Problems for Lecture 5

3.

The LDL^T forms are:

$$\begin{bmatrix}1 & 0\\ b & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 9-b^2\end{bmatrix}\begin{bmatrix}1 & b\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & c-8\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ b/c & 1\end{bmatrix}\begin{bmatrix}c & 0\\ 0 & (c^2-b^2)/c\end{bmatrix}\begin{bmatrix}1 & b/c\\ 0 & 1\end{bmatrix}$$

The first matrix is positive definite for $-3 < b < 3$, the second for $c > 8$, and the third both $c > 0$ and $c^2 - b^2 > 0$. The latter can be combined into $c > |b|$

14.

$$S = 4\begin{bmatrix}1 & -1 & 2\\ -1 & 1 & -2\\ 2 & -2 & 4\end{bmatrix}$$

$Rank = 1$. the pivots are $4, 0, 0$ the eigenvalues are $24, 0, 0$ and the determinant is 0

15.

$det_1 = 2\, det_2 = 6\, det_3 = 30$

$pivot_1 = 2\, pivot_2 = \frac{6}{2} = 3\, pivot_3 = \frac{30}{6} = 5$