often framed as a probabilistic barrier: a leap from a “random chemical soup” to a minimal protocell of  $10^9$  bits within Earth’s finite timespan. Prevailing models, grounded in rate–distortion theory and Kolmogorov complexity, treat this transition as an information-theoretic bottleneck, requiring persistent retention of vanishingly rare configurations. This perspective is argued to be deeply incomplete. It neglects the structuring role of mineral catalysis, planetary-scale energy flows, and tidal dynamics, which collectively transform early Earth into a distributed chemical reactor—a planetary-scale autocatalytic system. Rather than a sudden jump in complexity, life likely emerged through hierarchical autocatalysis, where localized catalytic sets merge upward, producing incremental selection of homeostatic subsystems long before lipid membranes enclosed the first protocells.

This hypothesis is formalized using the Relativistic Scalar–Vector Plenum (RSVP) framework, introducing entropy fields  $S$  to describe the directional flow of free energy and information across mineral surfaces and tidal cycles. Hierarchical autocatalytic closure is modeled via merging operators in a symmetric monoidal category, with emergent modules represented as colimits under functorial composition. A corresponding PDE formalism captures the scaling law of effective search compression as a function of catalytic surface area and network connectivity. By contrasting this planetary-reactor model with Jeremy England’s dissipative adaptation theory and Sara Walker’s causal information framework, deep commonalities in constraint-driven emergence are highlighted.

Finally, an analogy to artificial intelligence is drawn: scaling the surface area of reaction parallels the hierarchical optimization of strategies in large-scale models, where local constraints rapidly merge into global architectures via selective pruning and modular compression. Both systems exploit distributed search under structured energy gradients, collapsing combinatorial complexity into functional order. This synthesis reframes abiogenesis as a scaling phenomenon rather than a fluke, linking life’s emergence, entropy-driven dynamics, and the architecture of intelligence under a single mathematical umbrella.

## Keywords

Autocatalysis, RSVP Theory, Prebiotic Chemistry, Rate–Distortion, Category Theory, AI Emergence

## Preface: Why Scaling Laws of Life and Intelligence Are the Same Problem

The scaling laws governing biological evolution and artificial intelligence share profound similarities, rooted in hierarchical processes of selection and compression. This monograph explores these parallels through the lens of planetary autocatalysis, demonstrating how Earth’s geophysical environment facilitated the emergence of complexity. By integrating geochemical evidence, mathematical formalisms, and computational analogies, a unified framework is provided for understanding life’s origins and its continuity with intelligent systems.

## 0.1 Introduction

The origin of life remains one of science’s most enduring puzzles. Traditional narratives portray a leap from a “random chemical soup” to the first protocell as a rare statistical event—a dramatic phase transition requiring the spontaneous assembly of informational complexity on the order of  $10^9$  bits. Recent frameworks borrow from rate–distortion theory, Kolmogorov complexity, and computational analogies to quantify this improbability, arguing that abiogenesis demands vast timescales and highly favorable conditions. While these approaches offer mathematical elegance, they suffer from a fatal simplification: they model prebiotic chemistry as an unstructured combinatorial lottery.

This monograph challenges that view. The early Earth was not a homogeneous pool of molecules—it was a structured, dynamic, energy-driven system, shaped by geological and planetary-scale processes. Minerals such as montmorillonite and kaolinite acted as selective catalysts, promoting template-directed polymerization of RNA-like oligomers [1]. Wet–dry cycles at tidal flats, driven by Earth’s unusually large moon, provided periodic concentration gradients, favoring condensation reactions essential for peptide

and nucleotide assembly. Mechanical trituration, hydrothermal fluxes, and mineral templating collectively expanded the reactive surface area by orders of magnitude, transforming the entire planet into a distributed, open chemical network. Under these conditions, hierarchical autocatalytic systems could emerge gradually, long before lipid membranes enclosed the first protocells.

This perspective reframes abiogenesis as a scaling phenomenon rather than an isolated improbability. Complexity does not arise in a single leap but through a cascade of autocatalytic mergers, in which local catalytic sets combine into higher-order structures under persistent energy flux and constraint. Instead of a random search through chemical state space, early Earth implemented a biased search algorithm, where mineral scaffolds, tidal forcing, and chemical feedback loops served as natural priors guiding the trajectory of molecular evolution.

To formalize this hypothesis, the Relativistic Scalar-Vector Plenum (RSVP) framework is introduced, which represents the emergence of order as a flow in coupled scalar, vector, and entropy fields  $\Phi$ ,  $\mathbf{v}$ ,  $S$ . These fields capture the distribution of free energy, directional fluxes, and informational density across catalytic landscapes. Building on this, a category-theoretic model of hierarchical autocatalysis is constructed, where chemical species and catalytic sets form objects, interactions define morphisms, and emergent modules arise as colimits in a symmetric monoidal category. The process of compositional closure is interpreted through sheaf-theoretic gluing, ensuring local-to-global coherence under distributed constraints.

This approach is contrasted with two influential frameworks: Jeremy England’s dissipative adaptation theory, which models self-organization as a consequence of entropy production under driven conditions, and Sara Walker’s causal information framework, which emphasizes the role of top-down constraints in the origin of life. While these models share important conceptual territory, neither fully accounts for the planetary-scale autocatalytic architecture described here, nor do they connect emergence in prebiotic chemistry to hierarchical optimization phenomena observed in artificial intelligence.

Indeed, the analogy to AI is instructive. Just as early Earth increased the “surface area” of catalytic interactions through tidal cycling and mineral evolution, modern AI systems scale through parallel search and hierarchical pruning, discarding vast numbers of strategies to refine architectures under resource constraints. Both processes instantiate a universal law of constraint-driven emergence, whereby local exploration and selective amplification lead to global order. This suggests a deep continuity between life’s origin and the evolution of intelligence—natural or artificial—within a shared mathematical substrate.

In what follows, this argument is developed in seven parts. The shortcomings of the probabilistic bottleneck view are examined and a planetary-reactor model grounded in geochemistry and mineral catalysis is introduced (Part I and II). The dynamics of hierarchical autocatalysis are then formalized through RSVP entropy fields, PDE systems, and category-theoretic operators (Part III and IV). Part V situates this framework relative to existing theories, while Part VI explores its implications for artificial intelligence and technological evolution. The monograph concludes with testable predictions for prebiotic chemistry and scaling laws for adaptive systems, suggesting that the emergence of complexity—whether biochemical or computational—is neither an accident nor a miracle, but the natural trajectory of systems that harness energy gradients under structured constraints.



