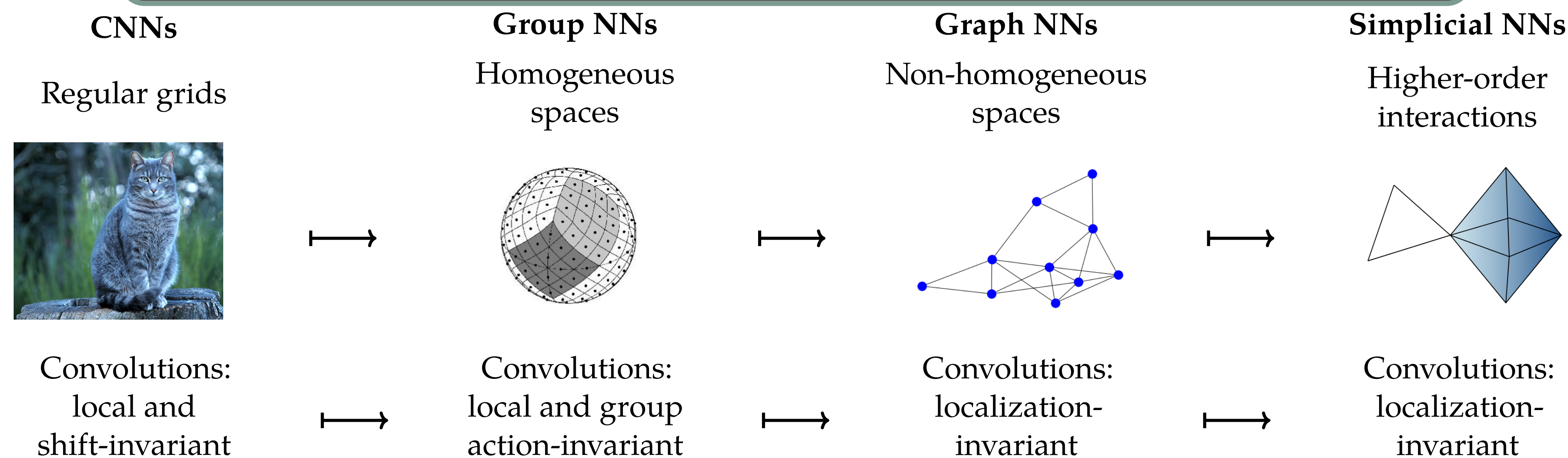


SIMPLICIAL NEURAL NETWORKS

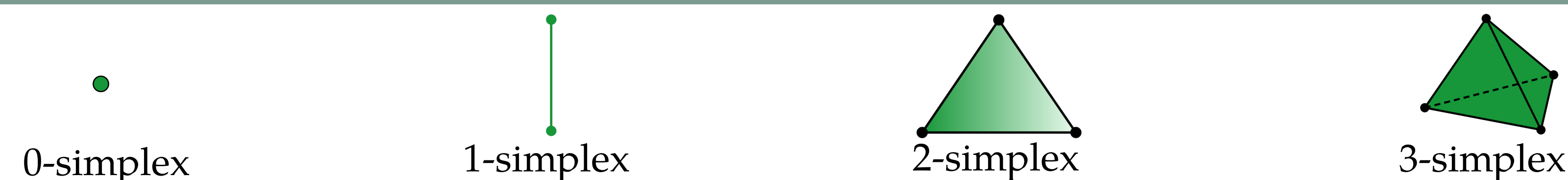
Stefania Ebli, Michaël Defferrard, Gard Spreemann

EPFL

Convolution: a way to exploit the space's structure



Basic building blocks of a space: simplices



Simplicial Neural Networks

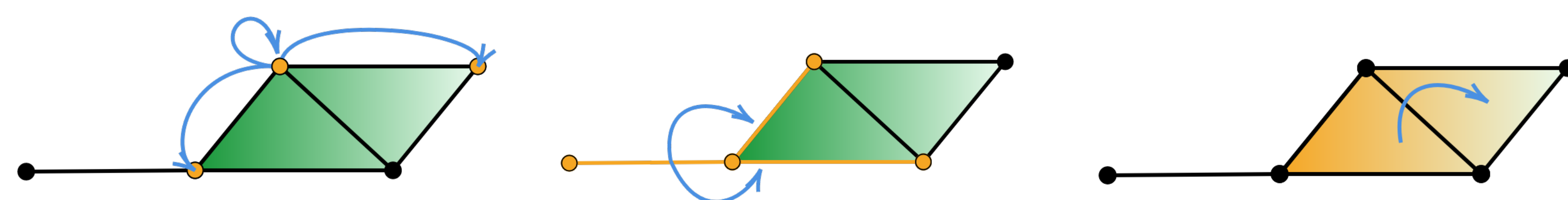
Laplacians for simplicial complexes

The graph Laplacian can be extended to Laplacians for simplices of any dimension k [1]. The k -Laplacian can be interpreted as a function propagating values of functions on the k -simplices. These functions are called k -cochains, x_k .

