

[13011]

DERIVE BAYES INFERENCE

$$LH(x) * Prior(x) = \left(\frac{2}{\sigma_{LH} \sqrt{2\pi}} \right) \cdot \left(\frac{1}{\sigma_P \sqrt{2\pi}} \right) \cdot e^{-\frac{1}{2} \left(\frac{x - \mu_{LH}}{\sigma_{LH}} \right)^2} \cdot e^{-\frac{1}{2} \left(\frac{x - \mu_P}{\sigma_P} \right)^2}$$

$$\hookrightarrow \frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\frac{1}{2} \left(\frac{x}{\sigma_P} \right)^2} \quad \text{if } \mu_P = 0$$

$$\hookrightarrow \frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\left[\frac{1}{2} \left(\frac{x - \mu_{LH}}{\sigma_{LH}} \right)^2 - \frac{1}{2} \left(\frac{x}{\sigma_P} \right)^2 \right]}$$

* as $\exp(x) \cdot \exp(y) = \exp(x+y)$

$$\hookrightarrow \frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\left[\frac{(x - \mu_{LH})^2}{2\sigma_{LH}^2} - \frac{x^2}{2\sigma_P^2} \right]}$$

$$\hookrightarrow \frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\left[\frac{(x - \mu_{LH})^2}{2\sigma_{LH}^2} - \frac{x^2}{2\sigma_P^2} \right]}$$

$$*(a-b)^2 = a^2 - 2ab + b^2$$

$$\frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\left[\frac{(x^2 - 2x\mu_{LH} + \mu_{LH}^2)}{2\sigma_{LH}^2} - \frac{x^2}{2\sigma_P^2} \right]}$$

$$\frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\left[\frac{-x^2 + 2x\mu_{LH} - \mu_{LH}^2}{2\sigma_{LH}^2} - \frac{x^2}{2\sigma_P^2} \right]}$$

$$\frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\frac{-2\sigma_P^2 \cdot x^2 + 4\sigma_P^2 \cdot \mu_{LH} - 2\sigma_P^2 \cdot \mu_{LH}^2 - 2\sigma_{LH}^2 \cdot x^2}{4\sigma_{LH}^2 \cdot \sigma_P^2}}$$

$$\frac{1}{2\pi \cdot \sigma_{LH} \cdot \sigma_P} \cdot e^{-\frac{-(2 \cdot x^2) + 4\mu_{LH} - 2\mu_{LH}^2 - (2 \cdot x^2) \cdot \frac{\sigma_{LH}^2}{\sigma_P^2}}{4 \cdot \sigma_{LH}^2}}$$

$$\frac{36}{144} = \frac{1}{4}$$

GENERAL

if $\mu_P \neq 0$

$$\mu_{post} = \mu_L \cdot \frac{c^2 \left(\frac{\sigma}{\sigma_P} \right)^2 \cdot \frac{\mu_P}{\sigma_P^2}}{\left(\frac{\sigma}{\sigma_P} \right)^2 + c^2}$$

if $\mu_P = 0$; $\mu_{post} = \frac{\mu_L \cdot \frac{c^2}{\sigma_P^2}}{1 + \frac{c^2}{\sigma_P^2}}$

$$\mu_{post} = \frac{\frac{\mu_L}{\sigma_L^2} + \frac{\mu_P}{\sigma_P^2}}{\frac{1}{\sigma_L^2} + \frac{1}{\sigma_P^2}}$$

$$\hookrightarrow \mu_L \cdot \frac{c^2}{K^2 + c^2} = \mu_{post}$$

OK

$\sigma = \text{dev}$

$\mu_L = \mu_{real}$

$K = \frac{\sigma}{\sigma_{prior}}$

$$\hookrightarrow \frac{\frac{\mu_L}{\left(\frac{\sigma}{c} \right)^2} + \frac{\mu_P}{\sigma_P^2}}{\frac{1}{\left(\frac{\sigma}{c} \right)^2} + \frac{1}{\sigma_P^2}} = \frac{\frac{\mu_L}{\frac{\sigma^2}{c^2}} + \frac{\mu_P}{\sigma_P^2}}{\frac{1}{\frac{\sigma^2}{c^2}} + \frac{1}{\sigma_P^2}} = \frac{\left(\frac{1}{\frac{\sigma^2}{c^2}} \right) \cdot \mu_L + \frac{\mu_P}{\sigma_P^2}}{\left(\frac{1}{\frac{\sigma^2}{c^2}} \right) \cdot 1 + \frac{1}{\sigma_P^2}}$$

$$\hookrightarrow \frac{\mu_L + \frac{\mu_P \cdot \left(\frac{\sigma^2}{c^2} \right)}{\sigma_P^2}}{1 + \frac{\left(\frac{\sigma^2}{c^2} \right)}{\sigma_P^2}} = \mu_L \cdot \frac{1 + \frac{\mu_P \cdot \left(\frac{\sigma^2}{c^2} \right)}{\mu_L \cdot \sigma_P^2}}{1 + \frac{\left(\frac{\sigma^2}{c^2} \right)}{\sigma_P^2}} = \mu_L \cdot \frac{1 + \frac{\left(\frac{\sigma}{\sigma_P} \right)^2 \cdot \frac{1}{c^2}}{1 + \left(\frac{\sigma}{\sigma_P} \right)^2 \cdot \frac{1}{c^2}}}{1 + \left(\frac{\sigma}{\sigma_P} \right)^2 \cdot \frac{1}{c^2}}$$