

```

> with(LinearAlgebra) :
> with(plots) :
> with(MTM) :
> with(geom3d) :
> with(ArrayTools) :
> with(VectorCalculus) :
>
> A := Matrix([ [1.782, 1.7115, -14.2455], [1.8315, 1.7895, -14.1585], [1.779, 1.962,
-14.0775], [1.6905, 1.977, -14.151],
[1.638, 1.98, -14.2455], [1.6395, 2.0115, -14.2455], [0.57, 2.9715, -14.2455], [0.4605,
2.9325, -14.2455], [0.228, 2.823, -14.2455],
[0.141, 2.7585, -14.2455], [0.015, 2.6535, -14.2455], [-0.1005, 2.5335, -14.2455], [-0.147,
2.4735, -14.2455], [-0.156, 2.4615, -14.2455],
[-0.2475, 2.37, -14.2455], [-0.4395, 2.1225, -14.2455], [-0.45, 2.1135, -14.2455], [
-0.5955, 1.9395, -14.2455], [-1.281, 1.389, -14.2455],
[-1.3095, 1.389, -14.223], [-1.308, 1.3545, -14.2455], [-1.389, 1.2405, -14.2455], [
-1.4235, 1.1985, -14.2455], [-1.4625, 1.149, -14.2455],
[1.683, 1.7775, -14.2455]]);

```

$$A := \begin{bmatrix} 1.782 & 1.7115 & -14.2455 \\ 1.8315 & 1.7895 & -14.1585 \\ 1.779 & 1.962 & -14.0775 \\ 1.6905 & 1.977 & -14.151 \\ 1.638 & 1.98 & -14.2455 \\ 1.6395 & 2.0115 & -14.2455 \\ 0.57 & 2.9715 & -14.2455 \\ 0.4605 & 2.9325 & -14.2455 \\ 0.228 & 2.823 & -14.2455 \\ 0.141 & 2.7585 & -14.2455 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

25 × 3 Matrix

(1)

```

>
> Apoints := [point(Pt0, 1.782, 1.7115, -14.2455), point(Pt1, 1.8315, 1.7895, -14.1585),
point(Pt2, 1.779, 1.962, -14.0775), point(Pt3, 1.6905, 1.977, -14.151),
point(Pt4, 1.638, 1.98, -14.2455), point(Pt5, 1.6395, 2.0115, -14.2455), point(Pt6, 0.57,
2.9715, -14.2455), point(Pt7, 0.4605, 2.9325, -14.2455),
point(Pt8, 0.228, 2.823, -14.2455), point(Pt9, 0.141, 2.7585, -14.2455), point(Pt10, 0.015,
2.6535, -14.2455), point(Pt11, -0.1005, 2.5335, -14.2455),
point(Pt12, -0.147, 2.4735, -14.2455), point(Pt13, -0.156, 2.4615, -14.2455), point(Pt14,
-0.2475, 2.37, -14.2455), point(Pt15, -0.4395, 2.1225, -14.2455),
point(Pt16, -0.45, 2.1135, -14.2455), point(Pt17, -0.5955, 1.9395, -14.2455), point(Pt18,
-1.281, 1.389, -14.2455), point(Pt19, -1.3095, 1.389, -14.223),
point(Pt20, -1.308, 1.3545, -14.2455), point(Pt21, -1.389, 1.2405, -14.2455), point(Pt22,

```

```

-1.4235, 1.1985, -14.2455), point(Pt23, -1.4625, 1.149, -14.2455),
point(Pt24, 1.683, 1.7775, -14.2455)];
Apoints := [Pt0, Pt1, Pt2, Pt3, Pt4, Pt5, Pt6, Pt7, Pt8, Pt9, Pt10, Pt11, Pt12, Pt13, Pt14, Pt15,
Pt16, Pt17, Pt18, Pt19, Pt20, Pt21, Pt22, Pt23, Pt24]

```

(2)

```

> centroid(Ctroid, Apoints);

```

*Ctroid*

(3)

```

> coordinates(Ctroid)