L_0 : Graph Laplacian
 $y_0 = L_0 x_0$

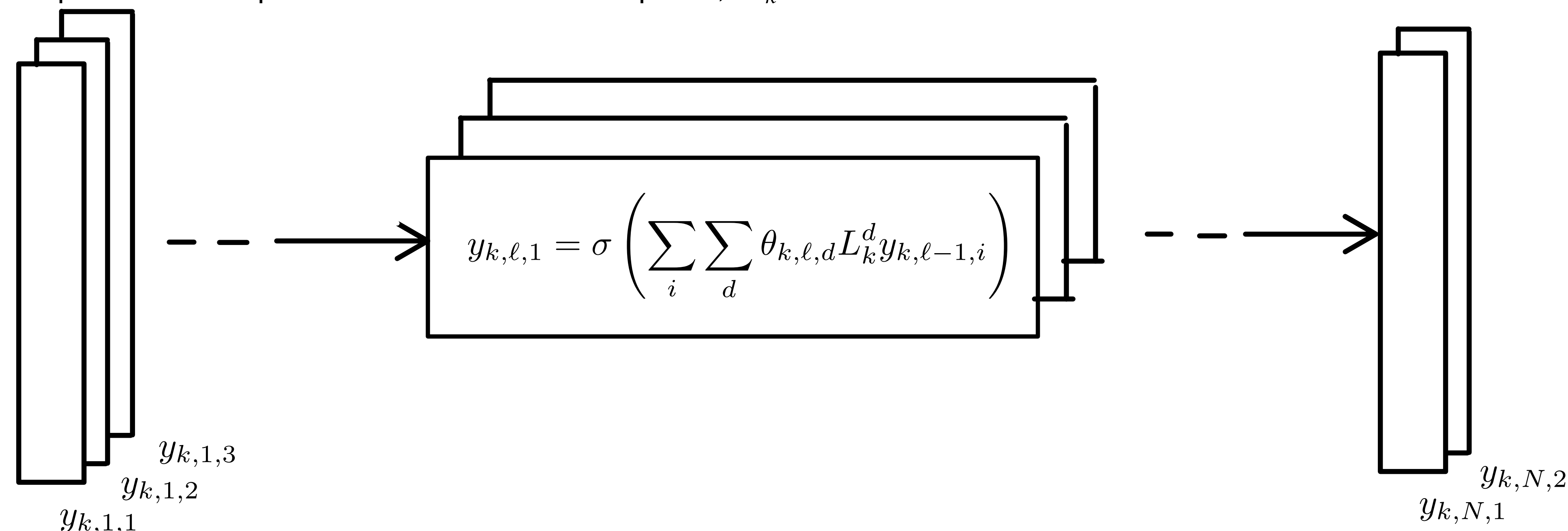
L_1 : 1-Laplacian
 $y_1 = L_1 x_1$

L_2 : 2-Laplacian
 $y_2 = L_2 x_2$



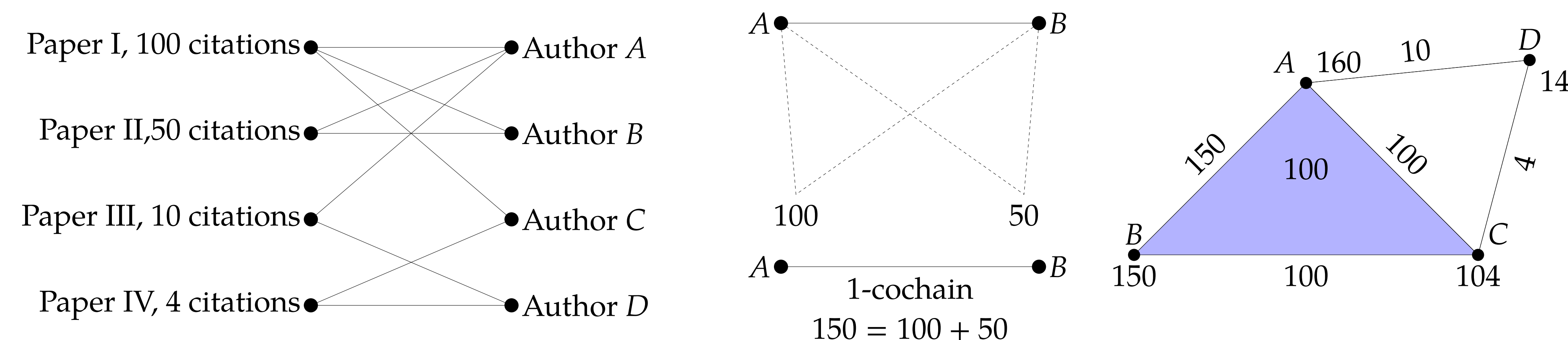
Simplicial Neural Networks (SNNs)

- The convolutional filters are low-degree polynomials in the Laplacian with learnable coefficients.
- The inputs are multiple k -cochains on the k -simplices, W_k .



