Lecture 11: Power rankings

Skidmore College, MA 251

Goals

- Power rankings in sports
- Models for paired comparisons
- Tools: Elo, Bradley-Terry

Set-up:

Let's rank NBA Western Conference teams at the end of the 2019 regular season. Judging criteria: Who would win a game played tomorrow on a neutral site?

Rank	Team
1	Golden State
2	Houston
3	OKC? Utah?

Paired comparisons

In a league with n_t number of teams, there are $n_t!$ possible allocations of power rankings. Ultimately, each rank comes down to a set of decisions called **paired comparisons**, where each player or team is compared to another player or team.

Ex: If $P(OKC > Utah) > 0.5$, then OKC ranks 3 (assuming OKC also better than all other teams besides Golden State, Houston.)

Note: $P(OKC > Utah) > 0.5$ is the probability of Oklahoma City beating Utah

What assumptions are we making in doing several paired comparisons?

Idea of paired comparisons

One participant versus another (see more here):

1. Players, teams in sport
2. Consumer products in market research
3. Images in psychology

Most common paired comparison model: **Bradley-Terry (BTM)**

Notation, BTM

- Players (or teams) i and j
- Assume $P(i \text{ beats } j) = \frac{\alpha_i}{\alpha_i + \alpha_j}$
- α_i and α_j reflect player **abilities** for i and j, respectively
  - $\alpha_i > 0$ and $\alpha_j > 0$
- Odds $(i \text{ beats } j) = \frac{P(i \text{ beats } j)}{P(j \text{ beats } i)} = \frac{\alpha_i/(\alpha_i+\alpha_j)}{\alpha_j/(\alpha_i+\alpha_j)} = \frac{\alpha_i}{\alpha_j}$

Example, BTM:

Rank	Team	α
1	Golden State	5.3
2	Houston	4.7
3	OKC	2.9

Estimate

1. P(Golden State $>$ OKC)
2. Odds(Golden State $>$ OKC)
3. P(Houston $>$ Golden State)
4. Odds(Houston $>$ Golden State)

Notation, BTM

- logit(P(i beats j)) $=$ log(Odds (i beats j)) $=$ log(α_i) - log(α_j) $= \lambda_i$ - λ_j
  - $\lambda_i = log(\alpha_i)$ for all i
  - $\alpha_i = e^{\lambda_i}$ for all i

Example, BTM:

Rank	Team	λ
1	Skidmore	1.2
2	Vassar	0
3	RIT	-0.9

Questions

1. P(Skidmore $>$ RIT)
2. What does it mean to have a λ of 0?

How to find BTM parameter estimates?

```r
library(BradleyTerry2); library(tidyverse)
data("baseball", package = "BradleyTerry2")
head(baseball)
```

```
##   home.team away.team home.wins away.wins
## 1 Milwaukee   Detroit         4         3
## 2 Milwaukee   Toronto         4         2
## 3 Milwaukee  New York         4         3
## 4 Milwaukee    Boston         6         1
## 5 Milwaukee Cleveland         4         2
## 6 Milwaukee Baltimore         6         0
```

The model

```
baseballModel1 <- BTm(cbind(home.wins, away.wins), home.team, away.team,
  data = baseball, id = "team")
library(broom)
tidy(baseballModel1)
```

```
## # A tibble: 6 x 5
##   term          estimate std.error statistic    p.value
##   <chr>            <dbl>     <dbl>     <dbl>      <dbl>
## 1 teamBoston       1.11      0.334      3.32 0.000908
## 2 teamCleveland    0.684     0.332      2.06 0.0393
## 3 teamDetroit      1.44      0.340      4.23 0.0000234
## 4 teamMilwaukee    1.58      0.343      4.61 0.00000409
## 5 teamNew York     1.25      0.336      3.71 0.000203
## 6 teamToronto      1.29      0.337      3.84 0.000121
```

Where's Baltimore?

