Relational Learning

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CS 760: Machine Learning

Spring 2019

Goals for the lecture

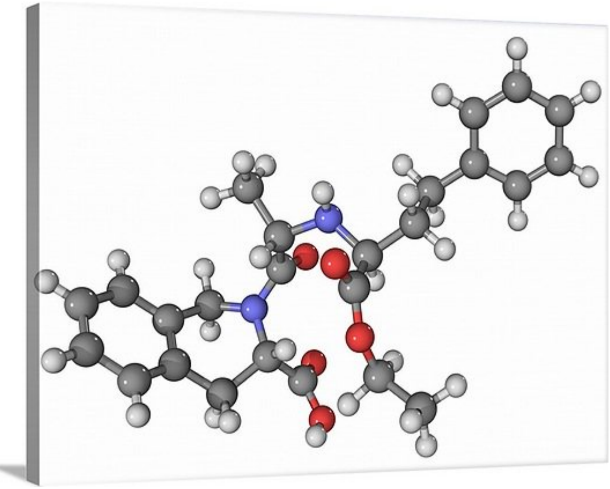
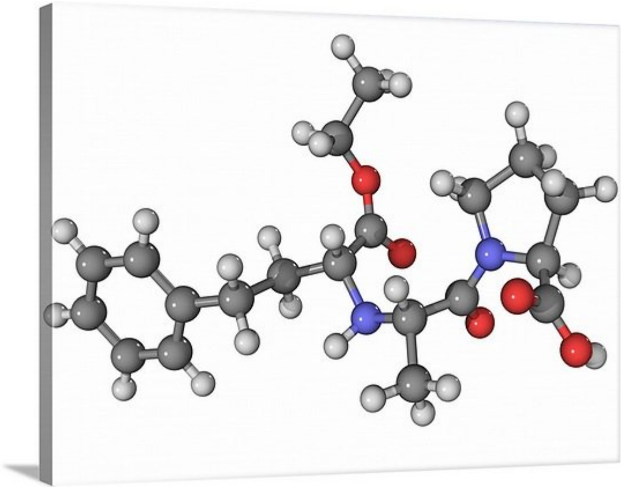
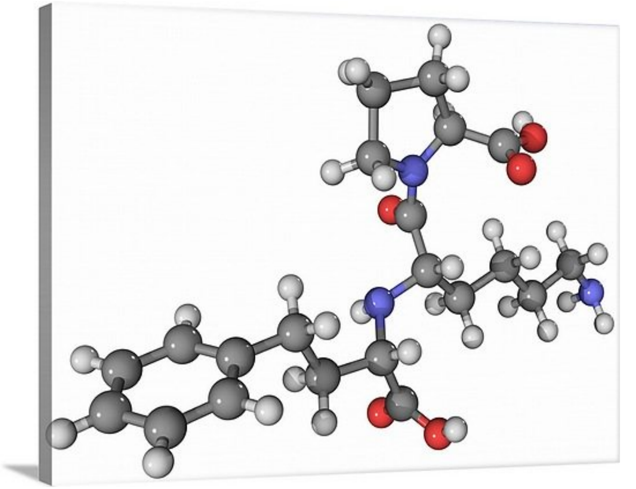
you should understand the following concepts

- relational learning
- the FOIL algorithm
- Markov logic networks
- plate models

Relational learning example

consider the task of learning a *pharmacophore*: the substructure of a molecule that interacts with a target of interest

- instances for this task consist of interacting (+) and non-interacting molecules (-)

to represent each instance, we'd like to describe

- the (variable # of) atoms in the molecule
- the possible conformations of the molecule
- the bonds among atoms
- distances among atoms
- etc.

Relational learning example

[Finn et al., *Machine Learning* 1998]

a multi-relational representation for molecules

Molecule	Target_1	...	Target_n
mol1	inactive		i nactive
mol2	active		inactive
•			
•			
•			

Molecular Bioactivity

Molecule	Bond_ID	Atom_1_ID	Atom_2_ID	Bond_Type
mol1	bond1	a1	a2	aromatic
•				
•				
•				

Bonds

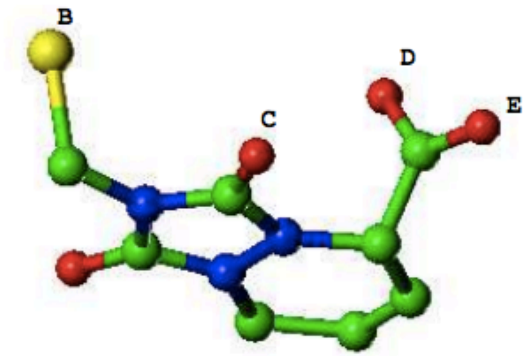
Molecule	Conformer	Atom_ID	Atom_Type	X_Coordinate	Y_Coordinate	Z_Coordinate
mol1	conf1	a1	carbon	2.58	−1.23	0.69
•						
•						
•						

3D Atom Locations

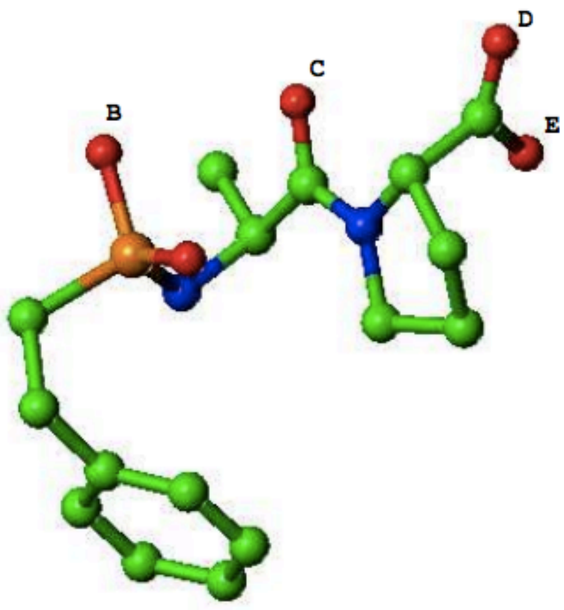
Relational learning example

[Finn et al., *Machine Learning* 1998]

a learned relational rule characterizing ACE inhibitors




```
Molecule A is an ACE inhibitor if
for some conformer Conf of A:
   molecule A contains a zinc binding site B;
   molecule A contains a hydrogen acceptor C;
   the distance between B and C in Conf is 7.9 +/- .75;
   molecule A contains a hydrogen acceptor D;
   the distance between B and D in Conf is 8.5 +/- .75;
   the distance between C and D in Conf is 2.1 +/- .75;
   molecule A contains a hydrogen acceptor E;
   the distance between B and E in Conf is 4.9 +/- .75;
   the distance between C and E in Conf is 3.1 +/- .75;
   the distance between D and E in Conf is 3.8 +/- .75.
```



Relational representation

ACE_inhibitor(A) ← has_zinc_binding_site(A, B) $\wedge$
has_hydrogren_acceptor(A, C) $\wedge$
distance(B, C, 7.9, 0.75) $\wedge$
has_hydrogen_acceptor(A, D) $\wedge$
distance(B, D, 8.5, 0.75) $\wedge$
distance(C, D, 8.5, 0.75) $\wedge$
has_hydrogen_acceptor(A, E) $\wedge$
distance(B, E, 4.9, 0.75) $\wedge$
distance(C, E, 3.1, 0.75) $\wedge$
distance(D, E, 3.8, 0.75)

To learn an equivalent rule with a feature-vector learner, what features would we need to represent?

has_zinc_binding_site
has_hydrogen_acceptor
zinc_binding_site_and hydrogen_acceptor_distance
hydrogen_acceptor_hydrogen_acceptor_distance
…

can easily encode distance between a pair of atoms; but this pharmacophore has 4 important atoms with 6 relevant distances among them

Relational learning example

[Page et al., *AAAI* 2012]

- Data from electronic health records (EHRs) is being used to learn models for risk assessment, adverse event detection, etc.
- A patient's record is described by multiple tables in a relational DB

demographics

PatientID	Gender	Birthdate
P1	M	3/22/63

diagnoses

PatientID	Date	Physician	Symptoms	Diagnosis
P1	1/1/01	Smith	palpitations	hypoglycemic
P1	2/1/03	Jones	fever, aches	influenza

labs

PatientID	Date	Lab Test	Result
P1	1/1/01	blood glucose	42
P1	1/9/01	blood glucose	45

genetics

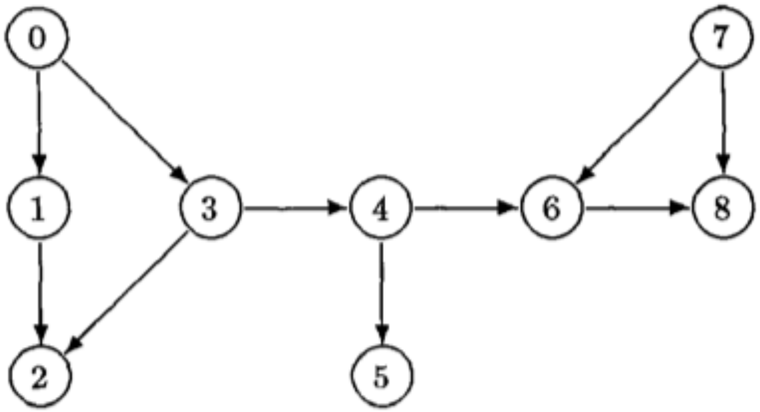
PatientID	SNP1	SNP2	…	SNP500K
P1	AA	AB		BB
P2	AB	BB		AA

drugs

PatientID	Date Prescribed	Date Filled	Physician	Medication	Dose	Duration
P1	5/17/98	5/18/98	Jones	prilosec	10mg	3 months