```

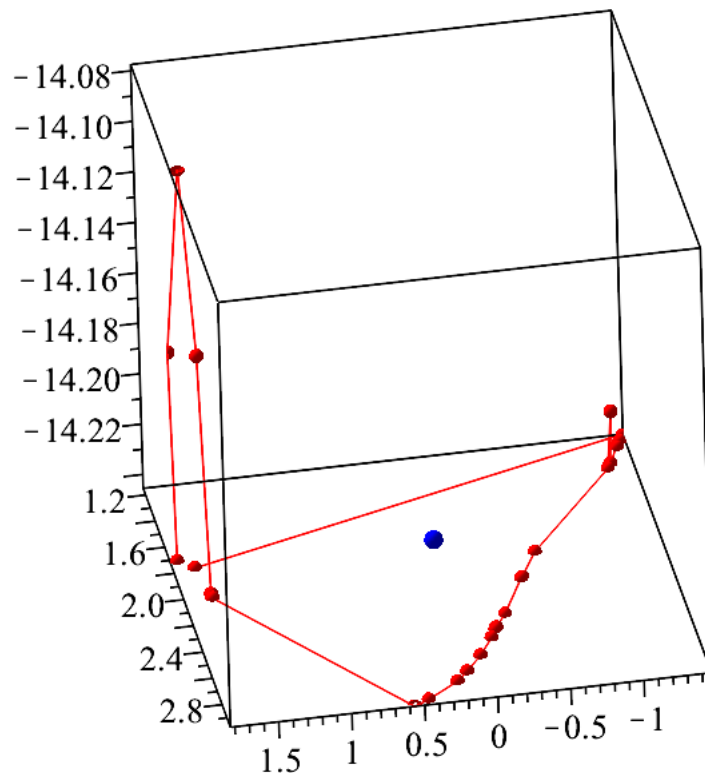
[0.1259400000, 2.043300000, -14.23062000]

(4)

```

> plots:-display(dataplot(A,'points', color = "Red"), pointplot3d(coordinates(Ctroid), color
= "Blue", symbol = solidsphere, symbolsize = 20) )

```



```

> offset_mat := Matrix( [coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid)] ) :
> Atranslated := A - offset_mat :

```

```
> U, S, V := svd(Atranslated);
```

```
U, S, V :=
```

-0.267124103829850	0.252654858545715	0.328447469968134	-0.15199060486160
-0.278281688516017	0.227433325151404	-0.197587045528460	-0.14453188065518
-0.275448627226135	0.158266017129718	-0.711781611797461	-0.37722903195020
-0.260887382790726	0.145093770019475	-0.283222584763066	0.89716864708441
-0.251918495221568	0.139045278408323	0.277142870588197	0.005290718766934
-0.253225386532467	0.127115559552382	0.273311194178767	0.005125891454597
-0.105660747627881	-0.321264428040743	0.0239209693004040	0.008937184495412
-0.0859550521179952	-0.314680110369689	0.0158646361315640	0.009993990961885
-0.0432203640454571	-0.290494308363608	0.00215667031378094	0.012367493851759
-0.0264415107261525	-0.272449222286947	0.0000221465914143751	0.013369870059143
⋮	⋮	⋮	⋮

```
,
```

5.83573167663451	0	0
0	2.56442884010182	0
0	0	0.166599286849082
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
⋮	⋮	⋮

```
,
```

25 × 3 Matrix

-0.980600238396847	0.195014596598809	0.0198110971289227
-0.195421163915420	-0.980490062964459	-0.0212086096171715
-0.0152885954224181	0.0246686753060395	-0.999578768936523

```
=
```

