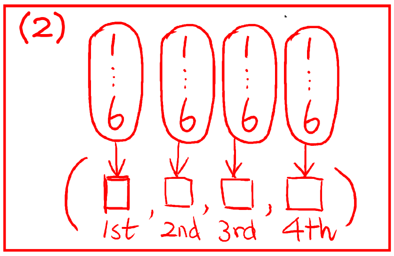
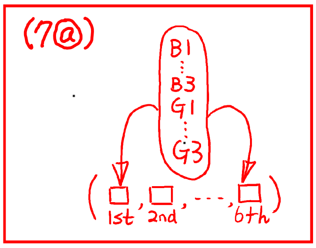
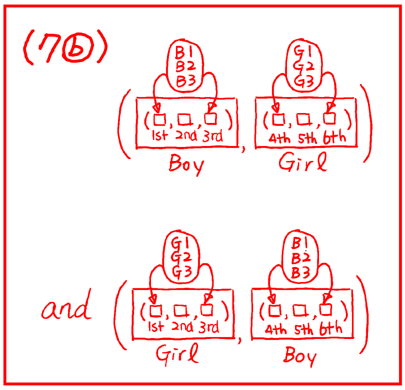
2、每次骰骰子有可能的結果：{1, 2, 3, 4, 5, 6}，且這次和下次結果並不相關，所以共有 $6\times6\times6\times6 = 1296$ 種情況.







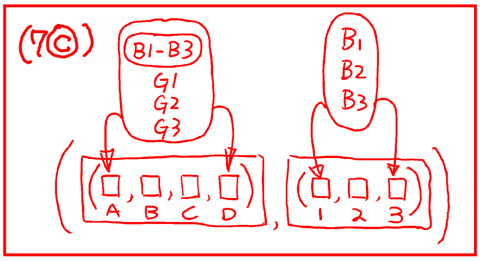
7. (a) 3男3女隨意坐：$6! = 720$ 種坐法

(b) (1) 先分別排男生、女生：男：$3!$
女：$3!$ ⇒ ans: $2\cdot3!\cdot3!$

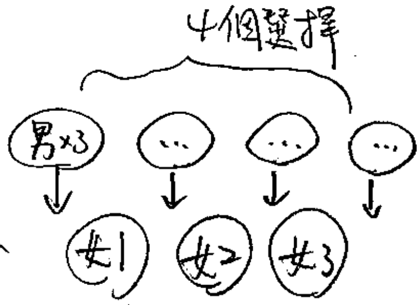
(2) 排男生先 or 女生先：2

(c) (1) 排男：$3!$

(2) 排女：$3!$



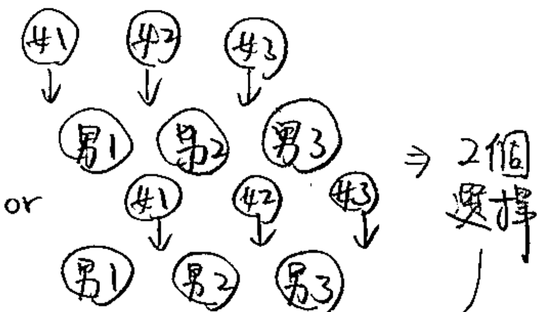
(3) 選擇3個男生要排在那個位置，圖示



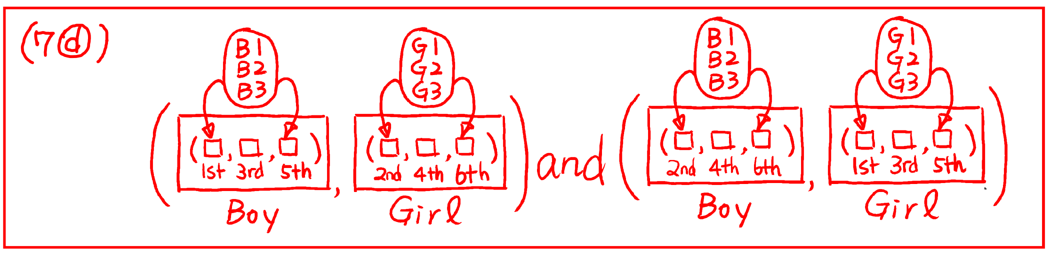
ans = $\underset{\text{選擇}}{4}\cdot\underset{\text{排男}}{3!}\cdot\underset{\text{排女}}{3!}$

