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LAGA_chapter06.05problem6.5.5.wxm (LAGA examples)
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      A test document for Geometric Algebra with wxMaxima
      contains...
      Initialization
      Loading of functions (intrinsic and GA specific)
       Pseudoscalar definition (specifies the space dimension) and
      calculation of the inverse pseudoscalar used to generate the dual of a multivector
      Enumeration of the standard basis for the specified dimension
      Product Properties in the geometric algebra, G3
      Reference book...Linear and Geometric Algebra (LAGA)
      by Alan Macdonald
      Initialization
    (%i54) ext:["wxm"]$
                                        file_type_maxima:append(ext,file_type_maxima)$
                                         batchload("initialize_fns")$
      the pseudoscalar and its inverse
      the lowest useable dimension pseudoscalar should be \{e1,e2\} i.e. Plen = 2
      e.g. for four dimensions edit Pseudos:{e1,e2,e3}$ to Pseudos:{e1,e2,e3,e4}$
    (%i1) Pseudos:{e1,e2,e3}$
                                    Pvar:listofvars(Pseudos)$
                                    Plen:length(Pvar)$
                                    I:Pseudos$
                                    ni:(Plen-1)*Plen/2$
                                    Ii:(-1)^ni*I$
                                    kill(ni)$
                                    ldisplay(Pvar)$
             (\%t8) Pvar = [e1, e2, e3]
    (%i9) batchload("initialize_lsts")$
             (\%t9) lstblds = [\{e1\}, \{e2\}, \{e3\}\}, \{\{e1\}, \{e1\}, \{e1\}, \{e3\}, \{e2\}, \{e3\}\}, \{\{e2\}, e3\}\}, \{\{e1\}, \{e2\}, \{e3\}\}, \{\{e3\}, \{e3\}, \{e4\}, \{e
        (\%t10) allblds = [{e1},{e2},{e3},{e1},{e2},{e1},{e2},{e1},{e2},{e2},{e2},{e3},{e1},{e2},{e3}]
        (\%t11) invblds = [\{e1\}, \{e2\}, \{e3\}, -\{e1, e2\}, -\{e1, e3\}, -\{e2, e3\}, -\{e1, e2, e3\}]
      end of Initialization
      Problem 6.5.5
      page 114
      first use function makelistgrademv() to make two bivectors;
    (%i12) lstgA:[2]$
                                         nameA:"A"$
                                         makelistgrademv(nameA,lstgA)$
                                         ldisplay(A)$
                                         IstgB:[2]$
                                         nameB:"B"$
                                         makelistgrademv(nameB,lstgB)$
                                         ldisplay(B)$
       (\%t15) A = a_{2.3} * \{e2,e3\} + a_{2.2} * \{e1,e3\} + a_{2.1} * \{e1,e2\}
        (\%t19) B = b_{2,3} * \{e2,e3\} + b_{2,2} * \{e1,e3\} + b_{2,1} * \{e1,e2\}
      display both geometric products
    (%i20) M1:A&*B$
                                         M2:B&*A$
                                         ldisplay(M1,M2)$