**Part I**

**Theoretical Foundations**





# Chapter 1

## The Puzzle of Life's Origins

The statistical improbability narrative: phase transitions and complexity thresholds.

Critique of the informational jump model [2].

Ignorance of:

Clay mineral catalysis [1].

Tidal trituration by Earth's moon (wet-dry cycles, surface area expansion).

Distributed planetary-scale homeostatic systems prior to lipid membranes.

Thesis: The Earth was a giant open cell before protocells.



## Chapter 2

# From Geochemistry to Catalytic Landscapes

Robert Hazen's mineral evolution: how minerals create structured catalytic spaces.

Cooperative catalysis on layered silicates and oxide surfaces.

Evidence for wet-dry cycling and mineral templating in RNA polymerization.

Kauffman's autocatalytic sets and Stuart Kauffman's work on combinatorial closure.



## Part II

# A Planetary Reactor Model



## Chapter 3

# Tidal Triturations and Global Autocatalysis

Modeling tidal fracturing and grinding as a surface area multiplier.

Wet-dry cycles: evaporative concentration, polymer formation.

Geological timescale argument: planetary-scale parallelism vastly accelerates search.





## Chapter 4

# Distributed Proto-Metabolism

Homeostatic networks without membranes:

Energy gradients across mineral scaffolds.

Chemical reservoirs in tidal flats as primitive compartments.

The Earth as a distributed autocatalytic system: a giant open reactor.

### 4.1 From Random Soup to Structured Reactor

Traditional origin-of-life models often depict early Earth as a well-mixed, unstructured chemical soup, with abiogenesis framed as a rare stochastic event. Such models implicitly treat molecular interactions as independent and spatially homogeneous. In contrast, geological, chemical, and planetary evidence suggests that prebiotic chemistry was structured, heterogeneous, and energy-driven.

Minerals such as clays, zeolites, and pyrites provided not only surfaces for molecular adsorption but also template-directed catalysis, stabilizing nascent polymers and biasing reactions toward functional configurations [1]. These surfaces dramatically increase the effective reactive surface area, creating local autocatalytic networks that persist far longer than isolated molecules in solution.

### 4.2 Tidal Forces and Mechanical Trituration

Earth's unusually large moon introduced periodic tidal cycles, generating wet-dry cycles at coastal and intertidal regions. These cycles concentrated solutes, promoted condensation reactions, and repeatedly exposed molecular assemblies to differential stress and partial degradation, enabling selection of more robust chemical structures.

Mechanical trituration from tidal agitation, hydrothermal circulation, and sedimentary movements further amplified the interaction network, effectively turning the entire planetary surface into a distributed, hierarchical chemical reactor. This environmental structuring suggests that early Earth itself functioned as a massive protocell, where local autocatalytic sets could form, merge, and stabilize over geological timescales.

### 4.3 Autocatalytic Sets and Hierarchical Emergence

Following Kauffman's notion of autocatalytic sets, it is recognized that self-sustaining chemical networks can arise long before membrane encapsulation. Reactions within these networks obey closure under catalysis, meaning every reaction in the set is catalyzed by some member of the set itself.

By nesting these sets hierarchically across mineral surfaces and geophysical gradients, the system can:

1. Increase information retention, as locally stabilized molecules persist through repeated cycles.
2. Promote modular assembly, where small autocatalytic networks combine into larger, higher-order modules.
3. Enhance robustness, as redundancy and distributed interactions mitigate local failure modes.

These processes resemble hierarchical optimization in AI, where most candidate strategies are discarded immediately, and only productive configurations propagate to higher levels of abstraction.

## 4.4 Temporal Constraints and Phase Transitions

Abiogenesis occurred within finite temporal windows, constrained by the early Earth’s geochemical and astronomical timeline:

- Formation of liquid water:  $\sim 4.404$  Gy ago

- First sterilizing impacts (Theia and Moneta):  $\sim 4.51$ – $4.47$  Gy ago

- Earliest microfossils:  $\sim 3.465$  Gy ago

Hierarchical autocatalysis allows for a progressive accumulation of complexity, mitigating the need for improbable instantaneous assembly. In this framework, a phase transition in chemical order can occur when local autocatalytic modules reach sufficient density and connectivity, effectively bootstrapping pre-Darwinian evolution across the planetary network.

## 4.5 Integration with Mineral and Environmental Factors

Robert Hazen’s work emphasizes mineral evolution as a driver of chemical complexity, showing that new mineral phases created feedback loops that progressively stabilized functional molecules. By integrating tidal trituration, wet–dry cycling, and mineral-catalyzed autocatalysis, a planetary-scale scaffold for prebiotic chemistry is proposed:

- Local interactions: catalysis and templating on mineral surfaces

- Regional scaling: merging of autocatalytic sets along energy gradients

- Global integration: distributed networks stabilized by environmental periodicities

Under this view, lipid membranes are a later-stage innovation, emerging to encapsulate already functional autocatalytic modules rather than being the initial limiting factor in the origin of life.

## 4.6 Summary

Part II reframes the early Earth not as a chaotic “soup” but as a structured, hierarchical, and energy-driven system. Tidal forces, mineral surfaces, wet–dry cycles, and hierarchical autocatalysis collectively form a planetary-scale reactor in which prebiotic information accumulates, persists, and propagates. This perspective establishes the physical and chemical context required for the formalism introduced in Part III, linking entropy, information flow, and hierarchical merging across scales.

## Part III

# Mathematical Formalisms



## Chapter 5

# RSVP Entropy Fields and Emergent Structure

Review RSVP variables:

Scalar potential field  $\Phi$ .

Vector flow field  $\mathbf{v}$ .

Entropy density  $S$ .

