An introduction to SeisTomoPy

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Contents

1 What is SeisTomoPy

SeisTomoPy is a new Python tool that facilitates the use of a suite of tomographic models available to the public, with a single programme. SeisTomoPy provides six tools that allow to visualize tomographic models, compare them and extract information for further scientific purposes. The tool comes with a graphical interface with intuitive buttons and simple parameters but the same information can also be gained by using the Python class that can be run routinely in Python scripts. SeisTomoPy is suited for global and spherical tomographic models and is provided with a default list of recent tomographic models. However, the user can also upload additional models if desired.

SeisTomoPy includes several recent tomographic models that are available to the community. In Table 1 we summarize the characteristics of the different models included in the first version of SeisTomoPy. For detailed information on each model the reader is referred to the original publication.

Associated to this first library of models, we provide six tools that will aid extracting information about each of these models:

- Cross section for generating cross sections and extracting values of V_s, V_p and density as input for AXISEM to generate synthetic seismograms,
- Map for producing global tomographic maps at a given depth,
- Spectrum for computing the amplitude spectrum at any depth,
- Correlation for computing the correlation at any depth between the different models,
- Path for plotting wave paths on the top of tomographic models,
- Travel Time for computing travel times of body waves for the different tomographic models.

SeisTomoPy has been developed for saving the generated results from each of the functions in output files or figures that can be used for further scientific studies.

It is also possible to upload a model if desired. A package is provided in order to create new model files. Our choice was to develop models in spherical harmonics which:

- 1) enables to store the models quite efficently, and avoids to deal with huge model files,
- 2) is an efficient way to compute of any section, map, spectrum, etc, in a reasonable amount of time.

This means that certain global models will not be suitable to be included in the tool, in particular those with irregular parametrization and regional models.

Table 1: Summary of the characteristics of every tomographic model included in SeisTomoPy. Abbreviations in the table: SW - Surface waves, BW - body waves, NM - normal modes, T - period, Sph. Harm. - spherical harmonics, V_s, V_p, V_{sh}, V_{sv} - shear velocity, compressional velocity, horizontal shear velocity, vertical shear velocity, ν_α, ν_ρ - scaling factors such that $d\ln(V_p) = \nu_\alpha d\ln(V_s)$ and $d\ln(\rho) = \nu_\rho d\ln(V_s)$.

Model	Data	Inverted parameter	Lateral param.	Radial param.	Theory used	Reference model	Scaling laws	Crustal model	Global or upper mantle
S40RTS [Ritsema et al., 2011]	SW phase velocities BW travel times NM splitting coefficients	V_s	Sph. Harm. up to degree 40	21 cubic spline functions	Ray theory	PREM iso.	ν_α varies from 2 (surf) to 3 (CMB) $\nu_\rho = 0.5$	Corrected CRUST2.0 [Bassin et al., 2000]	Global
S362WMANI+M [Moulik & Ekström, 2014]	SW phase velocities Long period BW waveforms BW travel times NM splitting coefficients	V_{sh}, V_{sv}	Spherical splines	16 cubic splines discontinuous across 650 km	Ray theory	STW105 [Kustowski et al., 2008]	$\nu_\alpha = 0.55$ $\nu_\rho = 0$	Corrected CRUST2.0	Global
SEMUCB-WM1 [French & Romanowicz, 2014]	SW wavforms (T > 60 s) BW waveforms (T > 36 s and T > 32 s)	V_{sh}, V_{sv}	Spherical splines	20 cubic splines	3-D synthetics and NACT	their own model [French & Romanowicz, 2014]	$\nu_\alpha = 0.5$ $\nu_\rho = 0$	Corrected smoothed version CRUST2.0	Global
SGLOBE-rani [Chang et al., 2015]	SW phase velocities BW travel times	V_{sh}, V_{sv}	Sph. Harm. up to degree 35	20 cunic splines	Ray theory	PREM aniso.	$\nu_\alpha = 0.5$ $\nu_\rho = 0.4$	Corrected CRUST2.0	Global
SEISGLOB1 [Durand et al., 2016]	SW phase velocities NM splitting and coupling coefficients	V_s	Sph. Harm. up to degree 20	21 cubic spline functions	Ray theory	PREM iso.	$\nu_\alpha = 0.55$ $\nu_\rho = 0.2$	Inverted	Global
SP12RTS [Koelemeijer et al., 2016]	SW phase velocities BW travel times NM splitting coefficients	V_s, V_p	Sph. Harm. up to degree 12	21 cubic spline functions	Ray theory	PREM iso.	$\nu_\rho = 0.5$	Corrected CRUST2.0	Global
SEISGLOB2 [Durand et al., 2017b]	SW phase velocities NM splitting coefficients NM splitting and coupling coefficients	V_s	Sph. Harm. up to degree 40	21 cubic spline functions	Ray theory	PREM iso.	$\nu_\alpha = 0.55$ $\nu_\rho = 0.2$	Corrected CRUST2.0	Global
3D2016-09S [?]	SW phase velocities	V_{sv}, V_{sh}	Correlation length 200 km down to 750 km increases to 800 km down to 1000 km	Correlation length 50 km	Path average	PREM aniso.		Corrected 3SMAC [Nataf & Ricard, 1993]	Upper mantle

2 Requirements

SeisTomoPy has a number of dependencies listed below.

- gfortran : GNU Fortran (MacPorts gcc48 4.8.5_0) 4.8.5
- Python 2.7, 3.5, 3.6
- iPython 5.3.0
- matplotlib 2.0.2
- numpy 1.11.0
- obspy 1.0.1
- pyqt 5.6.0
- scipy 0.17.0

For installing the Python dependencies, please run :
`conda install -c conda-forge matplotlib numpy obspy pyqt scipy pyqtgraph basemap`

If there is any problem with the compilation of fortran source files, we recommand to install fortran using:
https://gcc.gnu.org/wiki/GFortranBinaries

3 Installation

Clone the git repository and install in an editable fashion:

```
$ git clone https://github.com/stephaniedurand/SeisTomoPy_Valpha.git
$ cd SeisTomoPy_Valpha
$ pip install -v -e .
```

4 How to use SeisTomoPy class

Lauch iPython. The first thing to do is to import SeisTomoPy so that you will be able to use the various SeisTomoPy functions.

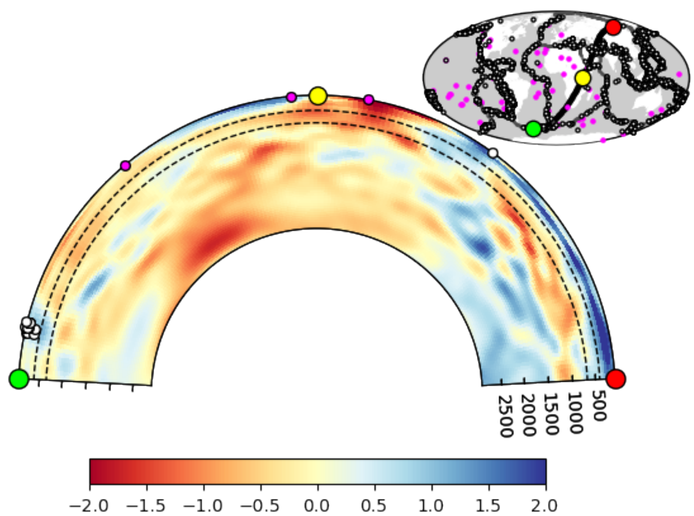
```python
import SeisTomoPy
```

4.1 Cross sections

The user can generate cross sections through V_p, V_s or ρ variations anywhere on Earth in some default tomographic models provided with the tool using `SeisTomoPy.cross_section_plot`. This generates directly the plot of the cross section. However, the user could also be interested in getting the values of the cross section and then perform some further calculations. This can also be achieved running `SeisTomoPy.cross_section`. The latter function returns as outputs the matrix containing the cross section (Z) as well as the angle (th) and radius vectors (r) that the user would need if he desires to perform further calculations with this cross section.

Below is an example of how to get a cross section beneath Africa using both class. Pink circles show hotspot locations, white circles denote earthquake locations and green, red and yellow circles the starting, ending and mid-point, respectively, along the profile.

```
# Setting parameters
# Model to be plotted
# Choose between:
# SEISGLOB2, S40RTS, SEMUCBWM1, S362WMANIM, SEISGLOB1, SP12RTS, SGLOBE, 3D2016, MYMODEL
model = "SEISGLOB2"
# Parameter to be plotted
# Choose between: VS, VP, RHO
para = "VS"
# Latitutde of the starting point of the cross section
elat = -60
# Longitude of the starting point of the cross section
elon = -49
# Latitutde of the ending point of the cross section
slat = 60
# Longitude of the ending point of the cross section
slon = 119
# Depth of the cross section
depth = 2890
# Spherical harmonic degrees to be used
NSmax = 40
# Maximal velocity perturbations for the colorbar
Vmax = 2

# Running cross_section_plot
SeisTomoPy.cross_section_plot(model,para,elat,elon,slat,slon,depth,NSmax,Vmax)

# Running cross_section
Z, th, r = SeisTomoPy.cross_section(model,para,elat,elon,slat,slon,depth,NSmax)
```

