Wave Equation

This lecture is about the third of the great trio of partial differential equations. Laplace's equation was #1. That's called an **elliptic equation**. The heat equation was #2. That's called a **parabolic equation**. Now we reach the wave equation. That's #3, and it's called **hyperbolic equation**. So somehow the three equations remind us of ellipses, parabolas and hyperbolas. They have different types of solutions. Laplace's equation, you solve it inside a circle or inside some closed region. The heat equation and the wave equation, time enters, and you are going forward in time. The heat equation is first order in time. And the wave equation, the full-scale wave equation, is second order in time.

$$u_{tt} = c^2 u_{xx}$$

And it matches the second derivative in space with a velocity coefficient c^2. We are in 1-dimensional space. If we were in three dimensions, where we really have sound waves and light waves and all the most important things in life, then there would be a u_{xx}, u_{yy}, u_{zz}.

So what are the differences. first of all, between the heat equation and wave equation?

The heat, the signal travels infinitely fast. And the wave equation, the signal travels with finite velocity, and that velocity is c.

The solution is

$$u(t, x) = f(x - ct) + g(x + ct)$$

We can solve it with initial condition $u(0, x)$ and $\partial u/\partial t(0, x)$. And it gives a formula called d'Alembert's formula named after d'Alembert.