

Phase Plane Pictures Spirals and Centers

This lecture is about the case when the roots are complex. Now we are going to have curves and spirals, because of the complex part. So here are the three possibilities now.

$$Ay'' + By' + Cy = 0$$

Complex roots are $s = a \pm i\omega$ for $B^2 < 4AC$

- Spiral source $a > 0, B < 0$
- Center $a = 0$, no damping. neutrally stable.
- Spiral sink $a < 0$, underdamping and stable.

Let start with that, the neutrally stable.

$$y'' + \omega^2 y = 0$$

It's pure oscillation, a spring going up and down, and LC circuit going back and forth. And we see the solutions.

$$y = c_1 \cos \omega t + c_2 \sin \omega t$$

And

$$y' = -c_1 \omega \sin \omega t + c_2 \omega \cos \omega t$$

So for every t , it's going to be an easy figure. When the $\omega = 1$. We simply go around a circle in the phase plane. And it will be an ellipse with the ω equals other values. If you choose a small initial condition, it will be a small ellipse and it is called **center**.

Let's the spiral source or sink.