Problems for Lecture 6

1.

$x^T x = (c_1 x_1 + \cdots + c_n x_n) \cdot (c_1 x_1 + \cdots + c_n x_n)$

and you can get the first formula because of the property of the orthonormal eigenvectors.

For the second formula, We know

$$Sx = c_1 S v_1 + \cdots + c_n S v_n = c_1 \lambda_1 v_1 + \cdots c_n \lambda_n v_n$$

and do the same as the first formula we get the second one.

6.

The singular value of A is $3\sqrt{5}, \sqrt{5}$.

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$$