Second Order Equations

It is time to move on to second order equations. First order equations, we have done pretty carefully. Second order equations are a step harder. But they come up in nature, they even come in evert application because they include an acceleration -- a second derivative.

$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + cy = 0$$

this would be a second order equation, because of that second derivative. I am often going to have constants A, B, C , we have enough difficulties to it without allowing those to change. So A, B, C constants, and we will do the null solution to start with. Later, there will be a forcing term on the right-hand side.

The point is, now, what's new is that there are two null solutions.

$$y_{null} = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

so we have two constants, c_1, c_2 in the null solutions and we need two initial conditions to determine those constants. So previously, for a first order equation, we were give $y(0)$. Now, when we have acceleration, we give the initial velocity -- $y'(0)$.

Let's see some examples. The first example -- the most basic equation of motion in physics and engineering -- It called **harmonic motion** . And $B = 0$. It is Newton's law.

$$my'' + ky = 0$$

No y' term. It is like a spring going up and down, or a clock pendulum going back and forth. We want to solve that equation.

$$\boxed{y_n = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right)}$$

That is free harmonic motion. Something is just oscillating. In rotation problems, something is just going around a circle at a constant speed. And notice, these are not the same as $y_{null} = c_1 e^{s_1 t} + c_2 e^{s_2 t}$. Cosines are related to exponentials, but not identical. So we could write the answer using cosine and sine or exponentials with Euler's great formula.

Let's introduce $\omega_n = \sqrt{\frac{k}{m}}$,'n' here stands for the natural frequency. We can rewrite the above equation to make it simple

$$\boxed{y'' + {\omega_n}^2 y = 0}$$

We can figure out what C_1, C_2 are, coming from the initial conditions.

$$C_1 = y(0) \qquad C_2 = y'(0)/\omega_n$$

That tells the motion, forever and ever. Energy is constant. Potential energy plus kinetic energy. That is the best example, the simplest example and the first example.