Homework/Programming Assignment #2 **Homework/midterm Due: 04/06/2023- 5:00 PM**

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I/We have followed the rules in completing this Assignment.

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Question	Points	Total
HA 1	25	
HA 2	25	
HA 3	25	
HA 4	25	
PA	100	
PA. k (Bonus)	15	
PA. m (Bonus)	30	
Presentation* (Bonus)	20	

Instruction:

1. Remember that this is a graded assignment. It is the equivalent of a **midterm take-home exam**.
2. *** You should present the results of the PA in the class** and receive extra bonus depending on the quality of your presentation!
3. **For PA questions**, you need to write a report showing how you derived your equations, describes your approach, test functions, and discusses the results. You should show your test results for each function.

3. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
4. It is open book, notes, and web. But you should cite any references you consult.
5. Unless I say otherwise in class, it is due before the start of class on the due date mentioned in the Assignment.
6. **Sign and append** this score sheet as the first sheet of your assignment.
7. Remember to submit your assignment in Canvas.

THA Homework Assignment #2

ME 397 ASBR, Sp 23, Dr. Farshid Alambeigi

Jared Rosenbaum and Steven Swanbeck

1) **Exercise 4.8**: The spatial RRRRPR open chain of Figure **4.14** is shown in its zero position, with fixed and end-effector frames chosen as indicated. Determine the end-effector zero position configuration M, the screw axes S_i in $\{0\}$, and the screw axes B_i in $\{b\}$.

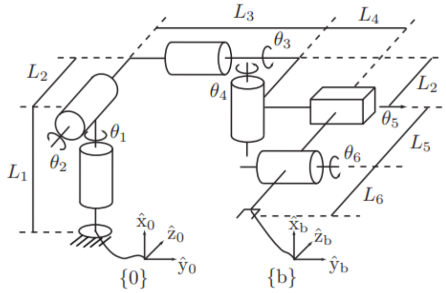

Figure 4.14: A spatial RRRRPR open chain.

$$v = \; - w \times q$$

M = [1, 0, 0, L_1;
0, 1, 0, L_3+L_4;
0, 0, 1, $-(L_5+L_6)$;
0, 0, 0, 1]

Fixed Frame:

i	w_i	v_i
1	(1, 0, 0)	(0, 0, 0)
2	(0, 0, -1)	(0, L_1, 0)
3	(0, 1, 0)	($-L_2$, 0, L_1)
4	(1, 0, 0)	(0, 0, $-L_3$)
5	(0, 0, 0)	(0, 1, 0)
6	(0, 1, 0)	(L_5, 0, L_1)

Body Frame: (assuming $L_4 = \theta_5$)

i	w_i	v_i
1	(1, 0, 0)	(0, L_5+L_6, L_4+L_3)
2	(0, 0, -1)	(L_4+L_3, 0, 0)
3	(0, 1, 0)	(-L_2-L_5-L_6. 0, 0)
4	(1, 0, 0)	(0, L_5+L_6, L_4)
5	(0, 0, 0)	(0, 1, 0)
6	(0, 1, 0)	(-L_6, 0 0)

2) **Exercise 4.11**: The spatial RPRRR open chain of Figure **4.17** is shown in its zero position. Determine the end-effector zero position configuration M, the screw axes S_i in $\{0\}$, and the screw axes B_i in $\{b\}$.

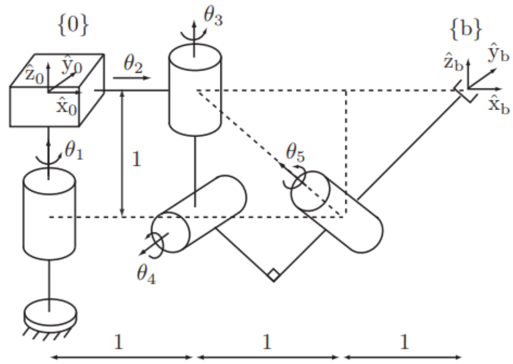

Figure 4.17: An RPRRR spatial open chain.

$$v = -w \times q$$

M = [1, 0, 0, 3;
0, 1, 0, 0;
0, 0, 1, 0;
0, 0, 0, 1]

Fixed Frame:

i	w_i	v_i
1	(0, 0, 1)	(0, 0, 0)
2	(0, 0, 0)	(1, 0, 0)
3	(0, 0, 1)	(0, -1, 0)
4	(0, -1, 0)	(-1, 0, -1)
5	$(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$	$(0, -\frac{\sqrt{2}}{2}, 0)$

Body Frame:

i	w_i	v_i
1	(0, 0, 1)	(0, 3, 0)
2	(0, 0, 0)	(1, 0, 0)
3	(0, 0, 1)	(0, 2, 0)
4	(0, -1, 0)	(-1, 0, 2)
5	$(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$	$(0, \sqrt{2}, 0)$

3) Exercise 5.12: The RRRP chain of Figure **5.22** is shown in its zero position. Let p be the coordinates of the origin of {b} expressed in {s}.

