Examples of Fourier Series

For δ function. We can get that

$$\delta(x) = \frac{1}{2\pi} + \frac{1}{\pi}(\sum_{k=1}^{\infty} \cos kx)$$

with the special properties of delta function.

And we will see more examples of Fourier series. We will start with a function that's odd.

$$f(x) = \begin{cases} 1 & 0 \le x < \pi \\ -1 & -\pi \le x < 0 \end{cases}$$

So we can get the coefficient by

$$b_k = \frac{2}{\pi} \int_0^{\pi} f(x) \sin kx \, dx = \frac{2}{\pi} \frac{-\cos kx}{k} |_0^{\pi}$$

We can see some slow decay in the Fourier coefficients.

$$f(x) = \frac{4}{\pi}(\sin x + \frac{\sin 3x}{3} + \cdots + \frac{\sin nx}{n} + \cdots)$$

Smooth function connects with fast decay.

Rule for derivatives:

$$\frac{df}{dx} = \sum ikc_k e^{ikx}$$

$$f(x-d) = \sum c_k e^{ik(x-d)}$$

Those are two good rules that show why you can use Fourier series in differential equations and in difference equations.