LINEARNA ALGEBRA

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- RJEŠENJA ZADATAKA -

1. (a) Za $A \in M_n$ i sve $i,j \in \{1,2,\dots,n\}$ vrijedi:

$$\det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij} \qquad \text{(razvoj po } i\text{-tom retku)}$$

$$= \sum_{i=1}^{n} (-1)^{i+j} a_{ij} M_{ij}, \qquad \text{(razvoj po } j\text{-tom stupcu)}$$

pri čemu je M_{ij} minora elementa a_{ij} (determinanta dobivena uklanjanjem i-tog retka i j-tog stupca matrice A).

(b) Tvrdnja slijedi uzastopnim Laplaceovim razvojem po zadnjem retku:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{nn} \end{vmatrix} \leftarrow =$$

$$= \underbrace{(-1)^{n+n}}_{=1} a_{nn} \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1\,n-1} \\ 0 & a_{22} & a_{23} & \cdots & a_{2\,n-1} \\ 0 & 0 & a_{33} & \cdots & a_{3\,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{n-1\,n-1} \end{vmatrix} \leftarrow$$

$$= \dots = \underbrace{(-1)^{i+i}}_{=1} a_{ii} \cdots a_{nn} \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1\,i-1} \\ 0 & a_{22} & a_{23} & \cdots & a_{2\,i-1} \\ 0 & 0 & a_{33} & \cdots & a_{3\,i-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{i-1\,i-1} \end{vmatrix} = \dots = a_{11} a_{22} \cdots a_{nn}$$

(c) Za $A=[a_{ij}]\in M_n$ neka je $A'\in M_n$ matrica dobivena zamjenom i-tog i j-tog retka od A. Imamo

$$\det A+\det A' = \begin{vmatrix} & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \end{vmatrix} + \begin{vmatrix} & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \end{vmatrix}$$

$$= \begin{vmatrix} & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \end{vmatrix} + \underbrace{\begin{vmatrix} & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \end{vmatrix}}_{=0}$$

$$+ \begin{vmatrix} & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \end{vmatrix} + \underbrace{\begin{vmatrix} & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \end{vmatrix}}_{=0}$$

$$= \begin{vmatrix} & \vdots & \\ a_{i1} & a_{i2} \cdots & a_{in} \\ & \vdots & \\ a_{i1}+a_{j1} & a_{i2}+a_{j2} \cdots & a_{in}+a_{jn} \\ & \vdots & \end{vmatrix} + \begin{vmatrix} & \vdots & \\ a_{j1} & a_{j2} \cdots & a_{jn} \\ & \vdots & \\ a_{i1}+a_{j1} & a_{i2}+a_{j2} \cdots & a_{in}+a_{jn} \\ & \vdots & \end{vmatrix}$$

$$= \begin{vmatrix} & & \vdots & \\ a_{i1}+a_{j1} & a_{i2}+a_{j2} & \cdots & a_{in}+a_{jn} \\ & & \vdots & \\ a_{i1}+a_{j1} & a_{i2}+a_{j2} & \cdots & a_{in}+a_{jn} \\ & & \vdots & \end{vmatrix} = 0$$

$$\Rightarrow \det A + \det A' = 0 \quad \Rightarrow \det A' = -\det A$$

(d) $\begin{vmatrix} x^2 & (x+1)^2 & (x+2)^2 \\ x & x+1 & x+2 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} 1\cdot(-x^2) \\ + \\ 1\cdot(-x) \end{matrix} = \begin{vmatrix} 0 & 2x+1 & 4x+4 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$

$$= \begin{vmatrix} 2x+1 & 4x+4 \\ 1 & 2 \end{vmatrix} = \cancel{4x}+2-\cancel{4x}-4 = -2$$

2.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 3 & \lambda & 3 \\ 1 & \lambda & 3 & 2 \end{array}\right] \begin{matrix} |\cdot(-2) \\ + \\ |\cdot(-1) \\ + \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & \lambda-1 & 4 & 1 \end{array}\right] \begin{matrix} + \\ |\cdot(-1) \\ |\cdot(1-\lambda) \\ + \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -\lambda-3 & 0 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & -\lambda^2-\lambda+6 & -\lambda+2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -\lambda-3 & 0 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & (-\lambda+2)(\lambda+3) & -\lambda+2 \end{array}\right]$$

Razlikujemo slučajeve:

1° $-\lambda+2=0 \iff \lambda=2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} \Rightarrow x = 5z \\ \Rightarrow y = 1-4z \\ \end{matrix}$$

Stavljanjem $z=t$, $t\in\mathbb{R}$, dobivamo da sustav ima beskonačno mnogo rješenja oblika

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}, \quad t\in\mathbb{R}.$$

2° $\lambda+3=0 \iff \lambda=-3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 5 \end{array}\right] \longrightarrow$$ iz ovog retka vidimo da sustav u ovom slučaju nema rješenja

3° $(-\lambda+2)(\lambda+3)\neq 0$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\lambda-3 & 0 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & (-\lambda+2)(\lambda+3) & -\lambda+2 \end{array}\right] \begin{matrix} \\ \\ |:(-\lambda+2)(\lambda+3) \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 0 & -\lambda-3 & 0 \\ 0 & 1 & \lambda+2 & 1 \\ 0 & 0 & 1 & \frac{1}{\lambda+3} \end{array}\right]$$

U ovom slučaju sustav ima jedinstveno rješenje $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{\lambda+2} \\ \frac{1}{\lambda+3} \end{bmatrix}$.