       (\%t22)/R/M1 = (b_{2,1}*a_{2,2}-b_{2,2}*a_{2,1})*\{e2,e3\} + (-b_{2,1}*a_{2,3}+b_{2,3}*a_{2,1})*\{e1,e3\} + (-b_{2,1}*a_{2,3}+b_{2,3}*a_{2,1})*(e1,e3) + (-b_{2,1}*a_{2,2}+b_{2,3}*a_{2,1})*(e1,e3) + (-b_{2,1}*a_{2,2}+b_{2,3}*a_{2,1})*(e1,e3) + (-b_{2,1}*a_{2,2}+b_{2,3}*a_{2,1})*(e1,e3) + (-b_{2,1}*a_{2,2}+b_{2,3}*a_{2,1})*(e1,e3) + (-b_{2,1}*a_{2,2}+b_{2,3}*a_{2,1})*(e1,e3) + (-b_{2,1}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,3}*a_{2,2}+b_{2,2}+b_{2,2}*a_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b_{2,2}+b
(b_{2,2}*a_{2,3}-b_{2,3}*a_{2,2})*\{e1,e2\}-b_{2,3}*a_{2,3}-b_{2,2}*a_{2,2}-b_{2,1}*a_{2,1}
       (\%t23)/R/M2 = (a_{2,1}*b_{2,2}-a_{2,2}*b_{2,1})*\{e2,e3\}+(-a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*\{e1,e3\}+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e1,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e2,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,3}+a_{2,3}*b_{2,1})*(e3,e3)+(a_{2,1}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}*b_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2}+a_{2,2
(a_{2,2}*b_{2,3}-a_{2,3}*b_{2,2})*\{e1,e2\}-a_{2,3}*b_{2,3}-a_{2,2}*b_{2,2}-a_{2,1}*b_{2,1}
      use the function grader() to find the bivector content of the two products,
      noting that the denominator is held in the cell grarr[Plen+1]=grarr[4]
    (%i24) grarr:grader(M1)$
                                         B1:grarr[2]/grarr[4]$
                                          grarr:grader(M2)$
                                          B2:grarr[2]/grarr[4]$
                                         Idisplay(B1,B2)$
       (\%t28)/R/B1 = (-a_{2,1}*b_{2,2}+b_{2,1}*a_{2,2})*\{e2,e3\} + (a_{2,1}*b_{2,3}-b_{2,1}*a_{2,3})*\{e1,e3\} + (a_{2,1}*b_{2,3}-b_{2,1}*a_{2,3})*(e1,e3) + (a_{2,1}*b_{2,3}-b_{2,3}-b_{2,3})*(e1,e3) + (a_{2,1}*b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3})*(e1,e3) + (a_{2,1}*b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2,3}-b_{2
(-a_{2,2}*b_{2,3}+b_{2,2}*a_{2,3})*{e1,e2}
      (\%t29)/R/B2 = (a_{2,1}*b_{2,2}-b_{2,1}*a_{2,2})*\{e2,e3\} + (-a_{2,1}*b_{2,3}+b_{2,1}*a_{2,3})*\{e1,e3\} + (-a_{2,1}*b_{2,3}+b_{2,1}*a_{2,3})*\{e1,e3\} + (-a_{2,1}*b_{2,3}+b_{2,3}+b_{2,3})*(e1,e3) + (-a_{2,1}*b_{2,3}+b_{2,3}+b_{2,3})*(e1,e3) + (-a_{2,1}*b_{2,3}+b_{2,3}+b_{2,3})*(e1,e3) + (-a_{2,1}*b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3})*(e1,e3) + (-a_{2,1}*b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_{2,3}+b_
(a_{2,2}*b_{2,3}-b_{2,2}*a_{2,3})*{e1,e2}
      now compare the two sides of the equation
    (%i30) lhs:M1-M2$
                                         rhs:B1-B2$
                                         is(equal(lhs,rhs))$
                                         ldisplay(lhs,rhs,%)$
      (\%t33)/R/lhs = (-2*a_{2,1}*b_{2,2}+2*b_{2,1}*a_{2,2})*{e2,e3}+(2*a_{2,1}*b_{2,3}-2*b_{2,1}*a_{2,3})*{e1,e3}
  +(-2*a_{2,2}*b_{2,3}+2*b_{2,2}*a_{2,3})*{e1,e2}
     (\%t34)/R/ \text{ rhs} = (-2*a_{2,1}*b_{2,2}+2*b_{2,1}*a_{2,2})*\{e2,e3\} + (2*a_{2,1}*b_{2,3}-2*b_{2,1}*a_{2,3})*\{e1,e3\}
   +(-2*a_{2,2}*b_{2,3}+2*b_{2,2}*a_{2,3})*{e1,e2}
       (\%t35)\% = true
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Created with wxMaxima.