Fourier Series Solution of Laplace Equation

This is lecture is about using for Fourier series. You will see how Fourier series comes in in the Laplace's equation.

$$\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

That's the way heat, temperature. distributes itself when you leave it alone. In this problem we are going to put a source of heat at boundary point. So it will be point source. A delta function. And on the rest of the boundary, temperature is 0. So the boundary function is a delta function with a spike at that one point and 0 elsewhere. And our problem is to solve the Laplace's equation inside the circle. And we use polar coordinates because we have got a circle.

$$u(r, \theta) = \sum_{n=0}^{\infty} a_n r^n \cos n\theta + \sum_{n=1}^{\infty} b_n r^n \sin n\theta$$

And

$$u(1, \theta) = \delta(\theta) = \frac{1}{2\pi} + \frac{1}{\pi}(\cos\theta + \cos 2\theta + \cdots)$$

$$u(r, \theta) = \frac{1}{2\pi} \frac{1 - r^2}{1 + r^2 - 2r\cos\theta}$$

For most cases, we can't solve the Laplace's equation with nice form. We would use **Laplace's difference equation** (Laplace's five-point scheme). That's an important problem in numerical analysis.