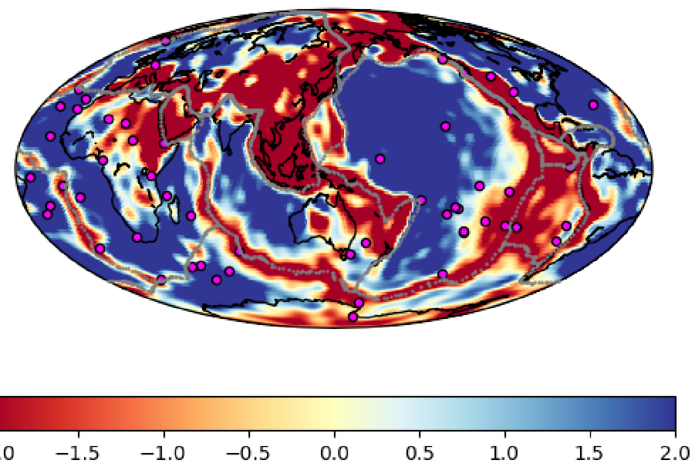


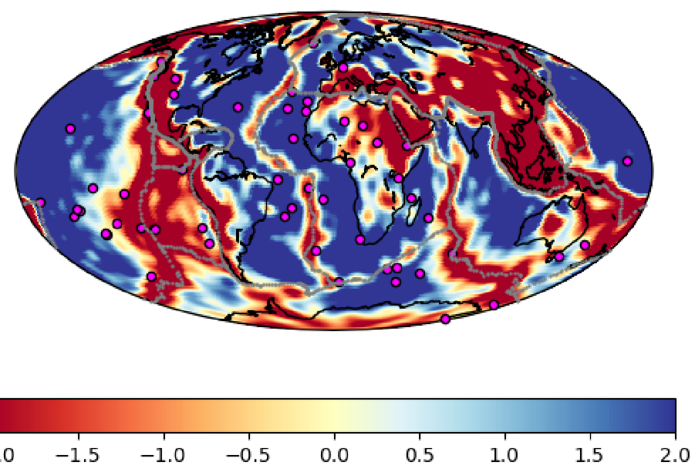
4.2 Maps

The user can create maps at a given depth for the whole globe using `SeisTomoPy.tomomap_plot`. This generates directly the plot of the map. It is also possible to obtain the values of the map and then perform some further calculations using `SeisTomoPy.tomomap`

Below is an example of how to obtain a map for the tomographic model 3D2016-S09 at 50 km depth using both class. Pink circles show hotspot locations and grey lines the plate boundaries.

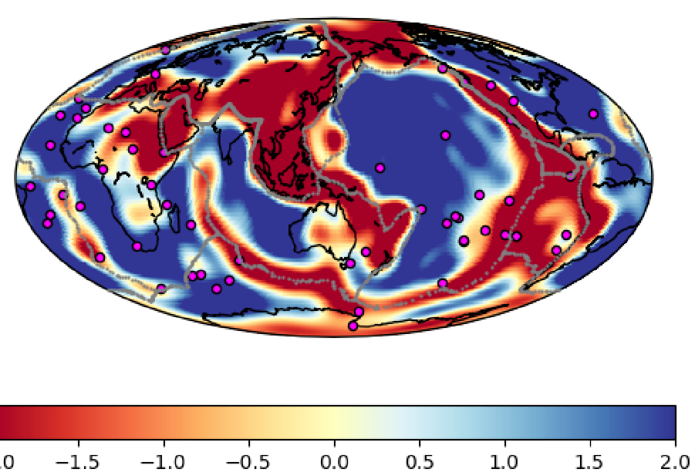
```
# Setting parameters
# Model to be plotted
model = "3D2016"
# Parameter to be plotted
para = "VS"
# Depth of the map to be plotted
depth = 50
# Spherical harmonic degrees to be used
NSmax = 60
# Central longitude
# default value 140 degrees
lon0 = 140

# Maximal velocity perturbations for the colorbar
Vmax = 2

# Running tomomap_plot
SeisTomoPy.tomomap_plot(model,para,depth,NSmax,Vmax,lon0)

# Changing the central longitude
lon0 = 0
SeisTomoPy.tomomap_plot(model,para,depth,NSmax,Vmax,lon0)

# Running tomomap
Z, lat, lon = SeisTomoPy.tomomap(model,para,depth,NSmax)
```





It is possible to change $NSmax$ to a value below 60 in order to filter the model.

```
# Filtering the model
NSmax = 18
SeisTomoPy.tomomap_plot(model,para,depth,NSmax,Vmax)
```



These first tools can also be used with an uploaded tomographic model (see section 7 for details). The user would then refer to the new model by setting:

```
model = "MYMODEL"
```

4.3 Spectrum

The user can compute the amplitude spectrum, $S(X,z,l)$, for a given model at a given depth, z, for a given parameter, X, and for a certain spherical harmonic degree, l, using `SeisTomoPy.spectrum` and `SeisTomoPy.spectrum_fromfile`. The first one computes the spectrum for the models included by default in SeisTomoPy while the second one enables the user to compute the spectrum in another model as long as the correct input file is uploaded. Please refer to section 9 for details about these files.

The spectrum is defined by:

$$S\left(X,z,l\right)=\sqrt{\frac{1}{4\pi}\sum_{m=-l}^{l}\left(\frac{\delta X}{X}(z)\right)_{lm}\left(\frac{\delta X}{X}(z)\right)^{\star}_{lm}}, \tag{1}$$

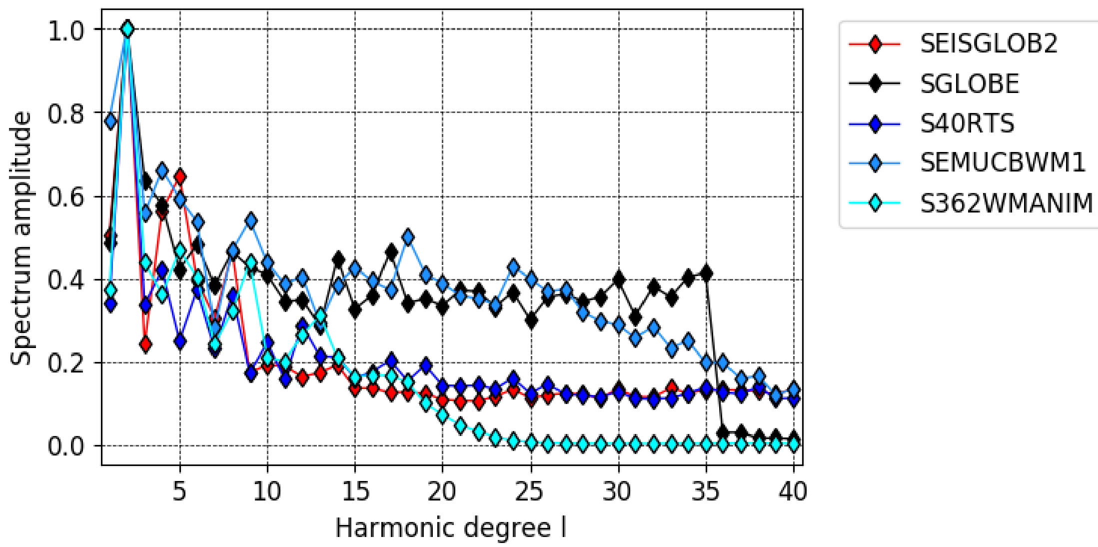
where the exponent "⋆" denotes the complex conjugate and $\left(\frac{\delta X}{X}(z)\right)_{lm}$ the spherical harmonic coefficients of the considered parameter X for a given tomographic model at depth z. X stands for V_s, V_p or density.

Below is first an example of obtained spectrum for various models at 520 km depth, for parameter V_s and up to spherical harmonic degree 40. We then show how to get the spectrum for another model not included in SeisTomoPy.

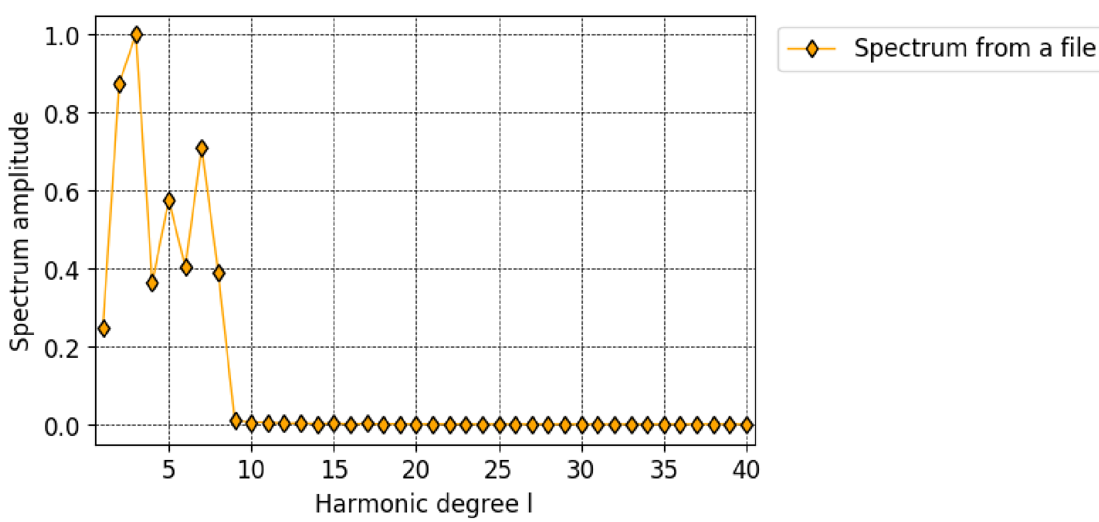
```
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
print(" Example of spectrum computed in the default tomographic models ")
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")

# Setting parameters
# Models to be used for computing the spectrum
model1 = "SEISGLOB2"
model2 = "SGLOBE"
model3 = "S40RTS"
model4 = "SEMUCBWM1"
model5 = "S362WMANIM"
# Depth at which the spectrum is computed
depth = 520
# Parameter used for computing the spectrum
para = "VS"
# Maximum spherical harmonic degree up to which the spectrum is computed
NSmax = 40

# Running spectrum
sp1 = SeisTomoPy.spectrum(model1,para,depth,NSmax)
sp2 = SeisTomoPy.spectrum(model2,para,depth,NSmax)
sp3 = SeisTomoPy.spectrum(model3,para,depth,NSmax)
sp4 = SeisTomoPy.spectrum(model4,para,depth,NSmax)
sp5 = SeisTomoPy.spectrum(model5,para,depth,NSmax)
```

```python
# Plotting the results
plt.plot(sp1[:,0], sp1[:,1]/np.amax(sp1[:,1]), linewidth=1.0,color="red", ...
        marker="d",markeredgecolor="k", label=model1)
plt.plot(sp2[:,0], sp2[:,1]/np.amax(sp2[:,1]), linewidth=1.0,color="k", ...
        marker="d",markeredgecolor="k", label=model2)
plt.plot(sp3[:,0], sp3[:,1]/np.amax(sp3[:,1]), linewidth=1.0,color="blue", ...
        marker="d",markeredgecolor="k", label=model3)
plt.plot(sp4[:,0], sp4[:,1]/np.amax(sp4[:,1]), linewidth=1.0,color="dodgerblue",...
        marker="d",markeredgecolor="k", label=model4)
plt.plot(sp5[:,0], sp5[:,1]/np.amax(sp5[:,1]), linewidth=1.0,color="cyan",...
        marker="d",markeredgecolor="k", label=model5)

plt.legend(bbox_to_anchor=(1.45, 1))
plt.xlabel("Harmonic degree l")
plt.ylabel("Spectrum amplitude")
plt.xlim([0.5, NSmax+0.5])
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.show()

print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
print("  Example of spectrum computed in another model")
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")

# Running spectrum_fromfile
filename = "SeisTomoPy_notebook/files/input_file_spectrum.xyz"
sp_fromfile = SeisTomoPy.spectrum_fromfile(filename,NSmax)

# Plotting the results
plt.plot(sp_fromfile[:,0], sp_fromfile[:,1]/np.amax(sp_fromfile[:,1]), ...
         linewidth=1.0,color="orange",marker="d",markeredgecolor="k", ...
        label="Spectrum from a file")
plt.legend(bbox_to_anchor=(1.55, 1))
plt.xlabel("Harmonic degree l")
plt.ylabel("Spectrum amplitude")
plt.xlim([0.5, NSmax+0.5])
plt.grid(color="k", linestyle="--",linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.show()
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
   Example of spectrum computed in the default tomographc models
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```