(d) (1) 先排男 $3!$

(2) 排女且兩同性不坐一起，圖示：



ans: $\underset{\text{選擇}}{2}\cdot\underset{\text{排女}}{3!}\cdot\underset{\text{排男}}{3!}$



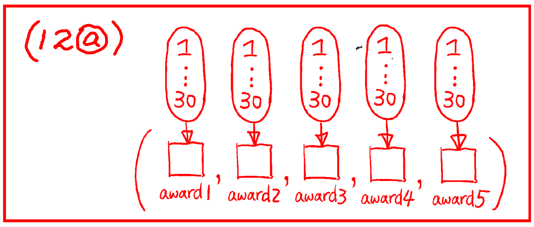
made by 廖芳翊, 劉必翔 助教

12: (a) 獎1, 獎2 ... 獎5

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$

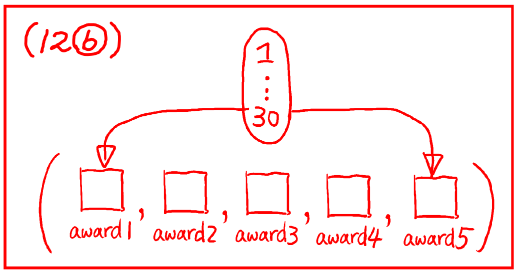
30個人都有機會拿到 : : : :

ans: $30^5 = 24300000$ ※



(b) 30個人選5人，由5個人分獎1~5

$C^{30}_5 \cdot 5! = 17100720$ 種 ※

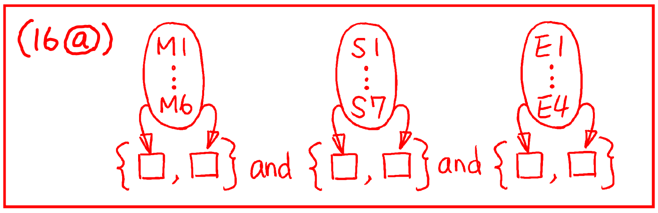


16: (a) 從 math 中選2本 C^6_2

從 eco 中選2本 C^4_2

從 sci 中選2本 C^7_2

ans: $C^6_2 + C^4_2 + C^7_2 = 42$ ※



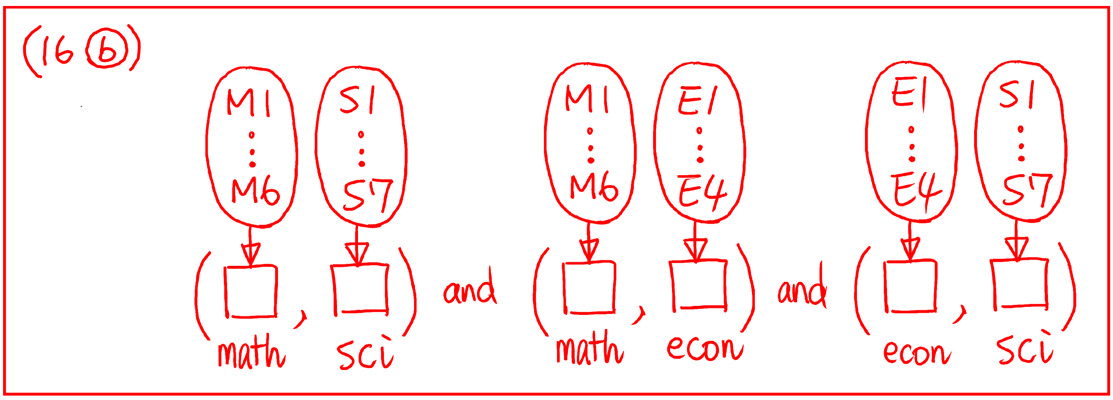
(b) 選2個科目再各選一本書.

