

Problems for Lecture 22

1.

$$F'(X) = -\frac{1}{x_{11}} \quad F''(X) = \frac{1}{x_{11}^2}$$

6.

The least squares problem of minimizing $f(x) = \frac{1}{2} \|Ax - b\|^2$ can be solved using gradient descent. Here, A is a matrix, x and b are vectors of appropriate dimensions, and $\|\cdot\|$ denotes the Euclidean norm.

The gradient of $f(x)$ is given by:

$$\nabla f(x) = A^T(Ax - b)$$

To perform gradient descent, we start with an initial guess x_0 and iteratively update it using the gradient and a step size s_k as follows:

$$x_{k+1} = x_k - s_k \nabla f(x_k)$$

Substituting the expression for $\nabla f(x)$, we get:

$$x_{k+1} = x_k - s_k A^T(Ax_k - b)$$

This is the gradient descent equation for the least squares problem of minimizing $f(x)$.