Vortex Distribution in Superfluid Dark Matter Halos

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I. INTRODUCTION

Despite the success of the Lambda Cold Dark Matter (ΛCDM) paradigm on cosmological scales, the explicit field theoretic description of dark matter has remained elusive. One theory, superfluid dark matter, has received renewed attention with recent work [1, 2] in merging the behavior of ΛCDM on cosmological scales and Modified Newtonian Dynamics on galactic scales. Furthermore, superfluid models have rich phenomenology, including work on solving the core-cusp problem [3], effects on gravitational lensing [4], baryogenesis [5], and unifying dark matter and dark energy [6].

One important property, which may lead to observational differences between superfluid and particle dark matter models, is the irrotational circulation of the superfluid, resulting in the formation of vortices when the superfluid rotates above a critical angular velocity [7]. The superfluid density within a vortex vanishes over a characteristic (healing) length ξ, which typically results in a maximum of $\mathcal{O}(100)$ vortices within a typical halo [8–11]. Effects of vortices on the superfluid halo structure and dynamics have been previously considered [10–14], where the vortices are assumed to have no curvature and form a uniform lattice. However, both theoretical [15–18] and experimental [19–21] research in two-dimensional systems suggests that the vortex spatial distribution can deviate from a uniform lattice in inhomogeneous rotating superfluids. Yet this nonuniformity has been largely ignored when applied to superfluid dark matter. As shown in [9, 12], the inhomogeneity of vortices within the dark matter halo can significantly change the galactic rotation curves. Further, Silverman and Mallet [8] originally speculated that the expected abundance of vortex lines within the superfluid dark matter halo may have noticeable effects on gravitational lensing and polarization of distant background sources. Thus, a theoretical understanding of the distribution of vortices within a halo must be found in order to quantify the expected magnitude and form of these gravitational effects.

In this article, we consider the effects of the inhomogeneous superfluid dark matter on the expected vortex distribution and the shape of vortex lines, following a similar method to [15, 16]. [**Outline of paper**]

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II. BACKGROUND EQUATIONS AND ENERGY FUNCTIONAL

We will be interested in a self-interacting (dark matter) field, which in the nonrelativistic limit can be described by the Gross-Pitaevskii equation [22]. In this context, the field's "trapping" potential $V(\mathbf{r})$ is taken as the gravitational potential given by the Poisson equation[1]. Explicitly written, the Gross-Pitaevskii equation and Poisson equations describing the system are given as:

$$i\dot{\Psi} = \left(-\frac{1}{2m}\nabla^2 + V(\mathbf{r}) + g|\Psi|^2\right)\Psi, \quad (1)$$

$$\nabla^2 V = 4\pi G m^2 |\Psi|^2, \quad (2)$$

where the dot represents a time derivative, m is the mass of Ψ, and g is the self-coupling constant. Redefining the field Ψ by the Madelung representation [23],

$$\Psi = \sqrt{\rho_s}\exp(i\theta), \quad (3)$$

the Gross-Pitaevskii equation decomposes into the fluid-like equations

$$\dot{\rho}_s + \boldsymbol{\nabla} \cdot (\rho_s \mathbf{v}) = 0, \quad (4)$$

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v} = -\frac{1}{m}\boldsymbol{\nabla}\left(V + g\rho_s - \frac{\nabla^2\sqrt{\rho_s}}{2m\sqrt{\rho_s}}\right), \quad (5)$$

where $\mathbf{v} = \boldsymbol{\nabla}\theta/m$ is defined as the field's flow velocity field, which satisfies the irrotational condition $\boldsymbol{\nabla} \times \mathbf{v} = 0$.

Despite this condition, the $\mathbf{v}$ field attains nonzero circulation in the presence of vortices by being singular on the vortex axis. Requiring that Ψ is single-valued leads to a condition that circulation of velocity field must be quantized,

$$\oint_C v\,dl = \frac{h}{m}\Delta\phi = l\frac{h}{m} \quad (6)$$

Vortex size is given by the healing length ξ,

$$\xi = \sqrt{\frac{\hbar^2}{2mgn}} \quad (7)$$

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[1] Additional forms of energy density, such as baryonic matter, can also be included in the Poisson equation without changing the following analysis.

Superfluid density drops as a function of the distance from the vortex axis as ρ^l.

In general, the dark matter halo will contain angular momenta, which can lead to time-dependence in the potential. Instead of using the fluid-like equations in Eqs. (4) and (5), we will work in a frame rotating with frequency $\boldsymbol{\Omega}$ so that the trapping potential is static. Furthermore, we will look for solutions where the vortex distribution is stationary. In this rotating frame, the energy functional can be written as

$$E = \int_{\mathcal{V}} \left[\frac{\left(\boldsymbol{\nabla}\sqrt{\rho_s}\right)^2}{2m} + (V(\mathbf{r}) - \mu)\rho_s + \frac{g}{2}\rho_s^2 \right. $$
$$\left. + \frac{i}{2}(\boldsymbol{\Omega} \times \mathbf{r}) \cdot \boldsymbol{\nabla}\rho_s \right] + E_v, \qquad (8)$$

where we have introduced the chemical potential μ, and

E_v is the energy associated with the vortices:

$$E_v = \frac{1}{2m} \int_{\mathcal{V}} \rho_s \left[(\boldsymbol{\nabla}\theta)^2 - 2m(\boldsymbol{\Omega} \times \mathbf{r}) \cdot \boldsymbol{\nabla}\theta \right],$$
$$= \frac{m}{2} \int_{\mathcal{V}} \rho_s \left[(\mathbf{v} - \boldsymbol{\Omega} \times \mathbf{r})^2 - (\boldsymbol{\Omega} \times \mathbf{r})^2 \right]. \qquad (9)$$

We note, minimization of the total energy with respect to $\rho_s, \mathbf{v}$ is equivalent to the solving the time-independent fluid-like equations in a rotated frame.

Since ρ_s can be approximately solved without considering the backreaction of the vortices on the fluid density, the vortex energy E_v can be minimized with respect to $\mathbf{v}$ using the previously found ρ_s. Discreteness of vortices will introduce an important contribution to the energy that will be described in the following section.

Need to explain why the backreaction on ρ is negligible

III. VORTEX DISTRIBUTION IN AN INHOMOGENEOUS SUPERFLUID

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