Relational learning example

- suppose we want to learn the general concept of can-reach in a graph, given a set of training instances describing a particular graph



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$\ominus$: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩ ⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩ ⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩ ⟨5,1⟩ ⟨5,2⟩ ⟨5,3⟩ ⟨5,4⟩ ⟨5,5⟩ ⟨5,6⟩ ⟨5,7⟩ ⟨5,8⟩ ⟨6,0⟩ ⟨6,1⟩ ⟨6,2⟩ ⟨6,3⟩ ⟨6,4⟩ ⟨6,5⟩ ⟨6,6⟩ ⟨6,7⟩ ⟨7,0⟩ ⟨7,1⟩ ⟨7,2⟩ ⟨7,3⟩ ⟨7,4⟩ ⟨7,5⟩ ⟨7,7⟩ ⟨8,0⟩ ⟨8,1⟩ ⟨8,2⟩ ⟨8,3⟩ ⟨8,4⟩ ⟨8,5⟩ ⟨8,6⟩ ⟨8,7⟩ ⟨8,8⟩

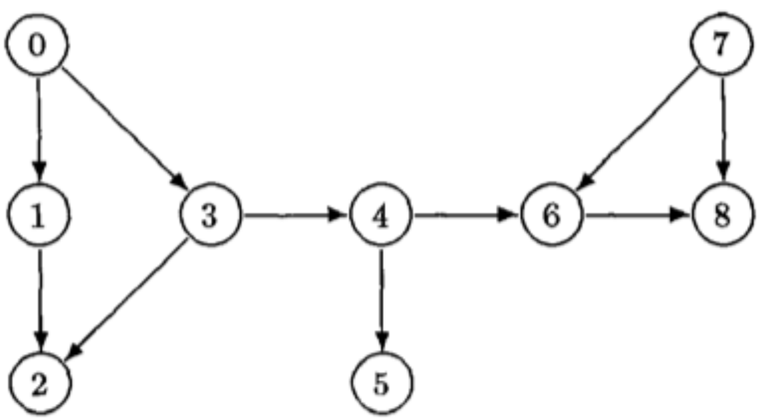
- how would you represent this task to a learner?

Relational learning example

- a relational representation, such as first-order logic, can capture this concept succinctly and in a general way

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_2)$$

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_3) \wedge \text{can-reach}(X_3, X_2)$$



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⟨8,7⟩ ⟨8,8⟩

The FOIL algorithm for relational learning

[Quinlan, *Machine Learning* 1990]

given:

- tuples (instances) of a target relation
- *extensionally* represented background relations

do:

- learn a set of rules that (mostly) cover the positive tuples of the target relation, but not the negative tuples

Input to FOIL

- instances of target relation

$\oplus$: $\langle 0,1\rangle$ $\langle 0,2\rangle$ $\langle 0,3\rangle$ $\langle 0,4\rangle$ $\langle 0,5\rangle$ $\langle 0,6\rangle$ $\langle 0,8\rangle$ $\langle 1,2\rangle$ $\langle 3,2\rangle$ $\langle 3,4\rangle$
$\langle 3,5\rangle$ $\langle 3,6\rangle$ $\langle 3,8\rangle$ $\langle 4,5\rangle$ $\langle 4,6\rangle$ $\langle 4,8\rangle$ $\langle 6,8\rangle$ $\langle 7,6\rangle$ $\langle 7,8\rangle$

$\ominus$: $\langle 0,0\rangle$ $\langle 0,7\rangle$ $\langle 1,0\rangle$ $\langle 1,1\rangle$ $\langle 1,3\rangle$ $\langle 1,4\rangle$ $\langle 1,5\rangle$ $\langle 1,6\rangle$ $\langle 1,7\rangle$ $\langle 1,8\rangle$
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$\langle 8,7\rangle$ $\langle 8,8\rangle$

- extensionally defined background relations

$$\mathit{linked\text{-}to} = \{\langle 0,1\rangle, \langle 0,3\rangle, \langle 1,2\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,5\rangle, \langle 4,6\rangle, \langle 6,8\rangle, \langle 7,6\rangle, \langle 7,8\rangle\}$$

The FOIL algorithm for relational learning

FOIL uses a covering approach to learn a set of rules

LEARNRULESET(set of tuples T of target relation, background relations B)
{
    S = { }
    repeat
        $R \leftarrow$ LEARNRULE(T, B)
        $S \leftarrow S \cup R$
        $T \leftarrow T$ – positive tuples covered by R
    until there are no (few) positive tuples left in T
    return S
}

The FOIL algorithm for relational learning

LEARNRULE(set of tuples T of target relation, background relations B)
{
    $R = \{\ \}$
    repeat
        $L \leftarrow$ best literal, based on T and B, to add to right-hand side of R
        $R \leftarrow R \cup L$
        $T \leftarrow$ new set of tuples that satisfy L
    until there are no (few) negative tuples left in T
    return R
}

Literals in FOIL

- Given the current rule $R(X_1, X_2, \ldots X_k) \leftarrow L_1 \wedge L_2 \wedge \ldots \wedge L_n$
  FOIL considers adding several types of literals

$X_j = X_k$

$X_j \neq X_k$

both X_j and X_k either appear in the LHS of the rule, or were introduced by a previous literal

$Q(V_1, V_2, \ldots V_a)$

$\neg Q(V_1, V_2, \ldots V_a)$

at least one of the V_i's has to be in the LHS of the rule, or was introduced by a previous literal

where Q is a background relation

Literals in FOIL (continued)

$X_j = c$

where c is a constant

$X_j \neq c$

$X_j > a$

$X_j \leq a$

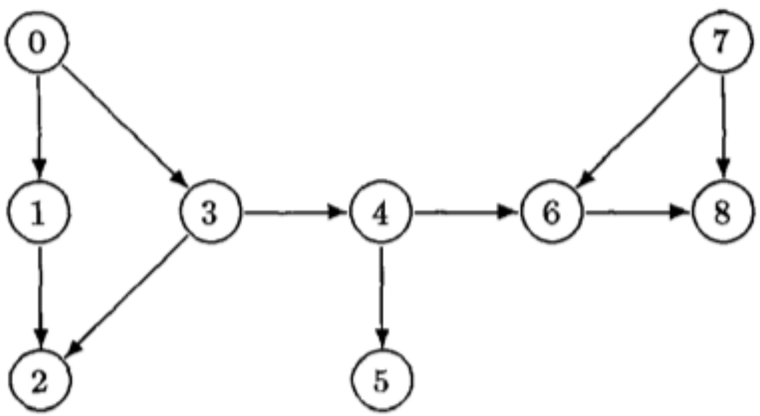
$X_j > X_k$

$X_j \leq X_k$

where X_j and X_k are numeric variables and a is a numeric constant

FOIL example

- suppose we want to learn rules for the target relation can-reach(X_1, X_2)
- we're given instances of the target relation from the following graph



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- and instances of the background relation linked-to

$$\textit{linked-to} = \{\langle 0,1\rangle, \langle 0,3\rangle, \langle 1,2\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,5\rangle, \langle 4,6\rangle, \langle 6,8\rangle, \langle 7,6\rangle, \langle 7,8\rangle\}$$

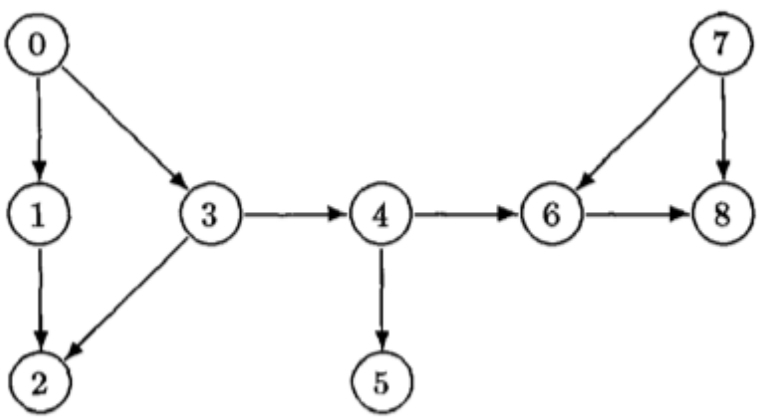
FOIL example

- the first rule learned covers 10 of the positive instances

  $\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_2)$

- the second rule learned covers the other 9 positive instances

  $\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_3) \wedge \text{can-reach}(X_3, X_2)$



