

## Problems for Lecture 19

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3.

Let  $E$  denote the integers can be divided by 3, and  $F$  denote the integers can be divided by 7. Then

$$P(E \cup F) = P(E) + P(F) - P(EF) = \frac{1}{3} + \frac{1}{7} - \frac{1}{21} = \frac{3}{7}$$

8.

To prove the identity, we start with the definition of variance:

$$\sigma^2 = E[(X - E[X])^2]$$

Expanding the square, we get:

$$\sigma^2 = E[X^2 - 2XE[X] + E[X]^2]$$

Since  $E[X]$  is simply the mean  $m$ , we can write this as:

$$\sigma^2 = E[X^2] - 2mE[X] + m^2$$

Now, we can use the definition of the expected value:

$$E[X] = \sum_i p_i x_i$$

$$E[X^2] = \sum_i p_i x_i^2$$

Substituting these expressions into the equation for  $\sigma^2$ , we get:

$$\sigma^2 = \sum_i p_i x_i^2 - 2m \sum_i p_i x_i + m^2$$

Using the expression for  $m$ , we get:

$$\sigma^2 = \sum_i p_i x_i^2 - 2(\sum_i p_i x_i)^2 + (\sum_i p_i x_i)^2 = \sum_i p_i x_i^2 - m^2$$