Response to Complex Exponential

Complex numbers are going to come in to this lecture. We are going to solve the problem

$$\begin{aligned}\frac{dy}{dt} &= ay + \cos\omega t + i\sin\omega t \\ &= ey + e^{i\omega t}\end{aligned}$$

We assume the solution is like

$$y(t) = Ye^{i\omega t}$$

we will find the Y by substituting this into the equation.

$$i\omega Ye^{i\omega t} = aYe^{i\omega t} + e^{i\omega t}$$

we divide everything by $e^{i\omega t}$. We get

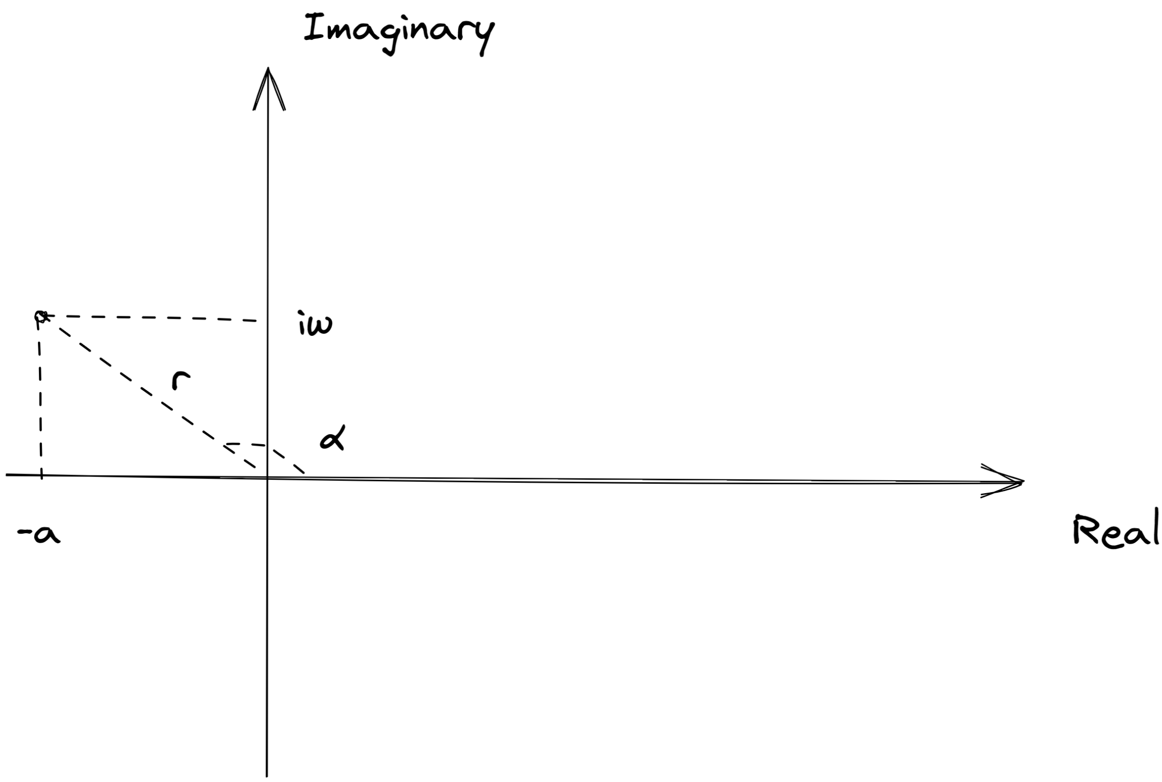
$$Y = \frac{1}{i\omega - a}$$

<u>our idea is to use this complex solution to find two real solution.</u>

We will take the real part of this complex solution and the imaginary part of this solution.

Note that $Y = \frac{1}{i\omega - a}$ is a really a awkward quantity that we have to get into a good form -- **polar form**

$$i\omega - a = re^{i\alpha}$$



That is called putting the number into its polar form.

$$r = \sqrt{a^2 + \omega^2}$$

$$y(t)=\frac{1}{\sqrt{a^2+\omega^2}}e^{-i\alpha}e^{i\omega t}=\frac{1}{\sqrt{a^2+\omega^2}}e^{i(\omega t-\alpha)}$$

Now we are ready to take the real part and the imaginary part.

Using the Euler's formula, we get the real part is

$$y(t)=\frac{1}{\sqrt{a^2+\omega^2}}\cos(\omega t-\alpha)$$

that is exactly what we get in the previous lecture. And the Imaginary part is substitute $\cos$ with $\sin$.