Simplicial Neural Networks

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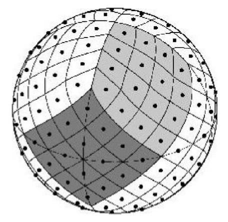
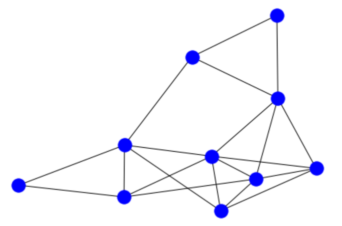
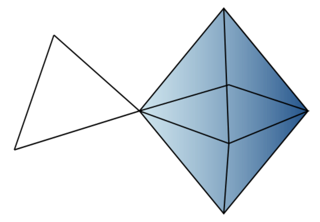
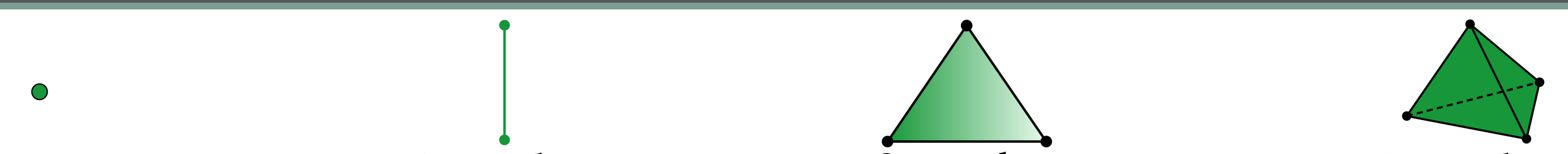
Convolution: a way to exploit the space's structure

CNNs	**Group NNs**	**Graph NNs**	**Simplicial NNs**
Regular grids	Homogeneous spaces	Non-homogeneous spaces	Higher-order interactions
			
Convolutions: local and shift-invariant	Convolutions: local and group action-invariant	Convolutions: localization-invariant	Convolutions: localization-invariant