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- note that these rules generalize to other graphs

Evaluating literals in FOIL

- FOIL evaluates the addition of a literal L to a rule R by

$$FOIL\_Gain(L,R) = t\left(\log_2 \frac{p_1}{p_1 + n_1} - \log_2 \frac{p_0}{p_0 + n_0}\right)$$

- where

  p_0 = # of positive tuples covered by R

  n_0 = # of negative tuples covered by R

  p_1 = # of positive tuples covered by $R \wedge L$

  n_1 = # of negative tuples covered by $R \wedge L$

  t = # of positive of tuples of R also covered by $R \wedge L$

- like information gain, but takes into account
  - we want to cover positives, not just get a more “pure” set of tuples
  - the size of the tuple set grows as we add new variables

Evaluating literals in FOIL

$$FOIL\_Gain(L,R) = t\left(Info\left(R_0\right) - Info\left(R_1\right)\right)$$

- where R_0 represents the rule without L and R_1 is the rule with L added
- $Info(R_i)$ is the number of bits required to encode a positive in the set of tuples covered by R_i

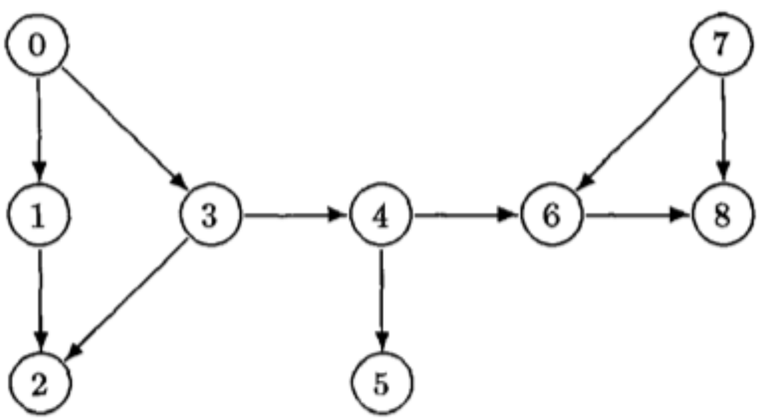
$$Info\left(R_i\right) = -\log_2\left(\frac{p_i}{p_i + n_i}\right)$$

Recall this example

- Definition of can-reach:

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_2)$$

$$\text{can-reach}(X_1, X_2) \leftarrow \text{linked-to}(X_1, X_3) \wedge \text{can-reach}(X_{3,} , X_2)$$



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FOIL example

- consider the first step in learning the second clause

can-reach(X_1, X_2) ←

can-reach(X_1, X_2) ← linked-to(X_1, X_3)

$$FOIL\_Gain(L,R) = 9\left(\log_2 \frac{18}{18+54} - \log_2 \frac{9}{9+62}\right)$$

$$= 8.8$$

⊕: [redacted] ⟨0,2⟩ [redacted] ⟨0,4⟩ ⟨0,5⟩ ⟨0,6⟩ ⟨0,8⟩ [redacted] [redacted] [redacted]
⟨3,5⟩ ⟨3,6⟩ ⟨3,8⟩ [redacted] [redacted] ⟨4,8⟩ [redacted] [redacted] [redacted]

⊖: ⟨0,0⟩ ⟨0,7⟩ ⟨1,0⟩ ⟨1,1⟩ ⟨1,3⟩ ⟨1,4⟩ ⟨1,5⟩ ⟨1,6⟩ ⟨1,7⟩ ⟨1,8⟩
⟨2,0⟩ ⟨2,1⟩ ⟨2,2⟩ ⟨2,3⟩ ⟨2,4⟩ ⟨2,5⟩ ⟨2,6⟩ ⟨2,7⟩ ⟨2,8⟩ ⟨3,0⟩
⟨3,1⟩ ⟨3,3⟩ ⟨3,7⟩ ⟨4,0⟩ ⟨4,1⟩ ⟨4,2⟩ ⟨4,3⟩ ⟨4,4⟩ ⟨4,7⟩ ⟨5,0⟩
⟨5,1⟩ ⟨5,2⟩ ⟨5,3⟩ ⟨5,4⟩ ⟨5,5⟩ ⟨5,6⟩ ⟨5,7⟩ ⟨5,8⟩ ⟨6,0⟩ ⟨6,1⟩
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⟨8,7⟩ ⟨8,8⟩

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⟨0,6,3⟩ ⟨0,8,1⟩ ⟨0,8,3⟩ ⟨3,5,2⟩ ⟨3,5,4⟩ ⟨3,6,2⟩ ⟨3,6,4⟩
⟨3,8,2⟩ ⟨3,8,4⟩ ⟨4,8,5⟩ ⟨4,8,6⟩

⊖: ⟨0,0,1⟩ ⟨0,0,3⟩ ⟨0,7,1⟩ ⟨0,7,3⟩ ⟨1,0,2⟩ ⟨1,1,2⟩ ⟨1,3,2⟩
⟨1,4,2⟩ ⟨1,5,2⟩ ⟨1,6,2⟩ ⟨1,7,2⟩ ⟨1,8,2⟩ ⟨3,0,2⟩ ⟨3,0,4⟩
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⟨4,0,6⟩ ⟨4,1,5⟩ ⟨4,1,6⟩ ⟨4,2,5⟩ ⟨4,2,6⟩ ⟨4,3,5⟩ ⟨4,3,6⟩
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⟨7,4,8⟩ ⟨7,5,6⟩ ⟨7,5,8⟩ ⟨7,7,6⟩ ⟨7,7,8⟩

Another Approach – Extend Markov Networks… Recall:

$$\Pr\left(\vec{V}\right) = \frac{1}{Z} \exp \sum_i w_i f_i\left(\vec{V}\right)$$

- In this formulation, the w's are just weights and the f's are just features
- As such, we can throw the graph out if we want – we have everything we need in the w_is and f_is
- In this view, parameter learning is just weight learning

Assume: Data set D consists of $d_1, ..., d_m$

Features are $f_1, ..., f_n$

Weights on features are $W = \langle w_1, ..., w_n \rangle$

Let $n_i(D)$ denote the number of times (data points where) f_i is true in D

With slight abuse of notation, let $E_W(n_i(m))$ denote the expected number of times f_i is true in m examples under the distribution defined by W

Let ll denote the log-likelihood of W given D: $\ln P(D|W)$

$$ll = \sum_j \ln \frac{1}{Z} e^{\sum_i w_i f_i(d_j)}$$

$$ll = \sum_{i,j} w_i f_i(d_j) - \ln Z$$

$$\frac{\partial ll}{\partial w_i} = \sum_j f_i(d_j) - \frac{1}{Z} \sum_V e^{\sum_i w_i f_i(V)} f_i(V)$$

$$\frac{\partial ll}{\partial w_i} = n_i(D) - E_W(n_i(m))$$

Assume: Data set D consists of $d_1, ..., d_m$
Features are $f_1, ..., f_n$
Weights on features are $W = \langle w_1, ..., w_n \rangle$
Let $n_i(D)$ denote the number of times (data points where)
f_i is true in D
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of times f_i is true in m examples under the distribution defined by W
Let ll denote the log-likelihood of W given D: $\ln P(D|W)$

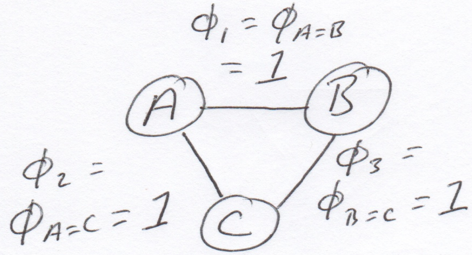
$$ll = \sum_j \ln \frac{1}{Z} e^{\sum_i w_i f_i(d_j)}$$

$$ll = \sum_{i,j} w_i f_i(d_j) - \ln Z$$

$$\frac{\partial ll}{\partial w_i} = \sum_j f_i(d_j) - \frac{1}{Z} \sum_V e^{\sum_i w_i f_i(V)} f_i(V)$$

$$\frac{\partial ll}{\partial w_i} = \underbrace{\boxed{n_i(D)}}_{\text{Target value}} - \underbrace{\boxed{E_W(n_i(m))}}_{\text{To be changed}}$$

Example: Ising Model



$$Z = \sum_{\vec{v}} e^{\sum_i \phi_i(\vec{v})} \approx 56.2$$

$\vec{v}$ A	B	C	$e^{\sum_i \phi_i(\vec{v})}$	$P_{\vec{\phi}}(\vec{v})$
0	0	0	$e^3 \approx 20$	.35
0	0	1	$e \approx 2.7$	.05
0	1	0	.	.
0	1	1	.	.
1	0	0	.	.
1	0	1	.	.
1	1	0	.	.
1	1	1	$e^3 \approx 20$	.35

Data

A	B	C
1	1	0
0	1	1
1	1	1
0	0	0
1	0	1

$\#_{A=B} = 3$

$\#_{B=C} = 3$

$\#_{A=C} = 3$

$$E_{\vec{\phi}, |D|=5}[\#_{A=B}] = .8(5) = 4$$

same for $\#_{A=C}$, $\#_{B=C}$

Gradient: $\langle -1, -1, -1 \rangle$

New $\vec{\phi}$: $\langle 1, 1, 1 \rangle + \eta \langle -1, -1, -1 \rangle$

(e.g., .05) $= \langle .95, .95, .95 \rangle$

Alternative: Markov logic

- a logical knowledge base is a set of **hard constraints** on the set of possible worlds
- let's make them **soft constraints**: when a world violates a formula, it becomes less probable, not impossible
- give each formula a weight (higher weight → stronger constraint)

$$\mathrm{P(world)} \propto \exp\left(\sum \text{weights of formulas it satisfies}\right)$$

MLN definition

- a *Markov Logic Network* (MLN) is a set of pairs (F, w) where
  F is a formula in first-order logic
  w is a real number

- together with a set of constants, it defines a Markov network with
  - one node for each grounding of each predicate in the MLN
  - one feature for each grounding of each formula F in the MLN, with the corresponding weight w

MLN example: friends & smokers

Smoking causes cancer.

