Chapter 1

Introduction

Chapter 2

BRDF

2.1 Radiometry Introduction

2.1.1 Important Quantities

Symbol	Quantity	Unit
Q	Radiant Energy	j
Φ	Radiant flux	W
I	Radiant intensity	Wsr^{-1}
E	Irradiance (incident)	Wm^{-2}
L	Radiance	$Wm^{-2}sr^{-1}$

Table 2.1: Important Radiometry Quantities

2.2 BRDF

The *Bidirectional Reflectance Distribution Function*, BRDF, is the mathematic tool describing the reflection of light encounters an surface. ω_i is the incident lighting direction, ω_o is the direction in which the reflected light leaving from the surface, and n is the normal vector at the location p on the surface. Given the incident radiance $L_i(p, \omega_i)$, we can find the outgoing radiance to the viewer, $L_o(p, \omega_o)$.

The BRDF, f_r, defines the relationship between differential reflected radiance and differential irradiance:

$$f_r = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i)(\omega_i \cdot n)d\omega i} \tag{2.1}$$

To compute the outgoing radiance given a surface normal, incident light direction and radiance and a BRDF, we have:

$$L_o(p,\omega_o) = \int_{\Omega} f_r(p,\omega_i,\omega_o)L_i(p,\omega_i)(\omega_i \cdot n)d\omega_i \tag{2.2}$$

2.3 Microfacet Reflection Model

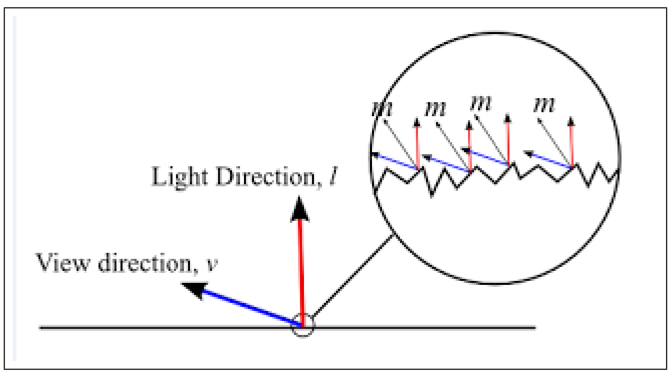

Figure 2.1: a surface is composed of many micro-facets and each micro-facet will only reflect light in a single direction according to their normal(m).

So, in the above diagram, for light coming from direction l to be reflected to viewing direction v, the micro-facet normal m must be equals to the half vector between l and v.

A microfacet BRDF has the following form:

$$f_{microfacet}(l,v) = \frac{F(l,h)G(l,v,h)D(h)}{4(n \cdot l)(n \cdot v)} \tag{2.3}$$

Where

- F is fesnel relectace term.
- G is geometry term of shadowing masking between microfacet.

- $D(h)$ is normal distribution term describing how microfacet normals is distributed around direction half-vector h
- l is the light direction.
- v is the view direction.
- n is the surface normal.
- h is the half vector between l and v

Two interpretations of BRDF: (1) Given a incident direction of a ray of light, the BRDF gives the relative distribution of reflected and scattered light over all outgoing directions. (2) Given a view direction, the BRDF gives the relative contribution of light from each incoming direction to the outgoing light. Both interpretations are illustrated in figure 2.2.

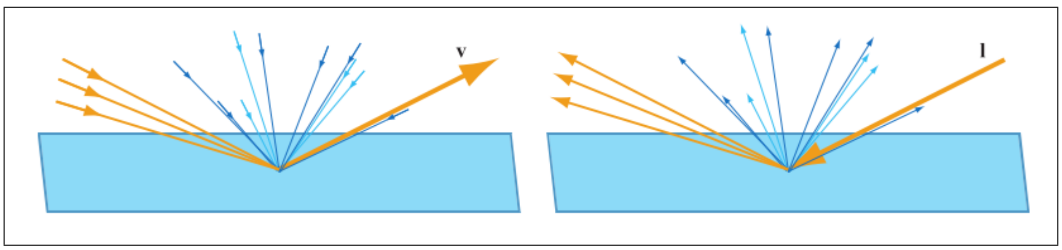

Figure 2.2: Left side is (2), right side is (1).

2.3.1 Fresnel Term

A common used *Schlick* approximation to the Fresnel equaiton is:

$$F(f_0) = f_0 + (1 - f_0)(1 - l \cdot h)^5 \tag{2.4}$$

Where

- f_0 is fresnel relectance term.
- l is the incident light direction.
- v is the view direction.

- h is the half vector between l and v.

and f_0 can be calculated by:

$$f_0 = (\frac{1-n}{1+n})^2 \tag{2.5}$$

2.3.2 Distribution Term

Distribution term is used to describe how the microfacet normal distributed around a given direction (roughness). In the demo, I used two distribution function: Blinn-Phong and Beckmann distribution function.

$$D_{BlinnPhong}(h) = \frac{\alpha+2}{2\pi}(n \cdot h)^{\alpha} \tag{2.6}$$

2.3.3 Geometry Term

Geometry term is used for describing how much the microfacet is blocked by other microfacet. In games we usually use the implicit form:

$$G_{implicit}(n, h, v, l) = (n \cdot l)(n \cdot v) \tag{2.7}$$

2.3.4 Rendering Equation in Games Rendering

Put all the terms above together, we have the microfacet specular BRDF,

$$L_o(V) = \frac{\alpha+2}{8}(n \cdot h)^{\alpha} F(f_0, l, h) \otimes c_{light}(n \cdot l) \tag{2.8}$$

where f_0 is fresnel term that can be replaced with specular color c_{spec}.

Add diffuse term, we have the final rendering equation:

$$L_o(V) = (c_{diff} + \frac{\alpha+2}{8}(n \cdot h)^{\alpha} F(c_{spec}, l, h)) \otimes c_{light}(n \cdot l) \tag{2.9}$$

2.3.5 Energy between diffuse and specular reflection