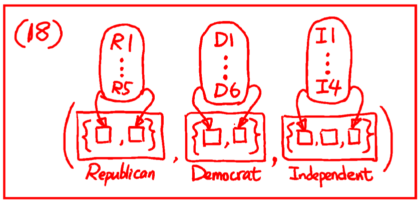
(1) if 未選 math, choice: $C^7_1 C^4_1 = 28$

(2) if 未選 sci, choice: $C^6_1 C^4_1 = 24$

(3) if 未選 eco, choice: $C^7_1 C^6_1 = 42$

ans: 94 ※

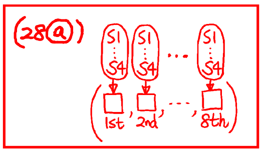




Problem 18

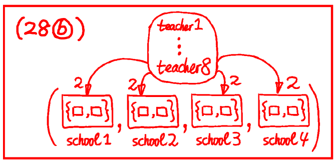
Choose 2 from 5 Republicans; 2 from 6 Democrats; 3 from 4 Independents:

$$\binom{5}{2} \times \binom{6}{2} \times \binom{4}{3} = 600.$$



Problem 28

- Every teacher has 4 school to choose: 4^8.
- Name 4 schools as A,B,C,D. Choose 2 teachers from 8 to A; choose 2 from the rest 6 teachers to B; choose 2 from the rest 4 teachers to C; the last 2 go to D: $\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = 2520$.



Problem 34

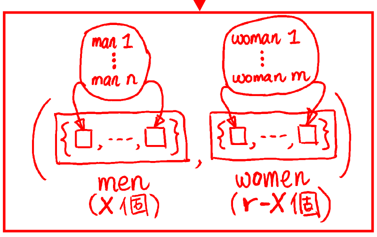
- Consider that we have 10 fish and want to count the number of each types, thus we change the question to find the number of non-negative integer solution of $x_1+x_2+x_3+x_4+x_5 = 10$, the answer will be: $\binom{5+10-1}{10} = 1001$.
- Use the similar method as previous part, we may rewrite the question as $x_2+x_3+x_4+x_5 = 7$, the answer will be: $\binom{4+7-1}{7} = 120$.
- Consider the opposite case, there are at most 1 trout caught. Case 0 trout: $\binom{13}{3} = 286$; Case 1 trout: $\binom{12}{3} = 220$. We know that there are total 1001 possibilities, hence, the answer we want is: $1001 - 286 - 220 = 495$.

Theoretical Exercises 8

Prove by double counting:

Left-hand side: Choose r people group from n men and m women, which derive $\binom{n+m}{r}$.

Right-hand side: Consider the r people group, the group has X men and r-X women, where X ranges from 0 to r. For each X, the number of possibilities is $\binom{n}{X}\binom{m}{r-X}$. Now, sum it over X, we get $\sum_{X=0}^{r}\binom{n}{X}\binom{m}{r-X}$, which is the form of right-hand side.



alternative solution:
$x_1 + \cdots + x_4 + x_5 = 10$, where
$x_1 \geq 2$, $x_2, \ldots, x_5 \geq 0$
Let $x_1' = x_1 - 2$. Then,
$x_1' + x_2 + \cdots + x_5 = 8$, and
$x_1', x_2, \ldots, x_5 \geq 0$.
the number of integer solutions for $(x_1', x_2, \ldots, x_5)$
is $\binom{8+4}{4} = \binom{12}{4} = 495$