Impulse Response and Step Response

This is a lecture in which we go for second-order equations, constant coefficients. We look for the impulse response, the key function in this whole business and the step response, too.

$$g'' + Bg' + Cg = \delta(t)$$

we are going to call g -- that will be the impulse response, where the right-hand side is a delta function, and impulse , a sudden force at the moment $t = 0$. We want a formula for it.

Then the another situation.

$$r'' + Br' + Cr = H(t)$$

is when the right-hand side is a step function. start from $r(0) = r'(0) = 0$. The step response starts from rest. Then action happens when clicking a switch of a machine at $t = 0$ and then $r(t)$ will rise to a constant. Very important solutions. We will focus especially on the first one.

The solution is

$$g(t) = \frac{e^{s_1 t} - e^{s_2 t}}{s_1 - s_2}$$

That's the impulse response -- a null solution that satisfies there special initial conditions. In mathematics, that's sometimes called the fundamental solution. It's a solution from which you can create all solutions. It's really the mother of solution to this second-order differential equation. Because if I have another forcing function, this tells us that growth rate. It's just like to e^{at} for the first-order equation.

We need to get more insight on that for particular cases. So let's see the same function when no damping. $B = 0$.

Now that's see the step function.

$$r(t) = 1 + \frac{s_2 e^{s_1 t} - s_1 e^{s_2 t}}{s_1 - s_2}$$

It starts from 0, and it rises to 1.Asymptotically it's 1.

The impulse response is like responsible for everything. How is $r(t)$ connected to $g(t)$?

The step function is the integral of the delta function. So that is the two key solution, you could say. The impulse response important in theory and in practice. The step response extremely important in practice because turning on a switching is so basic an operation in engineering.