Variation of Parameters

This lecture is a specific way to solve linear differential equation. We will take a second ode as an example. This way is called variation of parameters, and it will lead us to formula for the answer, an integral. So what's the idea?

We are looking for a particular solution. We have to know two null solutions to get started. And the idea is to multiply them by functions.

$$y(t) = c_1(t)y_1(t) + c_2(t)y_2(t)$$

Plug into the equation and we can get the result.

$$\begin{cases} c_1'(t)y_1 + c_2'(t)y_2 = 0 \\ c_1'(t)y_1' + c_2'(t)y_2' = f(t) \end{cases}$$

c_1', c_2' depend on t. So the answer is

$$y(t) = y_1(t) \int \frac{-y_2 f(t)}{W(t)}\, dt + y_2(t) \int \frac{y_1 f(t)}{W(t)}\, dt$$

$W(t)$ is the Wronskian.