GNU General Public License 2019 Stephen Athel Abbott. An application document for Geometric Algebra using wxMaxima
Ref: The Survey, paragraph 4.1.2 for flat spacetime The intrinsic functions for curved spacetime have been applied within the GAwxM environment,
Without using the GAwxM extrinsic functions, in the hope that it might be useful someday Use of ctensor functions christof() and uricci() to calculate the Ricci tensor, Ricci scalar and hence the covariant Einstein tensor from a user defined covariant metric tensor
Initialization (%i1) ext:["wxm"]\$
file_type_maxima:append(ext,file_type_maxima)\$ batchload("initialize_fns")\$ the pseudoscalar and its inverse
the lowest useable dimension pseudoscalar should be {e1,e2} i.e. Plen = 2 e.g. for four dimensions edit Pseudos: {e1,e2,e3}\$ to Pseudos: {e1,e2,e3,e4}\$ (%i1) Pseudos: {e1,e2,e3,e4}\$ Pyar: listofyars (Pseudos)\$
Pvar:listofvars(Pseudos)\$ Plen:length(Pvar)\$ I:Pseudos\$ ni:(Plen-1)*Plen/2\$ Ii:(-1)^ni*I\$
kill(ni)\$ ldisplay(Pvar)\$ (%t8) Pvar=[e1,e2,e3,e4]
<pre>(%i9) batchload("initialize_lsts")\$ (%t9) lstblds=[[{e1},{e2},{e3},{e4}],[{e1,e2},{e1,e3},{e1,e4},{e2,e3},{e2,e4}], ,{e3,e4}],[{e1,e2,e3},{e1,e2,e4},{e1,e3,e4},{e2,e3,e4}],[{e1,e2,e3,e4}]]</pre>
(%t10) allblds = [{e1},{e2},{e3},{e4},{e1,e2},{e1,e3},{e1,e4},{e2,e3},{e2,e4},{e2,e4},{e3,e4},{e1,e2,e3},{e1,e2,e3},{e1,e2,e3},{e1,e2,e3},{e1,e2,e4},{e3,e4},{e1,e2,e3},{e1,e2,e3},{e1,e2,e4},{e1,e2,e3},-{e1,e2,e3},-{e2,e3},-{e2,e3},-{e2,e3},-{e2,e3},-{e2,e3},-{e2,e3},-{e2,e3},-{e3,e4},-{e3,e4},-{e3,e4},-{e3,e4},-{e3,e4},-{e1,e2,e3},-{e3,e4},-{e1,e2,e3},-{e3,e4}]
end of Initialization set derivabbrev:false\$
(%i12) derivabbrev:false\$ (%i13) ratprint:false\$
The Survey, para.4.1.3
show the flat spacetime required for the imitation of $G(1,3)$, where we have used the fourth axis, e4, for the time axis and the intrinsic maxima imaginary, %i, for the space axes (%i14) $g1:\%i*\{e1\}$ \$
g2:%i*{e2}\$ g3:%i*{e3}\$ g4:{e4}\$ the spacetime coordinate vector
(%i18) xb:x*g1+y*g2+z*g3+t*g4; (%o18) %i*{e3}*z+%i*{e2}*y+%i*{e1}*x+{e4}*t
The Survey, paragraph 4.1.2, suggests a spacetime split using the geometric product, here that is xb&*g4; we could call this the split spacetime coordinate, splxb
(%i19) splxb:xb&*g4; (%o19)/R/ {e3,e4}*%i*z+{e2,e4}*%i*y+{e1,e4}*%i*x+t
for the actual split into time and space (%i20) realpart(splxb); imagpart(splxb);
(%020) t (%021) $\{e3,e4\}*z+\{e2,e4\}*y+\{e1,e4\}*x$ in order to unsplit the split spacetime we postmultiply by the inverse of g4 (that also
occurs in the global list, invblds[] as {e4}) (%i22) invg4:g4/normod(g4)^2;
(%022) { e4 } to recover the spacetime coordinate vector
<pre>(%i23) splxb&*invg4; is(equal(%,xb)); (%o23)/R/ %i*{e3}*z+%i*{e2}*y+%i*{e1}*x+{e4}*t</pre>
(%024) true for any 3D position vector, rb, in a flat spacetime (%i25) rb; v* (a1) +v* (a2) +r* (a3) \$
(%i25) rb:x*{e1}+y*{e2}+z*{e3}\$ normrb:normod(rb)\$ ldisplay(normrb)\$ (%t27) normrb = $\sqrt{z^2 + y^2 + x^2}$
Start of tensor analysis for a nearly-flat spacetime
<pre>(%i28) load(ctensor)\$ init_ctensor()\$ (%i30) ct_coords:[x,y,z,t]; dim;</pre>
(%o30)[x,y,z,t] (%o31)4
(%i32) declare(a,real,b,real); (%o32) done
GR seems to be a theory using differential equations to find a solution near a particular point in space-time. With that in mind, and modern statements that the universe seems to be very nearly flat, we can choose the covariant metric tensor below for functions $a(t)$ and $b(t)$ such that $0 < a < 1$ and $0 < b < 1$
the covariant metric tensor
$ \begin{aligned} & \text{lg:matrix}([\text{gs},0,0,0],[0,\text{gs},0],[0,0,\text{gs},0],[0,0,0,\text{gt}]); \\ & \begin{bmatrix} -a(\texttt{t})-1 & 0 & 0 & 0 \\ 0 & -a(\texttt{t})-1 & 0 & 0 \end{bmatrix} \end{aligned} $
$ \begin{pmatrix} (\%035) \\ 0 & 0 & -a(t)-1 & 0 \\ 0 & 0 & 0 & 1-b(t) \end{pmatrix} $
(%i36) matrixp(lg); (%o36) true the contravariant metric tensor
(%i37) ug:invert_by_lu(lg);
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
(%i38) matrixp(ug); (%o38) true
(%i39) derivabbrev:true\$ the Christoffel tensor of the second kind
rational simplification (%i40) ratchristof:true\$
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