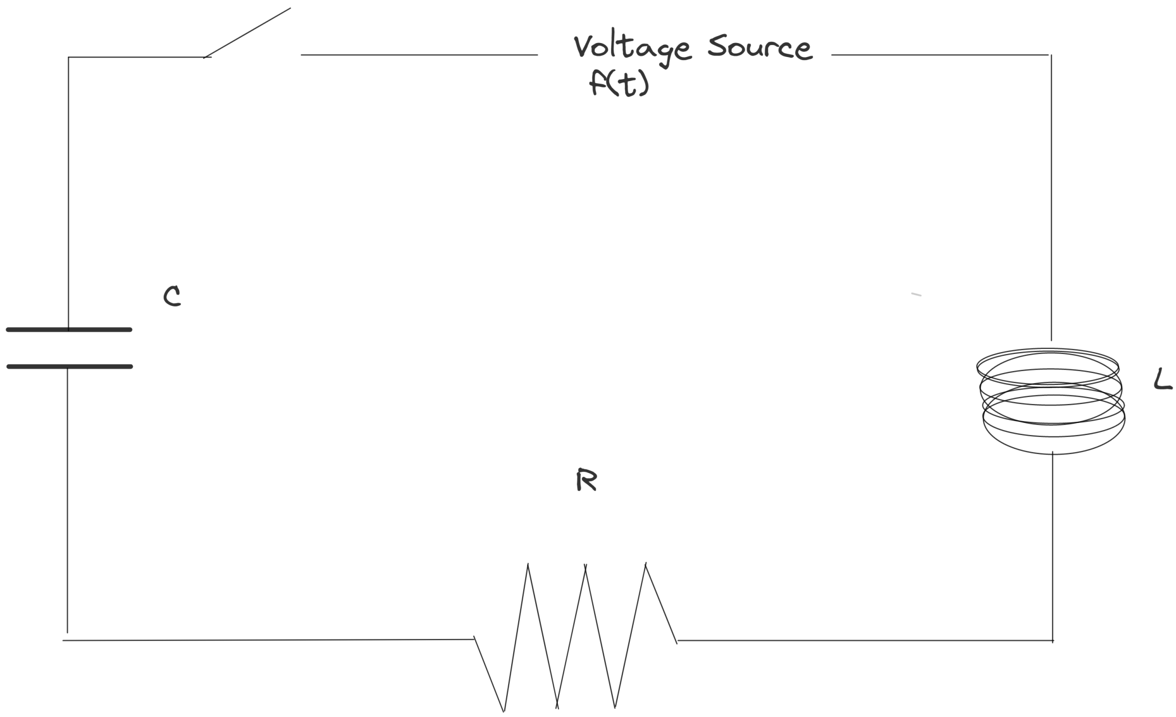
Electrical Networks Voltages and Currents

This lecture is about one of the key applications of ordinary differential equations to electrical flow, flow of currents in a network.

Here is a simple network. It called an RLC loop. The R stands for resistance to the flow. The L stand for an inductance. And C is the Capacitance



Those are the three elements of a simple linear constant coefficient problem associated with one loop. And then there is a switch, which we can close and there is a voltage source, so like a battery or alternating current.

$$f(t) = Ve^{i\omega t}$$

We want find the current $I(t)$. And we saw our differential equation will have that unknown $I(t)$, rather than usual y. Let's see the second order differential equation.

$$L\frac{dI}{dt} + RI + \frac{1}{C}\int I\,dt = Ve^{i\omega t}$$

You remember Ohm's law. That the voltage is the current times the resistance. We solve this equation by the standard idea which applies when we have constant coefficients and a pure exponential forcing term. We look for a solution that is a multiple of that exponential.

$$I(t) = We^{i\omega t}$$

Suppose we take the derivative of every term just to make it look really familiar.

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = i\omega e^{i\omega t}$$

So it's just a standard second order constant coefficient linear differential equation. And in fact, if you are a mechanical engineer, you would look at that and say, well, I don't know what L, R, and $1/C$ stand for. But I know that I should see the mass, the damping, and the stiffness there. So we have a complete parallel between two important fields of engineering, the electric engineering with L, R, and $1/C$, mechanical engineering with M, B for damping, and K for stiffness. And actually, that parallel allowed **analog computers** -- which came before digital computers and lost out in that competition. An analog computer was just solving this linear equation by actually imposing the voltage and measuring the current. So an analog computer actually solved the equation by creating the model and measuring the answer. But we are not creating an analog computer here. Here we just plug in and get the W.

$$(i\omega L + R + \frac{1}{i\omega C})W = V$$

and solve for W.

$$W = \frac{V}{R + (i\omega L - \frac{i}{\omega C})}$$

The whole thing with R, L, C is called the **impedance**. W is a complex number. And we can get the magnitude of the $R + (i\omega L - \frac{i}{\omega C})$

$$R^2 + (\omega L - \frac{1}{\omega C})^2$$

So how do we proceed with a full scale circuit with many nodes, many resistors, many conductors, many edges? Well, we have a big decision to make. We can use Kirchhoff's current law at the nodes and solve for the voltages at those nodes.