Friends have similar smoking habits.

MLN example: friends & smokers

$$\forall x \; Smokes(x) \Rightarrow Cancer(x)$$
$$\forall x, y \; Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$$

MLN example: friends & smokers

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x,y) \Rightarrow \big(Smokes(x) \Leftrightarrow Smokes(y)\big)$

MLN example: friends & smokers

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \big(Smokes(x) \Leftrightarrow Smokes(y)\big)$

Two constants: **Anna** (A) and **Bob** (B)

MLN example: friends & smokers

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
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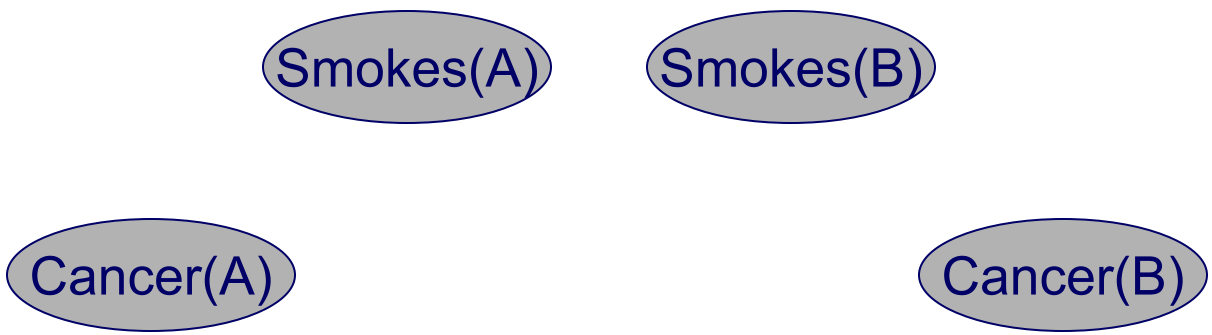
Two constants: **Anna** (A) and **Bob** (B)



MLN example: friends & smokers

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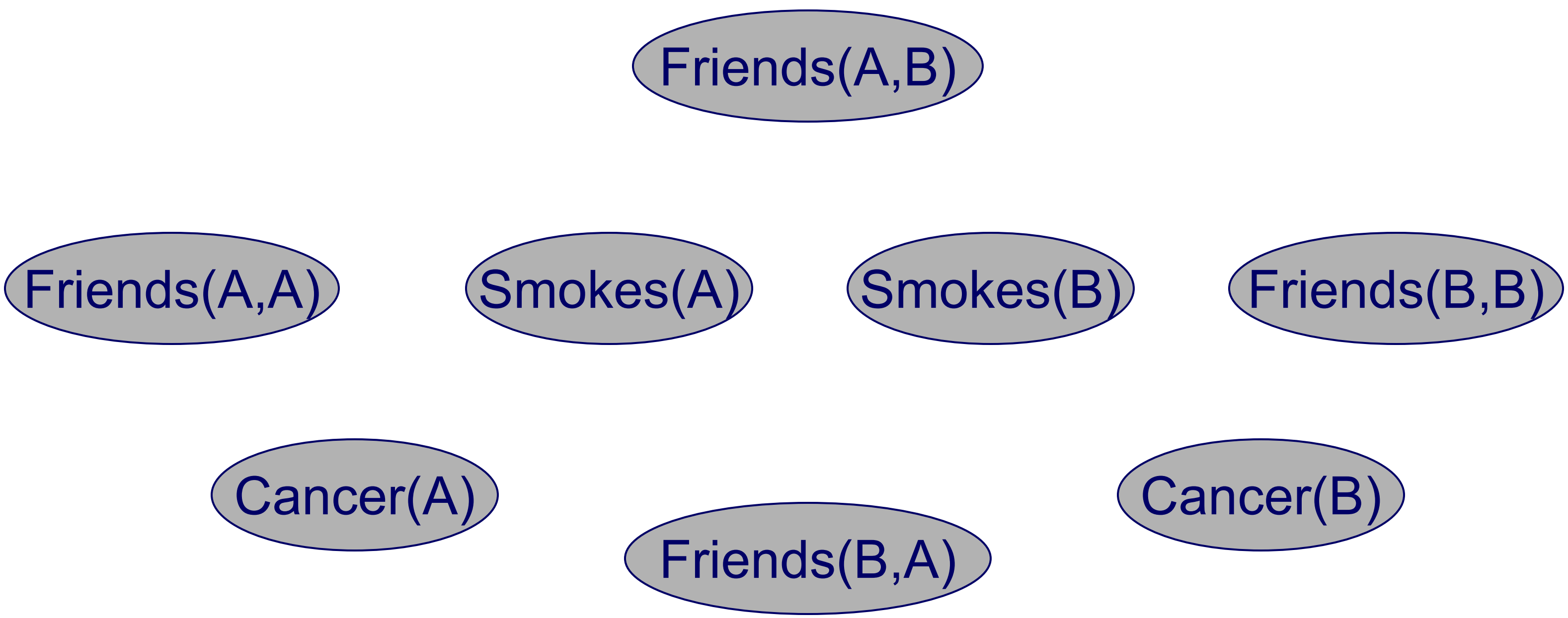

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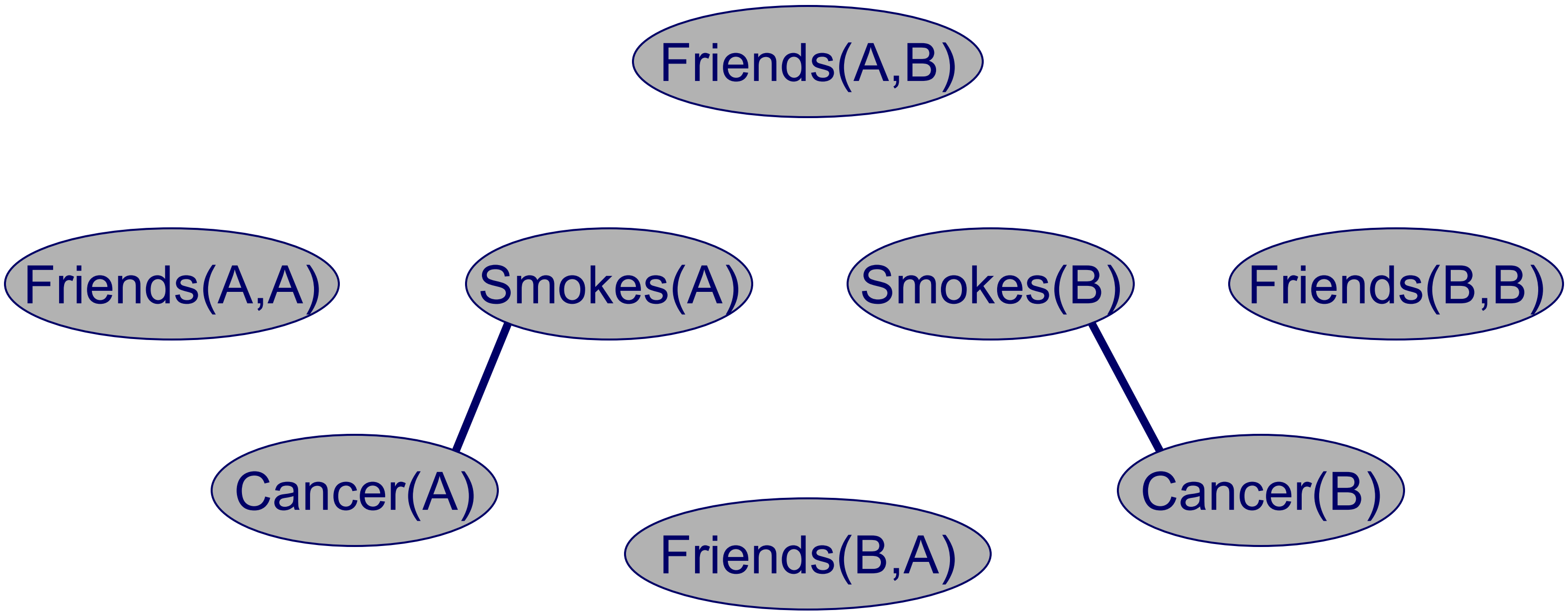