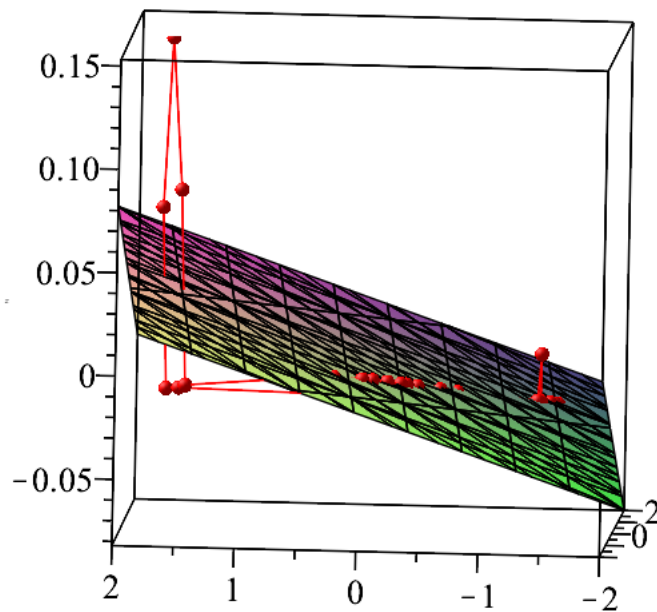
```
> pln := 0.0198110971289227 * (x - 0) + -0.0212086096171715 * (y - 0) +  
-0.999578768936523 * (z - 0) = 0;
```

```
pln := 0.0198110971289227 x - 0.0212086096171715 y - 0.999578768936523 z = 0
```

(6)

```
=
```

```
> plots:-display(dataplot(Atranslated,'points', color = "Red"), implicitplot3d( pln, x=-2..2, y=-2  
..2, z=-0.25..0.25, axes = boxed ) )
```



```

>
>
> A := Matrix( [[ 1.971, -2.3655, -14.0625], [2.004, -2.064, -14.1735], [1.977, -1.8885,
-14.2455], [2.01, -1.887, -14.2455],
[2.157, -1.9635, -14.205], [2.1375, -1.8705, -14.2455], [2.232, -1.845, -14.2455], [2.562,
-1.3935, -14.2455],
[2.5605, -1.308, -14.2455], [2.5545, -1.2495, -14.2455], [1.6845, -1.854, -14.2455],
[1.5825, -1.9335, -14.202], [1.602, -2.1285, -14.1225], [1.71, -2.382, -14.031], [1.8165,
-2.382, -14.0445]] ) :
>
> Apoints := [point(P0, 1.971, -2.3655, -14.0625), point(P1, 2.004, -2.064, -14.1735),
point(P2, 1.977, -1.8885, -14.2455),
point(P3, 2.01, -1.887, -14.2455), point(P4, 2.157, -1.9635, -14.205), point(P5, 2.1375,
-1.8705, -14.2455),
point(P6, 2.232, -1.845, -14.2455), point(P7, 2.562, -1.3935, -14.2455), point(P8, 2.5605,
-1.308, -14.2455),
point(P9, 2.5545, -1.2495, -14.2455), point(P10, 1.6845, -1.854, -14.2455), point(P11,
1.5825, -1.9335, -14.202),
point(P12, 1.602, -2.1285, -14.1225), point(P13, 1.71, -2.382, -14.031), point(P14, 1.8165,
-2.382, -14.0445) ] :
>
> centroid(Ctroid, Apoints);