- Convolutions can be implemented by sparse matrix-vector multiplications: the computational cost is $\mathcal{O}(\xi|W_k|)$.
- The number of weights to be learned is reduced from $\mathcal{O}(|W_k|)$ to $\mathcal{O}(1)$.
- The operation is d -localizing: if two simplices are more than d hops apart, there is no interaction between them.

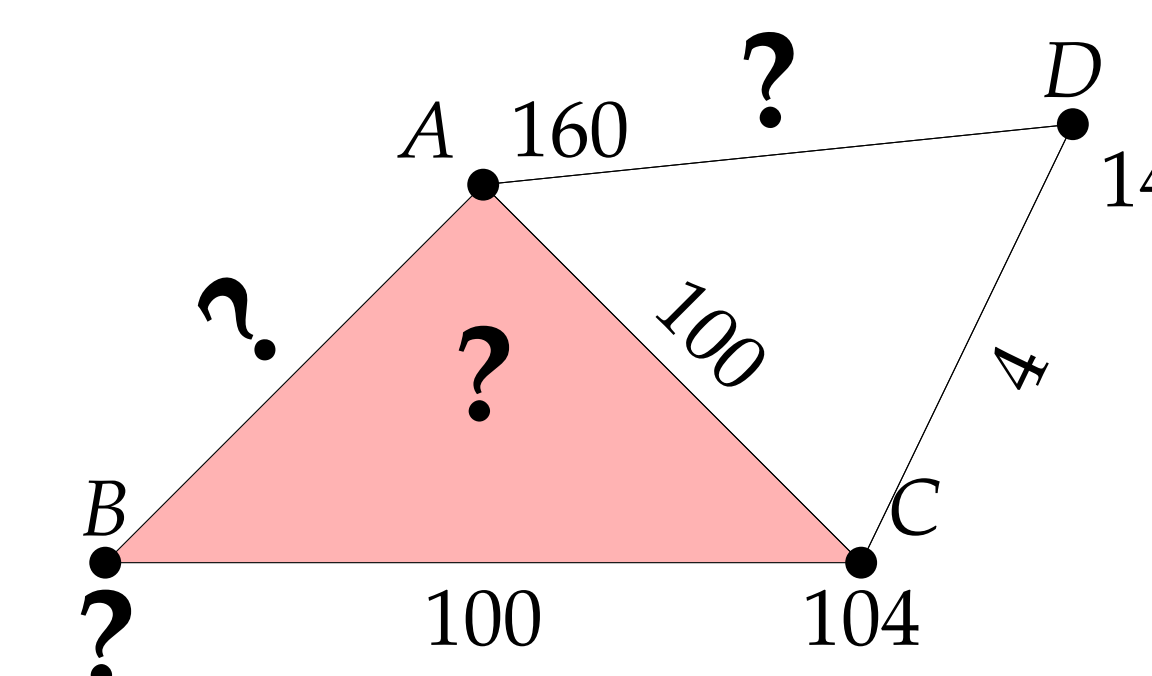
Coauthorship complex: from a bipartite graph to a complex



