

# Fourier Series

---

We are going to explain Fourier series. We have a function and we want to write it as a combination of cosines and sines.

$$f(x) = \sum_{n=0}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx = \sum_{-\infty}^{\infty} c_n e^{inx}$$

So how to find  $a_n, b_n$ ? The key is orthogonality. So that's the first central idea here in Fourier series, is the idea of orthogonality.

$$\int_{-\pi}^{\pi} (\cos nx)(\cos kx) dx = 0$$

So we multiply  $\cos kx$  on the both sides of the first equation and integrate. We can get

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx dx$$
$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx \quad (\text{The average of } f(x))$$

same with  $b_k$ .