General entropy balance equation with catalytic amplification:

$$\partial_t S + \nabla \cdot \mathbf{J}_S = \sigma - \sum_k \kappa_k \dot{I}^{(k)}.$$



## Chapter 6

# Hierarchical Autocatalysis and PDEs

Population PDE for strategy densities:

$$\partial_t \rho_s = D_s \nabla^2 \rho_s + \sum_{s'} M_{ss'} \rho_{s'} + \Gamma_s[\rho] - \Lambda_s[\rho].$$

$$\partial_t \rho_S \supset \sum_{A \subset S} \kappa_{A \rightarrow S} \prod_{s \in A} \rho_s.$$

$$N_{\text{eff}}^{(k+1)} \sim \alpha N_{\text{eff}}^{(k)}.$$





## Chapter 7

# Rate–Distortion and Information Bottlenecks

Rate–distortion inequality for hierarchical levels:

$$\mathcal{R}^{(k)}(x, t) = \eta^{(k)} \frac{H_{\text{avail}}^{(k)}}{D^{(k)}} \geq r_*^{(k)}.$$

Lower effective entropy cost per bit at higher levels.

Formal analogy to information bottleneck optimization in AI.

### 7.1 Overview

To move beyond conceptual arguments about planetary-scale autocatalysis, a formal framework is introduced that captures the emergence and flow of informational order in prebiotic systems. This approach builds on the Relativistic Scalar–Vector Plenum (RSVP) framework, extending it to represent coupled chemical, energetic, and informational dynamics in a distributed catalytic environment. Hierarchical autocatalysis is further encoded within a symmetric monoidal category, allowing description of compositional closure, merging of catalytic modules, and the scaling of complexity under structured constraints.

The key idea is to model the early Earth as a dynamical field, where molecules, catalytic sets, and environmental gradients interact according to both physical and informational laws. Complexity arises not from rare stochastic events but from directed flows of energy and information, constrained by planetary geometry, mineral surfaces, and tidal cycles.

### 7.2 RSVP Entropy Fields

Let  $\Phi$  denote the scalar field representing local free energy density available to chemical reactions at spatial position  $\mathbf{x}$  and time  $t$ . Let  $\mathbf{v}$  denote the vector field representing directional flux of molecules or catalytic intermediates along energy gradients, and let  $S$  denote the entropy field, measuring local informational disorder.

A coupled dynamical system is posited:

$$\begin{aligned} \frac{\partial \Phi}{\partial t} + \nabla \cdot (\Phi \mathbf{v}) &= -\lambda S + Q_\Phi \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\nabla \Phi + \nu \nabla^2 \mathbf{v} + \mathbf{F}_{\text{catalysis}} \\ \frac{\partial S}{\partial t} &= \alpha \|\nabla \Phi\|^2 - \beta S + \mathcal{I}_{\text{autocatalysis}} \end{aligned}$$

Where  $\alpha, \beta, \lambda, \nu$  are scaling coefficients encoding energetic dissipation, molecular diffusion, and entropy decay.

$Q_\Phi$  represents external energy input (e.g., tidal cycles, solar irradiation, hydrothermal fluxes).

$\mathbf{F}_{\text{catalysis}}$  encodes biasing forces introduced by mineral templates and catalytic surfaces.

$\mathcal{I}_{\text{autocatalysis}}$  captures the information amplification arising from hierarchical closure of chemical modules, analogous to error-corrected accumulation in AI search spaces.

This system formalizes the intuition that chemical order arises along directed energy flows, with entropy reduction occurring locally as autocatalytic modules stabilize and merge.

### 7.3 Hierarchical Autocatalysis as Category-Theoretic Composition

Chemical species and catalytic sets are modeled as objects  $\mathcal{M}$  in a symmetric monoidal category  $\mathcal{C}$ , where  $\otimes$  denotes combinatorial merging of species or modules into higher-order assemblies.

Morphisms  $f : \mathcal{M} \rightarrow \mathcal{N}$  encode chemical transformations or autocatalytic interactions.

Emergent autocatalytic modules correspond to colimits of subdiagrams:

$$\mathcal{M} = \varinjlim (\mathcal{D} \subset \mathcal{C})$$

Intuitively,  $\mathcal{M}$  represents a stable module arising from the convergence of compatible catalytic networks. Functorial mappings  $F : \mathcal{C} \rightarrow \text{Obs}$  encode observable properties such as stability, replication potential, or entropy reduction.

### 7.4 Entropy-Respecting Merge Operators

To formalize information preservation under hierarchical autocatalysis, a merge operator  $\oplus$  acting on catalytic modules is defined:

$$\oplus : (\mathcal{M}_i, S_i) \times (\mathcal{M}_j, S_j) \mapsto (\mathcal{M}_k, S_k)$$

where  $S_k \leq (S_i + S_j)/2$ , enforcing entropy-respecting combination. This ensures that the merger of two modules does not increase disorder, mirroring rate-distortion constraints in prebiotic chemistry:

$$R(D) \geq I_{\text{module}}/\tau_{\text{effective}}$$

where  $\tau_{\text{effective}}$  is the effective persistence time for functional molecules in the merged module.

### 7.5 PDE–Category Coupling

By coupling the RSVP fields with the category-theoretic formalism, autocatalytic growth is described as a field-dependent colimit formation:

$$\frac{d\mathcal{M}}{dt} = \varinjlim (F_{\Phi}(\mathcal{C}_i), F_{\Psi}(\mathcal{C}_i), F_S(\mathcal{C}_i))$$

Here,  $F_{\Phi}, F_{\Psi}, F_S$  are functors mapping field values to merging probabilities of catalytic sets. This allows calculation of emergent stability, information throughput, and effective search compression across the planetary network.

### 7.6 Scaling Laws and Hierarchical Compression

Let  $A$  denote the effective catalytic surface area (enhanced by tidal cycles, mineral fractality, and hydrothermal flows). Then the information accumulation rate scales sublinearly with area due to redundancy and local constraints:

$$R_{\text{eff}} \sim \eta \frac{H_{\text{prebiotic}}}{D} \left( \frac{A}{A_0} \right)^{\gamma}, \quad 0 < \gamma < 1$$

where  $\eta$  is the efficiency of retention and  $\gamma$  encodes hierarchical overlap between catalytic modules. This mirrors strategy compression in AI, where local explorations are selectively merged into global architectures while discarding most low-value pathways.

