Separable Equations

Separable equations are, in principle, the easiest to solve. They include nonlinear equations but they have a special feature that makes them easy and approachable. And that special feature is that the right hand of the equation separates into some function of t divide by or multiplied by some function of y

$$\frac{dy}{dt} = \frac{g(t)}{f(y)}$$

Suppose $f(y)$ is 1, then we have this simplest differential equation. Suppose there was no t, just $1/f(y)$ then we can get $\int f(y)\,dy = \int 1\,dt$

Here is the case when there is both a $g(t)$ and $f(y)$.

$$\int_{y(0)}^{y(t)} f(y)\,dy = \int_0^t g(s)\,ds$$

Let's see examples.

$$\frac{dy}{dt} = \frac{t}{y}$$

we combine those to

$$y\,dy = t\,dt$$

integrating both sides.

$$\frac{1}{2}(y^2(t) - y^2(0)) = \frac{1}{2}t^2$$

The equation is solved. But we have not found it in the form $y(t) = \cdot$

For this example, we can easily get

$$y(t) = \sqrt{y^2(0) + t^2}$$

In addition, it's essential to begin to look for dangerous points -- singular points where things are not quite right.

Here the singular point is clearly $y = 0$. If I start at $y_0 = 0$ then I am not sure what is the solution to that equation.

What happens strangely is there are other solutions.

Let's see another example about logistic equation

$$\frac{dy}{dt} = y - y^2$$

That is separable because $g(t)$ part is 1. We can easily get the solution with **partial fractions** (we will discuss later with the method of **Laplace**)