

Table of Distributions

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Discrete Distributions

| Distribution | Parameters | $f(x)$ | Value domain | F(x) | E(X) | Var(X) | $M_x(t)$ | $P_x(t)$ |
|--|---|--|-----------------------|---|-----------------|-----------------|---|----------------------|
| Bernoulli $Bernoulli(p)$ | $p \in [0, 1]$ | $\begin{cases} q, & \text{if } k=0 \\ p, & \text{if } k=1 \end{cases}$ | $x = \{0, 1\}$ | $\begin{cases} 0, & \text{if } k < 0 \\ q, & \text{if } 0 \leq k < 1 \\ 1, & \text{if } k \geq 1 \end{cases}$ | p | pq | $p(e^t + q)$ | $(pt + q)$ |
| Binomial $B(n, p)$ | $n \in \{0, 1, \dots\}$ $p \in [0, 1]$ | $\binom{n}{k} p^k q^{n-k}$ | $x = 0, 1, \dots, n$ | | np | npq | $p(e^t + q)^n$ | $(pt + q)^n$ |
| Poisson $Poisson(\lambda)$ | $\lambda > 0$ | $e^{-\lambda} \frac{\lambda^x}{x!}$ | $x = 0, 1, \dots$ | – | λ | λ | $e^{\lambda(e^t - 1)}$ | $e^{\lambda(t - 1)}$ |
| Geometric 0 $G_0(p)$ | $p \in [0, 1]$ | $q^x p$ | $x = 0, 1, 2, \dots$ | $1 - q^{x+1}$ | $\frac{q}{p}$ | $\frac{q}{p^2}$ | $\frac{p}{1 - qe^t}$ | |
| Geometric 1 $G_1(p)$ | $p \in [0, 1]$ | $q^{x-1} p$ | $x = 1, 2, \dots$ | $1 - q^x$ | $\frac{1}{p}$ | $\frac{q}{p^2}$ | $\frac{pe^t}{1 - qe^t},$ $t < -\ln(q)$ | |
| Negative Binomial 0 $NB_0(r, p)$ | | | | | | | | |
| Negative Binomial 1 $NB_1(r, p)$ | | | | | | | | |
| Hypergeometric $h(v, a, b)$ | a, b, n : pos. integers $n \leq a + b$ | $\frac{\binom{a}{x} \binom{b}{n-x}}{\binom{a+b}{n}}$ | $x=0, 1, 2, \dots, n$ | | $\frac{a}{a+b}$ | | – | – |
| Discrete Uniform $U(a, b)$ | a, b, n : pos. integers $n \leq a + b$ | $\frac{\binom{a}{x} \binom{b}{n-x}}{\binom{a+b}{n}}$ | $x=0, 1, 2, \dots, n$ | | $\frac{a+b}{2}$ | | – | – |

Continuous Distributions

| Distribution | Parameters | $f(x)$ | Value domain | F(x) | E(X) | Var(X) | $M_x(t)$ |
|---|---|--|---------------------------|--|---|--|---|
| Uniform $U(a, b)$ | $-\infty < a < b < +\infty$ | $\frac{1}{b-a}$ | $x \in [a, b]$ | $\begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$ | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ | $\frac{e^{tb}-e^{ta}}{t(b-a)}$ |
| Normal $N(n, p)$ | $n \in \{0, 1, \dots\}$ $p \in [0, 1]$ | $\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ | $x \in [-\infty, \infty]$ | | μ | σ^2 | $e^{\mu t + \frac{\sigma^2 t^2}{2}}$ |
| Standard Normal $N(0, 1)$ | – | $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ | $x = 0, 1, \dots$ | | 0 | 1 | |
| Exponential $Exp(\lambda)$ | $\lambda > 0$ | $\lambda e^{-\lambda x}$ | $x \in [0, \infty]$ | $1 - e^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | |
| Gamma $Gamma(k, \theta)$ | $k > 0$ $\theta > 0$ | $\frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}$ | $x \in [0, \infty]$ | | $k\theta$ | $k\theta^2$ | $(1 - \theta t)^{-k}, t < \frac{1}{\theta}$ |
| Beta $Beta(\alpha, \beta)$ | $\alpha > 0$ $\beta > 0$ | $\frac{x^{\alpha-1} (1-x)^{\beta-1}}{B(\alpha, \beta)}$ | $x \in [0, 1]$ | | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ | |
| Weibull $Wei(\lambda, k)$ | $\lambda \in (0, +\infty)$ $k \in (0, +\infty)$ | $\begin{cases} \frac{k}{\lambda} (\frac{x}{\lambda}) e^{-(\frac{x}{\lambda})^k}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ | $x \in [0, \infty]$ | $\begin{cases} 1 - e^{-(\frac{x}{\lambda})^k}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ | $\lambda \Gamma(1 + \frac{1}{k})$ | | |
| Erlang $Erl(k, \lambda)$ alt. $beta = \frac{1}{\lambda}$ | $k \in 1, 2, 3, \dots$ $\lambda \in (0, \infty)$ | $\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$ | $x \in [0, \infty]$ | | $\frac{k}{\lambda}$ | $\frac{k}{\lambda^2}$ | $(1 - \frac{t}{\lambda})^{-k}$ $t < \lambda$ |
| Pareto I $Pareto(\alpha, \beta)$ | $a > 0$ $b > 0$ | $\frac{\alpha\beta^\alpha}{(\beta+x)^{\alpha+1}}$ | $x \in [0, +\infty]$ | | $\frac{\beta}{\alpha-1},$ $\alpha > 1$ | $\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$ | – |
| Pareto II $Pareto$ | $a > 0$ $b > 0$ | | $x \in [0, +\infty]$ | | | | – |
| Lognormal $logN(\mu, \sigma^2)$ | $\mu \in (-\infty, +\infty)$ $\sigma > 0$ | $\frac{e^{-\frac{1}{2}(\frac{\log x - \mu}{\sigma})^2}}{x\sigma\sqrt{2\pi}}$ | $x \in (0, +\infty)$ | Compl. expression | $e^{\mu + \frac{\sigma^2}{2}}$ | $(e^{\sigma^2} - 1)(e^{2\mu + \sigma^2})$ | – |