## 7.7 Summary of Formalism

1. RSVP entropy fields  $\Phi, \mathbf{v}, S$  capture directional flow of free energy and information.
2. Chemical species and catalytic modules form objects and morphisms in a symmetric monoidal category, enabling hierarchical composition.
3. Entropy-respecting merge operators enforce physical constraints analogous to rate-distortion limits.
4. PDE–category coupling provides a dynamic framework linking local autocatalytic interactions to global emergent order.
5. Scaling laws demonstrate how planetary-level constraints compress combinatorial complexity, paralleling hierarchical optimization in AI.

This framework bridges geochemical plausibility, information theory, and category-theoretic structure, providing a quantitative scaffold for understanding planetary-scale autocatalysis and the emergence of life as a constraint-driven computation.



## Part IV

# Category Theory and Emergence



## Chapter 8

# Category-Theoretic Encoding of Autocatalytic Merging

Objects: modules (strategies).

Morphisms: catalytic interactions.

Colimits as emergent higher-level modules:

$$S = \varinjlim D.$$

Sheaf-theoretic gluing: distributed constraints integrating into global coherence.

### 8.1 Overview

Having developed a framework for planetary-scale autocatalysis (Part II) and a formal RSVP–category-theoretic description (Part III), this model is now situated within the broader theoretical landscape of origin-of-life research. In particular, it is contrasted with:

1. Jeremy England’s driven-dissipation theory, which emphasizes the thermodynamic favorability of life-like structures under sustained energy flows.
2. Sara Walker’s causal information and algorithmic frameworks, which conceptualize life as an emergent system of information processing constrained by physical causality.

The goal is to clarify points of alignment and divergence, highlighting how hierarchical autocatalysis on planetary scales extends, complements, or challenges these approaches.

### 8.2 England’s Driven-Dissipation Theory

England proposes that under continuous energy input, systems naturally evolve toward states that maximize entropy production through self-organization, effectively making life-like behavior statistically favored.

Alignment:

Both England’s framework and the hierarchical autocatalysis model recognize the centrality of energy fluxes in stabilizing complex structures. Tidal trituration, hydrothermal gradients, and mineral surfaces serve as physical drivers that maintain non-equilibrium states conducive to life.

The concept of entropy reduction via autocatalytic closure mirrors England’s argument that dissipation drives systems toward robust, organized states.

Divergence:

England’s approach is largely statistical and coarse-grained, focusing on general thermodynamic principles rather than explicit combinatorial and structural constraints.

Hierarchical autocatalysis emphasizes physical and chemical scaffolds, including mineral templating and modular chemical networks, which can generate specific functional outcomes and pre-Darwinian “memory” that England’s theory does not explicitly account for.

Mathematically, driven-dissipation can be expressed as:

$$\frac{dS_{\text{env}}}{dt} \geq \sum_i \frac{\dot{Q}_i}{T_i}$$

where  $S_{\text{env}}$  is environmental entropy and  $\dot{Q}_i$  is heat flux from energy sources. In RSVP terms, energy dissipation influences  $\Phi$  and  $\mathbf{v}$ , modulating autocatalytic growth rates  $\kappa$ .

### 8.3 Walker’s Causal Information Framework

Walker emphasizes that life is defined by causal information flows, where past states actively constrain future states to sustain functional processes. This aligns with rate-distortion considerations in prebiotic systems: information must persist long enough to propagate functional configurations.

Alignment:

Hierarchical autocatalysis explicitly models information retention and compression via mineral scaffolds, wet–dry cycles, and tidal feedback loops.

Both approaches highlight the importance of constraints and memory in emergent systems, rather than treating life as purely accidental.

Divergence:

Walker’s formalism is algorithmic and abstract, focusing on general causal networks without explicit reference to spatially extended chemical environments.

The model incorporates physical constraints (surface area, diffusion, degradation rates) that define the realizable causal pathways on early Earth, making the framework empirically grounded.

Mathematically, causal information flow can be expressed as:

$$I_{\text{causal}}(X_t \rightarrow X_{t+\Delta t}) = H(X_{t+\Delta t}) - H(X_{t+\Delta t}|X_t)$$

which parallels RSVP’s entropy field  $S$  and autocatalytic merge operators, representing local reduction of uncertainty in the chemical network.

### 8.4 Integrating England and Walker into the Hierarchical Autocatalysis Framework

1. Energy Flow  $\rightarrow$  Autocatalytic Activation: England’s driven dissipation can be mapped to  $\Phi$  and  $\mathbf{v}$ , providing the energetic drive for module formation.
2. Information Flow  $\rightarrow$  Stability of Modules: Walker’s causal information corresponds to  $S$  and the functorial evaluation of module persistence, quantifying how molecular configurations influence future network states.
3. Hierarchical Merging  $\rightarrow$  Emergent Complexity: By embedding both theories into a category-theoretic scaffold, it is formalized how local energy-driven order and causal memory accumulate into robust planetary-scale structures, bridging thermodynamic and information-theoretic perspectives.

### 8.5 Advantages of the Hierarchical Autocatalysis Model

Spatially Explicit: Captures mineral templates, tidal zones, and diffusion-limited interactions.

Hierarchically Scalable: Accounts for multi-level merging of autocatalytic sets, from molecular to planetary scales.

Information-Centric: Explicitly quantifies entropy reduction, retention, and module merging, linking prebiotic chemistry to computational analogs in AI.

Testable Predictions: Suggests specific environments (e.g., mineral-rich intertidal zones) as likely sites for early autocatalytic closure, guiding experiments.



## 8.6 Limitations and Open Questions

Quantitative parameters  $(\alpha, \beta, \eta)$  remain uncertain.

Coupling between thermodynamics, information flow, and planetary geometry requires further simulation and experimental validation.

Integration with evolving lipid membranes and protocellular encapsulation remains to be formalized in a full dynamical model.

## 8.7 Summary

Part IV situates planetary-scale hierarchical autocatalysis within the broader landscape of origin-of-life theory. By synthesizing England's thermodynamic drive and Walker's causal information metrics, a unified formalism is offered that is spatially explicit, information-aware, and hierarchically compositional. This framework clarifies how prebiotic chemistry could scale to robust, life-like complexity without invoking implausible statistical flukes, setting the stage for subsequent AI-assisted modeling and quantitative predictions in Part V.



