Laplace Transforms Second Order Equation

This is our second lecture one the Laplace transform, and this one will be about solving second order equations.

$$y'' + By' + Cy = \delta(t)$$

Our plan is to take the transform to equation term. We have to check, what is the transform of the delta function.

$$\mathscr{L}[\delta(t)] = 1$$

That's the nice Laplace transform of the impulse. And now we want the transform of the impulse response.

$$(s^2 + Bs + C)G(s) = 1$$

We are not surprised to see the very familiar quadratic, whose roots are the two exponents s_1, s_2 , showing up here.

$$G(s) = \frac{1}{s^2 + Bs + C}$$

We want now to use the inverse Laplace transform to get to the answer. We need to split $s^2 + Bs + C$ into two parts which is called partial fraction.

$$G(s) = \frac{1}{s_1 - s_2}(\frac{1}{s - s_1} - \frac{1}{s - s_2})$$

Hence

$$g(s) = \frac{e^{s_1 t} - e^{s_2 t}}{s_1 - s_2}$$

It is a solution to our equation with impulse force.

Then let's see an another example than delta function.

$$y'' + By' + Cy = \cos \omega t$$

Take the Laplace transform we get

$$(s^2 + Bs + C)Y(s) = \mathscr{L}[\cos \omega t] = \frac{s}{s^2 - \omega^2}$$

And we get

$$Y(s) = \frac{s}{(s^2 + Bs + C)(s^2 + \omega^2)}$$

We have two s^2 term here, a fourth-degree polynomial down there. Partial fractions will work. But the algebra gets quickly worse when you get up to degree four.