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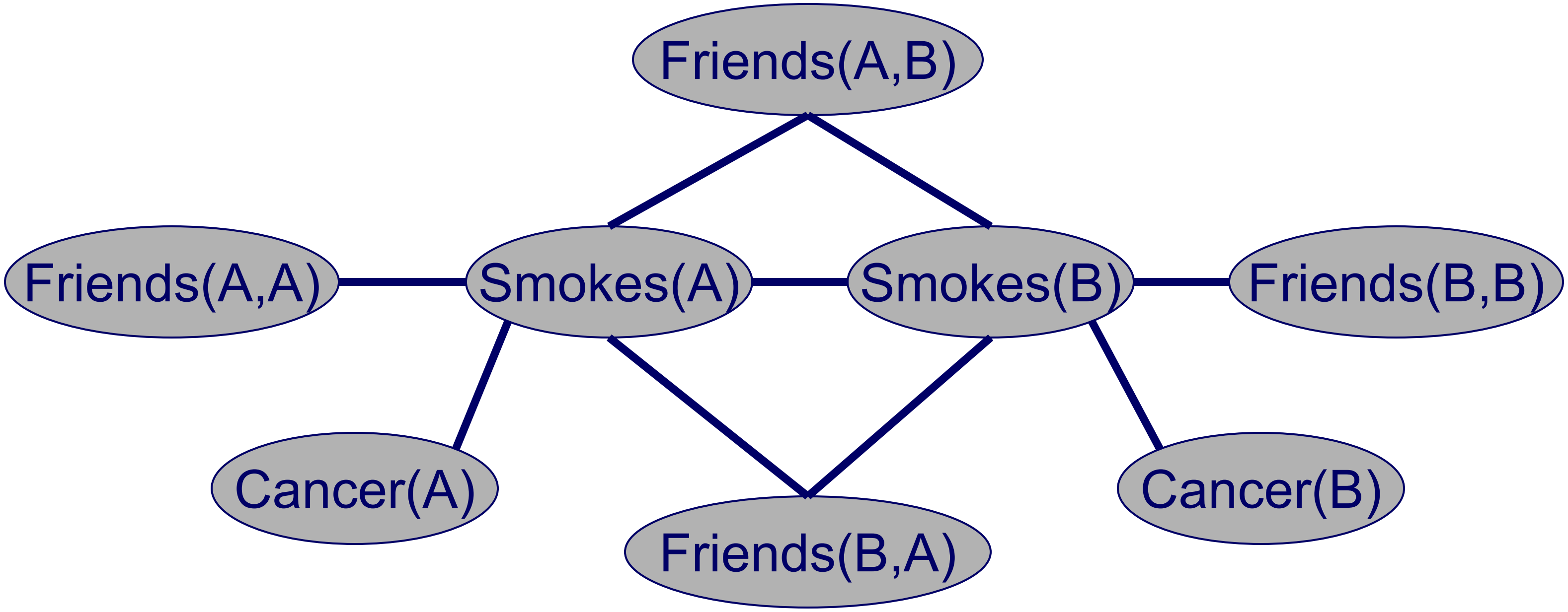


Markov logic networks

- a MLN is a **template** for ground Markov nets
  - the logic determines the form of the cliques
  - but if we had one more constant (say, **Larry**), we'd get a different Markov net

- we can determine the probability of a world $\boldsymbol{v}$ (assignment of truth values to ground predicates) by

$$P(\boldsymbol{v}) = \frac{1}{Z} \exp\left(\sum_i \underbrace{w_i}_{\text{weight of formula } i} \underbrace{n_i(\boldsymbol{v})}_{\text{\# of true groundings of formula } i \text{ in } \boldsymbol{v}} \right)$$

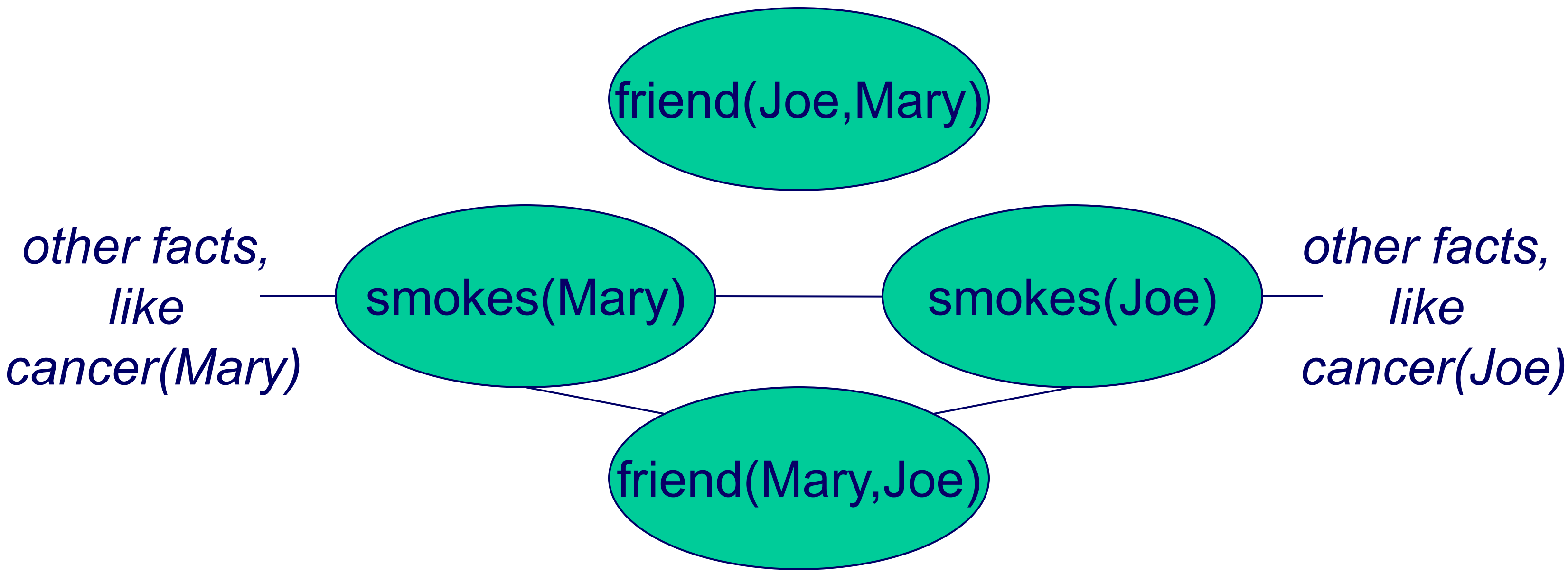
Example translated to Markov Network

- Facts:

$$\mathrm{smokes}(\mathrm{Mary})$$
$$\mathrm{smokes}(\mathrm{Joe})$$
$$\mathrm{friend}(\mathrm{Joe}, \mathrm{Mary})$$

- Rules:

Friend(x,y) ∧ Smokes(x) -> Smokes(y)



Computing weights

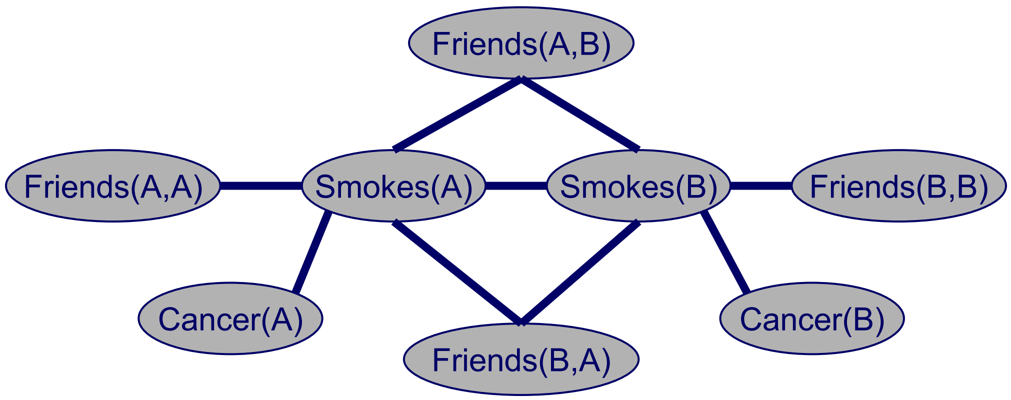
Consider the effect of rule R: Friends(x,y) ∧ Smokes(x) -> Smokes(y) **weight** **1.1**

	smokes(Mary)		¬smokes(Mary)	
	smokes(Joe)	¬smokes(Joe)	smokes(Joe)	¬smokes(Joe)
friends(Mary,Joe)	$e^{1.1}$	1	$e^{1.1}$	$e^{1.1}$
¬friends(Mary,Joe)	$e^{1.1}$	$e^{1.1}$	$e^{1.1}$	$e^{1.1}$