Next steps

```r
BTabilities(baseballModel1)
```

```
##              ability      s.e.
## Baltimore 0.0000000 0.0000000
## Boston    1.1076977 0.3338779
## Cleveland 0.6838528 0.3318764
## Detroit   1.4364084 0.3395682
## Milwaukee 1.5813559 0.3432557
## New York  1.2476178 0.3358606
## Toronto   1.2944851 0.3366691
```

Next steps

```r
exp(BTabilities(baseballModel1))
```

```
##            ability     s.e.
## Baltimore 1.000000 1.000000
## Boston    3.027380 1.396373
## Cleveland 1.981497 1.393581
## Detroit   4.205564 1.404341
## Milwaukee 4.861543 1.409529
## New York  3.482038 1.399144
## Toronto   3.649117 1.400276
```

Find estimated probability that (i) Boston defeats Cleveland and (ii) Baltimore defeats Boston

Home field advantage

```r
baseball$home.team <- data.frame(team = baseball$home.team, at.home = 1)
baseball$away.team <- data.frame(team = baseball$away.team, at.home = 0)
baseballModel2 <- update(baseballModel1, formula = ~ team + at.home)
tidy(baseballModel2)
```

```
## # A tibble: 7 x 5
##   term          estimate std.error statistic    p.value
##   <chr>            <dbl>     <dbl>     <dbl>      <dbl>
## 1 teamBoston       1.14      0.338      3.39 0.000710
## 2 teamCleveland    0.705     0.335      2.10 0.0354
## 3 teamDetroit      1.48      0.345      4.28 0.0000185
## 4 teamMilwaukee    1.62      0.347      4.66 0.00000313
## 5 teamNew York     1.28      0.340      3.76 0.000167
## 6 teamToronto      1.33      0.340      3.90 0.0000964
## 7 at.home          0.302     0.131      2.31 0.0210
```

Compare the fit of these two models:

```
AIC(baseballModel1)
```

```
## [1] 140.5186
```

```
AIC(baseballModel2)
```

```
## [1] 137.108
```

```
exp(baseballModel2$coeff)
```

```
##    teamBoston teamCleveland   teamDetroit teamMilwaukee  teamNew York
##      3.138681      2.023228      4.372597      5.050842      3.601464
##   teamToronto       at.home
##      3.770133      1.352914
```

Interpret the effect of HFA in baseball.

Challenges and final thoughts

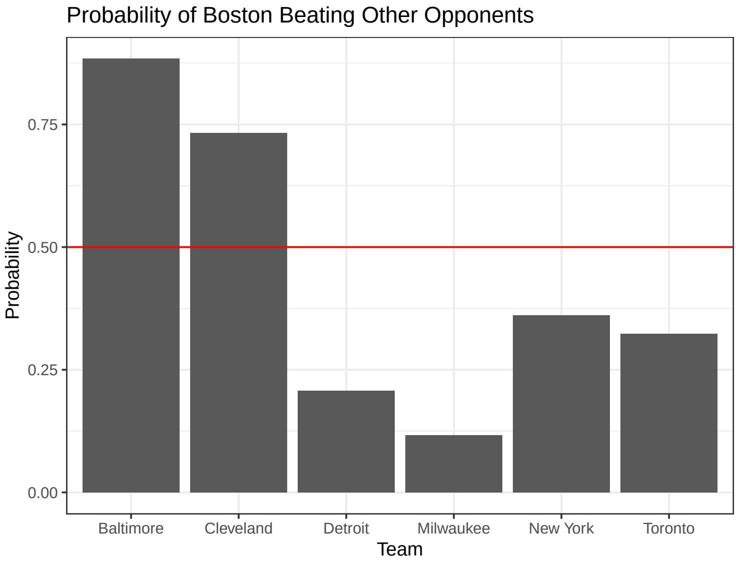
- Importance of data formatting for BTM
- Links to Elo
  - $\alpha_i = e^{S_i/k}$
  - k a sport-specific scale factor
  - S_i another team-level skill factor
  - Iterative process (can update after a game)
- Similarity to other systems (log 5, item-response)

Fun

```
df.predict <- data.frame(exp(BTabilities(baseballModel2)))
df.predict$Team <- rownames(df.predict)
boston_probs <- df.predict %>%
      filter(Team != "Boston") %>%
      mutate(p_boston = exp(3.03)/(exp(3.03) + exp(ability)))

p <- ggplot(boston_probs, aes(x = Team, y = p_boston)) +
  geom_col() +
  ylab("Probability") + theme_bw() +
  geom_hline(aes(yintercept = 0.5), color = "red") +
  ggtitle("Probability of Boston Beating Other Opponents")
```

Fun



Probability of Boston Beating Other Opponents
Probability
0.00
0.25
0.50
0.75
Baltimore
Cleveland
Detroit
Milwaukee
New York
Toronto
Team