3. (a) VEKTORSKI UMNOŽAK (produkt) vektora $\vec{a}, \vec{b} \in V^3$ je vektor $\vec{a} \times \vec{b} \in V^3$ sa svojstvima

(1) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin \angle(\vec{a}, \vec{b})$,

(2) $\vec{a} \times \vec{b} \perp \vec{a}$, $\vec{a} \times \vec{b} \perp \vec{b}$,

(3) uređena trojka $(\vec{a}, \vec{b}, \vec{a} \times \vec{b})$ čini desni sustav.

(b) Odredimo vektorski umnožak od $\vec{u}$ i $\vec{v}$:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & -2 & 3 \\ 2 & 4 & 5 \end{vmatrix} = -22\vec{\imath} + \vec{\jmath} + 8\vec{k}$$

$$\Rightarrow \vec{w} = \frac{1}{|\vec{u} \times \vec{v}|} \vec{u} \times \vec{v} = \frac{1}{\sqrt{549}} (-22\vec{\imath} + \vec{\jmath} + 8\vec{k})$$

(c) Vrijedi

$$\vec{u}_{\vec{v}} = |\vec{u}| \cdot \cos \angle(\vec{u}, \vec{v}) \cdot \frac{1}{|\vec{v}|} \vec{v} = \cancel{|\vec{u}|} \cdot \frac{\vec{u} \cdot \vec{v}}{\cancel{|\vec{u}|} \cdot |\vec{v}|} \cdot \frac{1}{|\vec{v}|} \vec{v}$$

$$= \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \vec{v} = \frac{2 - 8 + 15}{2^2 + 4^2 + 5^2} (2\vec{\imath} + 4\vec{\jmath} + 5\vec{k})$$

$$= \frac{2}{5} \vec{\imath} + \frac{4}{5} \vec{\jmath} + \vec{k}$$

4. Za vektore $E_{11}, E_{12}, E_{21}, E_{22}$ kanonske baze za $\mathcal{M}_2$ imamo

$$A(E_{11}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix},$$

$$A(E_{12}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix},$$

$$A(E_{21}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix},$$

$$A(E_{22}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix},$$

pa je matrični zapis od A u kanonskoj bazi

$$A(e) = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix}.$$

Odredimo $\operatorname{Im} A$. Za proizvoljnu matricu $M = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathcal{M}_2$ imamo

$$A(M) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x+2z & y+2w \\ 2x+4z & 2y+4w \end{bmatrix}$$

$$= x\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + y\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} + z\begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} + w\begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}$$

Reducirajmo skup $\left\{\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}\right\}$ do linearno nezavisnog skupa u $\mathcal{M}_2$.

Budući da je

$$\begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} = 2\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix},$$

a skup $\left\{\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}\right\}$ je linearno nezavisan u M_2

$$\alpha\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + \beta\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \alpha = \beta = 0,$$

slijedi da je taj skup (jedna) baza za A i $r(A) = 2$.

Neka je sada $M = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \operatorname{Ker} A$. Imamo

$$A(M) = 0$$

$$\Leftrightarrow \begin{bmatrix} x+2z & y+2w \\ 2x+4z & 2y+4w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} x + 2z = 0 \\ y + 2w = 0 \\ 2x + 4z = 0 \\ 2y + 4w = 0 \end{cases} \rightarrow \begin{array}{l} x = -2z \\ y = -2w \end{array}$$

Dakle, $M = \begin{bmatrix} -2z & -2w \\ z & w \end{bmatrix} = z\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} + w\begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix}$.

Budući da je skup $\left\{\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix}\right\}$ linearno nezavisan u M_2:

$$\alpha\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} + \beta\begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \alpha = \beta = 0,$$

taj je skup (jedna) baza za $\operatorname{Ker} A$ i $d(A) = 2$.

