Second Order Systems

Now, we are going to have systems of differential equations, so there will be matrices and vectors, using symmetric matrix. They will be second order

$$y'' + Sy = 0$$

That's the first time we have been prepared for the most fundamental equation of physics, of mechanics, oscillating springs -- so many applications -- rotating torques. It's very important in applications. The finite element, giant finite finite element codes are solving equations like that all the time. And we don't have a damping term here -- or a forcing term, so it's the null solutions that we are going to look for to match initial conditions. The real central equation always like

$$My'' + Ky = 0$$

--Newton's law. M is the diagonal matrix tell us the mass matrix. K is the stiffness matrix.

So, actually, in applications, the first job is to take the problem and create these matrices. Suppose we have got them. How do we solve them?

We look for, as we always do, solutions where time is separated from the vector x- $y = e^{i\omega t}x$ We substitute that into the equation, so we get

$$M(i\omega)^2 e^{i\omega t}x + Ke^{i\omega t}x = 0$$

And we see, we have an eigenvalue problem.

$$Kx = M\omega^2 x$$

So the solution is break down into some piece with $\sin \omega t$ and $\cos \omega t$. And the final solution is depend on the initial condition.