```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
    Example of spectrum computed in another model
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```



It is possible to run the spectrum calculations routinely, at various depths for instance. The example below can take some time to run, it can be reduced by changing

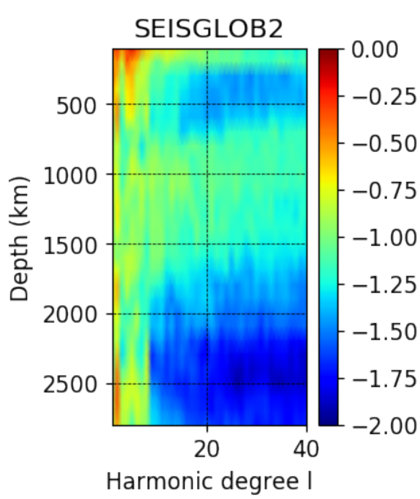
```
$ depths = np.arange(100,2900,100)
```

to

```
$ depths = np.arange(100,1100,100)
```

for instance.

```python
# Setting parameters
# Model to be used for computing the spectrum
model = "SEISGLOB2"
# Depth range at which the spectrum is computed
depths = np.arange(100,2900,100)
# Parameter used for computing the spectrum
para = "VS"
# Maximum spherical harmonic degree up to which the spectrum is computed
NSmax = 40

deg = np.arange(1,41,1)
sp1 = np.zeros([len(deg),len(depths)])

# Running spectrum routinely at various depths
for i in range(len(depths)):
    sp11 = SeisTomoPy.spectrum(model,para,depths[i],NSmax)
    for k in range(len(deg)):
sp1[k,i]=sp11[k,1]

# Plotting the result
X, Y = np.meshgrid(deg, depths)
plt.subplots_adjust(bottom=0.2, right=0.5, left=0.2, top=0.9)
plt.pcolormesh(X, Y, (np.log10(np.transpose(sp1))), ...
   shading="gouraud", cmap="jet", vmin=-2, vmax=0)
plt.gca().invert_yaxis()
plt.ylabel("Depth (km)")
plt.xlabel("Harmonic degree l")
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.title("SEISGLOB2")
plt.colorbar()
plt.show()
```



4.4 Correlations

The user can compute the correlation between two tomographic models, (1) and (2), using `SeisTomoPy.correlation`, `SeisTomoPy.correlation_fromfile` and `SeisTomoPy.correlation_fromfile2`. The first one computes the correlation between two models chosen from the included ones by default in SeisTomoPy, while the second and third ones enable the user to compute the correlation between either any model with one of the default ones or between any other models that the user wishes to use. In order to use the two last functions the user thus must provide the required input file. Please refer to section 9 for details about these files.

The correlation can be carried out for any parameter X_1 of model (1) and X_2 of model (2) and for the same depth between the two models ($z_1 = z_2$) or it could be for different depths ($z_1 \neq z_2$). Correlations are computed at a given spherical harmonic degree, l, following:

$$C(X_1, z_1, X_2, z_2, l) = \frac{\sum_{m=-l}^{l} \left(\frac{\delta X_1}{X_1}(z_1)\right)_{lm} \left(\frac{\delta X_2}{X_2}(z_2)\right)^{\star}_{lm}}{\sqrt{\sum_{m=-l}^{l} \left(\frac{\delta X_1}{X_1}(z_1)\right)_{lm} \left(\frac{\delta X_1}{X_1}(z_1)\right)^{\star}_{lm} \sum_{m=-l}^{l} \left(\frac{\delta X_2}{X_2}(z_2)\right)_{lm} \left(\frac{\delta X_2}{X_2}(z_2)\right)^{\star}_{lm}}}. \tag{2}$$

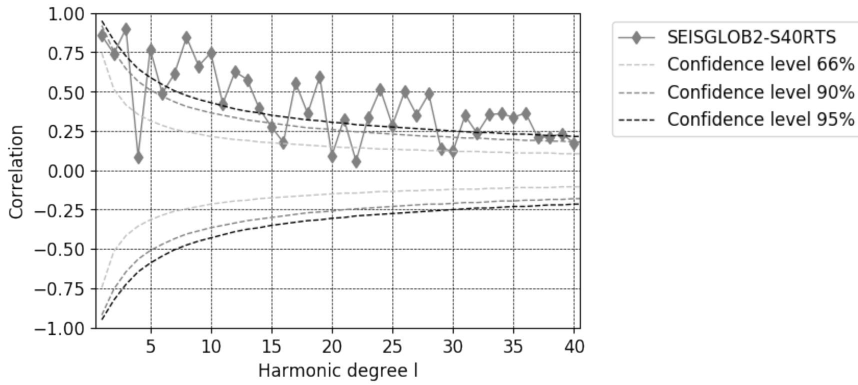
Below is an example of the correlation computed between SEISGLOB2 and S40RTS at 520 km depth, for parameter V_s and up to spherical harmonic degree 40. We then show how to find the correlation for any model not included in SeisTomoPy.

```
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
print("Example of correlation computed betwenn the default tomographic models")
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")

# Setting parameters
# Models to be used for computing the correlation
model1 = "SEISGLOB2"
model2 = "S40RTS"
# Depth at which the correlation is computed
depth = 520
# Parameters used for computing the correlation
para1 = "VS"
para2 = "VS"
# Maximum spherical harmonic degree up to which the correlation is computed
NSmax = 40

# Running correlation
corr = SeisTomoPy.correlation(model1,model2,depth,depth,para1,para2,NSmax)

plt.plot(corr[:,0],corr[:,1],linewidth=1.0,color="grey",marker="d", ...
        label=model1 + "-" + model2)
conf66 = np.loadtxt("conf66.dat")
conf90 = np.loadtxt("conf90.dat")
conf95 = np.loadtxt("conf95.dat")
plt.plot(conf66[:,0], conf66[:,1], linewidth=1.0,color="silver", ls="--", ...
```

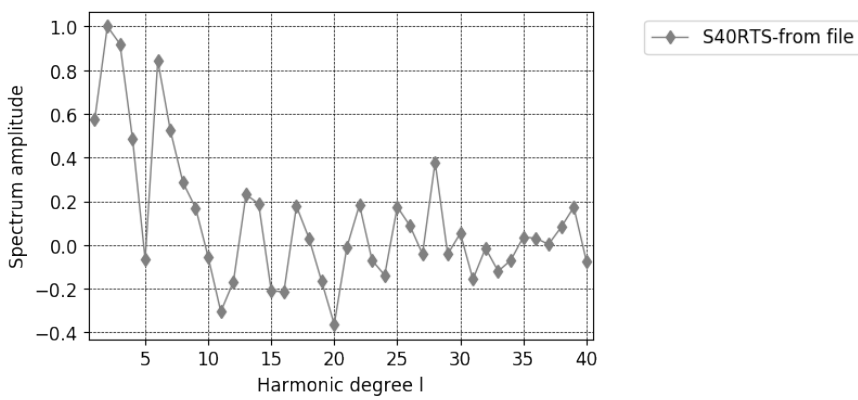
```
        label="Confidence level 66%")
plt.plot(conf90[:,0], conf90[:,1], linewidth=1.0,color="gray", ls="--", ...
label="Confidence level 90%")
plt.plot(conf95[:,0], conf95[:,1], linewidth=1.0,color="black", ls="--", ...
        label="Confidence level 95%")
plt.plot(conf66[:,0], -conf66[:,1], linewidth=1.0,color="silver", ls="--")
plt.plot(conf90[:,0], -conf90[:,1], linewidth=1.0,color="gray", ls="--")
plt.plot(conf95[:,0], -conf95[:,1], linewidth=1.0,color="black", ls="--")
plt.xlabel("Harmonic degree l")
plt.ylabel("Correlation")
plt.xlim([0.5, NSmax+0.5])
plt.ylim([-1, 1])
plt.legend(bbox_to_anchor=(1.6, 1))
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.show()

print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
print("   Example of correlation using other models   ")
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
# Setting new parameters
# Models to be used for computing the correlation
model1 = "S40RTS"
# Depth at which the correlation is computed
depth1 = 2800
# Parameters used for computing the correlation
para1 = "VS"
# Maximum spherical harmonic degree up to which the correlation is computed
NSmax = 40

# Running correlation_fromfile
filename = "SeisTomoPy_notebook/files/input_file_spectrum.xyz"
corr_fromfile = SeisTomoPy.correlation_fromfile(model1,depth1,para1,filename,NSmax)

# Plotting the results
plt.plot(corr_fromfile[:,0], corr_fromfile[:,1]/np.amax(corr_fromfile[:,1]), ...
        linewidth=1.0,color="grey",marker="d",label=model1 + "-from file")
plt.legend(bbox_to_anchor=(1.55, 1))
plt.xlabel("Harmonic degree l")
plt.ylabel("Spectrum amplitude")
plt.xlim([0.5, NSmax+0.5])
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.show()

# Running correlation_fromfile2
filename1 = "SeisTomoPy_notebook/files/input_file_spectrum.xyz"
filename2 = "SeisTomoPy_notebook/files/input_file_spectrum2.xyz"
corr_fromfile2 = SeisTomoPy.correlation_fromfile2(filename1,filename2,NSmax)

# Plotting the results
plt.plot(corr_fromfile2[:,0], corr_fromfile2[:,1]/np.amax(corr_fromfile2[:,1]), ...
        linewidth=1.0,color="grey",marker="d",label="from file1-from file2")
plt.legend(bbox_to_anchor=(1.55, 1))
```

