Eigenvalues and Stability 2 by 2 Matrix,

This is a good time to do 2×2 matrices, their eigenvalues and their stability. 2×2 eigenvalues are the easiest to do and understand. Good to separate out the 2×2 case from the later $n\times n$ eigenvalue problem.

Trace and Det. Stability means the eigenvalues $\lambda < 0$ when it is real or λ has the real part to be negative when it is a complex number. So the question is what's the test on the matrix. We can get that information from trace and determinant without going to finding the eigenvalues.

Suppose we have two negative eigenvalues then certainly, this would mean the trace would be negative and the determinant will give a positive number. That is the stability test. And actually, it works also for the complex situation. So the test for stability is

Trace < 0 and Det > 0.