## Table of Distributions

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## **Discrete Distributions**

Distribution	Parameters	f(x)	Value domain	F(x)	E(X)	Var(X)	$M_x(t)$	$P_x(t)$
Bernoulli Bernoulli( $p$ )	$p \in [0,1]$	$\begin{cases} q, & \text{if k=0} \\ p, & \text{if k=1} \end{cases}$	x = {0, 1}	$\begin{cases} 0, & \text{if } k < 0 \\ q, & \text{if } 0 < = k < 1 \\ 1, & \text{if } k > = 1 \end{cases}$	p	pq	$p(e^t + q)$	(pt+q)
$\begin{array}{c} \textbf{Binomial} \\ B(n,p) \end{array}$	$n \in \{0,1,\}$ $p \in [0,1]$	$\binom{n}{k} p^x q^{n-x}$	x = 0,1,,n	$\sum_{i=1}^{\lfloor x\rfloor} \binom{n}{i} p^i q^{n-1}$	np	npq	$p(e^t + q)^n$	$(pt+q)^n$
$\begin{array}{c c} \textbf{Poisson} \\ Poisson(\lambda) \end{array}$	$\lambda > 0$	$e^{-\lambda} \frac{\lambda^x}{x}$	x = 0,1,	$\sum_{k=1}^{n} e^{-\lambda} \frac{\lambda^x}{x}$	λ	λ	$e^{\lambda(e^t-1)}$	$e^{\lambda(t-1)}$
$\begin{array}{c} \textbf{Geometric 0} \\ G_0(p) \\ \text{\# success} \end{array}$	$p \in [0,1]$	$q^x p$	x = 0,1,2,	$1 - q^{x-1}$	$\frac{q}{p}$	$\frac{q}{p^2}$	$rac{p}{1-qe^t}$	$\frac{p}{1-qt}$
$\begin{array}{c} \textbf{Geometric 1} \\ G_1(p) \\ \text{\# until first succ.} \end{array}$	$p \in [0,1]$	$q^{x-1}p$	x = 1,2,	$1-q^x$	$\frac{1}{p}$	$rac{q}{p^2}$	$\begin{array}{c} \frac{pe^t}{1-qe^t}, \\ t < -ln(q) \end{array}$	$\frac{pt}{1-qt}$
$\begin{tabular}{ll} \bf Negative \ Binomial \ 0 \\ NB_0(r,p) \ \# \ successes \end{tabular}$	$r = 0, 1, 2, \dots$ $p \in [0, 1]$				$\frac{rq}{p}$	$\frac{rq}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^r$	$\left(\frac{p}{1-qt}\right)^r$
	$r = 1, 2, \dots$ $p \in [0, 1]$							
$ \begin{array}{c} \textbf{Hypergeometric} \\ h(v,a,b) \end{array} $	$a, b, n$ : pos. integers $n \le a + b$	$\frac{\binom{a}{x}\binom{b}{n-x}}{\binom{a+b}{n}}$	x=0,1,2,n	Complex expression	$\frac{a}{a+b}$	Complex expression	Complex expression	Complex expression
$\begin{array}{c} \textbf{Discrete Uniform} \\ U(a,b) \end{array}$	$a,b,n$ : pos. integers $b \ge a$ $n=b-a+1$	$\frac{1}{n}$	x=a,a+1,b-1,b	$\frac{\lfloor k \rfloor - 1 + a}{n}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$	$\frac{e^{at} - e^{(b+1)t}}{n(1 - e^t)}$	$\frac{t^a-t^{b-1}}{n(1-t)}$

## **Continuous Distributions**

Distribution	Parameters	f(x)	Value domain	F(x)	E(X)	Var(X)	$M_x(t)$
$\begin{array}{c} \textbf{Uniform} \\ U(a,b) \end{array}$	$-\infty < a < b < +\infty$	$\frac{1}{b-a}$	$x \in [a, b]$	$\begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
$\begin{array}{c} \textbf{Normal} \\ N(n,p) \end{array}$	$n \in \{0,1,,\}$ $p \in [0,1]$	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$x \in [-\infty, \infty]$		$\mu$	$\sigma^2$	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
$\begin{array}{c} \textbf{Standard Normal} \\ N(0,1) \end{array}$	$\mu = 0$ $\sigma = 1$	$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	x = 0,1,		0	1	$e^{rac{t^2}{2}}$
	$\lambda > 0$	$\lambda e^{-\lambda x}$	$x \in [0, \infty]$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}, t < \lambda$
$\begin{array}{c} \textbf{Gamma} \\ Gamma(k,\theta) \end{array}$	$k > 0$ $\theta > 0$	$\frac{x^{k-1}e^{-\frac{x}{\theta}}}{\theta^k\Gamma(k)}$	$x \in [0, \infty]$	$rac{\gamma(k,rac{x}{ heta})}{\Gamma(k)}$	$k\theta$	$k\theta^2$	$\left  (1 - \theta t)^{-k}, t < \frac{1}{\theta} \right $
Beta $Beta(lpha,eta)$	$\begin{array}{c} \alpha > 0 \\ \beta > 0 \end{array}$	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(a,b)}$	$x \in [0, 1]$	Complex expression	$\frac{\alpha}{\alpha+eta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	Complex expression
$\begin{array}{c} \textbf{Weibull} \\ Wei(\lambda,k) \end{array}$	$\lambda \in (0, +\infty)$ $k \in (0, +\infty)$	$\begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right) e^{-\left(\frac{x}{\lambda}\right)^k}, & x \ge 0\\ 0, & x < 0 \end{cases}$	$x \in [0, \infty]$	$\begin{cases} 1 - e^{-\left(\frac{x}{\lambda}\right)}, & x \le 0\\ 0, & x < 0 \end{cases}$	$ \lambda \Gamma(1 + \frac{1}{k}) $		
Erlang $Erl(k,\lambda)$ alt. $beta=\frac{1}{\lambda}$	$k \in 1, 2, 3, \dots$ $\lambda \in (0, \infty)$	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$	$x \in [0, \infty]$	$\frac{\gamma(k,\lambda x)}{(k-1)!}$	$rac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$(1 - \frac{t}{\lambda})^{-k} \\ t < \lambda$
Pareto I $Pareto(lpha,eta)$	a > 0 $b > 0$	$rac{lphaeta^lpha}{(eta+x)^{a+1}}$	$x \in [0, +\infty]$		$\alpha > 1$	$\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$	_
Pareto II Pareto	a > 0 $b > 0$		$x \in [0, +\infty]$				_
$\frac{\textbf{Lognormal}}{logN(\mu,\sigma^2)}$	$\mu \in (-\infty, +\infty)$ $\sigma > 0$	$\frac{e^{-\frac{1}{2}(\frac{\log x - \mu}{\sigma})}}{x\sigma\sqrt{2\pi}}$	$x \in (0, +\infty)$	Compl. expression	$e^{\mu + \frac{\sigma^2}{2}}$	$(e^{\sigma^2}-1)(e^{2\mu+\sigma^2})$	_