This is the only setting that does not satisfy R
Thus, it is given value 1, while the others are
Given value exp(weight(R))

Probability of a world in an MLN

$$\boldsymbol{v} = \begin{bmatrix} \text{Friends(A,A)} = \text{T} \\ \text{Friends(A,B)} = \text{T} \\ \text{Friends(B,A)} = \text{T} \\ \text{Friends(B,B)} = \text{T} \\ \text{Smokes(A)} = \text{F} \\ \text{Smokes(B)} = \text{T} \\ \text{Cancer(A)} = \text{F} \\ \text{Cancer(B)} = \text{F} \end{bmatrix}$$



$\forall x\ Smokes(x) \Rightarrow Cancer(x)$

$x = \text{A}$ T

$x = \text{B}$ F

$n_1(\boldsymbol{v}) = 1$

$\forall x, y\ Friends(x,y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

$x = \text{A}, y = \text{A}$ T

$x = \text{A}, y = \text{B}$ F

$x = \text{B}, y = \text{A}$ F

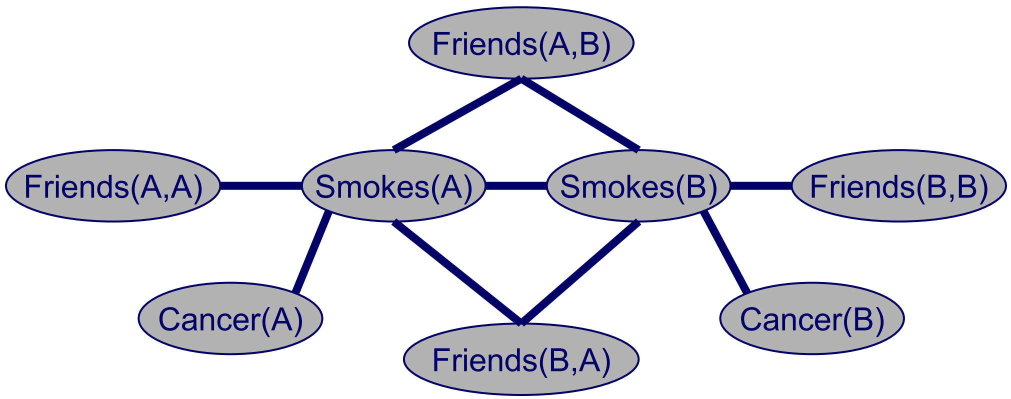
$x = \text{B}, y = \text{B}$ T

$n_2(\boldsymbol{v}) = 2$

# of true groundings of formula 1 in $\boldsymbol{v}$

Probability of a world in an MLN

$$
\boldsymbol{v} = \begin{bmatrix} \text{Friends(A,A)} = \text{T} \\ \text{Friends(A,B)} = \text{T} \\ \text{Friends(B,A)} = \text{T} \\ \text{Friends(B,B)} = \text{T} \\ \text{Smokes(A)} = \text{F} \\ \text{Smokes(B)} = \text{T} \\ \text{Cancer(A)} = \text{F} \\ \text{Cancer(B)} = \text{F} \end{bmatrix}
$$



$$
P(\boldsymbol{v}) = \frac{1}{Z} \exp\left(\sum_i w_i n_i(\boldsymbol{v}) \right)
$$

$$
= \frac{1}{Z} \exp\left(1.5(1) + 1.1(2)\right)
$$

For Learning, Input is Relational DB

- the input to the learning process is a relational database of ground atoms

Friends(x, y)	Smokes(x)	Cancer(x)
Anna, Anna	Bob	Bob
Anna, Bob		
Bob, Anna		
Bob, Bob		

- the closed world assumption is used to infer the truth values of atoms not present in the DB

Parameter learning

- parameters (weights on formulas) can be learned as in Markov network

$$\frac{\partial}{\partial w_i} \log P_w(\boldsymbol{v}) = n_i(\boldsymbol{v}) - E_w\left[n_i(\boldsymbol{v})\right]$$

\# of times clause i is true in data

Expected # times clause i is true according to MLN

- approximation methods may be needed to estimate *both* terms

MLN experiment

- testbed: a DB describing Univ. of Washington CS department
  - 12 predicates

    Professor($person$)

    Student($person$)

    Area($x, area$)

    AuthorOf($publication, person$)

    AdvisedBy($person, person$)

    etc.

  - 2707 constants

    publication (342)

    person (442)

    course (176)

    project (153)

    etc.

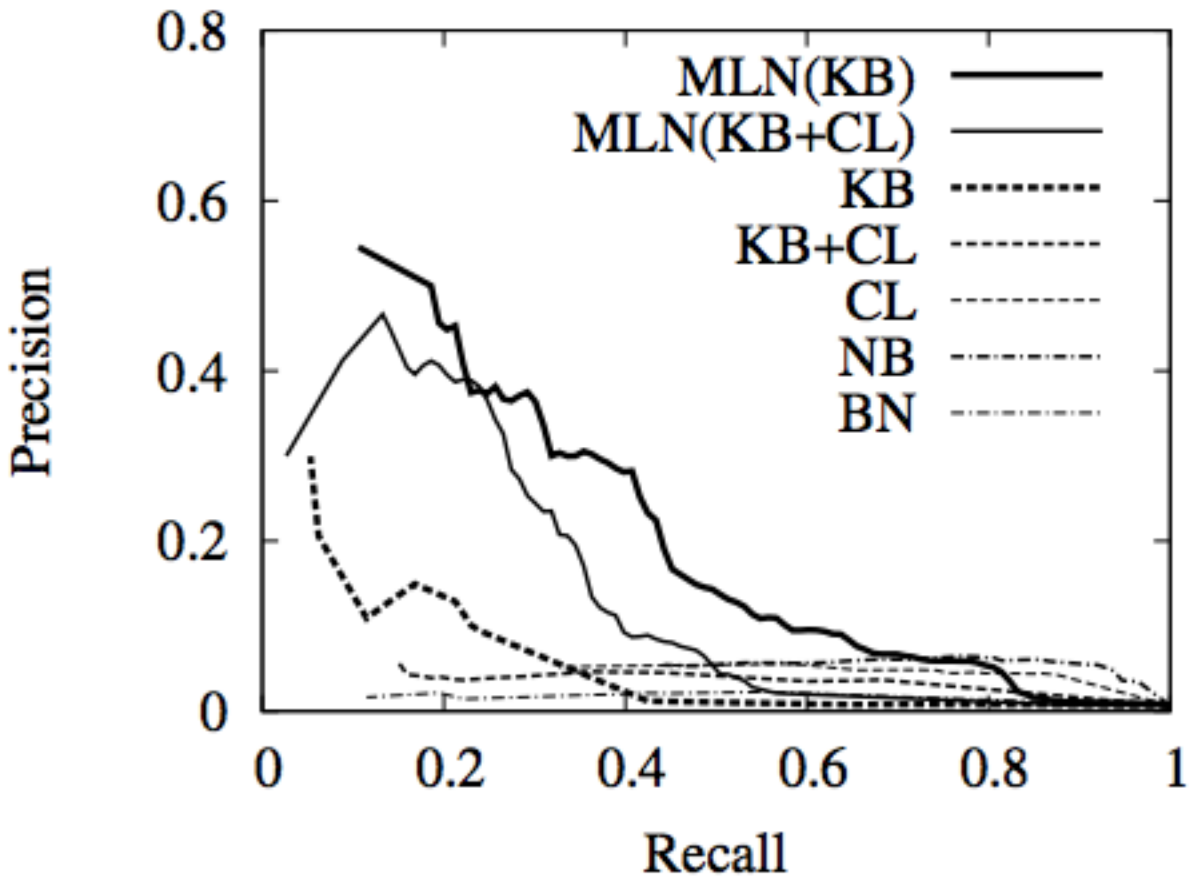
MLN experiment

- obtained knowledge base by having four subjects provide a set of formulas in first-order logic describing the domain

- the formulas in the KB represent statements such as
  - students are not professors
  - each student has at most one advisor
  - if a student is an author of a paper, so is her advisor
  - at most one author of a given publication is a professor
  - etc.

