Second Order Equations with Damping

About

$$Ay'' + By' + Cy = \cos \omega t = Re(e^{i\omega t})$$

Rectangular form

$$y = M \cos \omega t + N \sin \omega t$$

$$M = \frac{C - A\omega^2}{D}$$

$$N = \frac{B\omega}{D}$$

$$D = (C - A\omega^2)^2 + (B\omega)^2$$

That is the solution. We also have a better form solution -- polar form

$$y(t) = G \cos(\omega t - \alpha)$$

$$G = \frac{1}{\sqrt{D}} = \sqrt{M^2 + N^2}$$

$$\tan \alpha = \frac{N}{M} = \frac{BD}{C - A\omega^2}$$

Better notation instead of A, B, C

$$\omega_n = \sqrt{\frac{C}{A}} = \sqrt{\frac{k}{m}}$$

$$\text{damping ratio} \zeta = \frac{B}{\sqrt{4AC}} = \frac{b}{\sqrt{4mk}}$$