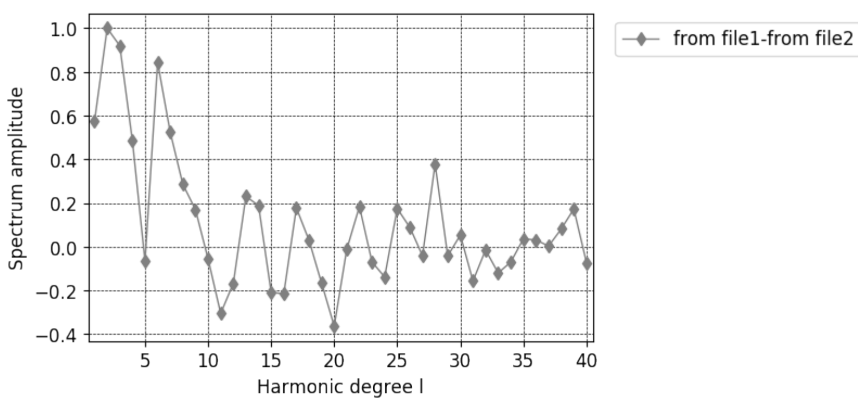
```
plt.xlabel("Harmonic degree l")
plt.ylabel("Spectrum amplitude")
plt.xlim([0.5, NSmax+0.5])
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.show()
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Example of correlation computed betwenn the default tomographic models
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```




```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
   Example of correlation using other models
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```





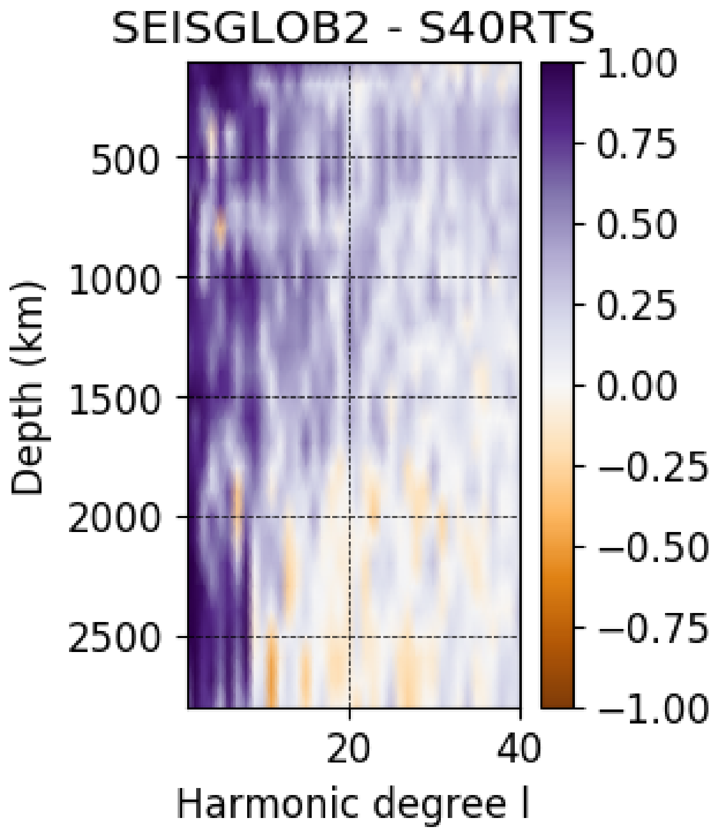
It is possible to run the correlation calculations routinely at various depths. The example below can take some time to run. One can reduce this time changing

$ depths = np.arange(100,2900,100)

to

$ depths = np.arange(100,1100,100).

```
# Setting parameters
# Models to be used for computing the correlation
model1 = "SEISGLOB2"
model2 = "S40RTS"
# Depth range at which the correlation is computed
depths = np.arange(100,2900,100)
# Parameters used for computing the correlation
para1 = "VS"
para2 = "VS"
# Maximum spherical harmonic degree up to which the correlation is computed
NSmax = 40

deg = np.arange(1,41,1)
corr1 = np.zeros([len(deg),len(depths)])

# Running correlation routinely at various depths
for i in range(len(depths)):
    corr11 = SeisTomoPy.correlation(model1,model2,depths[i],depths[i],para1,para2,NSmax)
    corr1[:,i] = corr11[:,1]

# Plotting the results
X, Y = np.meshgrid(deg, depths)
plt.subplots_adjust(bottom=0.2, right=0.5, left=0.2, top=0.9)
plt.pcolormesh(X, Y, ((np.transpose(corr1))), ...
   shading="gouraud", cmap="PuOr", vmin=-1, vmax=1)
plt.gca().invert_yaxis()
plt.ylabel("Depth (km)")
plt.xlabel("Harmonic degree l")
plt.title("SEISGLOB2 - S40RTS")
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.colorbar()
plt.show()
```



4.5 Paths

The user may want to check which seismic structures of the mantle are sampled by seismics waves. To do so the user can use `SeisTomoPy.path_plot` and `SeisTomoPy.path_plot_fromfile` to display seismic wave paths on top of cross sections made in the desired tomographic model coming from either one of the default ones included in SeisTomoPy or that the user provides.

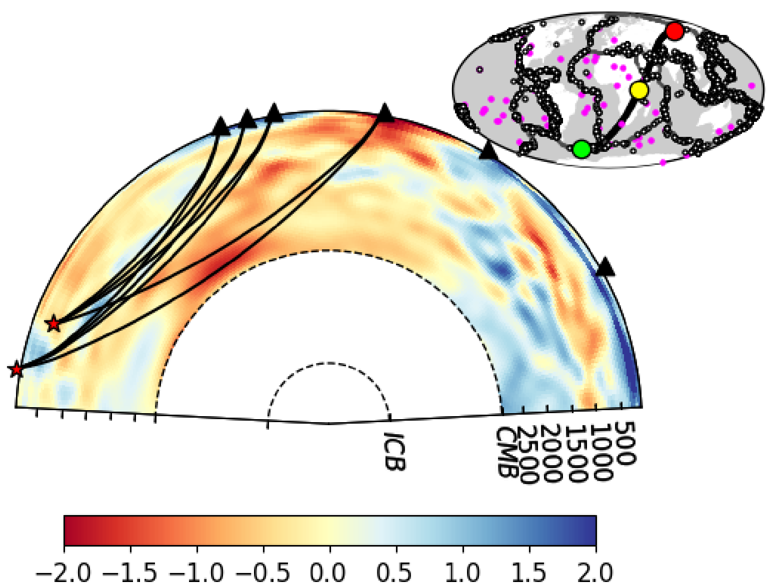
Below is an example with tomographic model SEISGLOB2, parameter V_s and seismic phases S, ScS, PKP, PKiKP and Sdiff. If it happens that the seismic phase does not exist for the distance range between the earthquake and the station, then it will simply be ignored.

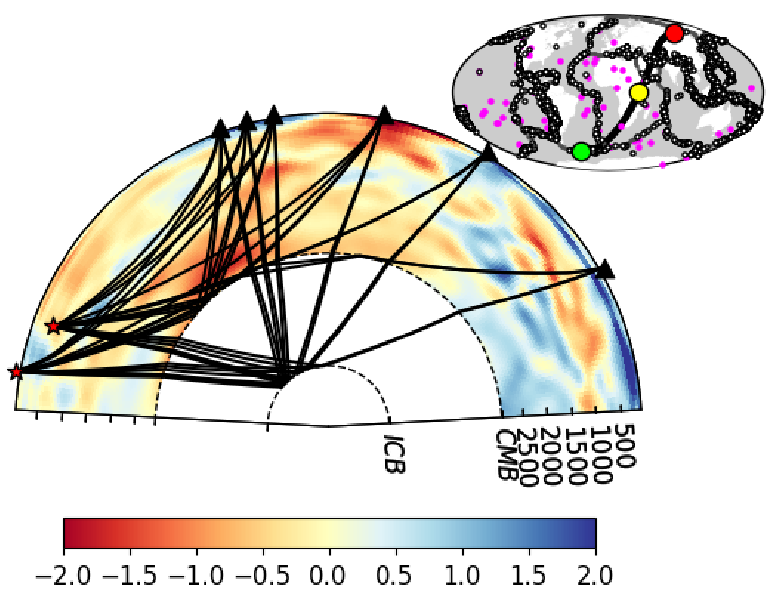
```
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
print("Two examples of path plots using the default tomographic models")
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
# Setting parameters
# Model to be plotted
model = "SEISGLOB2"
# Parameter to be plotted
para = "VS"
# Latitutde of the starting point of the cross section
elat = -60
# Longitude of the starting point of the cross section
elon = -49
# Latitutde of the ending point of the cross section
```

```
slat = 60
# Longitude of the ending point of the cross section
slon = 119
# Maximal velocity perturbations for the colorbar
Vmax = 2
# Liste of seismic phases
phlist = "S"
# Position of stations and event
EVT = np.loadtxt("SeisTomoPy_notebook/files/event.xy")
STA = np.loadtxt("SeisTomoPy_notebook/files/station.xy")

# Running path_plot
SeisTomoPy.path_plot(model,para,Vmax,elat,elon,slat,slon,EVT,STA,phlist)

# Liste of seismic phases
phlist = "S ScS PKP PKiKP Sdiff"

# Running path_plot
SeisTomoPy.path_plot(model,para,Vmax,elat,elon,slat,slon,EVT,STA,phlist)

print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")
print("  An example of path plot using a model file ")
print("!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!")

# Setting new parameters
# Latitutde of the starting point of the cross section
elat = -60
# Longitude of the starting point of the cross section
elon = -49
# Latitutde of the ending point of the cross section
slat = 60
# Longitude of the ending point of the cross section
slon = 130
# Parameter to be plotted
para = "VS"
# Angular size of the cross section
width = 180
# Sampling rate in theta direction
width_step = 1
# Theta vector
th = np.arange(90-(width/2), 90+(width/2)+width_step, width_step)
# Starting and ending radius of the cross section
r_min = 3481
r_max = 6281
# Sampling rate in radius direction
r_step = 50
# Radius vector
r = np.arange(r_min, r_max+r_step, r_step)

# Liste of seismic phases
phlist = "Pdiff"

# Running path_plot_from_file
filename = "SeisTomoPy_notebook/files/input_file_path.xyz"
```

