space_time_rotation.wxm

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An application document for Geometric Algebra using wxMaxima

Ref: The Survey, paragraph 4.1.2

Use of G(1,3) to rotate spacetime!

Initialization

(%i57) ext:["wxm"]$
        file_type_maxima:append(ext,file_type_maxima)$
        batchload("initialize_fns")$

the pseudoscalar and its inverse
the lowest useable dimension pseudoscalar should be {e1,e2} i.e. Plen = 2
e.g. for four dimensions edit Pseudos:{e1,e2,e3}$ to Pseudos:{e1,e2,e3,e4}$

(%i1)   Pseudos:{e1,e2,e3,e4}$
        Pvar:listofvars(Pseudos)$
        Plen:length(Pvar)$
        I:Pseudos$
        ni:(Plen-1)*Plen/2$
        Ii:(-1)^ni*I$
        kill(ni)$
        ldisplay(Pvar)$

  (%t8) Pvar=[e1,e2,e3,e4]

(%i9)   batchload("initialize_lsts")$

  (%t9) lstblds=[[{e1},{e2},{e3},{e4}],[{e1,e2},{e1,e3},{e1,e4},{e2,e3},{e2,e4},{
e3,e4}],[{e1,e2,e3},{e1,e2,e4},{e1,e3,e4},{e2,e3,e4}],[{e1,e2,e3,e4}]]
  (%t10) allblds=[{e1},{e2},{e3},{e4},{e1,e2},{e1,e3},{e1,e4},{e2,e3},{e2,e4},{e3,
e4},{e1,e2,e3},{e1,e2,e4},{e1,e3,e4},{e2,e3,e4},{e1,e2,e3,e4}]
  (%t11) invblds=[{e1},{e2},{e3},{e4},-{e1,e2},-{e1,e3},-{e1,e4},-{e2,e3},-{e2,e4}
,-{e3,e4},-{e1,e2,e3},-{e1,e2,e4},-{e1,e3,e4},-{e2,e3,e4},{e1,e2,e3,e4}]

end of Initialization

floating point print (display) precision

(%i12) fpprintprec:6$
        ratprint:false$
        ldisplay(fpprintprec,fpprec,ratprint)$

  (%t14) fpprintprec=6
  (%t15) fpprec=16
  (%t16) ratprint=false

The Survey, para.4.1.2

show the spacetime gammas required for the imitation of G(1,3), where, to avoid the use
of gamma_zero, we have used the fourth axis, e4, for the time axis and the intrinsic
maxima imaginary, %i, for the space axes

(%i17) g1:%i*{e1}$
        g2:%i*{e2}$
        g3:%i*{e3}$
        g4:{e4}$

the spacetime coordinate vector using the gammas

(%i21) x:x1*g1+x2*g2+x3*g3+t*g4;

(%o21) %i*{e3}*x3+%i*{e2}*x2+%i*{e1}*x1+{e4}*t

spacetime vector rotation for a simple velocity, vel*vhat (= vel*g1)

(%i22) vel:0.8$
        alpha:atanh(vel)$
        ahalf:alpha/2;

(%o24) 0.5493

form the rotation bivector

(%i25) vhat:g1$
        B:vhat&*g4*ahalf$
        ev(%,numer,expand);

(%o27) 0.5493*%i*{e1,e4}

form the left and right exponential multipliers

(%i28) mvexp(-B,13)$
        lexp:ev(%,numer,expand);

(%o29) 1.1547-0.5774*%i*{e1,e4}

(%i30) mvexp(+B,13)$
        rexp:ev(%,numer,expand);

(%o31) 0.5774*%i*{e1,e4}+1.1547

apply the rotation to the spacetime coordinate vector and find
the rotated spacetime vector

(%i32) xbar:lexp&*x&*rexp$
        ev(%,numer,expand)$
        collectterms(%,%i,e1,e4);

(%o34) %i*(1.0*{e3}*x3+1.0*{e2}*x2)+%i*{e1}*(1.66667*x1-1.33333*t)+{e4}*
(1.66667*t-1.33333*x1)