Basic building blocks of a space: simplices

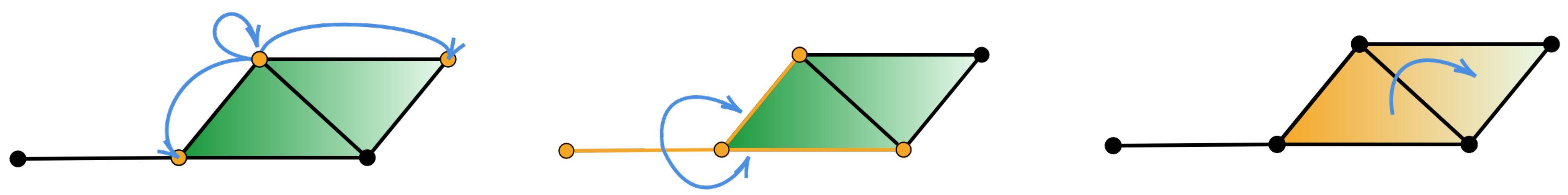


0-simplex 1-simplex 2-simplex 3-simplex

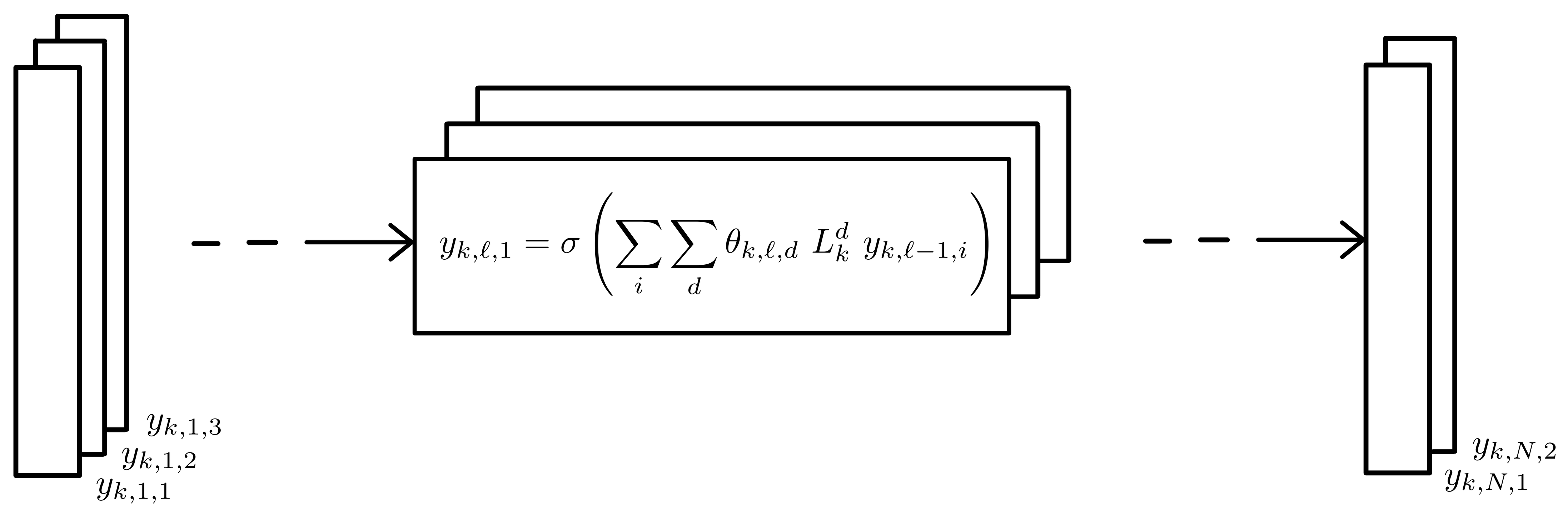
Laplacians for simplicial complexes

The graph Laplacian can be extended to Laplacians for simplices of any dimension k [**1**]. The k-Laplacian can be interpreted as a function propagating values of functions on the k-simplices. These functions are called k-cochains, x_k.

L_0: Graph Laplacian $y_0 = L_0 x_0$ | L_1: 1-Laplacian $y_1 = L_1 x_1$ | L_2: 2-Laplacian $y_2 = L_2 x_2$

