Exponential Response -- Possible Resonance

This lecture is the nice case, constant coefficient, linear equations, and the right hand side is an exponential.

$$Ay'' + By' + Cy = e^{st}$$

We look for a solution

$$y = Ye^{st}$$

We just plug into the equation to find that transfer function Y.

$$Y = \frac{1}{As^2 + Bs + C}$$

It gives the exponential response. Very nice formula. And you remember for second degree equations, our most important case is as squared $As^2 + Bs + C$. That is the solution almost every time. But one thing can go wrong. Suppose for the particular s, the particular exponent, in the forcing function is also one of the $s's$ in the null solutions. And that will make the denominator 0. This is called **resonance**. Those special $s's$, we could also call them **poles of the transfer function**.

We need a new $y(t)$. It's a typical case of L'Hopital rule from calculus when we approach this bad situation.

$$y_p = \frac{e^{st}}{As^2 + Bs + C}$$

Subtract a null solution.

$$y_p = \frac{e^{st} - e^{s_1 t}}{As^2 + Bs + C}$$

This is what we call a very particular solution.

$$y_{resonant} = \frac{te^{s_1 t}}{2As_1 + B}$$

If s_1 was a double root, then using L'Hopital back again.

We will use Laplace transform in latter lecture.