Systems of Equations

Many mathematical models involve high order derivatives. But the MATLAB ODE solvers only work with systems of first order ordinary differential equations. So we have to rewrite the models to just involve first order derivatives. Let's see how to do that with a very simple model, the harmonic oscillator

$$x'' + x = 0$$

This involves a second order derivative. So to write it as a first order system, we introduced the vector y. This is a vector with two components.

$$y = [x; x']$$

Then, we have

$$y' = [x'; x'']$$

So the differential equation now becomes

$$y_2' + y_1 = 0$$

And then

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} y_2 \\ -y_1 \end{bmatrix}$$

When we write this as an anonymous function for MATLAB, here's the function

```
F = @(t,y) [y(2);-y(1)]
```

Initial conditions, suppose the initial conditions are that

$$\begin{gathered} x(0) = 0 \quad x'(0) = 1 \\ y_1(0) = 0 \quad y_2(0) = 1 \\ y(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{gathered}$$

That implies the solution is $\sin t$ and $\cos t$

$$y(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

Let's bring then into MATLAB

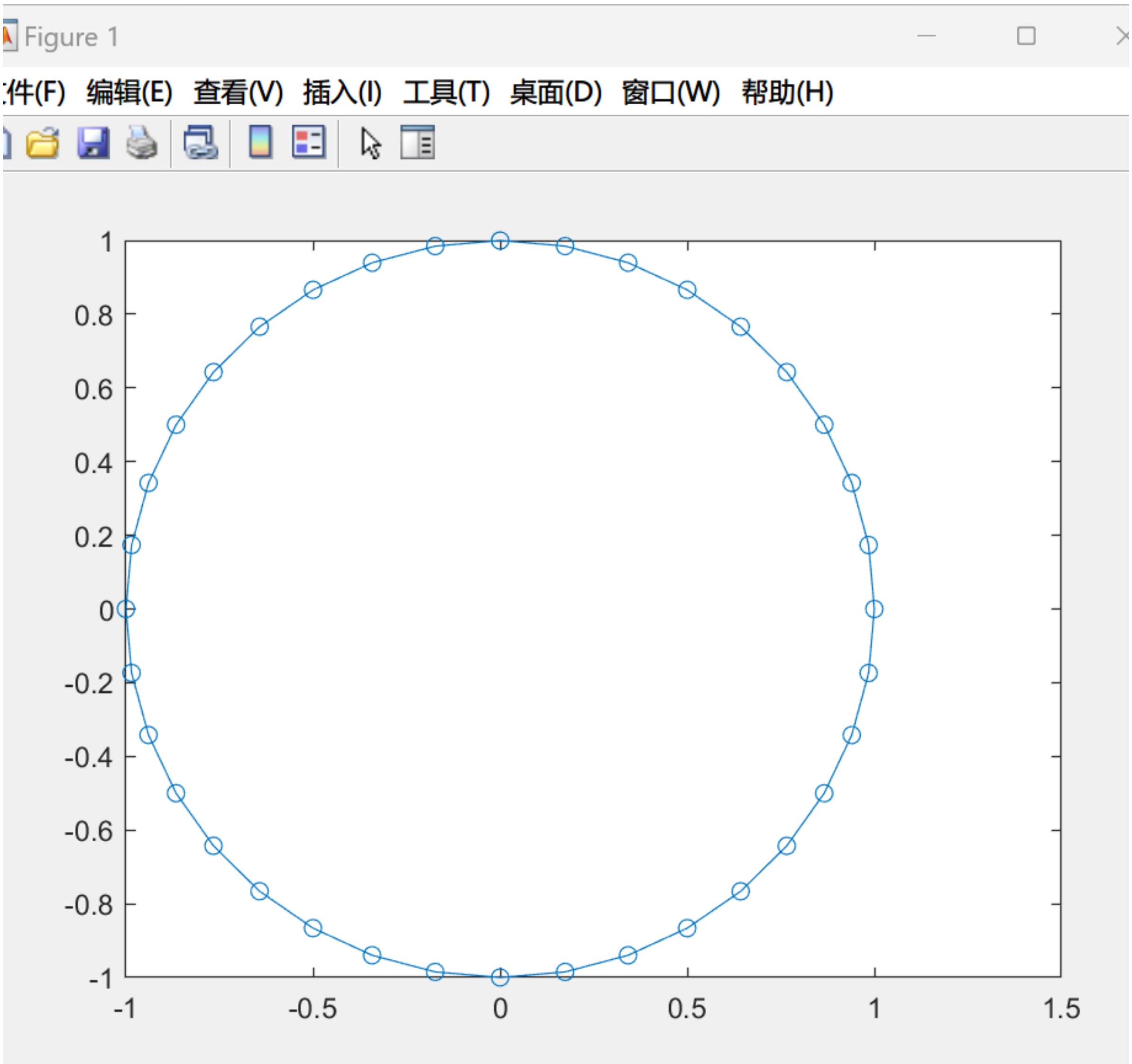
```
>> F = @(t,y) [y(2);-y(1)];
   tspan = (0:1/36:1)*2*pi;
   y0 = [0;1]
   ode45(F,tspan,y0)
```

Let ask to capture the output in t and y.

```
>> [t,y] = ode45(F,tspan,y0)
```

Now we can plot them in the phase plane

```
>> plot(y(:,1),y(:,2),'o-')
```



The Van der Pol oscillator was introduced in 1927 by Dutch electrical engineer, to model oscillations in a circuit involving vacuum tubes.

$$x'' - \mu(1-x^2)x' + x = 0$$
$$y = [x; x']$$
$$y' = [x'; x'']$$
$$y' = [y_2; \mu(1-y_1^2)y_2 - y_1]$$

```
F = @(t,y) [y(2);mu*(1-y(1)^2)*y(2)-y(1)]
```

It has a nonlinear damping term. It is since been used to model phenomena is all kinds of fields, including geology and neurology. It exhibits chaotic behaviour. We are interested in it for numerical analysis because, as the parameter μ increases, the problem becomes increasingly stiff. To write it as a first order system for use with the MATLAB ODE solvers, we introduce the vector y containing x and x'.