Lorenz Attractor and Chaos

The Lorenz strange attractor, perhaps the world's most famous and extensively studied ode's. They were discovered in 1963 by an MIT mathematician and meteorologist -- Edward Lorenz. They started the field of chaos. They are famous because they are sensitive to their initial conditions. Small changes in the initial conditions have a big effect on the solution. Lorenz is famous for talking about the butterfly effect. How flapping of butterflies wings can affect the weather. A butterfly flying in Brazil can cause a tornado and Texas is a flamboyant version of a talk he gave.

$$\begin{aligned} x' &= \sigma(y - x) \\ y' &= \rho x - y - xz \\ z' &= xy - \beta z \end{aligned}$$

The equations are almost linear. The equations come out of a model of fluid flow. The Earth's atmosphere is a fluid. But this range of parameters, the three parameters, σ, ρ, β , these are outside the range that actually represents the Earth's atmosphere. We are going to take a look at these parameters.

$$\sigma = 10, \rho = 28, \beta = \frac{8}{3}$$

These are the most commonly used parameters. But we are going to be interested in other values of ρ as well.

But we are matrix guy, so we like to write the equations in the following form.

$$\begin{aligned} y' &= Ay \\ y &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \\ A &= \begin{bmatrix} -\beta & 0 & y_2 \\ 0 & -\sigma & \sigma \\ -y_2 & \rho & -1 \end{bmatrix} \end{aligned}$$

This matrix form s convenient for finding the critical points. Put a parameter η in place of y_2. Try to make the matrix singular.

$$A = \begin{bmatrix} -\beta & 0 & \eta \\ 0 & -\sigma & \sigma \\ -\eta & \rho & -1 \end{bmatrix}$$

we get

$$\eta = \pm\sqrt{\beta(\rho - 1)}$$

and the null vector is the critical point. If we take this vector as the starting value of the solution, then the solution stays there. $y' = 0$. This is an unstable critical point. And values near this solution deviate the solution.

And there are beautiful animation here, but I don't setup the `Lorenzplot` toolbox. You can check that in the video.