Problems for Lecture 4

2.the eigenvalues of A are $2, 1$ and the corresponding eigenvectors are $[1, 1]^T$ and $[-2, 1]^T$ respectively.

the eigenvalues of A^{-1} are $\frac{1}{2}, -1$ and the corresponding eigenvectors are $[1, 1]^T$ and $[-2, 1]^T$ respectively.

A^{-1} has the <u>same</u> eigenvectors as A. When A has eigenvalues λ_1 and λ_2 its inverse has eigenvalues $\frac{1}{\lambda_1}$ and λ_2

11.The eigenvalues of A equal the eigenvalues of A^T. This is because $det(A - \lambda I)$ equals $det(A^T - \lambda I)$. That is true because $det[(A - \lambda I)^T] = det(A^T - \lambda I)$.

15.(a)

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

computing the eigenvalues of A, we get $1, 3$ (actually we do not need to compute, because it is a upper triangular matrix). Then computing the eigenvectors respectively, we gets $[1, 0]^T$ and $[1, 1]^T$ .Hence we know

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad X^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

(b) If $A = X\Lambda X^{-1}$ then $A^3 = X\Lambda^3 X^{-1}$ and $A^{-1} = X\Lambda^{-1}X^{-1}$