**Part V**

**Comparative Frameworks**



## Chapter 9

# Contrasting Theories of Origins

Critique of statistical-physics-first models:

The paper’s “complexity jump” argument.

Jeremy England’s dissipative adaptation:

Systems under energy flux self-organize toward states that dissipate energy more effectively.

Compare to planetary-scale autocatalytic selection.

Sara Walker’s causal information theory:

Origins as emergence of top-down causal constraints.

Integration with RSVP: fields encode causal regularities.

### 9.1 Critique of Endres (2025)

Endres (2025) proposes that life’s emergence is an extreme informational improbability, requiring algorithmic compression estimates and rate-distortion theory to justify the formation of a minimal protocell ( $\sim 10^9$  bits of complexity) within  $\sim 500$  Myr on early Earth. While this approach is mathematically elegant, it suffers from several critical limitations:

#### 1. Neglect of environmental autocatalysis:

The analysis largely treats Earth as a homogeneous “chemical soup,” ignoring the geochemically structured and energy-fluctuating environments that provide natural selection at the molecular level.

Clay minerals, hydrothermal gradients, and tidal trituration massively increase effective surface area, providing templating, stabilization, and repeated combinatorial sampling.

#### 2. Omission of pre-Darwinian homeostasis:

Networks of autocatalytic reactions can maintain persistent functional states long before lipid membranes or true cells exist.

The Earth itself acts as a planetary-scale cell, where environmental constraints select for robust chemical cycles.

#### 3. Unnecessary appeal to extraterrestrial intervention:

Directed panspermia or terraforming hypotheses are philosophically interesting but scientifically superfluous, violating Occam’s razor.

Empirical evidence from mineralogical catalysis, intertidal chemistry, and geochemically repeated cycles provides a sufficient mechanism for assembling complex prebiotic networks.

#### 4. Misestimation of informational thresholds:

Endres’ use of  $10^9$  bits assumes a fully compressed protocell, whereas actual prebiotic evolution proceeds via incremental accumulation of functional modules, each stabilized by environmental scaffolds and autocatalytic closure.

## 9.2 Hierarchical Autocatalysis as Prebiotic AI Analogy

A conceptual analogy is proposed: early Earth chemistry functions as a distributed, hierarchical AI, with strategies represented by autocatalytic sets:

1. Local constraints (mineral surfaces, wet-dry cycles, UV gradients) act like memory and error-filtering modules, rapidly discarding ineffective combinations.
2. Hierarchical merging occurs as small autocatalytic modules combine into larger networks, analogous to strategy compression and pruning in artificial intelligence.
3. This process scales surface area of interaction, maximizing exploration of chemical space while retaining functional information, without requiring implausible spontaneous assembly of a fully formed protocell.

Formally, in RSVP terms:

$$S(\mathbf{x}, t) \xrightarrow{\text{local autocatalysis}} \mathcal{M}(\mathbf{x}, t) \xrightarrow{\text{hierarchical merging}} \Phi(\mathbf{x}, t), \mathbf{v}(\mathbf{x}, t)$$

$S$ : entropy field of prebiotic chemistry

$\mathcal{M}$ : autocatalytic module field (local information retention)

$\Phi, \mathbf{v}$ : emergent scalar and vector fields representing global prebiotic structure

This formalizes how incremental information retention and environmental selection reduce the effective informational improbability, bypassing Endres’ extreme combinatorial assumptions.

## 9.3 Integration with England and Walker

1. England: Energy fluxes (tidal, geothermal) drive dissipation, feeding autocatalytic modules and stabilizing functional networks.
2. Walker: Causal information flow is instantiated in the retention and propagation of functional chemical motifs, analogous to “memory” in hierarchical modules.
3. RSVP framework: Provides a quantitative mapping from local module entropy to global emergent fields, linking thermodynamics, information flow, and hierarchical chemical structure.

## 9.4 Implications for AI and Prebiotic Modeling

AI models can simulate massive combinatorial spaces, mimicking early chemical exploration and module pruning.

Hierarchical, information-driven algorithms provide insight into how chemical networks can self-organize, offering testable predictions about which environmental niches are most likely to have yielded prebiotic complexity.

This approach reframes prebiotic Earth as a natural computation engine, with environmental constraints playing the role of algorithmic heuristics, rather than invoking extraterrestrial “designer interventions.”

## 9.5 Summary

Endres’ improbability framework overestimates the informational barrier by ignoring structured, geochemically-driven selection.

Hierarchical autocatalysis provides a natural, scalable, and testable mechanism for prebiotic self-organization.

AI analogies illustrate how local pruning and hierarchical merging can accelerate the accumulation of complexity.

No extraterrestrial origins are necessary; Earth’s environmental topology and energy flows suffice to generate the necessary prebiotic structures.

## Part VI

# AI and Artificial Autocatalysis





## Chapter 10

# Scaling Laws: Life vs AI

Hierarchical autocatalysis  $\approx$  hierarchical model compression.

Merging operators  $\approx$  distillation and modularization in neural nets.

Aggressive pruning in evolution  $\approx$  beam search and lottery ticket hypothesis.

Local  $\rightarrow$  global selection as a universal scaling law:

Speed of convergence  $\propto$  Surface area of local trials.



# Chapter 11

## Simulating the Analogy

Chemical–AI dual experiment design:

CRN emulator with merging and pruning.

Hierarchical evolutionary algorithm with colimit-based module formation.

Metrics:

Entropy decay.

Percolation thresholds for module formation.

Scaling of effective search dimensionality.

### 11.1 Overview

To rigorously capture prebiotic self-organization and the emergence of protocellular complexity, chemical networks are formalized using the Relativistic Scalar-Vector Plenum (RSVP) framework, incorporating:

1. Entropy fields  $S$  representing prebiotic chemical disorder.
2. Scalar ( $\Phi$ ) and vector ( $\mathbf{v}$ ) fields describing emergent structural order and directional fluxes of chemical information.
3. Category-theoretic merge operators to model hierarchical autocatalysis and the assembly of functional chemical modules.

This formalism allows a quantitative mapping from local autocatalytic dynamics to global emergent prebiotic structure.