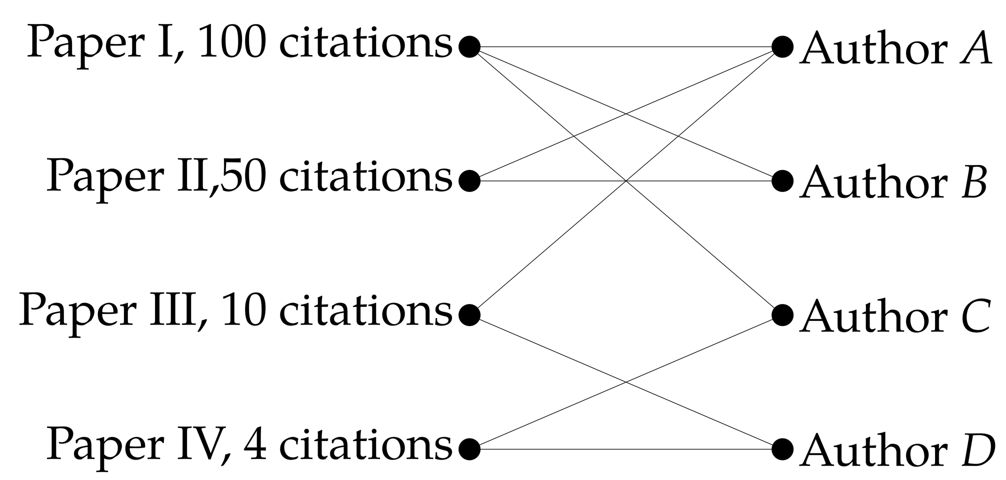


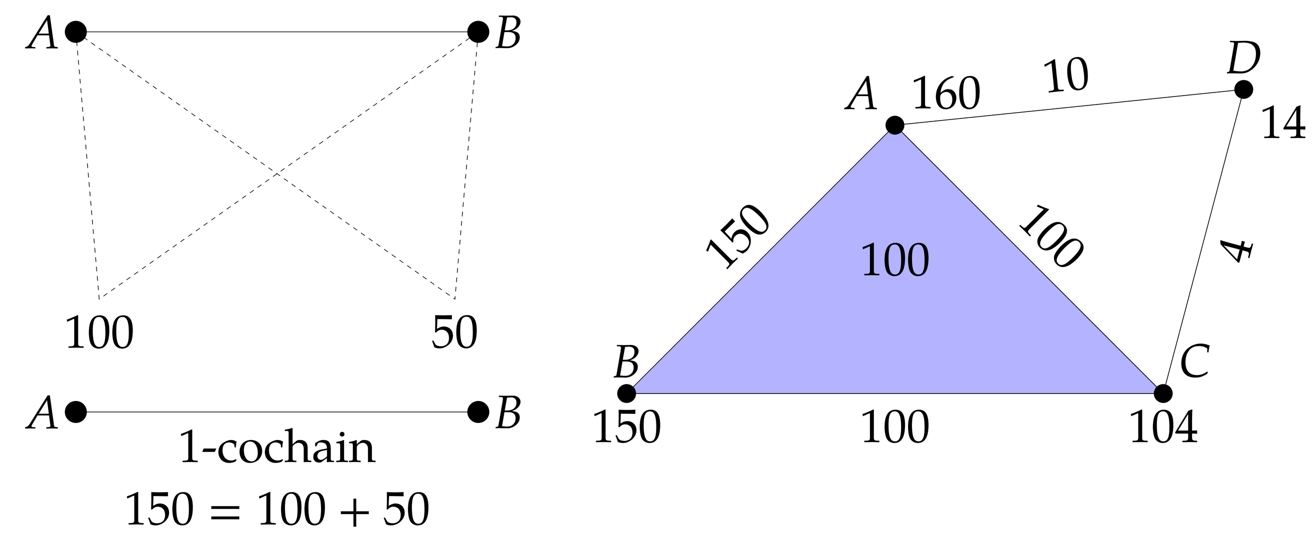
Simplicial Neural Networks (SNNs)



- Linear cost: convolutions are sparse matrix-vector multiplications.
- $O(1)$ weights to be learned.
- d-localizing: no interaction between simplices that are more than d hops apart.

