Solution for Any Input

We are going to solve the first order linear differential equation with a formula that works for any source term. And we want to understand the formula.

$$\frac{dy}{dt} = ay + q(t)$$

the ay can be seen as the interest added and the $q(t)$ can be seen as the new deposits.

$$\boxed{y(t) = y(0)e^{at} + \int_{s=0}^{s=t} e^{a(t-s)}q(s)\,ds}$$

The null solution that grows out of the initial condition and the particular solution that grows out of the source term.

We first wee the special condition $q(t) = C$ and then see $q(t)$ as e^{at} then $q(t) = \cos \omega t$ finally we will see the step function -- we do not make any deposits up until some time and then we start to go change to constant, and the delta function -- an impulse. (we will also see t, t^2 as well).

Back to the general formula, we should check it is correct. Intuitively, the input is in and grew, everything is linear, so we can just add the separate growth, separate results to find out what the balance was at the final time t.

the particular solution $y_p(t)$ satisfies

$$y_p(t) = e^{at}\int_{s=0}^{s=t} e^{-as}q(s)\,ds$$

This is all the stuff that depends on what time the deposit was made, time s. And what time we're looking at the balance, the later time ,t

the derivative of $y_p(t)$ will be the two terms in the differential equation.

$$\frac{dy_p}{dt} = ae^{at}\int_{s=0}^{s=t} e^{-as}q(s)\,ds + e^{at}e^{-at}q(t)$$

so we get $dy_p/dt = ay + q(t)$