

Table of Distributions

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Discrete Distributions

Distribution	Parameters	$f(x)$	Value domain	F(x)	E(X)	Var(X)	$M_x(t)$	$P_x(t)$
Bernoulli $Bernoulli(p)$	$p \in [0, 1]$	$\begin{cases} q, & \text{if } k=0 \\ p, & \text{if } k=1 \end{cases}$	$x = \{0, 1\}$	$\begin{cases} 0, & \text{if } k < 0 \\ q, & \text{if } 0 \leq k < 1 \\ 1, & \text{if } k \geq 1 \end{cases}$	p	pq	$p(e^t + q)$	$(pt + q)$
Binomial $B(n, p)$	$n \in \{0, 1, \dots\}$ $p \in [0, 1]$	$\binom{n}{k} p^k q^{n-k}$	$x = 0, 1, \dots, n$	$\sum_{i=1}^{\lfloor x \rfloor} \binom{n}{i} p^i q^{n-i}$	np	npq	$p(e^t + q)^n$	$(pt + q)^n$
Poisson $Poisson(\lambda)$	$\lambda > 0$	$e^{-\lambda} \frac{\lambda^x}{x!}$	$x = 0, 1, \dots$	$\sum_{k=1}^n e^{-\lambda} \frac{\lambda^k}{k!}$	λ	λ	$e^{\lambda(e^t - 1)}$	$e^{\lambda(t-1)}$
Geometric 0 $G_0(p)$ # success	$p \in [0, 1]$	$q^x p$	$x = 0, 1, 2, \dots$	$1 - q^{x+1}$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1 - qe^t}$	$\frac{p}{1 - qt}$
Geometric 1 $G_1(p)$ # until first succ.	$p \in [0, 1]$	$q^{x-1} p$	$x = 1, 2, \dots$	$1 - q^x$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1 - qe^t},$ $t < -\ln(q)$	$\frac{pt}{1 - qt}$
Negative Binomial 0 $NB_0(r, p)$ # successes	$r = 0, 1, 2, \dots$ $p \in [0, 1]$				$\frac{rq}{p}$	$\frac{rq}{p^2}$	$\left(\frac{p}{1 - qe^t}\right)^r$	$\left(\frac{p}{1 - qt}\right)^r$
Negative Binomial 1 $NB_1(r, p)$	$r = 1, 2, \dots$ $p \in [0, 1]$							
Hypergeometric $h(v, a, b)$	a, b, n : pos. integers $n \leq a + b$	$\frac{\binom{a}{x} \binom{b}{n-x}}{\binom{a+b}{n}}$	$x=0, 1, 2, \dots, n$	Complex expression	$\frac{a}{a+b}$	Complex expression	Complex expression	Complex expression
Discrete Uniform $U(a, b)$	a, b, n : pos. integers $b \geq a$ $n = b - a + 1$	$\frac{1}{n}$	$x=a, a+1, \dots, b-1, b$	$\frac{\lfloor x \rfloor - 1 + a}{n}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$	$\frac{e^{at} - e^{(b+1)t}}{n(1-e^t)}$	$\frac{t^a - t^{b-1}}{n(1-t)}$

Continuous Distributions

Distribution	Parameters	$f(x)$	Value domain	F(x)	E(X)	Var(X)	$M_x(t)$
Uniform $U(a, b)$	$-\infty < a < b < +\infty$	$\frac{1}{b-a}$	$x \in [a, b]$	$\begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal $N(n, p)$	$n \in \{0,1,\dots\}$ $p \in [0,1]$	$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$x \in [-\infty, \infty]$		μ	σ^2	$e^{\mu t + \frac{\sigma^2 t^2}{2}}$
Standard Normal $N(0, 1)$	$\mu = 0$ $\sigma = 1$	$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$	$x = 0,1,\dots$		0	1	$e^{\frac{t^2}{2}}$
Exponential $Exp(\lambda)$	$\lambda > 0$	$\lambda e^{-\lambda x}$	$x \in [0, \infty]$	$1 - e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}, t < \lambda$
Gamma $Gamma(k, \theta)$	$k > 0$ $\theta > 0$	$\frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}$	$x \in [0, \infty]$	$\frac{\gamma(k, \frac{x}{\theta})}{\Gamma(k)}$	$k\theta$	$k\theta^2$	$(1 - \theta t)^{-k}, t < \frac{1}{\theta}$
Beta $Beta(\alpha, \beta)$	$\alpha > 0$ $\beta > 0$	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$	$x \in [0, 1]$	Complex expression	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	Complex expression
Weibull $Wei(\lambda, k)$	$\lambda \in (0, +\infty)$ $k \in (0, +\infty)$	$\begin{cases} \frac{k}{\lambda} (\frac{x}{\lambda}) e^{-(\frac{x}{\lambda})^k}, & x \geq 0 \\ 0, & x < 0 \end{cases}$	$x \in [0, \infty]$	$\begin{cases} 1 - e^{-(\frac{x}{\lambda})^k}, & x \leq 0 \\ 0, & x < 0 \end{cases}$	$\lambda \Gamma(1 + \frac{1}{k})$		
Erlang $Erl(k, \lambda)$ alt. $beta = \frac{1}{\lambda}$	$k \in 1, 2, 3, \dots$ $\lambda \in (0, \infty)$	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$	$x \in [0, \infty]$	$\frac{\gamma(k, \lambda x)}{(k-1)!}$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$(1 - \frac{t}{\lambda})^{-k}$ $t < \lambda$
Pareto I $Pareto(\alpha, \beta)$	$a > 0$ $b > 0$	$\frac{\alpha\beta^\alpha}{(\beta+x)^{\alpha+1}}$	$x \in [0, +\infty]$		$\frac{\beta}{\alpha-1},$ $\alpha > 1$	$\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$	–
Pareto II $Pareto$	$a > 0$ $b > 0$		$x \in [0, +\infty]$				–
Lognormal $logN(\mu, \sigma^2)$	$\mu \in (-\infty, +\infty)$ $\sigma > 0$	$\frac{e^{-\frac{1}{2}(\frac{\log x - \mu}{\sigma})^2}}{x\sigma\sqrt{2\pi}}$	$x \in (0, +\infty)$	Compl. expression	$e^{\mu + \frac{\sigma^2}{2}}$	$(e^{\sigma^2} - 1)(e^{2\mu + \sigma^2})$	–