Counting Parameters in SVD, LU, QR, Saddle Points

We would begin with the end of the linear algebra part and move on to a little probability, a little more optimization, and a lot of deep learning. By way of review, to write down the big factorizations of a matrix. We are going to checking that the number of free parameters agrees with the number of parameters in A itself.

$$A = LU \quad QR \quad X\Lambda X^{-1} \quad Q\Lambda Q^T \quad QS \quad SVD$$

Matrix Form	# free parameter
L = triangular/diagonal	$\frac{1}{2}n(n-1)$
U = triangular	$\frac{1}{2}n(n+1)$
Q = orthogonal	$\frac{1}{2}n(n-1)$
Λ = diagonal	n
X = eigenvectors	$n^2 - n$

The diagonal elements of L is determined by A. But U is not.

Every eigenvector can be multiplied by a scalar. We can assume that the first element of each column is 1.

The number of $X\Lambda$'s free parameters is n^2

The orthogonal matrices should follow two rules -- symmetric and normal.

We can add up the number of the free parameters and get total n^2 -- the number of the free parameters of A.

For SVD, the number is $mr + nr - r^2$.

Saddles from

$$R(x) = \frac{x^T S x}{x^T x}$$

We have a symmetric matrix. It is called Rayleigh quotient. The maximum of $R(x)$ is the largest eigenvalue and corresponding x is the eigenvector. If we put in any x whatever and look at this number, it's smaller than λ_1. And the minimum of the ratio is λ_n, the smallest eigenvalue. The beautiful thing about this Rayleigh quotient is its derivative equals 0 right at the eigenvectors. And its value at the eigenvectors is the eigenvalue.