```

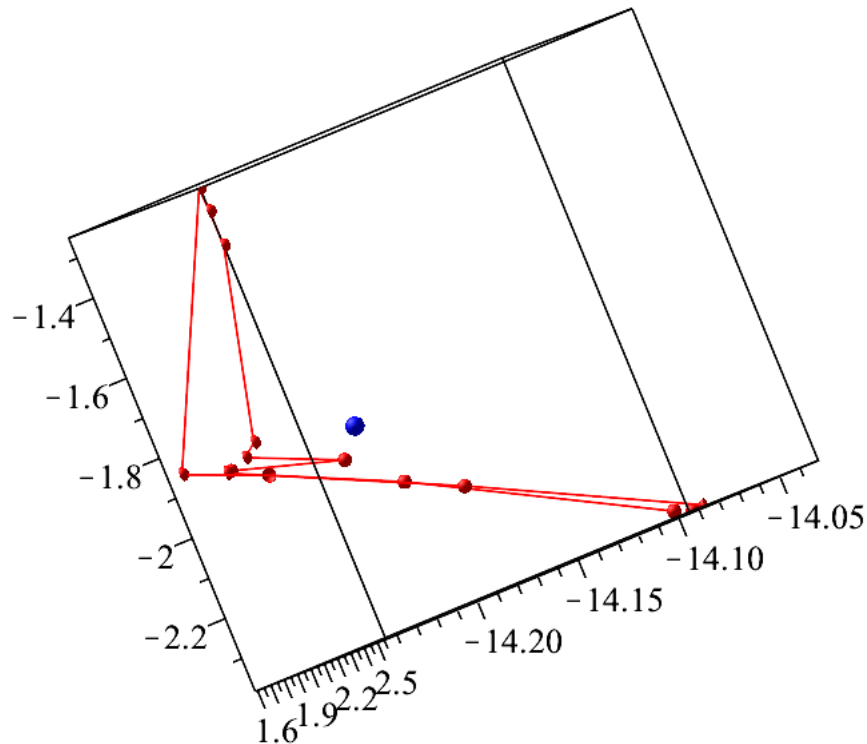
*Ctroid*

(7)

```
> coordinates(Ctroid);
[2.037400000, -1.901000000, -14.18700000]
```

(8)

```
>
> plots:-display(dataplot(A,'points', color = "Red"), pointplot3d(coordinates(Ctroid), color
= "Blue", symbol = solidsphere, symbolsize = 20) )
```



```
>
> offset_mat := Matrix( [coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid),
coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid), coordinates(Ctroid)] ) :
```

```
> Atranslated := A - offset_mat,
Atranslated :=
```

(9)

−0.0663999999999998	−0.4645000000000000	0.1244999999999999
−0.0333999999999999	−0.1630000000000000	0.0134999999999987
−0.0603999999999998	0.0125000000000000	−0.0585000000000004
−0.0274000000000001	0.0140000000000000	−0.0585000000000004
0.1196000000000000	−0.0625000000000000	−0.0180000000000007
0.1001000000000000	0.0305000000000000	−0.0585000000000004
0.1946000000000000	0.0560000000000000	−0.0585000000000004
0.5246000000000000	0.5075000000000000	−0.0585000000000004
0.5231000000000000	0.5930000000000000	−0.0585000000000004
0.5171000000000000	0.6515000000000000	−0.0585000000000004
⋮	⋮	⋮

15 × 3 Matrix

>  $U, S, V := \text{svd}(A_{\text{translated}});$

$U, S, V :=$	−0.229613490254240	0.443152193150669	0.0912426520517690	0.031465185397
	−0.0822506322673786	0.132723675714524	−0.138398344559418	−0.000471981347
	−0.0135715925199418	−0.106179533316223	−0.292990956640423	−0.3172969832
	−0.000348860099102781	−0.0680176046603755	−0.306532358830611	0.92343482835
	0.0206576072347672	0.202055391590444	−0.250504037636411	−0.05147391575
	0.0552396099909488	0.0683828145629245	−0.343574490889047	−0.07972556654
	0.102019988446585	0.155788929398902	−0.352090214603390	−0.07812994425
	0.417770236776334	0.0870908819775375	0.135443876464166	0.034999419939
	0.453145659243976	−0.00292057934447923	0.258177464376380	0.060466237268
	0.475452047881082	−0.0704922798827538	0.344516649869825	0.078205189912
	⋮	⋮	⋮	⋮

$$, \begin{bmatrix} 1.74720195052732 & 0 & 0 \\ 0 & 0.615886463101184 & 0 \\ 0 & 0 & 0.166785217097415 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix},$$

15 × 3 Matrix

$$\begin{bmatrix} 0.666693585047700 & 0.741105716151644 & -0.0792589499274576 \\ 0.734597046944590 & -0.635382245356244 & 0.238026429008094 \\ -0.126043017563576 & 0.216914083852148 & 0.968019337590965 \end{bmatrix}$$