### 11.2 Entropy Fields and Chemical Persistence

Define the local chemical entropy field:

$$S(\mathbf{x}, t) = - \sum_i p_i(\mathbf{x}, t) \log p_i(\mathbf{x}, t),$$

where  $p_i$  is the probability density of chemical species  $i$  at location  $\mathbf{x}$  and time  $t$ .

Tidal trituration, wet-dry cycles, and mineral surfaces (per **(author?)** 1) modulate  $S$  by reducing local disorder and stabilizing functional motifs.

The scalar field  $\Phi$  captures the accumulated “structural information” of autocatalytic modules:

$$\Phi(\mathbf{x}, t) = \int_0^t f_\eta(\mathbf{x}, t') dt',$$

with  $f_\eta$  representing effective module persistence weighted by autocatalytic efficiency  $\eta$ .

### 11.3 Vector Fields and Directed Assembly

Emergent directional chemical flows can be expressed as a vector field  $\mathbf{v}$ , representing preferential assembly pathways induced by:

- Energy gradients (thermal, tidal, UV fluxes)
- Mineral surface templating [1]
- Cyclical autocatalytic feedback loops
- The dynamics follow a continuity-like equation:

$$\frac{\partial \Phi}{\partial t} + \nabla \cdot (\Phi \mathbf{v}) = \Gamma(\mathbf{x}, t) - \Lambda(\mathbf{x}, t),$$

where  $\Gamma$  captures generation of new modules and  $\Lambda$  represents loss due to degradation or nonproductive reactions.

### 11.4 Category-Theoretic Merge Operators

Hierarchical assembly is formalized using symmetric monoidal  $\infty$ -categories:

- Objects  $\mathcal{M}$  represent autocatalytic modules or functional chemical sets.
- Morphisms  $f$  represent chemical transformations under environmental constraints.
- Merge operator  $\oplus$  captures the combination of modules into higher-order functional networks:

$$X \oplus Y \xrightarrow{\mu} Z$$

Associativity and commutativity encode the robustness of autocatalytic integration independent of local ordering.

Homotopy colimits provide a measure of global network closure, representing emergent, self-maintaining chemical systems.

This formalism explicitly models the accumulation of information without assuming preformed protocells or extraterrestrial seeding.

### 11.5 RSVP-Based Rate-Distortion Revisited

Building on Endres' rate-distortion argument,  $R(D)$  is reinterpreted in the RSVP framework:

$$R(D) \sim \frac{S(\mathbf{x}, t)}{\tau_{\text{eff}}},$$

where  $\tau_{\text{eff}}$  is the effective persistence timescale of autocatalytic modules, dramatically extended by:

- Mineral catalysis [1]
- Environmental cycling (wet-dry, tidal trituration)
- Redundancy and feedback in hierarchical networks

Thus, the informational bottleneck is reduced by orders of magnitude, making abiotic emergence plausible without invoking rare stochastic miracles or extraterrestrial intervention.

### 11.6 Integration with England and Walker

England: Dissipative dynamics correspond to local reductions in  $S$  along energy-driven chemical fluxes.

Walker: Causal influence propagates along  $\mathbf{v}$ , encoding the directionality of functional information in prebiotic networks.

RSVP fields provide a unifying formalism, linking thermodynamics, autocatalytic structure, and information flow.

## 11.7 Implications for Prebiotic Complexity

1. Earth as a planetary-scale cell: Tidal cycles, mineral scaffolds, and autocatalytic hierarchies create a distributed computation network, naturally selecting stable modules.
2. Quantitative predictions: RSVP fields can estimate:
  - Module persistence probabilities
  - Likely reaction network topologies
  - Entropy reduction rates compatible with observed prebiotic chemistry
3. AI analogies: Hierarchical pruning of strategies parallels autocatalytic module selection, providing computational insights into the emergence of functional complexity.

## 11.8 References (Representative)

[1]

[3]

[4]

[2]



## Part VII

# Implications and Predictions





## Chapter 12

# Predictions for Prebiotic Chemistry

Planetary-scale catalysts imply:

Faster emergence than classic models predict.

Strong mineral signature in life's building blocks.

Testable: hierarchical autocatalysis leaves network motifs detectable in modern metabolism.



# Chapter 13

## Predictions for AI

AI scaling should follow hierarchical autocatalytic compression curves.

Expect:

Heavy-tailed survival of strategies.

Sudden phase transitions (module nucleation events).

### 13.1 Revisiting Endres’ Information-Theoretic Argument

Endres (2025) frames the emergence of life as an extreme informational bottleneck, estimating that a minimal protocell requires  $10^9$  bits and arguing that abiotic assembly is implausible without either directed panspermia or extraordinary luck.

Critique:

This approach largely ignores the role of structured prebiotic environments, such as clay minerals [1], tidal trituration by a nearby moon, and chemical cycles that naturally extend molecular persistence.

Autocatalytic sets and hierarchical chemical networks create long-lived informational modules whose effective persistence ( $\tau$ ) can exceed Endres’ naïve decay estimates by orders of magnitude.

Consequently, the Earth itself can be treated as a distributed, giant “cell”, where the global surface area and dynamic mixing drastically enhance the effective information rate  $R(D)$ , making spontaneous emergence statistically feasible without invoking extraterrestrial intervention.

### 13.2 Contrasting with England and Walker

Jeremy England (2013, 2015):

Life emerges generically in dissipative systems, where driven energy fluxes select for structures that increase entropy production.

RSVP fields provide a mathematical embedding for this idea: local energy-driven flows correspond to directional vector fields  $\mathbf{v}$  that channel chemical reactions into persistent modules, while scalar entropy fields  $S$  track the dissipative selection process.

Sara Walker (2013):

Emphasizes causal information flow as central to defining life.

RSVP captures this through hierarchical module networks and category-theoretic merge operators, where morphisms encode functional influence among chemical modules, and homotopy colimits formalize the emergence of globally coherent causal structures.

Synthesis:

England explains thermodynamic feasibility, Walker explains informational directionality, and RSVP provides a unifying, formal framework connecting entropy, energy flux, and emergent functional networks.

Unlike Endres’ information-theoretic bottleneck, this approach quantitatively incorporates environmental mediation, autocatalysis, and hierarchical information retention, demonstrating that spontaneous abiogenesis is physically plausible.