Predicting missing citations on the coauthorship complex

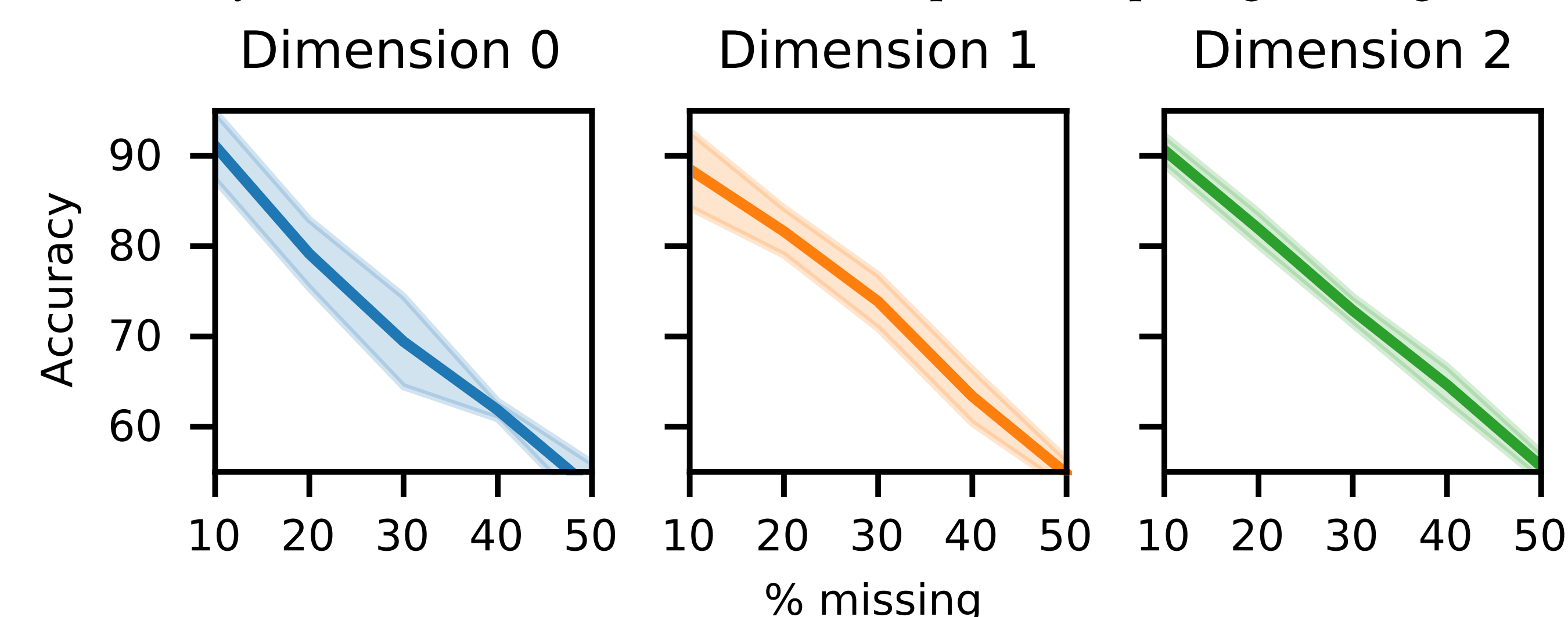
Data

Coauthorship complexes are built from the Semantic Scholar dataset [2] where missing citations are introduced at random on the k -cochains ($k = 1, 2, 3$) at five rates: 10%, 20%, 30%, 40%, and 50%.



First Results

Mean accuracy \pm standard deviation over 5 samples in imputing missing citations.



Performance of baselines: mean accuracy \pm standard deviation over 5 samples for 30% missing citations.

Method	Dimension 0	Dimension 1	Dimension 2
Global Mean	3.30 ± 0.82	5.75 ± 1.28	2.96 ± 0.49
Global Median	7.78 ± 2.70	10.44 ± 1.00	12.50 ± 0.63
Neighbors Mean	11.88 ± 5.29	24.15 ± 1.85	27.38 ± 1.18

Code: https://github.com/stefaniaebli/simplicial_neural_networks

References

- [1] D. Horak and J. Jost, *Spectra of combinatorial Laplace operators on simplicial complexes*, Adv. in Math. 2013.
- [2] W. Ammar et al., *Construction of the Literature Graph in Semantic Scholar*, <https://www.semanticscholar.org/paper/09e3cf5704bcb16e6657f6ceed70e93373a54618>.