Graphs

This lecture is a different direction. It will be about linear equations and not differential equations. A matrix is at the center of this lecture. And it's called the incidence matrix. And that incidence matrix tells us everything about a graph. What is the word graph meaning? It doesn't mean a graph of a function. The word graph is used in another way completely for some edges and some nodes.

We can transform a graph to a matrix with rows representing edges and column representing the node. The matrix can act on vectors.

For example, we can get voltage difference by multiplication. And we can get the flow by Kirchhoff's current law. It says that the total flow into a node equals the flow out. We are talking about equilibrium here. So if current is traveling around network and it's a stable equilibrium here. $A^T w = 0$, the incidence matrix is leading us to the fundamental equilibrium condition for flow in the network. Now one more law is needed. It ahs to connect voltage differences to flows, potentials to currents -- Ohm's Law.

And we won't be surprised that when the whole thing is put together and we have a final equation to solve. We end up with $A^T A$. And that magic combination is central to graph theory. It's called the graph Laplacian ad has a name and fame of its own.