(a) Determine the body Jacobian $J_b(\theta)$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$.

(b) Find $\dot{p}$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$ and $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = \dot{\theta}_4 = 1$.

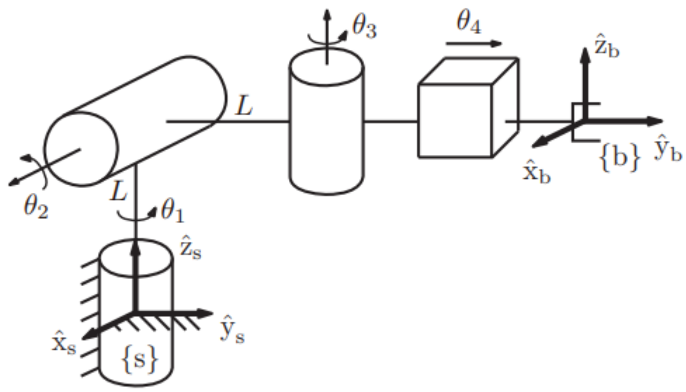

Figure 5.22: An RRRP spatial open chain.

To solve this problem, the screw axes were first defined analogous to the first two problems of this assignment. These were then implemented first into a longform script used to calculate the symbolic form of the Jacobian, then later confirmed using the symbolic options within the functions we developed for the programming portion of this assignment.

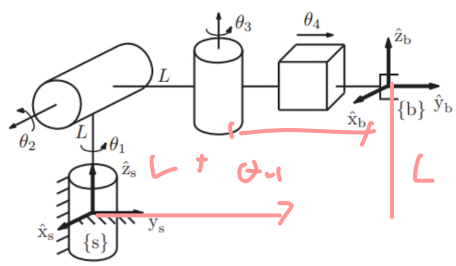

Figure 5.22: An RRRP spatial open chain.

i	w_i	q_i	v_i
1	0 0 1	$0, -L-\theta_4, -L$	$-L+\theta_4$ 0 0
2	1 0 0	$0, -L-\theta_4, 0$	0 0 $L+\theta_4$
3	0 0 1	$0, -\theta_4, 0$	$-\theta_4$ 0 0
4	0 0 0	—	0 1 0

a) The body Jacobian here was calculated using the shown script and later verified using the symbolic version of our *body_jacobian.m* function with these screw axes using the formula:

$$Ad_{T_{i-1}}(B_i), where\ T_{i-1} = e^{-[B_n]\theta_n} \ldots e^{-[B_{i+1}]\theta_{i+1}}$$

Script:

```
syms L t1 t2 t3 t4
L = sym('L', 'real');
t1 = sym('t1', 'real');
t2 = sym('t2', 'real');
t3 = sym('t3', 'real');
t4 = sym('t4', 'real');
Screws = [0 0 1 -L 0 0; 1 0 0 0 0 L; 0 0 1 0 0 0; 0 0 0 0 1 0];
Thetas = [0; 0; pi/2; L];
T1 = S2T(-transpose(Screws(1,:)),Thetas(1));
T2 = S2T(-transpose(Screws(2,:)),Thetas(2));
T3 = S2T(-transpose(Screws(3,:)),Thetas(3));
T4 = S2T(-transpose(Screws(4,:)),Thetas(4));
Adj3 = Adjoint_Matrix(T4);
Adj2 = Adjoint_Matrix(T4*T3);
Adj1 = Adjoint_Matrix(T4*T3*T2);
J4 = transpose(Screws(4,:));
J3 = Adj3*transpose(Screws(3,:));
J2 = Adj2*transpose(Screws(2,:));
J1 = Adj1*transpose(Screws(1,:));
answer = simplify([J1 J2 J3 J4])
qdot = answer*[1;1;1;1]
```

The body jacobian at this configuration is

```
J_b =
[ 0,  0,  0,     0]
[ 0, -1,  0,     0]
[ 1,  0,  1,     0]
[-2L,  0, -2L, 0]
[ -3L,  0,  0,  1]
[ 0,  L,  0,     0]
```

Where the last column is indeed the screw axis of the first joint, as expected for the space Jacobian.

b) Using $\dot{q} = J\dot{\theta}$, $J \in \mathbb{R}^{m\times n}$ (as shown in the above script)

```
Pdot =
   0
  -1
   2
 -4*L
 1 - 3L
   L
```

4) Exercise 5.13: For the 6R spatial open chain of Figure **5.23**,

(a) Determine its space Jacobian $J_s(\theta)$.