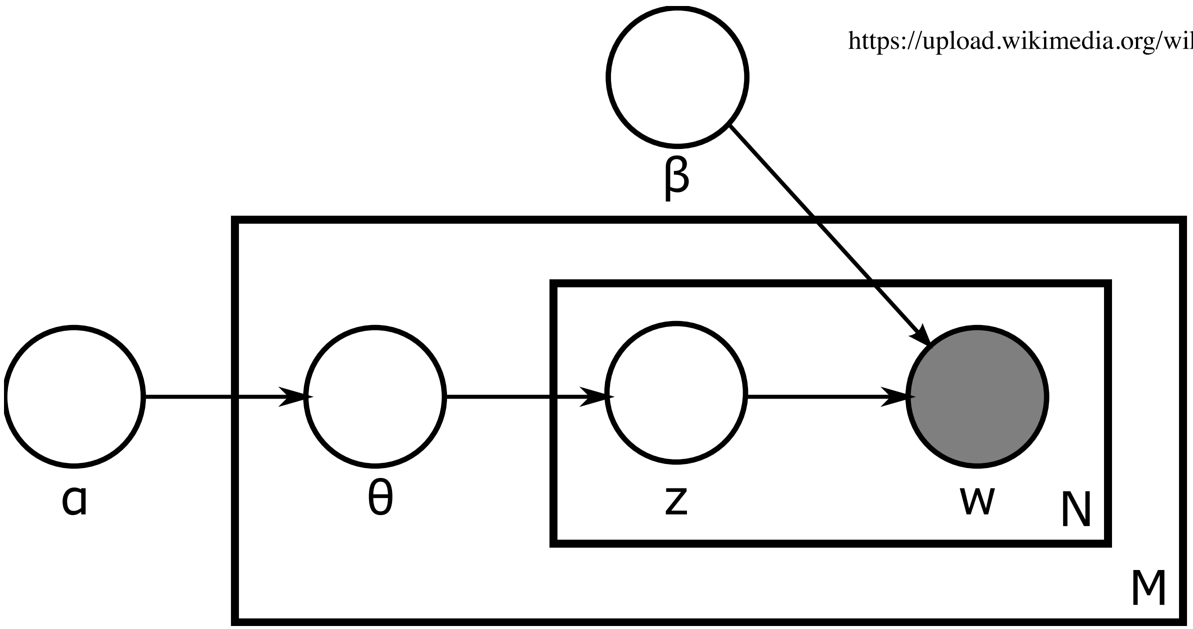
- note that the KB is not consistent

Learning to predict the AdvisedBy(x, y) relation



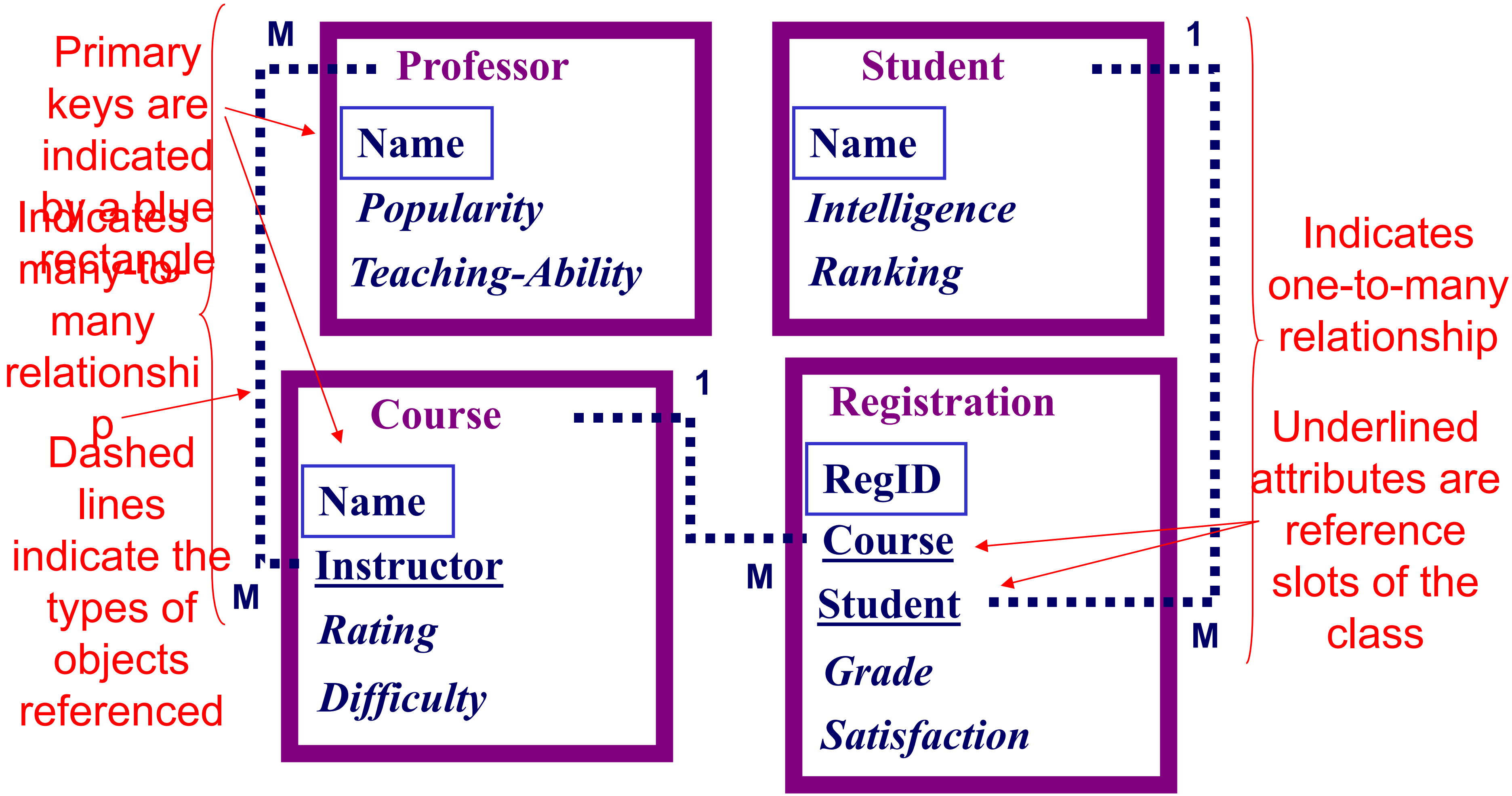
MLN w/ original KB
MLN w/ KB + learned rules
KB alone
KB + learned rules
learned rules only
naïve Bayes
Bayes net learner

Alternative for Relational Learning: Plates

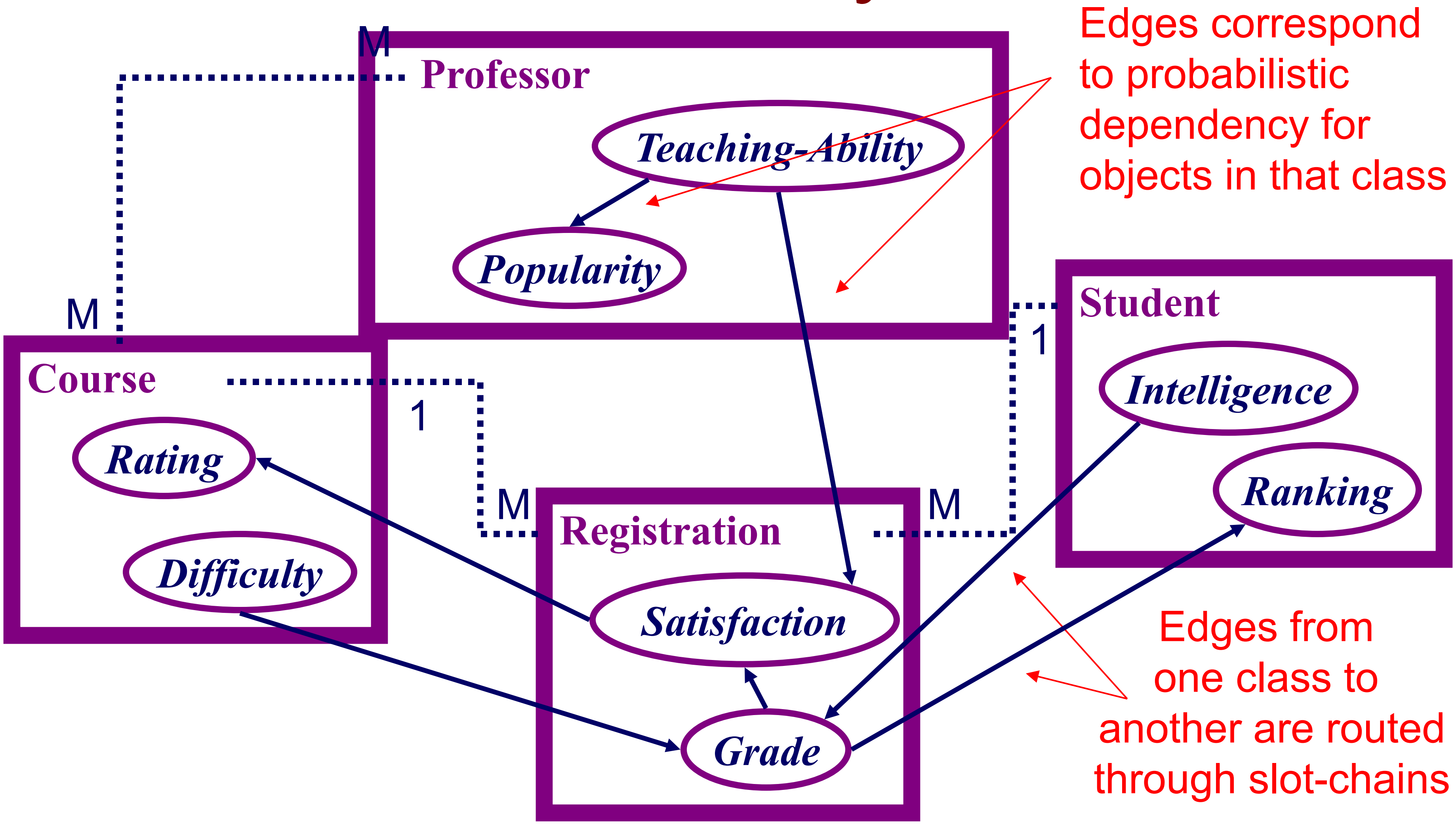


A rectangle labeled “N” denotes N copies of the Bayes net inside.
Typically arcs go *into* rectangles, but we relax to allow outgoing next…

