Response to Oscillating Input

$$\frac{dy}{dt} = ay + \cos\omega t \qquad y = y(0) \text{ at } t = 0$$

Look for $y_p(t) = M\cos\omega t + N\sin\omega t$

Just substitute it in the equation to find M and N. We get

$$-aM + \omega N = 1$$

$$-\omega M - aN = 0$$

solve the system.

$$M = \frac{-a}{\omega^2 + a^2} \qquad N = \frac{\omega}{\omega^2 + a^2}$$

But we think the form is not beautiful enough. We believe it can be written in a different way.

$$y(t) = G\cos(\omega t - \alpha)$$

polar coordinates is the right way to think this. G and α, a magnitude and an angle. We match that with the form we already had.

Using a little trigonometry here, we can get

$$M = G\cos\alpha \qquad N = G\sin\alpha$$

since $\cos^2\alpha + \sin^2\alpha = 1$, we get

$$G = \sqrt{M^2 + N^2} \qquad \alpha = \arctan\frac{N}{M}$$

it is called **sinusoidal identity**. Sinusoid is a word for any mixture of sines and cosines of the same omega t.

However, There is a link -- the key fact about complex numbers -- Euler's great formula will give us a connection between $\cos t$ and $\sin t$ with e^{it} . Hence we will back to exponentials. This if one more example of a nice source function.