Phase Plane Pictures Source, Sink, Saddle

This is the second lecture for Chapter 3. And it's going to be pictures again. But it's pictures for a second order equation. And we will make them nice. We will know formulas here. These will be constant coefficient, linear second order equations. And we know that the solution

$$y = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

So we are talking about null equations, 0 on the right-hand side. And we just want to draw that picture that goes with solution like that. So here is the magic word, **phase plane**. We are going to draw the pictures in a plane. And the axes we will choose will be y and y', not t. You will see how t -- time, comes into the picture. But we have the two axes will be y and y'. So we had to figure out what y' was. It just brings down an s from last term.

$$y' = c_1 s_1 e^{s_1 t} + c_2 s_2 e^{s_2 t}$$

Let's see an example.

$$y'' - 3y' + 2y = 0$$

Notice that the damping term is negative. This will be unstable. Solution will go out to infinity. And we can find those solutions.

$$s^2 - 3s + 2 = 0$$

We find the $s_1 = 1, s_2 = 2$. And now we are ready for the phase plane picture.

At first. The s_1 wins while $t \to \infty$ s_2 wins. This is called **source**. Because the solution goes to infinity. When the damping term is positive, the solution is converging to $(0, 0)$.

When the eigenvalues have one positive and one negative. It's called **saddle**. When we have in one direction things are growing, but in the other, things are decreasing.