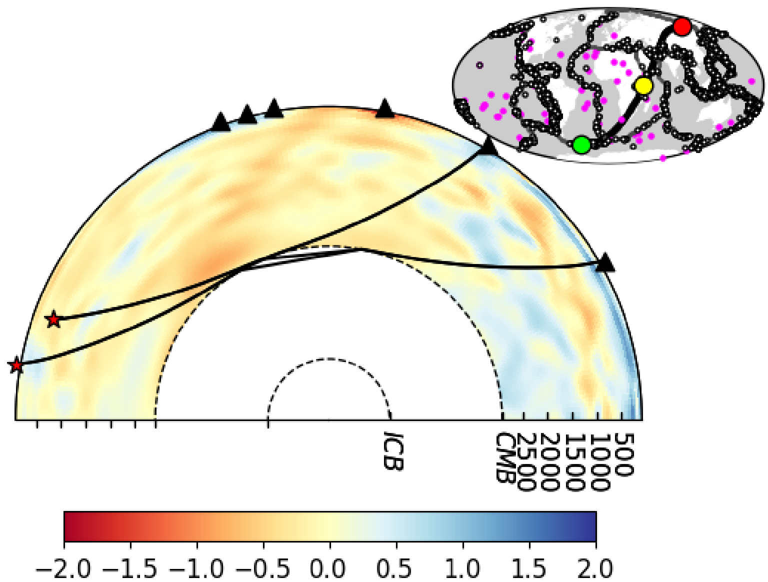
```
SeisTomoPy.path_plot_fromfile(filename,th,r,para,elat,elon,slat,slon,Vmax,EVT,STA,phlist)
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Two examples of path plots using the default tomographic models
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```





```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
  An example of path plot using a model file
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```



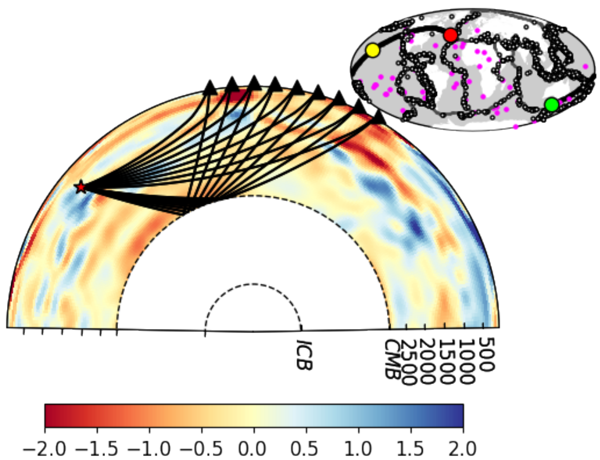
4.6 Travel times

The user can compute the travel time delays with respect to a reference model through a given tomographic model, δt^{3D}_{TomoPy}, for any given seismic phases and for any combinations of source and receiver provided by the user using `SeisTomoPy.get_travel_time`.

In the example below, we first plot the paths to see which part of the mantle the seismic waves are sampling. Then, we provide an example of delays computed in model SEISGLOB2 for S and ScS seismic phases in the same region.

```
# Setting parameters
# Model to be plotted
model = "SEISGLOB2"
# Parameter to be plotted
para = "VS"
# Latitutde of the starting point of the cross section
elat =   -43
# Longitude of the starting point of the cross section
elon = 140
# Latitutde of the ending point of the cross section
slat = 44
# Longitude of the ending point of the cross section
slon =  -42
# Maximal velocity perturbations for the colorbar
Vmax = 2
# Depth of the cross section
depth = 2890
# List of seismic phases
list = "S ScS"
```

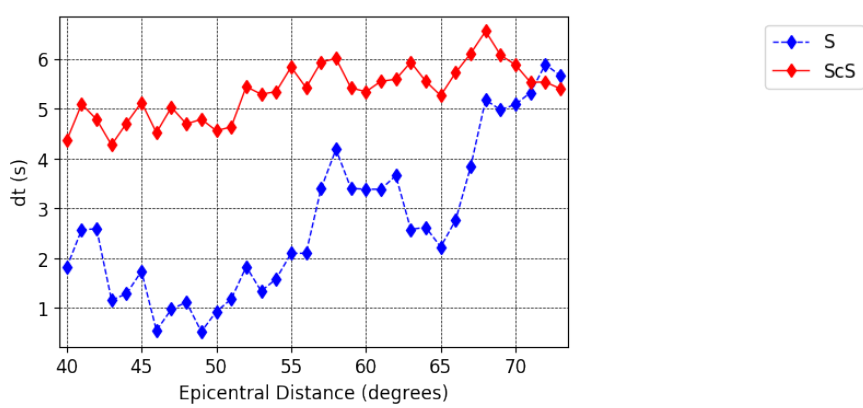
```
# Position of stations and event
EVT = np.loadtxt("SeisTomoPy_notebook/files/event_time.xy")
STA = np.loadtxt("SeisTomoPy_notebook/files/station_time.xy")

# Running path_plot
SeisTomoPy.path_plot(model,para,Vmax,elat,elon,slat,slon,EVT,STA,list)
```




```
# Setting parameters
# Model to be plotted
model = "SEISGLOB2"
# Latitutde of the event
elat = -21
# Longitude of the event
elon = -179
# Depth of the event
edepth = 610
# Position of stations
STA = np.loadtxt("SeisTomoPy_notebook/files/lat_lon_ttstation.txt")

tt2D = np.zeros(len(STA))
dtt2D = np.zeros(len(STA))
ttREF = np.zeros(len(STA))
dist = np.arange(40,74,1)
degmin = 40
degmax = 73

# Running TimePy routinely
for k in range(len(STA)):
    # List of seismic phases
    List = ["S"]
    tt2D[k],  dtt2D[k], ttREF[k], phase_name = ...
  SeisTomoPy.get_travel_time(model,elat,elon,edepth,STA[k,0],STA[k,1],List)
    file_str = str(k) + " " +  "S " + str(tt2D[k]) + " " + str(tt2D[k]+ttREF ...
  [k]) + " " + str(dtt2D[k]) + "\n"
    print(file_str)

plt.plot(dist,tt2D,marker="d",linewidth=1.0,color="blue", ls="--",label = "S")
```

```
for k in range(len(STA)):
    # List of seismic phases
    List = ["ScS"]
    tt2D[k], dtt2D[k], ttREF[k], phase_name = ...
  SeisTomoPy.get_travel_time(model,elat,elon,edepth,STA[k,0],STA[k,1],List)
    file_str = str(k) + " " + "ScS " + str(tt2D[k]) + " " + str(tt2D[k]+ttREF ...
  [k]) + " " + str(dtt2D[k]) + "\n"
    print(file_str)

# Plotting the results
plt.plot(dist,tt2D,marker="d",linewidth=1.0,color="red", ls="-",label = "ScS")
plt.xlabel("Epicentral Distance (degrees)")
plt.ylabel("dt (s)")
plt.xlim([degmin-0.5, degmax+0.5])
plt.grid(color="k", linestyle="--", linewidth=0.5)
plt.rcParams.update({"font.size": 12})
plt.legend(bbox_to_anchor=(1.6, 1))
plt.show()
```

```
0 S 1.80869197367 724.657340168 0.162310967555

1 S 2.57020639978 738.239623881 0.164944693174

2 S 2.58649315649 764.02806732 0.163141437663

3 S 1.15534945094 775.425672264 0.164111609055

4 S 1.28465831644 788.177963118 0.158493029883

5 S 1.73604927288 801.264323533 0.178782238942

6 S 0.545797945778 811.335660467 0.166015183453

...
```



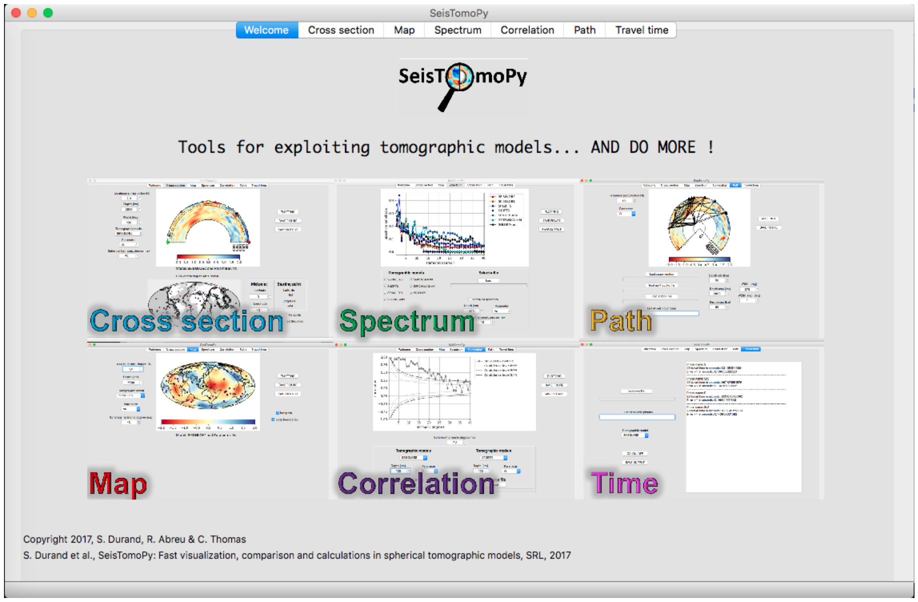


Figure 1: Welcoming screen of SeisTomoPy GUI interface.

