Laplace Transforms and Convolution

This is one more thing to tell about Laplace transforms, and introducing a new word -- convolution. And so we are going to find our old formula in new language, a new way. But the formula is familiar. And the problem is our basic problem, second order, linear, constant coefficient with a forcing term.

$$y'' + By' + Cy = f(t)$$

And we take zero boundary conditions. So the Laplace transform is just

$$(s^2 + Bs + C)Y(s) = F(s)$$

Then

$$Y(s) = G(s)F(s)$$

$G(s) = \frac{1}{s^2+Bs+C}$, Suppose my transform in one function of s times another function of s, what is the inverse transform?

The answer is the $g * f$. It is called convolution. Let's see what convolution is.

$$y = g * f = \int_0^t g(t-T)f(T)\,dT$$

Its transform is $G(s)F(s)$. Convolution grows the number of functions that we can deal with on Laplace transform. Because it tells us what to do with products.