Before my First Statistics Lecture I Should Know ...

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(Andreadakis et al., 2021; 2022; Auguie, 2017; Larsson, 2018; Wickham, 2016; Wickham et al., 2023)

I. Basic Identities

i. Algebraic

$$(a+b)^2 = a^2 + 2ab + b^2 \tag{1}$$
$$(a-b)^2 = a^2 - 2ab + b^2 \tag{2}$$
$$a^2 - b^2 = (a-b)(a+b) \tag{3}$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \tag{4}$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 \tag{5}$$

ii. Trigonometric Identities

$$sin^2x + cos^2x = 1 \tag{6}$$
$$tanx = \frac{sinx}{cosx} \tag{7}$$
$$cotx = \frac{cosx}{sinx} \tag{8}$$
$$sin^2x = \frac{tan^2x}{1+tan^2x} \tag{9}$$
$$-1 \le sinx \le 1, -1 \le cosx \le 1 \tag{10}$$

iii. Logarithmic & Exponential

$$log(xy) = log(x) + log(y) \tag{11}$$
$$log\left(\frac{x}{y}\right) = log(x) - log(y) \tag{12}$$
$$logx^k = k \cdot logx \tag{13}$$

II. Limits

i. Identities

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$$\lim_{x\to 0^+} lnx = +\infty \tag{14}$$
$$\lim_{x\to -\infty} lnx = -\infty \tag{15}$$
$$\lim_{x\to \infty} a^x = -\infty \tag{16}$$
$$\lim_{x\to -\infty} a^x = -\infty \tag{17}$$
$$\lim_{x\to \infty} e^x = +\infty \tag{18}$$
$$\lim_{x\to -\infty} e^x = 0 \tag{19}$$
$$\lim_{x\to 0} \frac{sinx}{x} = 1 \tag{20}$$
$$\lim_{x\to 0} \frac{cosx - 1}{x} = 0 \tag{21}$$

ii. Indeterminate forms

There are limits that cannot be evaluated:

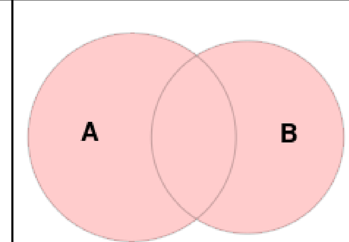
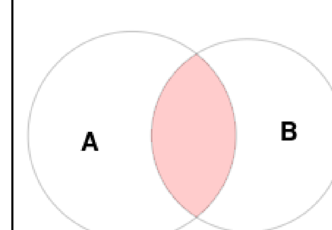
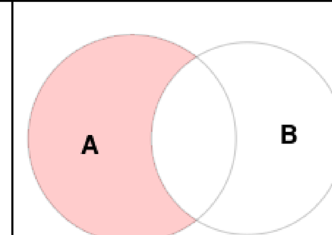
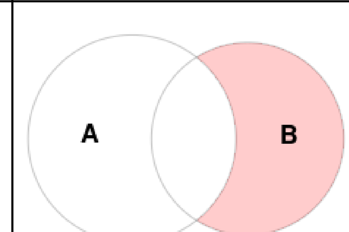
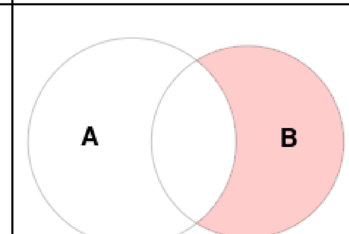
Form	Solving Approach
$\frac{0}{0}$	L' Hospital
$\frac{\infty}{\infty}$	L' Hospital
$0 \cdot \infty$	Not standard
$\infty - \infty$	Not standard
0^∞	Not standard
1^∞	$e^{ln(...)}$
∞^0	Not standard

L'Hospital's Rule can be applied when our limit has either the form of $\frac{\infty}{\infty}$ or $\frac{0}{0}$. In such a case DLH allows the calculation of the limit by using the derivative for both of nomimator and denominator :

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)} \tag{22}$$

III. Set Operations

i. Venn Diagrams

Set	Name	Venn Diagram
$A \cup B$	Union	
$A \cap B$	Intersection	
$A - B$	Difference	
$B - A$	Difference	
A'	Complement	

ii. Probability Identities

1. $Pr(A \cup B) = Pr)(A) + Pr(B) - Pr(A \cap B)$
2. $Pr(A \le B)$, if $A \subseteq B$
3. $Pr(A') = 1 - Pr(A)$

IV. Derivatives

i. Symbolism

There are 3 ways to depict derivatives:

- ***Lagrange Notation***: $f'(x), f''(x), ... f^{(n)}(x)$
- ***Leibniz Notation:*** $\frac{dy}{dx}, \frac{d^2y}{dx^2}, ... \frac{d^ny}{dx^n}$
- ***Newton Notation:*** $\dot{y}, \ddot{y}, ... f^{(n)}(x)$

ii. Common derivatives

$$(c)' = 0 \tag{23}$$
$$(x^n)' = n \cdot x^{n-1} \tag{24}$$
$$(sinx)' = cosx \tag{25}$$
$$(cosx)' = -sinx \tag{26}$$
$$(tanx)' = \frac{1}{cos^2x} \tag{27}$$
$$(cotx)' = -\frac{1}{sin^2x} \tag{28}$$
$$(lnx)' = \frac{1}{x} \tag{29}$$
$$(e^x)' = e^x \tag{30}$$

iii. Logarithmic Differentiation

V. Integration

i. By parts

$$\int f(x)g'(x)dx = f(x) \cdot g(x) - \int g(x)f'(x)dx \tag{31}$$

A general rule of thumb to choose $g(x)$ is ***LIATE*** approach. The options for $g(x)$ are ordered in descending order:

1. **L**ogarithmic function
2. **I**nverse Trigonometric function
3. **A**lgebraic function
4. **T**rigonometric function
5. **E**xponential function

In case our integral contains multiple functions of the same category the **LIATE** approach is not helpful. There are some tips to choose the best function as follows:
A. Set as dv the most complicated part
B. Set u as function whose derivative du will be simpler than u

ii. By substitution

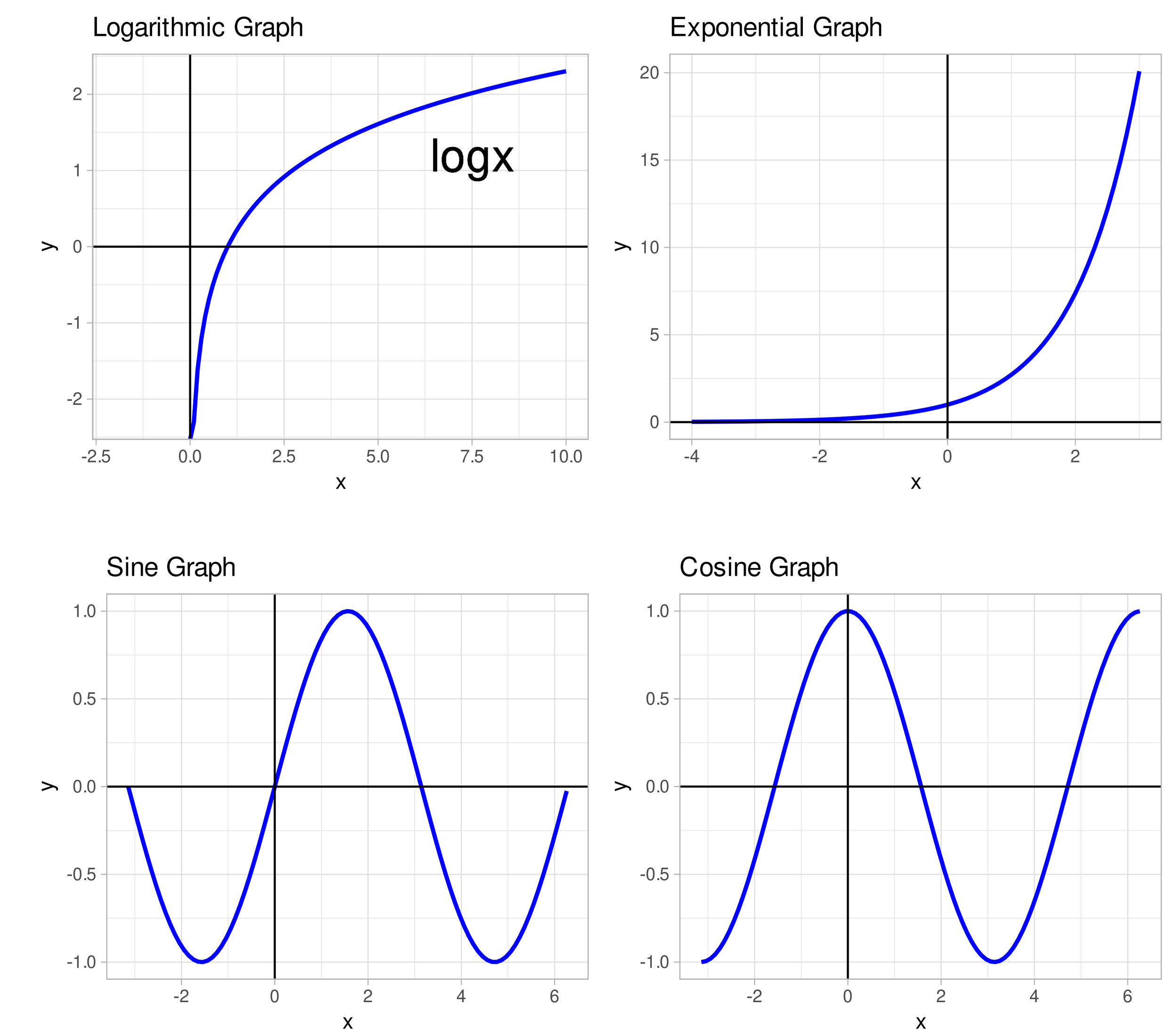
We consider:

$$\int f(g(x))\ g'(x)\ dx = \int f(u)du \tag{32}$$

$u = g(x)$ και $du = g'(x)dx$

VI. Appendix

i. Graphs



ii. Constants

- $\pi \approx 3.1415...$
- $e \approx 2.7182$

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