5 How to use SeisTomoPy GUI

It is possible to use SeisTomoPy tools via the GUI graphical interface. To do so the user has to launch the GUI interface (see Figure 1). With the top bar it is possible to navigate through the different tools. Each of them offer the possibility to save the plots as well as some output files.

The user must run:

```
ipython SeisTomPy_gui.py
```

5.1 Cross sections

Go on the tab Cross section. A screen with a map at the bottom and an empty space at the top appears. The parameters are already set to default values so that clicking on the button PLOTTING (top right button) computes a cross section.

Various options are available. First, it is possible to choose the location of the profile. To do so:

- 1) Click on the map at the bottom which will move the profile. This changes the mid-point of the profile (yellow dot). One can also manually enter the values of the latitude and longitude of the mid-point.
- 2) Change the azimuth of the profile.

Other parameters that can be changed include:

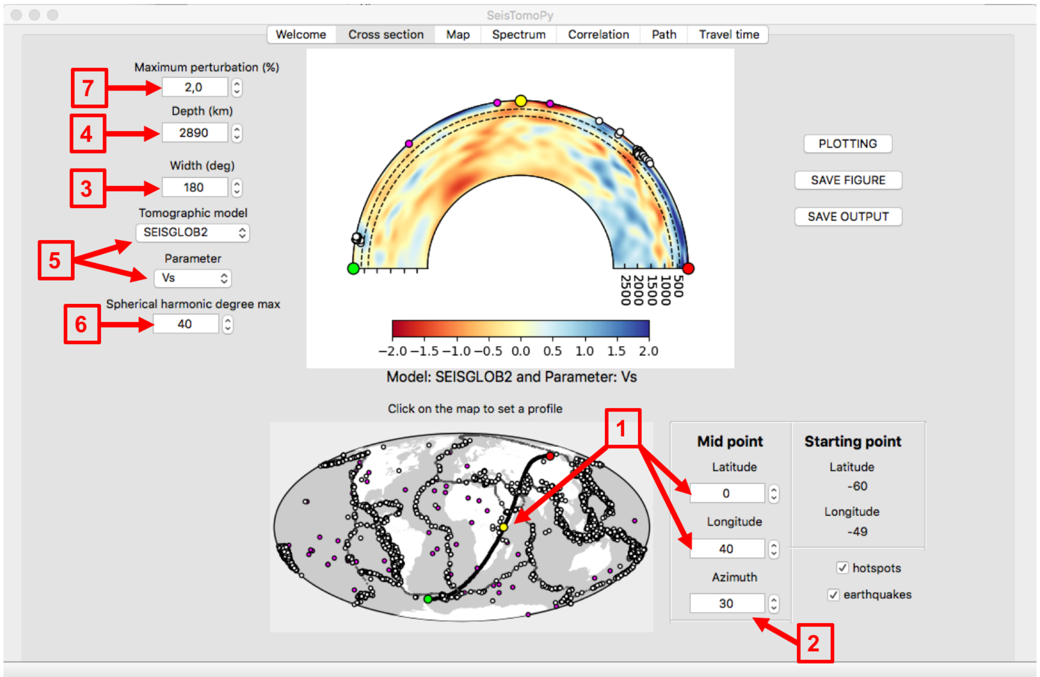


Figure 2: Summary of the different options in Cross section.

- 3) The width of the profile.
- 4) The maximum depth down to which the cross section will be plotted.
- 5) The tomographic model and parameter to be plotted.
- 6) The spherical harmonic degree up to which the model will be plotted. It can be at most 60.
- 7) The maximum values of the velocity perturbations used for the colorbar.

All these actions are summarized on Figure 2.

One can save the plots by clicking on SAVE FIGURE (top right button). This will produce two pdf files:

- `map_MODEL_PARA_LAT_LON_AZI.pdf`
- `crossec_MODEL_PARA_LAT_LON_AZI.pdf`

MODEL, PARA, LAT, LON and AZI, respectively, correspond to the tomographic model, parameter, latitude and longitude of the mid-point and azimuth used for generating the profile.

Some useful output files can also be saved by clicking on SAVE OUTPUT. A folder entitled `output_crossection_LAT_LON_AZI` will appear at the desired location. It contains 3 files:

- `output_cross_MODEL.out`: file containing all the values to reproduce the cross section. There are 6 columns: latitude, longitude, radius (km), $d\ln(V_p)$ (%), $d\ln(V_s)$ (%), $d\ln(\rho)$ (%).

- `MODEL_input_AxiSEM.sph`: input file for running synthetic seismograms with AxiSEM. **It is important to note that these files give the velocity perturbations with respect to the reference model used in every tomographic inversion so that if the user provides this file to AxiSEM, the good reference model file should also be provided. They are provided in** `Taup_models/AXISEM_REF`.

- `MODEL_PathPy.sph`: file that can be used in PathPy (see later).

5.2 Maps

On the tab Maps, one finds a screen with an empty space where the map will be plotted. The parameters are already set to default values so that one can already click on the button PLOTTING (top right button). After some seconds, the map will appear.

Parameters that can be changed include:

- 1) The depth of the map.

- 2) The tomographic model and parameter to be plotted.

- 3) The spherical harmonic degree up to which the model will be plotted. It can be at most 60.

- 4) The maximum values of the velocity perturbations used for the colorbar.

All these actions are summarized on Figure 3.

The plot can be saved by clicking on SAVE FIGURE (top right button). This will produce one pdf file:

- `map_MODEL_PARA_DEPTH.pdf`

MODEL, PARA and DEPTH, respectively, correspond to the tomographic model, parameter and depth used for generating the map.

Some useful output files can also be saved by clicking on SAVE OUTPUT. A folder entitled `output_map_DEPTH` will appear at the desired location. It contains 4 files:

- 3 files `map_NEW_MODEL_PARA.out`: files containing all the values to reproduce the map. There are 3 columns: latitude, longitude, $dln(PARA)$ (%).

- `output_map_MODEL.out`: summuray file with 5 columns: latitude, longitude, $dln(V_p)$ (%), $dln(V_s)$ (%), $dln(\rho)$ (%).

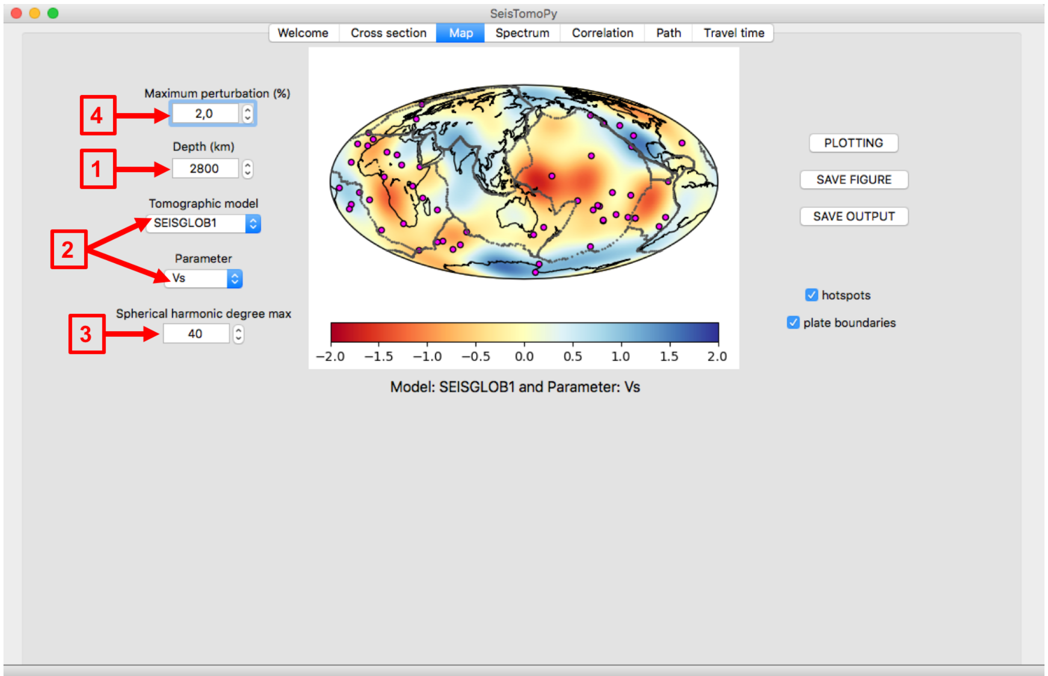


Figure 3: Summary of the different options in Map.

5.3 Spectrum

On the tab Spectrum, the spectrum will be calculated. The parameters are already set to default values so that clicking on the button PLOTTING (top right button) produces a spectrum.

Parameters that can be changed include:

- 1) The tomographic models to be used.
- 2) The depth at wich the spectrum is being computed.
- 3) The parameter.
- 4) The spherical harmonic degree up to which the spectrum will be computed. It can be at most 60.
- 5) It is also possible with this tool to compute the spectrum of any other model that the user wishes to use. Then the user must upload the file with the model at a given depth for a given parameter using the space entitle "Select a file" by clicking on the button "load". The file must have a specific format. Please refer to section 9 for details about these files.

All these actions are summarized on Figure 4.