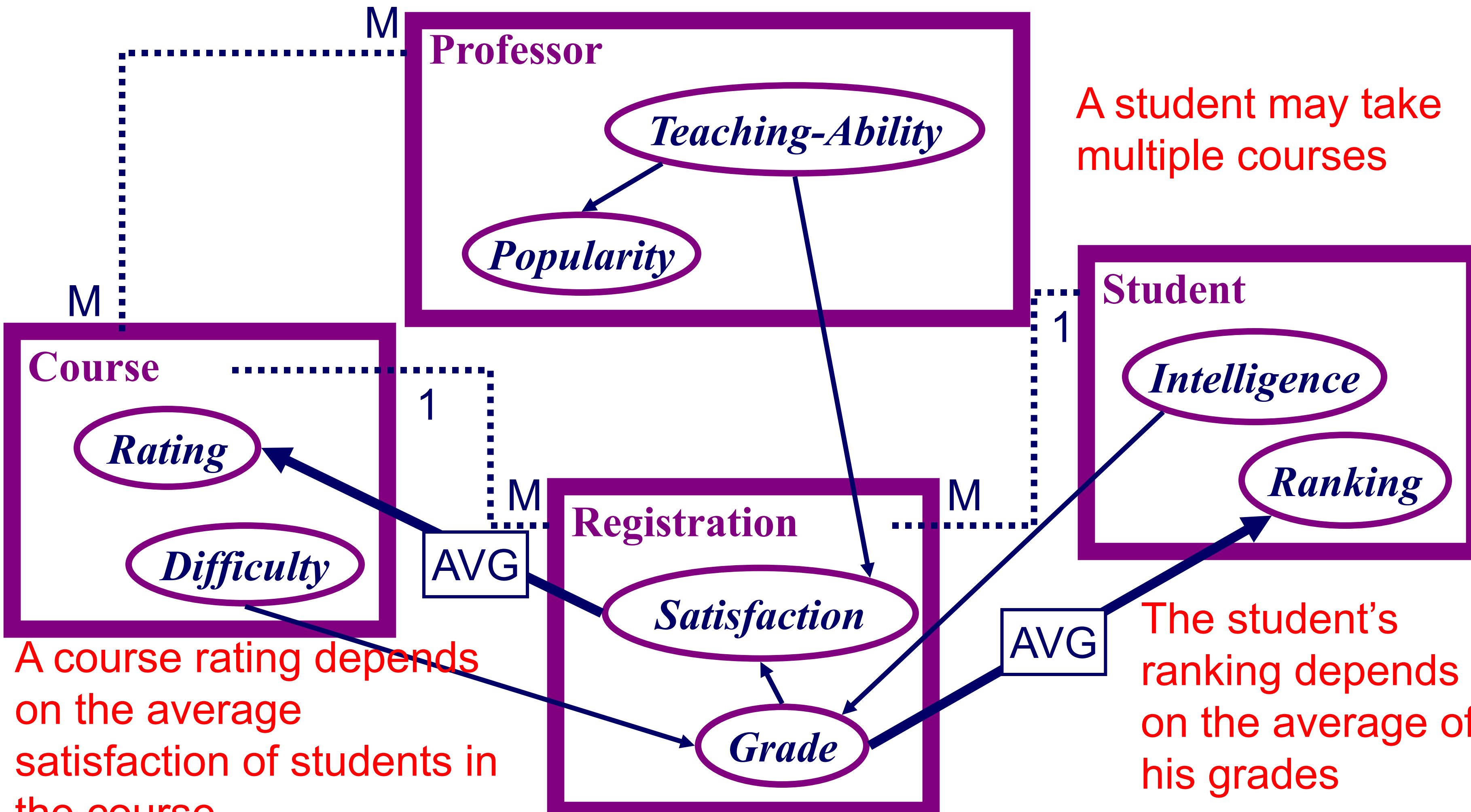
Alternative: Probabilistic Relational Models (PRMs)



PRM Dependency Structure for the University Domain



PRM Dependency Structure

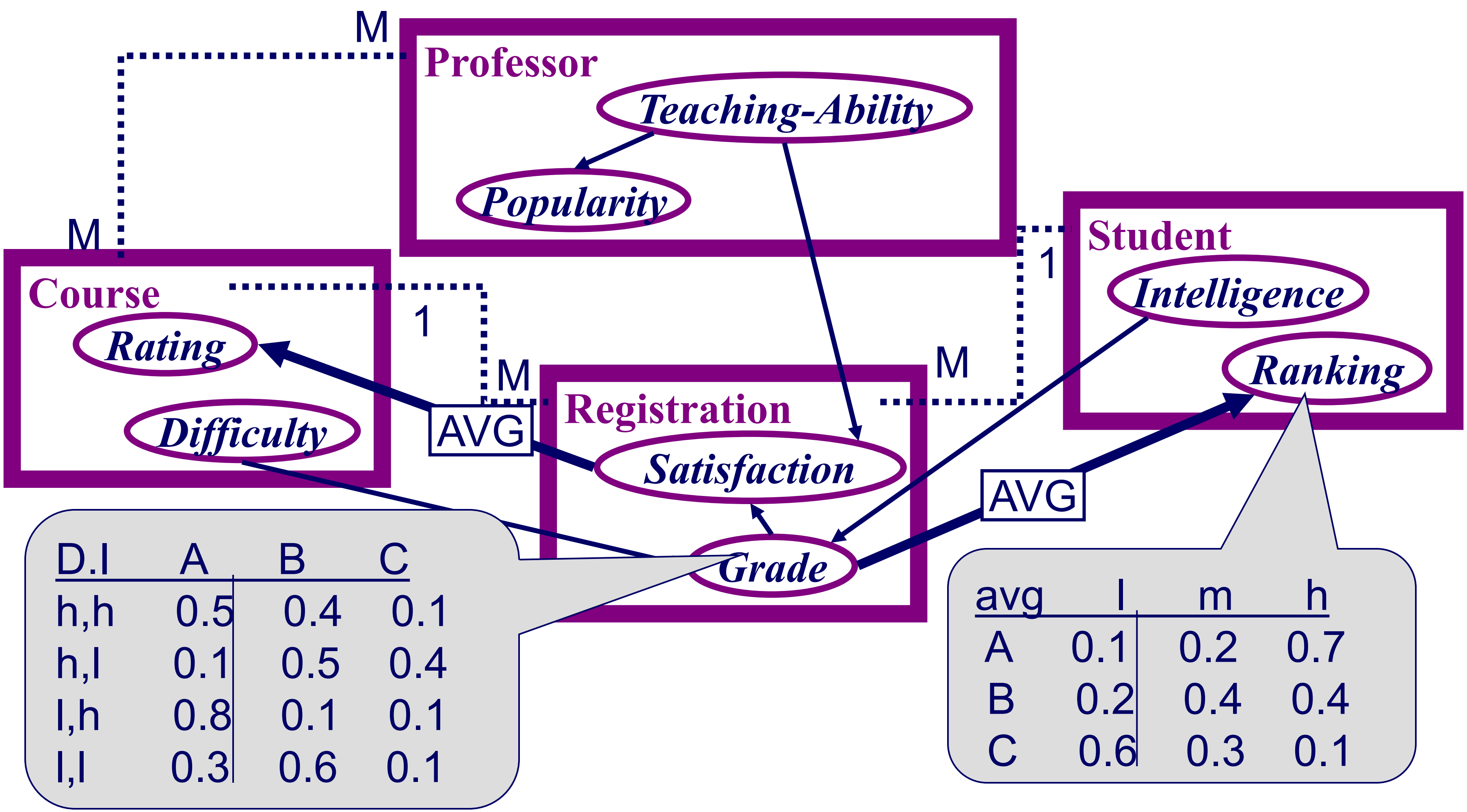


A student may take multiple courses

A course rating depends on the average satisfaction of students in the course

The student's ranking depends on the average of his grades

CPDs in PRMs



D.I	A	B	C
h,h	0.5	0.4	0.1
h,l	0.1	0.5	0.4
l,h	0.8	0.1	0.1
l,l	0.3	0.6	0.1

avg	l	m	h
A	0.1	0.2	0.7
B	0.2	0.4	0.4
C	0.6	0.3	0.1