(b) Find its kinematic singularities. Explain each singularity in terms of the alignment of the joint screws and of the directions in which the end-effector loses one or more degrees of freedom of motion.

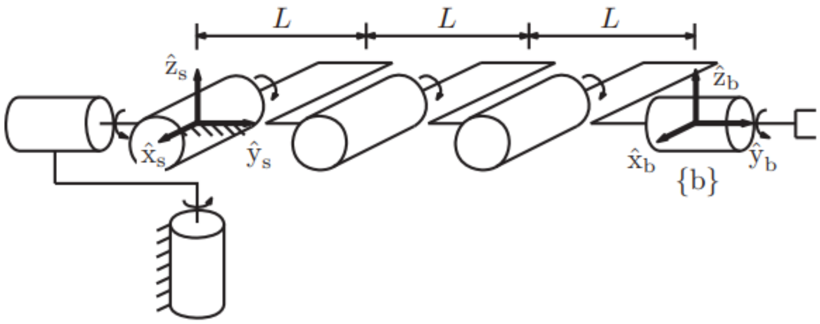

Figure 5.23: Singularities of a 6R open chain.

To solve this problem, the screw axes were first defined analogous to the first two problems of this assignment. These were then implemented first into a longform script used to calculate the symbolic form of the Jacobian, then later confirmed using the symbolic options within the functions we developed for the programming portion of this assignment.

(a) Determine its space Jacobian $J_s(\theta)$.

(b) Find its kinematic singularities. Explain each singularity in terms of the alignment of the joint screws and of the directions in which the end-effector loses one or more degrees of freedom of motion.

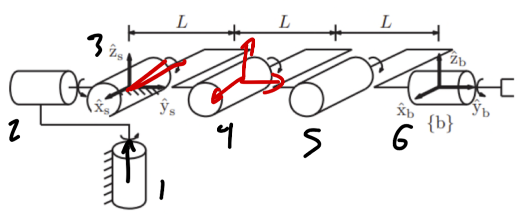

Figure 5.23: Singularities of a 6R open chain.

i	ω_i	v_i
1	0 0 1	0 0 0
2	0 1 0	0 0 -
3	-1 0 0	0 0 0
4	-1 0 0	0 0 L
5	-1 0 0	0 0 2L
6	0 1 0	0 0 0

{s}

a) The space Jacobian here was calculated using the shown script and later verified using the symbolic version of our *space_jacobian.m* function with these screw axes using the formula

$$Ad_{T_{i-1}}(S_i), where\ T_{i-1} = e^{[S_1]\theta_1} \dots e^{[S_{i-1}]\theta_{i-1}}$$

Script:

```
syms L t1 t2 t3 t4 t5 t6
L = sym('L', 'real');
t1 = sym('t1', 'real');
t2 = sym('t2', 'real');
t3 = sym('t3', 'real');
t4 = sym('t4', 'real');
t5 = sym('t5', 'real');
t6 = sym('t6', 'real');
Screws = [0 0 1 0 0 0; 0 1 0 0 0 0; -1 0 0 0 0 0; -1 0 0 0 0 L; -1 0 0 0
0 2*L; 0 1 0 0 0 0];
Thetas = [t1; t2; t3; t4; t5; t6];
T1 = S2T(transpose(Screws(1,:)),Thetas(1));
T2 = S2T(transpose(Screws(2,:)),Thetas(2));
T3 = S2T(transpose(Screws(3,:)),Thetas(3));
T4 = S2T(transpose(Screws(4,:)),Thetas(4));
T5 = S2T(transpose(Screws(5,:)),Thetas(5));
T6 = S2T(transpose(Screws(6,:)),Thetas(6));
Adj2 = Adjoint_Matrix(T1);
Adj3 = Adjoint_Matrix(T1*T2);
Adj4 = Adjoint_Matrix(T1*T2*T3);
Adj5 = Adjoint_Matrix(T1*T2*T3*T4);
Adj6 = Adjoint_Matrix(T1*T2*T3*T4*T5);
J1 = transpose(Screws(1,:));
J2 = Adj2*transpose(Screws(2,:));
J3 = Adj3*transpose(Screws(3,:));
J4 = Adj4*transpose(Screws(4,:));
J5 = Adj5*transpose(Screws(5,:));
J6 = Adj6*transpose(Screws(6,:));
answer = simplify([J1 J2 J3 J4 J5 J6])
detans = simplify((det(answer)))
```