One can save the plot by clicking on SAVE FIGURE (top right button). This will produce one pdf file:

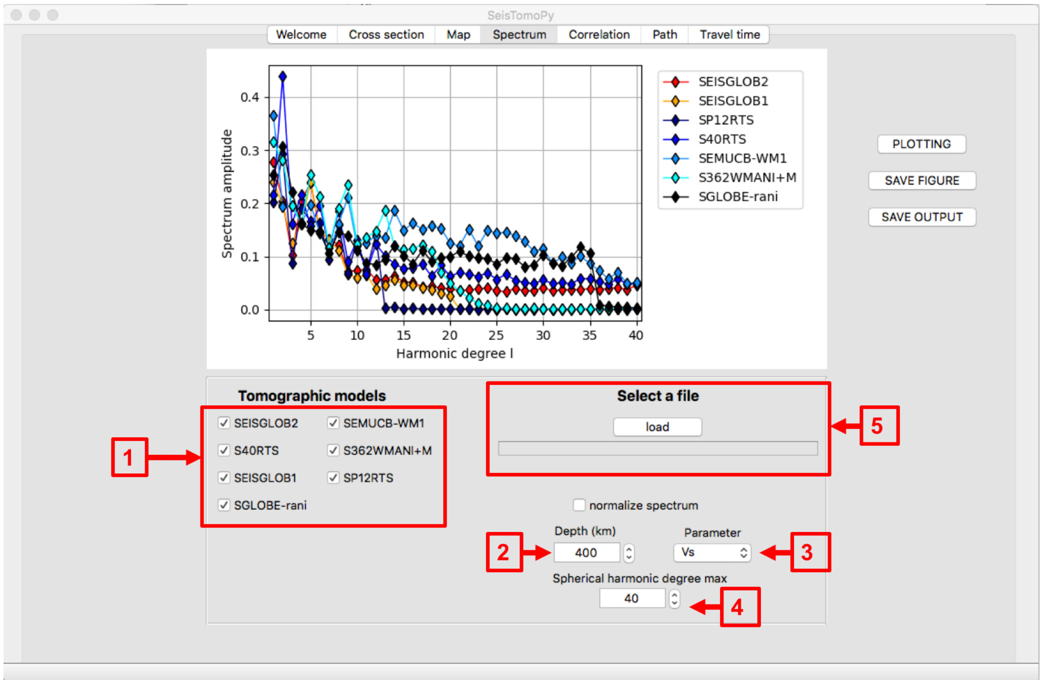


Figure 4: Summary of the different options in Spectrum.

- `spectre_DEPTH.pdf`

DEPTH corresponds to the depth at which the spectrum has been computed.

Some useful output files can be saved by clicking on SAVE OUTPUT. A folder entitled `output_spectre_DEPTH` will appear at the desired location. It contains as many files as the number of tomographic models used:

- `spectre_MODEL_PARA_DEPTH.out`: file containing all the values of the spectrum. There are 2 columns: harmonic degree, spectrum.

5.4 Correlations

Tab Correlation calculates correlations between models. The parameters are already set to default values so that clicking on the button PLOTTING (top right button) produces a correlation.

Parameters that can be changed include:

- 1) The two tomographic models.
- 2) The two depths.
- 3) The two parameters.

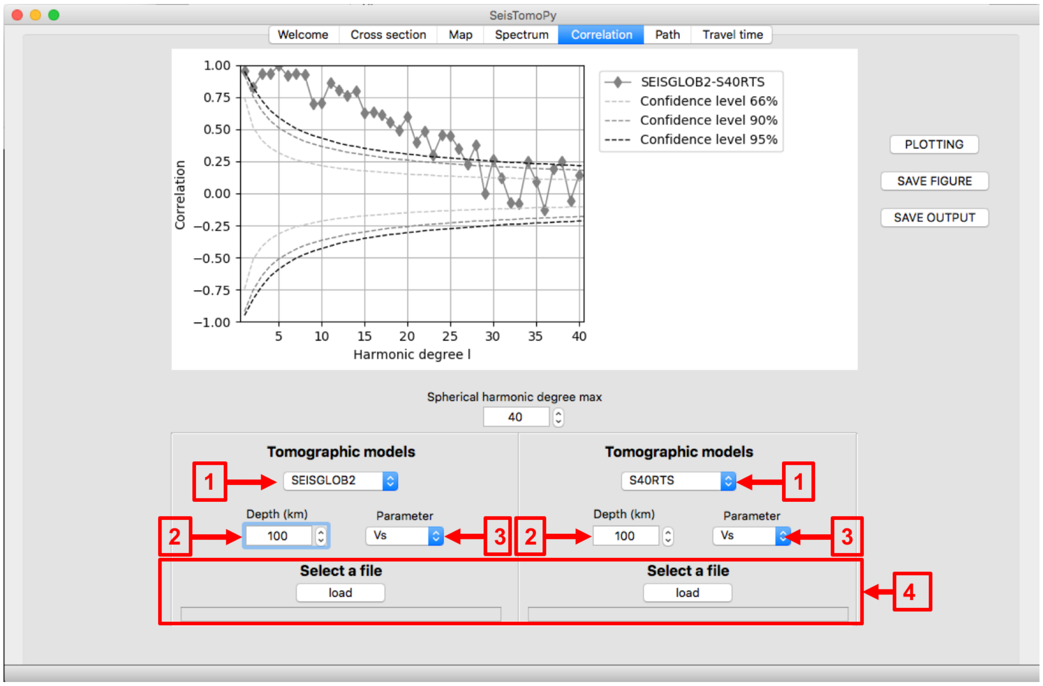


Figure 5: Summary of the different options in Correlation.

- 4) The spherical harmonic degree up to which the spectrum will be computed. It can be at most 60.
- 5) It is also possible with this tool to compute the correlation with any other model that the user wishes to use. The user must upload the file with the model at a given depth for a given parameter using the space entitled "Select a file" by clicking on the button "load". The file must have a specific format. Please refer to section 9 for details about these files. If this option is to be used, then the tomographic model should be set to "None".

All these actions are summarized on Figure 5.

The plot can be saved by clicking on SAVE FIGURE (top right button). This will produce one pdf file:

- `correlation.pdf`

Some useful output files can also be saved by clicking on SAVE OUTPUT. A folder entitled `output\_corr` will appear at the desired location. It contains 1 file:

- `corr_MODEL1_MODEL2.out`: file containing all the values of the correlation. There are 2 columns: harmonic degree, correlation.

5.5 Paths

Tab Path allows to calculate paths and plot them on top of tomographic models.

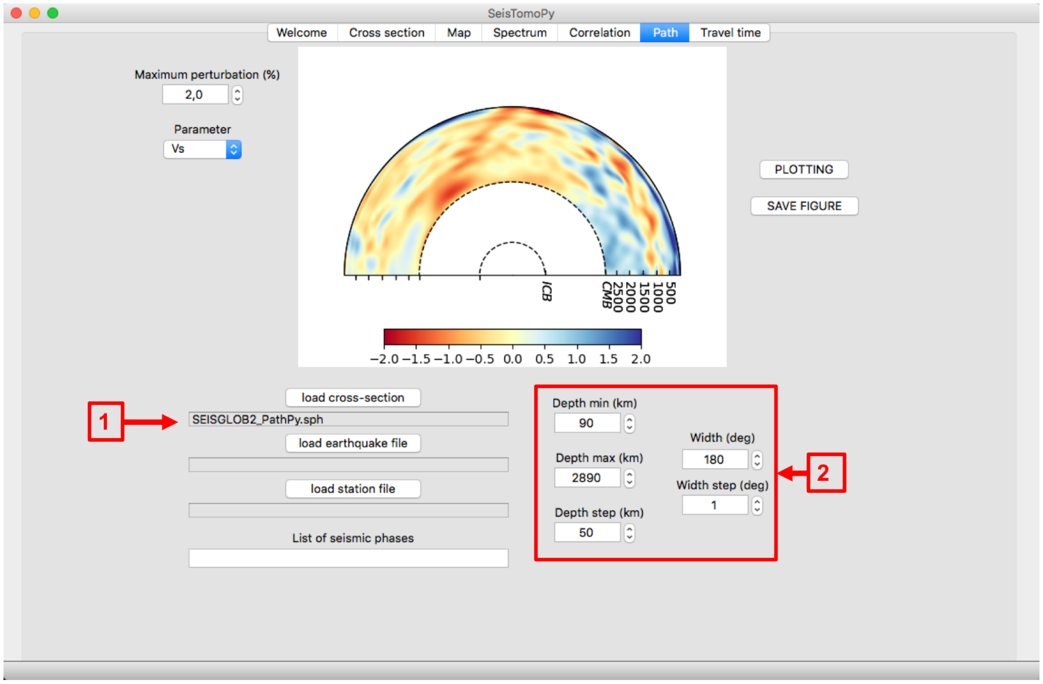


Figure 6: Summary of the different options in Path.

The first thing to do with Path is

- 1) to upload a cross section. This can be achieved by clicking on "load cross-section" and first using a file generated by the Cross section tool (`MODEL_PathPy.sph`).
- 2) Then in order to properly read this file the user must give the starting and ending depths in the file, as well as the depth step. The same for the width of the cross section. It is then already possible to test wether the file is properly read by clicking on PLOTTING. If yes, then a cross section is plotted.
- 3) Then one may want to plot some seismic wave paths on top of the cross section. To do so one has to upload earthquake and station files and provide a list of seismic phases. For the earthquake and station files, they must contain two columns with the angle (in degrees) and the depth (in km) of the earthquakes and stations. For the example Figure 6 the following files `event.xy`:

  ```
  0  0
  10  500
  ```

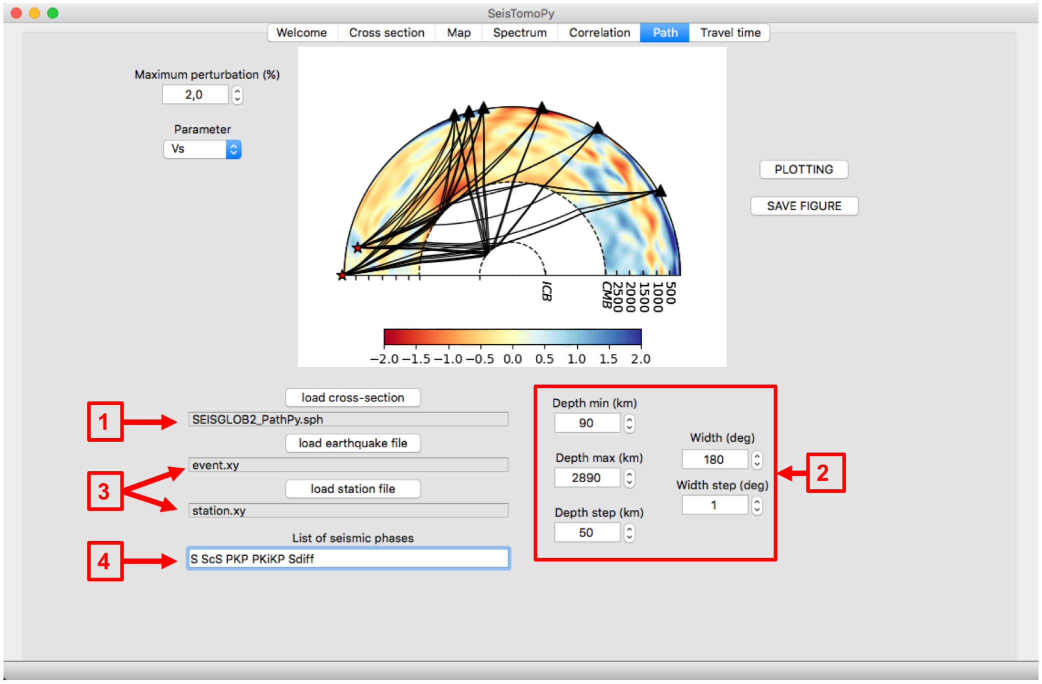


Figure 7: Summary of the different options in Path.

and `station.xy`:

```
70  0
75  0
80  0
100  0
120  0
150  0
```

have been used.

