space_time_para_4.1.3.wxm

An application document for Geometric Algebra using wxMaxima

Ref: The Survey, para.4.1.3

investigate the use of the fourth axis, e4, as a possible imitation of G(1,3)

Initialization

(%i38)
```
ext:["wxm"]$
file_type_maxima:append(ext,file_type_maxima)$
batchload("initialize_fns")$
```

the pseudoscalar and its inverse
the lowest useable dimension pseudoscalar should be {e1,e2} i.e. Plen = 2
e.g. for four dimensions edit Pseudos:{e1,e2,e3}$ to Pseudos:{e1,e2,e3,e4}$

(%i1)
```
Pseudos:{e1,e2,e3,e4}$
Pvar:listofvars(Pseudos)$
Plen:length(Pvar)$
I:Pseudos$
ni:(Plen-1)*Plen/2$
Ii:(-1)^ni*I$
kill(ni)$
ldisplay(Pvar)$
```

(%t8) Pvar=[e1,e2,e3,e4]

(%i9) batchload("initialize_lsts")$

(%t9) lstblds=[[{e1},{e2},{e3},{e4}],[{e1,e2},{e1,e3},{e1,e4},{e2,e3},{e2,e4},{e3,e4}],[{e1,e2,e3},{e1,e2,e4},{e1,e3,e4},{e2,e3,e4}],[{e1,e2,e3,e4}]]
(%t10) allblds=[{e1},{e2},{e3},{e4},{e1,e2},{e1,e3},{e1,e4},{e2,e3},{e2,e4},{e3,e4},{e1,e2,e3},{e1,e2,e4},{e1,e3,e4},{e2,e3,e4},{e1,e2,e3,e4}]
(%t11) invblds=[{e1},{e2},{e3},{e4},-{e1,e2},-{e1,e3},-{e1,e4},-{e2,e3},-{e2,e4},-{e3,e4},-{e1,e2,e3},-{e1,e2,e4},-{e1,e3,e4},-{e2,e3,e4},{e1,e2,e3,e4}]

end of Initialization

set derivabbrev:false$

(%i12) derivabbrev:false$

(%i13) ratprint:false$

The Survey, para.4.1.3

show the spacetime gammas required for the imitation of G(1,3)

(%i14)
```
g1:%i*{e1}$
g2:%i*{e2}$
g3:%i*{e3}$
g4:{e4}$
```

(%i18)
```
g1&.g1;
g2&.g2;
g3&.g3;
g4&.g4;
```

(%o18)/R/ -1
(%o19)/R/ -1
(%o20)/R/ -1
(%o21)/R/ 1

the spacetime coordinate vector using the gammas

(%i22) x:x1*g1+x2*g2+x3*g3+t*g4;

(%o22) %i*{e3}*x3+%i*{e2}*x2+%i*{e1}*x1+{e4}*t

choose a simple rotation angle for partial verification of the hyperbolic identity

(%i23)
```
alpha:%pi/3$
ahalf:alpha/2;
```

(%o24) $\frac{\pi}{6}$

the velocity magnitude as a fraction of the speed of light

(%i25)
```
tanh(alpha)$
vel:ev(%,numer);
```

(%o26) 0.78071443535927

choose a simple (imaginary ugh!) unit vector, vhat for the unit velocity

(%i27)
```
vhat:g2$
vhat&*vhat;
```

(%o28)/R/ -1

show the rotation plane and the rotation bivector

(%i29)
```
Plane:vhat&*g4$
B:Plane*ahalf$
ldisplay(Plane,B)$
```

(%t31)/R/ Plane={e2,e4}*%i
(%t32)/R/ B=$\frac{\%i*\pi*\{e2,e4\}}{6}$

form a rotation exponential, with accuracy limited using mvexp(,13)

(%i33)
```
mvexp(B,13)$
ev(%,numer,expand);
```

(%o34) 0.54785347388804*%i*{e2,e4}+1.140238321076428

verify that the intrinsic hyperbolic functions are consistent with function mvexp()
while imitating G(1,3) with the intrinsic imaginary, %i

(%i35)
```
cosh(ahalf)+Plane*sinh(ahalf)$
trigsimp(%)$
Rv:ev(%,numer,expand);
```

(%o37) 0.54785347388804*%i*{e2,e4}+1.140238321076429

numerical comparison of spacetime vector rotation with the Lorentz transformation
e.g. for a simple velocity, vel*vhat (= vel*g1)

(%i38)
```
vel:0.8$
alpha:atanh(vel)$
ahalf:alpha/2;
```

(%o40) 0.54930614433406

form the rotation bivector

(%i41)
```
vhat:g1$
B:vhat&*g4*ahalf$
ev(%,numer,expand);
```

(%o43) 0.54930614433405*%i*{e1,e4}

form the left and right exponential multipliers

(%i44)
```
mvexp(-B,13)$
lexp:ev(%,numer,expand);
```

(%o45) 1.154700538379249-0.57735026918963*%i*{e1,e4}

(%i46)
```
mvexp(+B,13)$
rexp:ev(%,numer,expand);
```

(%o47) 0.57735026918963*%i*{e1,e4}+1.154700538379249

apply the rotation to a spacetime coordinate vector "parallel" to the velocity

(%i48) x:x1*g1+t*g4;

(%o48) %i*{e1}*x1+{e4}*t

find the rotated spacetime vector

(%i49)
```
xbar:lexp&*x&*rexp$
ev(%,numer,expand)$
collectterms(%,%i,e1,e4);
```

(%o51) %i*{e1}*(1.666666666666659*x1-1.33333333333333*t)+{e4}*(1.666666666666659*t-1.33333333333333*x1)

compare the spacetime rotation result with the Lorentz transformation factors

(%i52) L:1/sqrt(1-vel^2);

(%o52) 1.666666666666667

(%i53) Lv:vel*1/sqrt(1-vel^2);

(%o53) 1.333333333333334

the Lorentz space and time

(%i54) x1bar:(-Lv*t+L*x1)$

(%i55) tbar:(+L*t-Lv*x1)$

the Lorentz spacetime vector

(%i56) x1bar*g1+tbar*g4;

(%o56) %i*{e1}*(1.666666666666667*x1-1.333333333333334*t)+{e4}*(1.666666666666667*t-1.333333333333334*x1)

Created with wxMaxima.