

Midpoint method, ODE2

The cost of a numerical method for solving ODE's is measured by the number of times it evaluates the function f per step.

Euler's method evaluates f once per step. Here is a new method that evaluates it twice per step.

If f is evaluated once at the beginning of the step to give a slope s_1 and then s_1 is used to take Euler's step halfway across the interval, the function is evaluated in the middle of the interval to give the slope s_2 and s_2 is used to take the step. For obvious reasons, this is called the midpoint method

$$\begin{aligned}s_1 &= f(t_n, y_n) \\ s_2 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}s_1\right) \\ y_{n+1} &= y_n + hs_2\end{aligned}$$