- 4) Finally, a list of seismic phases should be provided. The list must contain the name of the seismic phases. If there is more than one they must be separated by a blank (for instance S ScS Sdiff ...). It is possible to run the calclations by clicking on PLOTTING, and the cross section with the wave paths appears (see Figure 7).

The plot can be saved by clicking on SAVE FIGURE and a `path.pdf` file will appear at the chosen location.

5.6 Travel times

The last tool is Travel time. For this the user needs to

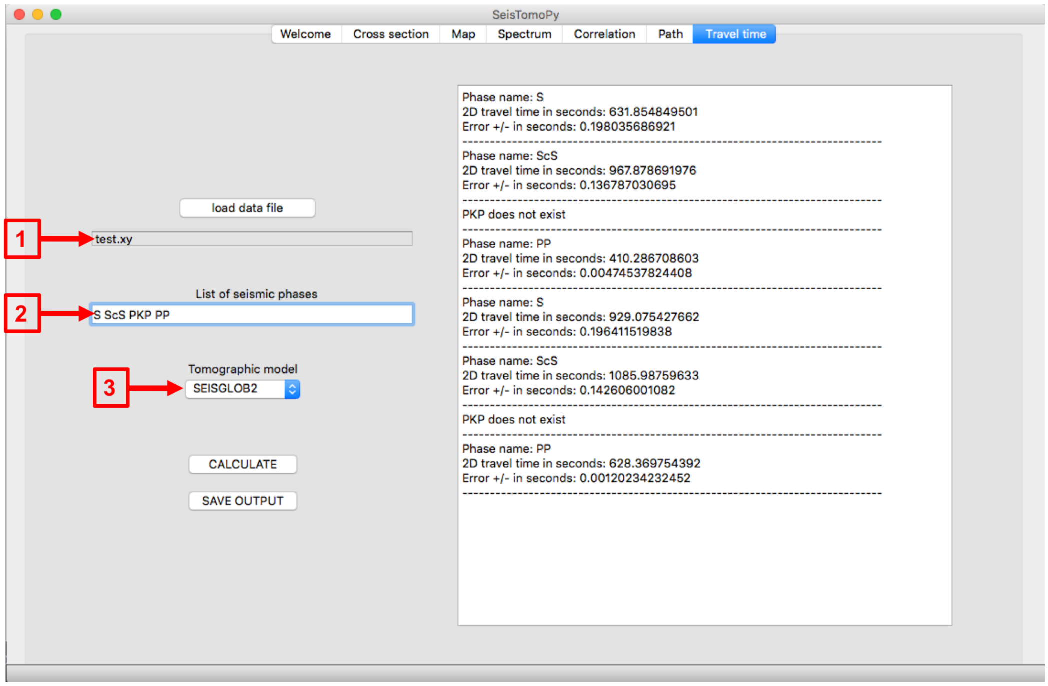


Figure 8: Summary of the different options in Travel time.

- 1) upload a file with the earthquake and station locations. This file contains 5 columns with event latitude, event longitude, event depth, station latitude and station longitude. For the example Figure 8 we used the file `test.xy`:

```
0 0 200 0 30
0 0 200 0 50
```

- 2) Then the user must specify the seismic waves to be computed. If more than one is given the different phases must be separated by blanks (for instance S ScS Sdiff).
- 3) Finally a tomographic model must be chosen. The travel time will be computed by clicking on CALCULATE. The summary of the calculations will appear in the empty space on the right.

The travel times can be saved by clicking on SAVE OUTPUT which will create a folder travel_times_output.

6 Model and parameter options

For using the tools, the tomographic models and parameters must be chosen. Here is the list of the acronyms to be used for the models:

```
"SEISGLOB2"
"S40RTS"
"SEMUCBWM1"
"S362WMANIM"
"SEISGLOB1"
"SP12RTS"
"SGLOBE"
"3D2016"
"MYMODEL"
```

and for the parameters

```
"VS"
"VP"
"RHO"
```

7 Creating a new model file

It is possible to upload a new model in SeisTomoPy. For this the user has to generate the model file. To do so we provide a package. The user will find a directory `Create_model_file` with a readme file explaining the procedure.

First the user must generate `vs.DEPTH.xyz` files at every kilometer depth and store them in the diectory `Create_model_file/model` and then go to `Create_model_file/src` and run

```
$ make clean
$ make all
$ ./main
```

This will generate two model files `YOURMODEL_5km.sph` and `YOURMODEL_1km.sph` that should be copied in `SeisTomoPy/fortran_files/models`. Then one can use all the tools of SeisTomoPy refering to the new model using

```
"MYMODEL"
```

8 Changing plate boundary model and hotspots and earthquake catalogues

The plate boundaries of model NUVEL-1A [*DeMets et al.*, 1990], the hotspot locations [*Müller et al.*, 1993] and a catalogue of earthquakes can be added. The catalogue of earthquakes is not exhaustive, it has been built with a minimum of earthquakes such that plate boundaries can be seen. We used events deeper than 100 km depth with magnitudes greater than 5.9 in the time range 1976-1998 and we added all events with magnitudes greater than 5.0 in the time range 2010-2011. Magnitudes were chosen to be larger than 5.0 to keep the dataset small.

However, the user may want to use another plate boundary model, or hotspot and earthquake catalogues. To that aim it is possible to provide new files with the same format as those already provided and with the same name.

8.1 Plate boundary model

The file must be named `plate_boundaries.xy` and contains in the first column the longitude and in the second column the latitude of the plate boundaries. Every segment of boundary should be separated by NaN values as illustrated below.

```
...
    10.30000   -53.2000
     11.5000   -53.0000
     13.0000   -52.7000
     14.2000   -52.4000
     15.0000   -52.2000
     15.4000   -51.9000
NaN NaN
     15.3000   -51.9000
     16.1000   -51.9000
     16.4000   -52.1000
     17.3000   -52.4000
...
```

8.2 Hotspot catalogue

The file must be named `points_chauds.xy` and contains in the first column the longitude of the hotspots and in the second column the latitude of the hotspots (see below).

```
...
42.0000 12.0000
-25.0000 -17.0000
-14.3700 -7.9500
-139.9699 -29.3722
-28.0000 38.0000
-113.0000 27.0000
164.7017 -67.3982
...
```

8.3 Earthquake catalogue

The file must be named `catalogue.xy` and contains in the first column the longitude of the event, in the second column the latitude of the event and in the third column the depth of the event (see below).

```
...
167.81 -15.97 174
120.07 -7.37 623
166.95 -14.91 103
129.74 -6.90 186
-177.61 -19.46 453
70.64 36.33 196
-68.51 -20.54 134
...
```

9 Input file formats for Spectrum, Correlations and Paths

It is possible to upload model files to be used in Spectrum, Correlation and Path.

For Spectrum and Correlation the files have the same format and it is the same format as the files `map_NEW_MODEL_PARA.out` obtained with Map tool, which have 3 columns: latitude (from -89 to 89 degrees, with a step of 1 degree), longitude (from 0 to 359 degrees, with a step of 1 degree) and parameter perturbations (%). It must thus contain in total 64440 lines. Below is an extract given as example.

```
         -89            0   0.73245523974336291
         -88            0   0.72104058555001316
         -87            0   0.71041631025040164
...
          87          359   0.49352141683917167
          88          359   0.51590110372203857
          89          359   0.54502724312040074
```

For Path, the file must contain 5 columns: the radius, the distance in degrees from the starting point, $d\ln(V_p)$, $d\ln(V_s)$ and $d\ln(\rho)$ in %. It is the same format as the files `MODEL_PathPy.sph` obtained with the Cross section tool. The user sets which parameter should be plotted in the input arguments of the function `SeisTomoPy.path_plot_fromfile` or with the parameters to set on the GUI. Below is an example.

```
        3481            0    2.7162811E-002   4.9385717E-002   9.8771474E-003
        3531            0    3.3263616E-002   6.0478839E-002   1.2099678E-002
        3581            0    4.0070403E-002   7.2856784E-002   1.4571265E-002
...
        6131          180    1.2205796        2.2192353       0.4438470
        6181          180    0.4952321        0.9005191       0.1801038
        6231          180    0.7806580        1.4193792       0.2833848
        6281          180    1.5027913        2.7322743       0.5464547
```

10 Future developments

Further functionalities will be added in the future:

- Displaying full anisotropic tomographic models.
- Displaying geodynamic models obtained from modelling codes such as Stag3D [*Tackley & Shunxing*, 2002], that can then be directly compared to tomographic models.

11 Reference

Please if you are using SeisTomoPy please refer to

S. Durand, R. Abreu, C. Thomas, 2017,
SeisTomoPy: Fast visualization, comparison and calculations in global tomographic models,
Seis. Res. Lett.,

References

Bassin, C., Laske, G., Masters, G., 2000, The current limits of resolution for surface wave tomography in North America, *EOS Trans AGU*, **81**, F897

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