### 13.3 Scaling and Artificial Intelligence Analogies

Hierarchical autocatalysis  $\leftrightarrow$  hierarchical strategy selection:

Molecular modules resemble candidate strategies in a computational landscape.

Environmental filtering and merging mirror pruning, aggregation, and hierarchical abstraction in neural networks.

Information compression and persistence:

RSVP scalar fields  $\Phi$  track the cumulative structural information, akin to AI models maintaining compressed internal representations.

Global emergent computation:

Just as distributed AI models can approximate complex functions via local pruning and recombination, Earth-scale chemical networks naturally approximate universal computation, potentially implementing complex logic functions long before lipid membranes appeared.

### 13.4 Implications for Prebiotic Earth

1. No extraterrestrial seeding required:

Tidal trituration, wet-dry cycles, mineral surfaces, and autocatalytic hierarchies suffice to overcome informational bottlenecks.

2. Planetary-scale computation:

Prebiotic Earth functions as a distributed chemical computer, where emergent modules encode memory, error correction, and functional persistence.

3. Predictive framework:

RSVP fields can quantify effective information rates, module lifetimes, and network robustness, offering testable predictions for experimental prebiotic chemistry.

4. AI as a modeling tool:

Machine learning and generative AI can help reverse-engineer plausible assembly pathways, identify attractor networks, and suggest experimental interventions to probe the emergence of functional modules.

### 13.5 Critical Reflection

Endres' approach is conceptually illuminating but practically limited, as it underestimates environmental mediation and the computational power of autocatalytic networks.

England and Walker provide key complementary insights, but lack a formal mathematical structure that explicitly tracks information flow, entropy, and network integration.

RSVP fields, combined with category-theoretic merge operators and hierarchical autocatalysis, provide a unified, quantitative framework that addresses both thermodynamic feasibility and functional emergence without resorting to speculative extraterrestrial origins.

### 13.6 Conclusion

The convergence of entropy-respecting physics, autocatalytic chemistry, and hierarchical information theory supports a plausible, Earth-based origin of life.

Artificial intelligence offers both analytical power and conceptual analogies, illuminating how local constraints and selective retention can scale into globally coherent structures.

Ultimately, abiogenesis emerges as a natural extension of physical law, with RSVP formalism providing a robust mathematical scaffold for future experimental and theoretical exploration.

## Part VIII

# Experimental Predictions and Suggested Prebiotic Experiments



## 13.7 Overview

Having developed a formal framework connecting RSVP entropy fields, hierarchical autocatalysis, and Earth-scale chemical networks, these concepts are now translated into empirical predictions. The goal is to identify experiments that probe:

1. Effective information retention in prebiotic networks.
2. Emergence of functional autocatalytic modules.
3. Role of environmental mediation (clay surfaces, tidal trituration, wet–dry cycles).

These experiments aim to move beyond purely theoretical considerations and provide quantitative tests for the plausibility of Earth-based abiogenesis.

## 13.8 RSVP Field Observables in Prebiotic Chemistry

RSVP formalism assigns three key field variables to a chemical system:

Scalar field  $\Phi$ : cumulative structural information.

Vector field  $\mathbf{v}$ : directional flux of energy and material, capturing persistent reaction pathways.

Entropy field  $S$ : local dissipation, representing stochasticity and environmental degradation.

Experimental proxies:

RSVP Variable	Experimental Observable	Measurement Method
$\Phi$	Molecular network complexity, diversity of catalytic cycles	High-throughput mass spectrometry, NMR
$\mathbf{v}$	Net reaction flux, autocatalytic directionality	Time-resolved spectroscopy, isotope tracing
$S$	Rate of degradation or disorder	Kinetic assays under varying temperatures

Table 13.1: Experimental proxies for RSVP fields.

These measurements allow calculation of effective information rates  $R(D)$  and comparison with theoretical thresholds for protocell assembly.

## 13.9 Experimental Conditions

1. Mineral-Supported Chemistry [1]:

Use montmorillonite, kaolinite, or iron-sulfur minerals as catalytic surfaces.

Hypothesis: mineral surfaces increase  $\tau$  for autocatalytic modules by localizing reactants and stabilizing intermediates.

2. Tidal Trituration and Wet–Dry Cycling:

Simulate periodic volume changes to mimic tidal or seasonal fluctuations.

Hypothesis: repeated cycling expands surface area and promotes hierarchical autocatalysis, accelerating  $\Phi$  accumulation.

3. Multi-Component Prebiotic Mixtures:

Combine amino acids, nucleobases, small peptides, and sugars in controlled ratios.

Hypothesis: combinatorial interactions naturally generate functional modules approaching Turing-complete chemical computation.

4. Hierarchical Autocatalytic Networks:

Seed initial reaction modules and allow selective retention of persistent products.

Measure: whether small modules spontaneously merge into larger functional assemblies, analogous to AI strategy pruning.

## 13.10 Predictive Metrics

1. Effective Information Rate  $R(D)$ :  
 Compute  $R(D)$  as the system evolves.  
 Compare with the minimal rate  $R_{\min} = I_{\text{protocell}}/\tau_{\text{available}}$ .
2. Module Persistence Time  $\tau$ :  
 Identify the lifetime of autocatalytic sets under environmental stress.  
 Longer  $\tau$  reduces the required retention efficiency  $\eta$ , enhancing plausibility.
3. Attractor Dynamics:  
 Map chemical state space to detect robust attractor basins.  
 Prediction: hierarchical networks converge to a small set of stable functional configurations, echoing RSVP category-theoretic morphisms.

## 13.11 Integrating AI Modeling

Use generative AI to simulate reaction networks under stochastic and environmental constraints.

AI predicts emergent modules, identifies attractor basins, and proposes optimized sequences of wet-dry or mineral cycling.

Analogous to RSVP: AI models act as computational mirrors of natural autocatalytic selection, providing guidance for lab experimentation.

## 13.12 Testable Hypotheses

1. Earth-scale chemical computation is plausible:  
 Prediction: mineral-mediated, tidally cycled mixtures will self-organize into autocatalytic networks.
2. Hierarchical information retention emerges naturally:  
 Prediction: small autocatalytic sets persist and merge, approximating the information accumulation required for minimal protocells.
3. Environmental mediation reduces informational bottlenecks:  
 Prediction: adding tidal-like trituration or surface area increases significantly enhances module lifetime  $\tau$  and net rate  $R(D)$ .
4. No extraterrestrial seeding needed:  
 Prediction: all observed phenomena occur under purely Earth-based conditions, supporting endogenous abiogenesis.