Coauthorship complex: from a bipartite graph to a complex

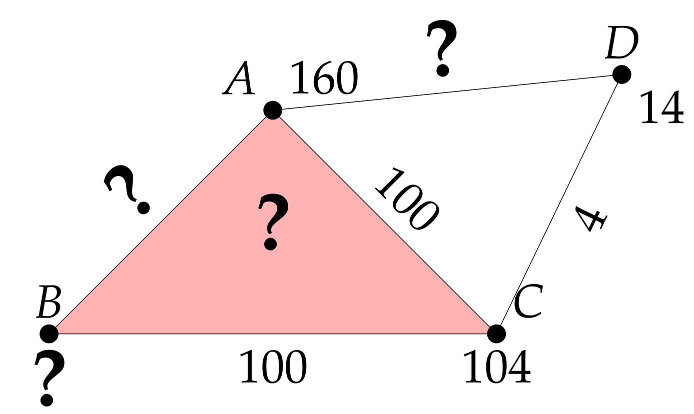




Predicting missing citations on the coauthorship complex

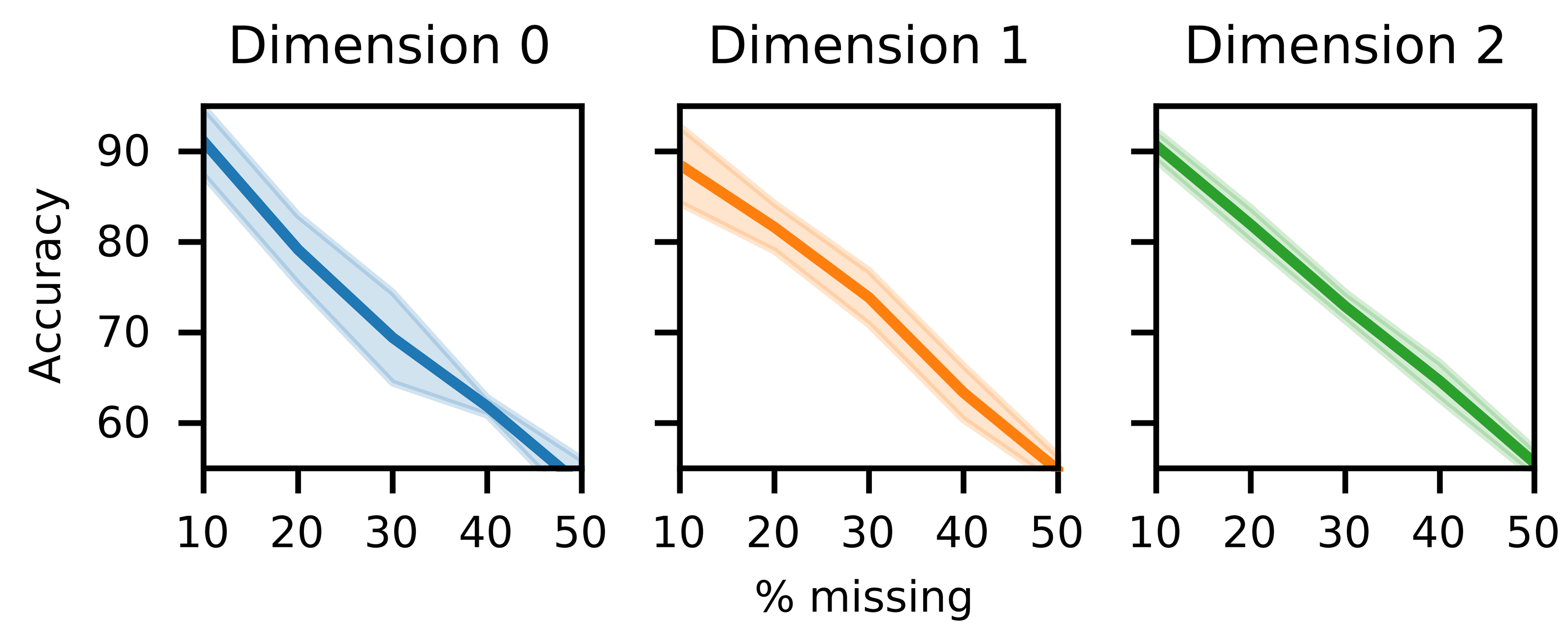
Data

Coauthorship complexes are built from the Semantic Scholar dataset [**2**] where missing citations are introduced at random on the k-cochains ($k = 1, 2, 3$) at five rates: 10%, 20%, 30%, 40%, and 50%.



First Results

Mean accuracy $\pm$ standard deviation over 5 samples in imputing missing citations.



Performance of baselines: mean accuracy $\pm$ standard deviation over 5 samples for 30% missing citations.

Method	Dimension 0	Dimension 1	Dimension 2
Global Mean	3.30 ± 0.82	5.75 ± 1.28	2.96 ± 0.49
Global Median	7.78 ± 2.70	10.44 ± 1.00	12.50 ± 0.63
Neighbors Mean	11.88 ± 5.29	24.15 ± 1.85	27.38 ± 1.18

Code: `https://github.com/stefaniaebli/simplicial_neural_networks`

References

[1] D. Horak and J. Jost, *Spectra of combinatorial Laplace operators on simplicial complexes*, Adv. in Math. 2013.

[2] W. Ammar et al. , *Construction of the Literature Graph in Semantic Scholar*, `https://www.semanticscholar.org/paper/09e3cf5704bcb16e6657f6ceed70e93373a54618`.

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