This produces the symbolic Jacobian:

```
J_s =
[0, -sin(t1), -cos(t1)*cos(t2),                              -cos(t1)*cos(t2),
-cos(t1)*cos(t2), sin(t5)*(cos(t4)*(sin(t1)*sin(t3) - cos(t1)*cos(t3)*sin(t2)) +
sin(t4)*(cos(t3)*sin(t1) + cos(t1)*sin(t2)*sin(t3))) - cos(t5)*(cos(t4)*(cos(t3)*sin(t1) +
cos(t1)*sin(t2)*sin(t3)) - sin(t4)*(sin(t1)*sin(t3) - cos(t1)*cos(t3)*sin(t2)))]
[0,  cos(t1), -cos(t2)*sin(t1),                              -cos(t2)*sin(t1),
-cos(t2)*sin(t1), cos(t5)*(cos(t4)*(cos(t1)*cos(t3) - sin(t1)*sin(t2)*sin(t3)) -
sin(t4)*(cos(t1)*sin(t3) + cos(t3)*sin(t1)*sin(t2))) - sin(t5)*(cos(t4)*(cos(t1)*sin(t3) +
cos(t3)*sin(t1)*sin(t2)) + sin(t4)*(cos(t1)*cos(t3) - sin(t1)*sin(t2)*sin(t3)))]
[1,        0,          sin(t2),                                     sin(t2),
sin(t2),
-sin(t3 + t4 + t5)*cos(t2)]
[0,        0,                0, L*cos(t1)*cos(t3)*sin(t2) - L*sin(t1)*sin(t3), L*cos(t1)*cos(t3)*sin(t2) -
L*sin(t1)*sin(t3) - L*cos(t3)*sin(t1)*sin(t4) - L*cos(t4)*sin(t1)*sin(t3) +
L*cos(t1)*cos(t3)*cos(t4)*sin(t2) - L*cos(t1)*sin(t2)*sin(t3)*sin(t4),
-L*cos(t1)*cos(t2)*(sin(t4 + t5) + sin(t5))]
[0,        0,                0, L*(cos(t1)*sin(t3) + cos(t3)*sin(t1)*sin(t2)), L*cos(t1)*sin(t3) +
L*cos(t1)*cos(t3)*sin(t4) + L*cos(t1)*cos(t4)*sin(t3) + L*cos(t3)*sin(t1)*sin(t2) +

L*cos(t3)*cos(t4)*sin(t1)*sin(t2) - L*sin(t1)*sin(t2)*sin(t3)*sin(t4),
-L*cos(t2)*sin(t1)*(sin(t4 + t5) + sin(t5))]
[0,        0,                0,                         L*cos(t2)*cos(t3),
L*cos(t2)*(cos(t3 + t4) + cos(t3)),
L*sin(t2)*(sin(t4 + t5) + sin(t5))]
```

Where the first column is indeed the screw axis of the first joint, as expected for the space Jacobian.

b) The robot is at singularity when the determinant of the Jacobian is equal to zero (works because the Jacobian is square).

As calculated using the above script, the determinant of the Jacobian is:

$- L^3 cos(\theta_2) * (cos(\theta_4) + 1) * (cos(\theta_4 + \theta_5) - cos(\theta_5))$

Which is equal to zero when:

$cos(\theta_2) = 0$ -> Joints 1 and 3 are collinear

$cos(\theta_4) + 1 = 0$ -> Joints 3 4 and 5 are coplanar and parallel (the joint axes are coplanar)

$cos(\theta_4 + \theta_5) = cos(\theta_5)$ -> Joints 2 and 6 are collinear

By definition:

This robot should be at singularity when joints 2 and 6 are collinear, when joints 1 and 3 are collinear

This robot should be at singularity when joints 3, 4, and 5 are coplanar and parallel