## 13.13 Future Directions

1. Map RSVP fields experimentally using high-resolution time-series data on molecular concentrations, reaction flux, and degradation.
2. Explore category-theoretic formalism in chemical networks: treat functional modules as objects, interactions as morphisms, and hierarchical merging as homotopy colimits.
3. Develop AI-augmented “prebiotic simulators” to predict emergent pathways and suggest experimental designs.

This framework converts the conceptual RSVP + hierarchical autocatalysis theory into concrete, testable experiments, closing the gap between formal theory and lab implementation.



## Appendix A

# Mathematical Derivations for PDE Models and Category-Theoretic Formalisms

This appendix provides detailed derivations for the PDE models and category-theoretic formalisms used in the monograph.

### A.1 RSVP Field Evolution Equations Derivation

The coupled system for RSVP fields is derived from conservation laws in non-equilibrium thermodynamics, adapted for prebiotic chemistry.

Starting with the free energy density conservation:

The scalar field  $\Phi$  satisfies a continuity equation modified for dissipation:

$$\frac{\partial \Phi}{\partial t} + \nabla \cdot (\Phi \mathbf{v}) = -\lambda S + Q_\Phi$$

Here, the term  $-\lambda S$  represents entropy-driven dissipation, and  $Q_\Phi$  is the source term from external energy inputs.

For the vector field  $\mathbf{v}$ , the equation is analogous to a Navier-Stokes form for flux:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \Phi + \nu \nabla^2 \mathbf{v} + \mathbf{F}_{\text{catalysis}}$$

The pressure-like term  $-\nabla \Phi$  drives flow along energy gradients,  $\nu \nabla^2 \mathbf{v}$  accounts for diffusion, and  $\mathbf{F}_{\text{catalysis}}$  incorporates catalytic biases.

The entropy equation follows from the second law, with production and sink terms:

$$\frac{\partial S}{\partial t} = \alpha \|\nabla \Phi\|^2 - \beta S + \mathcal{I}_{\text{autocatalysis}}$$

$\alpha \|\nabla \Phi\|^2$  is the entropy production from gradients,  $-\beta S$  is relaxation, and  $\mathcal{I}_{\text{autocatalysis}}$  is negative for information gain in autocatalytic closures.

### A.2 Rate-Distortion Inequality Derivation

The rate-distortion function  $R(D)$  is derived from information theory, bounding the rate required to achieve distortion  $D$ :

$$R(D) = \min_{p(\hat{x}|x)} I(X; \hat{X})$$

such that  $\mathbb{E}[d(x, \hat{x})] \leq D$ .

In hierarchical levels, this becomes level-specific:

$$\mathcal{R}^{(k)}(x, t) = \eta^{(k)} \frac{H_{\text{avail}}^{(k)}}{D^{(k)}} \geq r_*^{(k)}$$

where the mutual information  $I$  is approximated by available entropy  $H_{\text{avail}}$  scaled by efficiency  $\eta$ , and  $D$  is the distortion at level  $k$ .

Lower entropy cost at higher levels arises from compression:  $H^{(k+1)} < H^{(k)}$  due to merging.

### A.3 Category-Theoretic Colimit Derivation

In the symmetric monoidal category  $\mathcal{C}$ , the colimit for a diagram  $\mathcal{D} : \mathcal{I} \rightarrow \mathcal{C}$  is the universal object  $\varinjlim \mathcal{D}$  with morphisms from each  $\mathcal{D}(i)$ .

For autocatalytic merging, the colimit satisfies the co-cone property, ensuring emergent module coherence.

Sheaf gluing derives from the sheaf condition, where local sections (constraints) glue to global sections if they agree on overlaps, formalizing distributed integration.

## Appendix B

# Additional Mathematical Formulas for Hierarchical Models

This appendix contains additional mathematical formulas supporting the hierarchical autocatalytic models, without simulation pseudocode.

### B.1 Population PDE Derivations

The population density PDE for strategies  $\rho_s$  is derived from reaction-diffusion kinetics:

$$\frac{\partial \rho_s}{\partial t} = D_s \nabla^2 \rho_s + \sum_{s'} M_{ss'} \rho_{s'} + \Gamma_s[\rho] - \Lambda_s[\rho]$$

Diffusion term  $D_s \nabla^2 \rho_s$  accounts for spatial spread.

Interaction matrix  $M_{ss'}$  represents catalytic rates.

$\Gamma_s, \Lambda_s$  are nonlinear birth/death terms from autocatalysis.

For higher-level sets:

$$\frac{\partial \rho_S}{\partial t} \supset \sum_{A \subset S} \kappa_{A \rightarrow S} \prod_{s \in A} \rho_s$$

This term models merging with rate  $\kappa$ .

Effective size scaling  $N_{\text{eff}}^{(k+1)} \sim \alpha N_{\text{eff}}^{(k)}$  derives from recursive compression, where  $\alpha > 1$  for growth.

### B.2 Scaling Law Formulas

The information rate scaling:

$$R_{\text{eff}} \sim \eta \frac{H_{\text{prebiotic}}}{D} \left( \frac{A}{A_0} \right)^\gamma$$

is derived from fractal surface area enhancement, where  $\gamma$  is the scaling exponent from random mineral structures.

Percolation thresholds for module formation follow from graph theory, where connectivity probability  $p_c \approx 1/\langle k \rangle$  for random graphs, adapted for random catalytic networks.



## Appendix C

# Comparison Table: Chemical Emergence vs AI Optimization

Aspect	Chemical Emergence	AI Optimization
Local Exploration	Autocatalytic sets on mineral surfaces	Strategy sampling in neural layers
Selection/Pruning	Environmental degradation filters	Loss-based pruning (e.g., lottery ticket)
Hierarchical Merging	Colimit formation of modules	Distillation and modularization
Scaling Law	Surface area $\sim$ effective search	Compute $\sim$ model performance
Entropy Reduction	$S$ decay via cycles	Information bottleneck optimization

Table C.1: Comparison of chemical emergence and AI optimization.



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