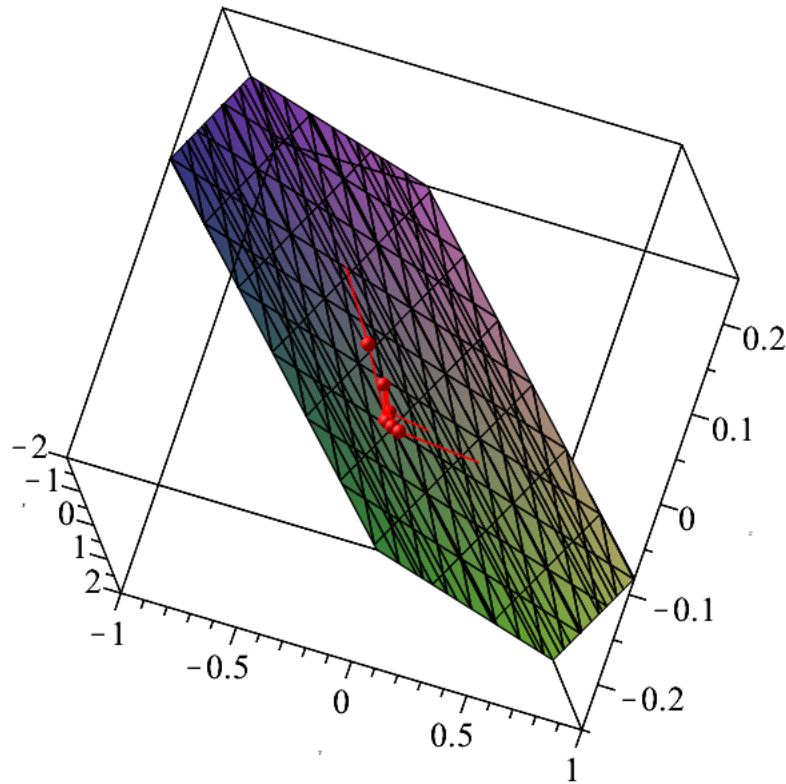
>

>  $pln := -0.0792589499274576 * (x - 0) + 0.238026429008094 * (y - 0) + 0.968019337590965 * (z - 0) = 0;$

$$pln := -0.0792589499274576 x + 0.238026429008094 y + 0.968019337590965 z = 0 \quad (11)$$

>

>  $plots:-display(dataplot(Atranslated, 'points', color = "Red"), implicitplot3d(pln, x = -2 .. 2, y = -1 .. 1, z = -0.25 .. 0.25, axes = boxed))$



```

>
> n := < -0.0792589499274576, 0.238026429008094, 0.968019337590965 >
n := (-0.0792589499274576)ex + (0.238026429008094)ey + (0.968019337590965)ez (12)

```

```

> d0 := < -0.06639999999999998, -0.4645000000000000, 0.1244999999999999 > . n;
d0 := 0.0152179255 (13)

```

```

>
>
> s := 0;
s := 0 (14)

```

```

> for i to 15 do s := s + ( evalf(DotProduct(n, <Atranslated[i, 1], Atranslated[ i, 2],
Atranslated[ i, 3]>)) ) · evalf(DotProduct(n, <Atranslated[i, 1], Atranslated[ i, 2],
Atranslated[ i, 3]>)) ) end do
s := 0.000231585257458205
s := 0.000764400818347824
s := 0.00315234153496199
s := 0.00576611390999460
s := 0.00751171325248059
s := 0.0107953637999447
s := 0.0142438065436843
s := 0.0147541162855535
s := 0.0166082957700491

```



$s := 0.0199099800352450$

$s := 0.0202152300508517$

$s := 0.0204056354783252$

$s := 0.0222370972719964$

$s := 0.0261395587557181$

$s := 0.0278173086422231$

**(15)**

$\text{rms\_error} := \text{sqrt}(s)$

$\text{rms\_error} := 0.166785217097389$

**(16)**