Euler, ODE1

These series of lecture is about solving ODE's in MATLAB.

We can begin by recalling the definition of derivative.

The derivative of a function is the slope of the tangent line to he graph at a point. Our numerical approximations will rely upon the slope of the secant to the graph. We know that as the interval goes to 0, the slope of the secant approaches the slope of tangent

$$y'(t_0) \approx \frac{y(t_0+h)-y(t_0)}{h}$$

put the h in the left hand side and we get

$$y(t_0+h) \approx y(t_0) + hy'(t_0)$$

This is the basic of **Euler's method**.

Let's see some examples.

The compound interest problem is just the interest rate times y

$$f(t,y) = 0.06y \qquad y(0) = 100$$

Here is the logistic equation.

$$a = 2, b = 3,$$

$$f(t,y) = ay - by^2 \qquad y(0) = 1$$

Euler's method actually is not a practical numerical method in general. We are just using it to get us started thinking about the ideas underlying numerical methods.

$$t_{n+1} = t_n + h \qquad y_n \approx y(t_n)$$

$$\frac{y(t_{n+1})-y(t_n)}{h} \approx f(t_n, y(t_n))$$

$$y_{n+1} = y_n + hf(t_n, y_n) \qquad \text{(Euler's Method)}$$

We are now ready for your first MATLAB program, ODE1

```
type ode1
```

It is called ODE1 because it's our first program and because it evaluates the function f that defines the differential equation once per step.

```
ode1(F,t0,t_final,y0)
```

There are five input arguments. The first if f, a function that defines the differential equation. This is something called an anonymous function. We will talk more about it in a moment. The other four are scalar numerical values. The first three define the interval of integration. We're going form t0 in step of h to t_final. The output is a vector.

The first argument to any of the MATLAB ODE solvers is the name of a function that specifies the differential equation. This is known as a function handle. The easiest way to get a function handle is to make use of an anonymous function created with the ampersand(&) or at sign(@)

```
% Function Hanles and Anonymous Functions
>> F=@(t,y) 0.06*y % interest problem
```

```
>> F(0,1)
ans = 0.00600
```

Example

$$
\begin{gathered}
y' = 2y \\
y(0) = 10 \\
0 \leq t \leq 3
\end{gathered}
$$

We define the anonymous function.

```
>> F=@(t,y) 2*y
```

The initial condition.

```
>> t0=0;
   t_final = 3;
   y0 = 10
```

step size

```
>> h=1
```

call the ode1

```
>> ode1(F,t0,h,tfinal,y0)
```

We have an animation that show these steps

```
>> euler_step
```