Summary of Distributions

List of common distributions and its characteristics

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Discrete Distributions

Distribution	Parameters	$f(x)$	Value domain	F(x)	E(X)	Var(X)	$M_x(t) = E(e^{tX})$	$P_x(t)$
Bernoulli Bernoulli(p)	$p \in [0, 1]$	$\begin{cases} q, & \text{if k=0} \\ p, & \text{if k=1} \end{cases}$	x = $\{0, 1\}$	$\begin{cases} 0, & \text{if k<0} \\ q, & \text{if 0<=k<1} \\ 1, & \text{if k>=1} \end{cases}$	p	pq	$p(e^t + q)$	$(pt + q)$
Binomial $B(n, p)$	$n \in$ {0,1,..,} $p \in$ [0,1]	$\binom{n}{k} p^x q^{n-x}$	x = 0,1,...,n	$\sum_{i=1}^{\lfloor x \rfloor} \binom{n}{i} p^i q^{n-1}$	np	npq	$p(e^t + q)^n$	$(pt + q)^n$
Poisson $Poisson(\lambda)$	$\lambda > 0$	$e^{-\lambda} \frac{\lambda^x}{x}$	x = 0,1,...	$\sum_{k=1}^{n} e^{-\lambda} \frac{\lambda^x}{x}$	λ	λ	$e^{\lambda(e^t - 1)}$	$e^{\lambda(t-1)}$
Geometric 0 $G_0(p)$ # success	$p \in [0, 1]$	$q^x p$	x = 0,1,2,...	$1 - q^{x-1}$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1-qe^t}$	$\frac{p}{1-qt}$
Geometric 1 $G_1(p)$ # until first succ.	$p \in [0, 1]$	$q^{x-1} p$	x = 1,2,...	$1 - q^x$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1-qe^t}$, $t < -ln(q)$	$\frac{pt}{1-qt}$
Negative Binomial 0 $NB_0(r, p)$	$r = 0, 1, 2, \ldots$ $p \in [0, 1]$	$\binom{x+r-1}{r-1} q^x p^r$	$x = 0, 1, 2, \ldots$	$\sum_{i=1}^{\lfloor k \rfloor} \binom{x+r-1}{r-1} q^x p^r$	$\frac{rq}{p}$	$\frac{rq}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^r$	$\left(\frac{p}{1-qt}\right)^r$
Negative Binomial 1 $NB_1(r, p)$	$r = 1, 2, \ldots$ $p \in [0, 1]$	$\binom{x-1}{r-1} q^{x-r} p^r$	$x = r, r+1, \ldots$	Complex expression	$\frac{r}{p}$	$\frac{rq}{p^2}$	$\left(\frac{pe^t}{1-qe^t}\right)^r$	$\left(\frac{pt}{1-qt}\right)^r$
Hypergeometric $h(v, a, b)$	a, b, n : pos. integers $n <= a + b$	$\frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$	x=0,1,2,...n	Complex expression	$\frac{a}{a+b}$	$\frac{abn(n-1)}{(a+b)^2(a+b-1)}$	Complex expression	Complex expression
Discrete Uniform $U(a, b)$	a, b, n : pos. integers $b \geq a$ $n = b - a + 1$	$\frac{1}{n}$	x=a,a+1..,b-1,b	$\frac{\lfloor k \rfloor - 1 + a}{n}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$	$\frac{e^{at} - e^{(b+1)t}}{n(1-e^t)}$	$\frac{t^a - t^{b-1}}{n(1-t)}$

Continuous Distributions

Distribution	**Parameters**	$f(x)$	**Value domain**	**F(x)**	**E(X)**	**Var(X)**	$M_x(t)$
Uniform $U(a,b)$	$-\infty < a < b < +\infty$	$\frac{1}{b-a}$	$x \in [a,b]$	$\begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal $N(n,p)$	$n \in$ {0,1,..,} $p \in$ [0,1]	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$x \in [-\infty, \infty]$		μ	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Standard Normal $N(0,1)$	$\mu = 0$ $\sigma = 1$	$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$	x = 0,1,...	$\Phi(x)$	0	1	$e^{\frac{t^2}{2}}$
Exponential $Exp(\lambda)$	$\lambda > 0$	$\lambda e^{-\lambda x}$	$x \in [0, \infty]$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}, t < \lambda$
Gamma $Gamma(k,\theta)$	$k > 0$ θ >0	$\frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}$	$x \in [0, \infty]$	$\frac{\gamma(k, \frac{x}{\theta})}{\Gamma(k)}$	$k\theta$	$k\theta^2$	$(1-\theta t)^{-k}$
Beta $Beta(\alpha,\beta)$	$\alpha > 0$ β >0	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(a,b)}$	$x \in [0,1]$	Complex expression	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	Complex expression
Weibull $Wei(c,\gamma)$	$c \in (0, +\infty)$ $\gamma \in (0, +\infty)$	$c\gamma x^{\gamma-1} e^{-cx^\gamma}$	$x \in [0, \infty]$	$1 - exp(-cx^\gamma)$	$c\,\Gamma(1+\frac{1}{\gamma})$	Complex expression	Complex expression
Erlang $Erl(k,\lambda)$ alt. $beta = \frac{1}{\lambda}$	$k \in 1,2,3,...$ $\lambda \in (0,\infty)$	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$	$x \in [0, \infty]$	$\frac{\gamma(k,\lambda x)}{(k-1)!}$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$(1-\frac{t}{\lambda})^{-k}$ $t < \lambda$
Pareto I $Pareto(\alpha,\beta)$	$a > 0$ $b > 0$	$\frac{\alpha\beta^\alpha}{(\beta+x)^{a+1}}$	$x \in [0, +\infty]$	$1 - \left(\frac{\beta}{x+\beta}\right)^\alpha$	$\frac{\beta}{\alpha-1}$, $\alpha > 1$	$\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$	–
Pareto II $Pareto$	$a > 0$ $b > 0$	$\frac{\alpha\beta^\alpha}{x^{a+1}}$	$x \in [0, +\infty]$	$1 - \left(\frac{\beta}{x}\right)^\alpha$	$\frac{\alpha\beta}{\alpha-1}$		–
Lognormal $logN(\mu,\sigma^2)$	$\mu \in (-\infty, +\infty)$ $\sigma > 0$	$\frac{e^{-\frac{(logx-\mu)^2}{2\sigma^2}}}{x\sigma\sqrt{2\pi}}$	$x \in (0, +\infty)$	$\Phi\left(\frac{lnx-\mu}{\sigma}\right)$	$e^{\mu+\frac{\sigma^2}{2}}$	$(e^{\sigma^2}-1)(e^{2\mu+\sigma^2})$	–
Burr $Burr(c,k)$	$c > 0$ $k > 0$	$\frac{ckx^{c-1}}{(1+x^c)^{k+1}}$	$x \in (0, +\infty)$	$1 - (1+x^c)^{-k}$	$\frac{\Gamma(1-\frac{1}{c}) \cdot \Gamma(1+\frac{1}{c})}{\Gamma(k)}$	–	–