Subjective Logic Algorithms

Subjective Logic is used for “prescriptive analytics.” Prescriptive analytics aims to project new situations where the past is only one indicator of the future and where outcomes are mostly embodied in counterfactuals (what could have happened but didn’t). In contrast, machine learning uses “predictive analytics,” which is a regression line based on past factuals. Counterfactuals are not present within data and are therefore unavailable to machine-learning, which represents the main weakness to ubiquitous machine-learning.

In general, combining opinions may be achieved in three main ways: Linear Opinion Pool, Independent Opinion Pool and Independent Likelihood Pool. An Independent Opinion Pool is also known as a Logarithmic Opinion Pool but is rarely used because it is extreme in reinforcing cumulative opinions (prior probabilities).

Linear Opinion Pool is the most popular class of aggregation methods for combining expert opinions, often using a weighted average. This technique has the advantages of appearing intuitive and requiring only moderate computation. With roots in Dempster-Shafer theory and Dempster's rule of combination, Subjective Logic Evidence Combination and Analysis of Competing Hypothesis (ACH) has formalised this area, which has otherwise lacked a solid theoretical foundation.

While an Independent Likelihood Pool approach also accumulates opinions, it avoids the problems of an Independent Opinion Pool. The Independent Likelihood approach aligns with the Bayesian Chain Rule that the joint probability distribution of the variables in a graph is the product of the factors of variables that are independent given their parent. Therefore the Independent Likelihood Pool approach has become the foundation of evidence combination in Bayes and Markov Networks and most machine learning.

Subjective Logic has resolved Laplace’s 200-year old objection to Bayes Rule that “don’t know” is different to “no.” This is best explained by a larger example: for the states of red, yellow and blue there is a notional fourth state of “don’t know.” The conceptual paradigm is similar to the three categories of knowledge in an analysts’ Johari Window (popularised by Donald Rumsfeld). These categories are the “known knowns,” the “known unknowns” and the “unknown unknowns.” In Subjective Logic, the “known knowns” are equivalent to the expectations of the states (i.e. the probability of red, the probability of yellow and the probability of blue), the known unknowns are the usual variances of the states, which is called first-order uncertainty, and the “unknown unknowns” are represented by a second or or “second-order” uncertainty.“

In many real world problems, the “answer may be bigger than the question we pose.” Second-order uncertainty represents the systemic risk that our conceptual model fails to encompass all the outcomes. Second-order uncertainty behaves quite differently to having an extra state. The topic of Subjective Logic is about the impact of second-order uncertainty across the states. In other words, Subjective Logic specialises in modelling second-order uncertainty to represent the “unknown unknowns” in conceptual models.

Subjective Logic Evidence Combination and Analysis of Competing Hypothesis

Subjective Logic evidence combination and Analysis of Competing Hypothesis (ACH) is a "judge in a box" for evaluating and combining expert opinions and mathematical evidence. It is primarily used in intelligence analysis and hypothesis selection (Pope et. al. 2007; Jøsang 2016). Subjective Logic provides a two-dimensional analysis of actionable belief and conceptual model systemic risk (i.e. the second-order uncertainty). Quantification of systemic risk is required by CIA Intelligence Community Directive 203 and is increasingly recognised in industries such as mining, pharmaceutical clinical trials and election polling.

Human intuition is often in error due to biases such as confirmation bias, which is Type 1 error that leads to false positives and expensive collateral damage; logical errors such as the base rate fallacy (also known as Prosecutor's Fallacy where humans often mistake the probability of the hypothesis for the probability of the evidence), data limitations and human cognitive limitations in the presence of complexity. For example, one might have 100-150 issues or evidence elements whereas humans can typically only handle up to 7 items (Miller, 1956, one of the most cited papers in psychological research).

Algorithmic decision making with Subjective Logic objectifies decisions to increase confidence and place less trust in intuition or "winging it." More formally, Subjective Logic is a disciplined approach to developing a specific multi-phase causal cognitive model, and follows with the constrained optimisation of an abstract objective hypothesis in the presence of defined prescriptive interventions. Actionable certainty greatly enhances the ability to move forward with an intervention. However, it doesn't change the fundamental perspective that only humans have the ability to "think about their thinking" to consider the consequences of what might happen after intervention before engaging.

Subjective Logic is particularly suited to forecasting from "small data" and combining uncertain & unknown inputs, outputs from big-data dashboards and human opinions. It has a mathematical elegance in that the probability distributions of each state in Subjective Logic, together with second-order uncertainty, may be represented by a Dirichlet distribution. Mathematically, the marginal opinions of the Dirichlet distribution are Beta distributions for each of the states, which are in-turn the primary class for Bayesian conjugate priors. This means that multivariate beliefs may be ingested as evidence into a Subjective Logic network, with Bayesian inversion applied to convert abductive evidence to deduction, with the outputs being Beta distributions for reassembly into Dirichlet distributions or beliefs and then concatenated into a true multi-phase belief network. The simplicity and convenience of Beta distributions renders the whole decision-science problem tractable in "closed-form calculus" which inherently embodies a full probabilistic approach and avoids Monte-Carlo sampling that can take hours.

Subjective Logic Markov Nets

Subjective Logic Markov Nets extend the principles of Subjective Logic into Markov Nets. While Subjective Logic Markov Nets and Subjective Logic Evidence Combination and Analysis of Competing Hypothesis are used for different purposes, they share the distinguishing features of actionable belief and quantification of systemic risk through second-order uncertainty.

Following Philip Dawid's work on probabilistic independence in the 1970s, Judea Pearl (1988) introduced Bayesian Networks as a graphical means of representing joint probability distributions and modelling complex systems. Bayesian Networks are also called belief networks, belief nets or causal networks.

There are now two main families of probabilistic graphical models: Bayesian Networks for directed acyclic graphs and Markov networks for undirected graphs. Markov networks are also known as Markov random fields (MRFs). Markov networks are more general purpose when it is not known which variables are specifically causal and when there are loops in logic flow.

Bayesian and Markov networks may be used for four types of reasoning: prescriptive, diagnostic, intercausal & combined reasoning.

The three main techniques for resolving these networks are: exact computation for small networks & either exact computation using message passing for larger networks or approximate computation using message passing for very large networks. These computations employ the SumProduct algorithm (or the SumMax algorithm for Maximum a posteriori (MAP) to select a single most likely hypothesis given data).

The networks model three types of domains: discrete domains where variables have states such as 1 for low and 2 for high or 1 for red, 2 for yellow and 3 for blue; continuous domains where variables might have Gaussian bell-curves; and Dirichlet distributions in the case of Subjective Logic networks.

As an example of the difference between Subjective Logic Markov Nets and Subjective Logic Evidence Combination and Analysis of Competing Hypothesis is the Bayesian or Markov network evaluation of two positive cancer tests. Entering two positives directly into a Bayesian or Markov network leads to a probability of cancer of nearly 100%, while the Subjective Logic Evidence Combination ACH model provides a probability of cancer of just 20%, which is the same as for one test alone.

While Bayes/Markov network methodology is intrinsically correct for the logic flow, the use of these networks to combine common evidence is incorrect (Minka 2002). Therefore Subjective Logic Markov Nets and Subjective Logic Evidence Combination and Analysis of Competing Hypothesis are two complementary and complete algorithms.

References

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