Table of Distributions

stesiam

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Discrete Distributions

Distribution	Parameters	$f(x)$	Value domain	F(x)	E(X)	Var(X)	$M_x(t)$	$P_x(t)$
Bernoulli Bernoulli(p)	$p \in [0,1]$	$\begin{cases} q, & \text{if k=0} \\ p, & \text{if k=1} \end{cases}$	x = $\{0,1\}$	$\begin{cases} 0, & \text{if k<0} \\ q, & \text{if 0<=k<1} \\ 1, & \text{if k>=1} \end{cases}$	p	pq	$p(e^t + q)$	$(pt + q)$
Binomial $B(n,p)$	$n \in \{0,1,...\}$ $p \in [0,1]$	$\binom{n}{k}p^x q^{n-x}$	x = 0,1,...,n	$\sum_{i=1}^{\lfloor x \rfloor} \binom{n}{i}p^i q^{n-1}$	np	npq	$p(e^t + q)^n$	$(pt + q)^n$
Poisson $Poisson(\lambda)$	$\lambda > 0$	$e^{-\lambda}\frac{\lambda^x}{x}$	x = 0,1,...	$\sum_{k=1}^{n} e^{-\lambda}\frac{\lambda^x}{x}$	λ	λ	$e^{\lambda(e^t - 1)}$	$e^{\lambda(t-1)}$
Geometric 0 $G_0(p)$ # success	$p \in [0,1]$	$q^x p$	x = 0,1,2,...	$1 - q^{x-1}$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}$	$\frac{p}{1-qt}$
Geometric 1 $G_1(p)$ # until first succ.	$p \in [0,1]$	$q^{x-1}p$	x = 1,2,...	$1 - q^x$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t},$ $t < -ln(q)$	$\frac{pt}{1-qt}$
Negative Binomial 0 $NB_0(r,p)$ X = # trials before r-successes	$r = 0,1,2,...$ $p \in [0,1]$	$\binom{x+r-1}{r-1}q^x p^r$	$x = 0,1,2,...$	Complex expression	$\frac{rq}{p}$	$\frac{rq}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^r$	$\left(\frac{p}{1-qt}\right)^r$
Negative Binomial 1 $NB_1(r,p)$ X = # trials before r-successes	$r = 1,2,...$ $p \in [0,1]$	$\binom{x-1}{r-1}q^{x-r}p^r$	$x = r, r+1, ...$	Complex expression	$\frac{r}{p}$	$\frac{rq}{p^2}$	$\left(\frac{pe^t}{1-qe^t}\right)^r$	$\left(\frac{pt}{1-qt}\right)^r$
Hypergeometric $h(v,a,b)$	a,b,n : pos. integers $n <= a + b$	$\frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$	x=0,1,2,...n	Complex expression	$\frac{a}{a+b}$	Complex expression	Complex expression	Complex expression
Discrete Uniform $U(a,b)$	a,b,n : pos. integers $b \geq a$ $n = b - a + 1$	$\frac{1}{n}$	x=a,a+1..,b-1,b	$\frac{\lfloor k \rfloor - 1 + a}{n}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$	$\frac{e^{at} - e^{(b+1)t}}{n(1-e^t)}$	$\frac{t^a - t^{b-1}}{n(1-t)}$

Continuous Distributions

Distribution	Parameters	$f(x)$	Value domain	F(x)	E(X)	Var(X)	$M_x(t)$
Uniform $U(a,b)$	$-\infty < a < b < +\infty$	$\frac{1}{b-a}$	$x \in [a,b]$	$\begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal $N(n,p)$	$n \in \{0,1,...\}$ $p \in [0,1]$	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$x \in [-\infty, \infty]$		μ	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Standard Normal $N(0,1)$	$\mu = 0$ $\sigma = 1$	$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	x = 0,1,...		0	1	$e^{\frac{t^2}{2}}$
Exponential $Exp(\lambda)$	$\lambda > 0$	$\lambda e^{-\lambda x}$	$x \in [0, \infty]$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}, t < \lambda$
Gamma $Gamma(k, \theta)$	$k > 0$ $\theta > 0$	$\frac{x^{k-1}e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}$	$x \in [0, \infty]$	$\frac{\gamma(k, \frac{x}{\theta})}{\Gamma(k)}$	$k\theta$	$k\theta^2$	$(1 - \theta t)^{-k}, t < \frac{1}{\theta}$
Beta $Beta(\alpha, \beta)$	$\alpha > 0$ $\beta > 0$	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(a,b)}$	$x \in [0,1]$	Complex expression	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	Complex expression
Weibull $Wei(\lambda, k)$	$\lambda \in (0, +\infty)$ $k \in (0, +\infty)$	$\begin{cases} \frac{k}{\lambda}(\frac{x}{\lambda})e^{-(\frac{x}{\lambda})^k}, & x \ge 0 \\ 0, & x < 0 \end{cases}$	$x \in [0, \infty]$	$\begin{cases} 1 - e^{-(\frac{x}{\lambda})}, & x \le 0 \\ 0, & x < 0 \end{cases}$	$\lambda\, \Gamma(1 + \frac{1}{k})$	Complex expression	Complex expression
Erlang $Erl(k, \lambda)$ alt. $beta = \frac{1}{\lambda}$	$k \in 1,2,3,...$ $\lambda \in (0, \infty)$	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$	$x \in [0, \infty]$	$\frac{\gamma(k, \lambda x)}{(k-1)!}$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$(1 - \frac{t}{\lambda})^{-k}$ $t < \lambda$
Pareto I $Pareto(\alpha, \beta)$	$a > 0$ $b > 0$	$\frac{\alpha\beta^\alpha}{(\beta+x)^{a+1}}$	$x \in [0, +\infty]$	$1 - \left(\frac{\beta}{x+\beta}\right)^\alpha$	$\frac{\beta}{\alpha-1},$ $\alpha > 1$	$\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$	–
Pareto II $Pareto$	$a > 0$ $b > 0$	$\frac{\alpha\beta^\alpha}{x^{a+1}}$	$x \in [0, +\infty]$	$1 - \left(\frac{\beta}{x}\right)^\alpha$	$\frac{\alpha\beta}{\alpha-1}$		–
Lognormal $logN(\mu, \sigma^2)$	$\mu \in (-\infty, +\infty)$ $\sigma > 0$	$\frac{e^{-\frac{(logx-\mu)^2}{2\sigma^2}}}{x\sigma\sqrt{2\pi}}$	$x \in (0, +\infty)$	Compl. expression	$e^{\mu+\frac{\sigma^2}{2}}$	$(e^{\sigma^2}-1)(e^{2\mu+\sigma^2})$	–