Laplace Equation

This lecture is talking about the first of the three great partial differential differential equations. So this one is called **Laplace's equation**, named after Laplace.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y} = 0$$

And you see partial derivatives. So we don't have time. This equation is in steady state. We have x, y, we are in the xy plane. And we have second derivatives in x and then y. So we are looking for solutions to that equation. And of course we are given some boundary values. The boundary is in the xy plane maybe a circle. Think about a circle in the xy plane. And on the circle, we know the solution u. So the boundary values around the circle are given. And we have to find the temperature u inside the circle. And the beauty is, it solves that basic partial differential equation.

So let's find some solutions. They might not match the boundary values, but we can use them. So $u = 1$ certainly solve the equation. $u = x$, $u = y$, $u = x^2 - y^2$. $u = 2xy$. Those are simple solutions. But those are only a few solutions and we need an infinite sequence because we are going to match boundary conditions. So is there a pattern here?

We hope for two cubic ones. And then we hope for two fourth degree ones. And that's the pattern.

$$x^3 - 3xy^2$$

It fits. Actually, we get these crazy polynomials from taking

$$(x + iy)^n$$

's real part and imaginary part. And we can take infinite combination of them.