5. (a) Neka su $\alpha, \beta \in \mathbb{R}$ te $(a_1,a_2,a_3,a_4), (b_1,b_2,b_3,b_4) \in X$, $(c_1,c_2,c_3,c_4), (d_1,d_2,d_3,d_4) \in Y$ proizvoljni. Imamo

$$(\alpha a_1+\beta b_1)+(\alpha a_2+\beta b_2) = \alpha \underbrace{(a_1+a_2)}_{=0}+\beta\underbrace{(b_1+b_2)}_{=0} = 0,$$

$$(\alpha a_3+\beta b_3)+(\alpha a_4+\beta b_4) = \alpha \underbrace{(a_3+a_4)}_{=0}+\beta\underbrace{(b_3+b_4)}_{=0} = 0,$$

pa slijedi $\alpha(a_1,a_2,a_3,a_4)+\beta(b_1,b_2,b_3,b_4) \in X$, tj. X je potprostor od $\mathbb{R}^4$.

Slično,

$$(\alpha c_1+\beta d_1)+(\alpha c_2+\beta d_2)+(\alpha c_3+\beta d_3)+(\alpha c_4+\beta d_4) =$$

$$= \alpha\underbrace{(c_1+c_2+c_3+c_4)}_{=0}+\beta\underbrace{(d_1+d_2+d_3+d_4)}_{=0} = 0,$$

pa $\alpha(c_1,c_2,c_3,c_4)+\beta(d_1,d_2,d_3,d_4) \in Y$, tj. i Y je potprostor od $\mathbb{R}^4$.

(b) Neka je $\vec{x}=(x_1,x_2,x_3,x_4) \in X$ proizvoljna uređena četvorka. Tada

$$\begin{cases} x_1+x_2=0 \Rightarrow x_2=-x_1, \\ x_3+x_4=0 \Rightarrow x_4=-x_3, \end{cases}$$

tj.

$$\vec{x} = x_1(1,-1,0,0)+x_3(0,0,1,-1).$$

Budući da je skup $\{(1,-1,0,0),(0,0,1,-1)\}$ linearno nezavisan u $\mathbb{R}^4$:

$$\alpha(1,-1,0,0)+\beta(0,0,1,-1) = (\alpha,-\alpha,\beta,-\beta) = (0,0,0,0) \Rightarrow \alpha=\beta=0,$$

slijedi da je taj skup baza za X i $\dim X = 2$.

Jednako tako, za $\vec{y} = (y_1, y_2, y_3, y_4) \in Y$ imamo

$$y_1 + y_2 + y_3 + y_4 = 0 \Rightarrow y_4 = -y_1 - y_2 - y_3 - y_4,$$

tj.

$$\vec{y} = (y_1, y_2, y_3, -y_1 - y_2 - y_3)$$
$$= y_1 (1,0,0,-1) + y_2 (0,1,0,-1) + y_3 (0,0,1,-1).$$

Budući da je skup $\{(1,0,0,-1), (0,1,0,-1), (0,0,1,-1)\}$ linearno nezavisan u $\mathbb{R}^4$:

$$\alpha(1,0,0,-1) + \beta(0,1,0,-1) + \gamma(0,0,1,-1) = (\alpha, \beta, \gamma, -\alpha-\beta-\gamma) = (0,0,0,0)$$
$$\Rightarrow \alpha = \beta = \gamma = 0,$$

taj je skup baza za Y i $\dim Y = 3$.

i. Matrični zapis bilo kojeg linearnog operatora $A : X \to Y$ u bilo kojem paru baza imat će $\dim Y = 3$ retka i $\dim X = 2$ stupca.

ii. Prema teoremu o rangu i defektu

$$\dim(\operatorname{Im} A) = \dim X - \dim(\operatorname{Ker} A) = 2 - 1 = 1.$$

6.

$$k_A(\lambda)=\det(\lambda I-A)=\begin{vmatrix}\lambda+3 & 12 & 0 & 0\\ -2 & \lambda-7 & 0 & 0\\ 0 & 0 & \lambda+5 & 3\\ 0 & 0 & -4 & \lambda-3\end{vmatrix}$$

$$=-3\cdot\begin{vmatrix}\lambda+3 & 12 & 0\\ -2 & \lambda-7 & 0\\ 0 & 0 & -4\end{vmatrix}+(\lambda-3)\begin{vmatrix}\lambda+3 & 12 & 0\\ -2 & \lambda-7 & 0\\ 0 & 0 & \lambda+5\end{vmatrix}$$

$$=12\begin{vmatrix}\lambda+3 & 12\\ -2 & \lambda-7\end{vmatrix}+(\lambda-3)(\lambda+5)\begin{vmatrix}\lambda+3 & 12\\ -2 & \lambda-7\end{vmatrix}$$

$$=\left[12+(\lambda-3)(\lambda+5)\right]\cdot\begin{vmatrix}\lambda+3 & 12\\ -2 & \lambda-7\end{vmatrix}$$

$$=\left[12+(\lambda-3)(\lambda+5)\right]\cdot\left[(\lambda+3)(\lambda-7)+24\right]$$

$$=(\lambda^2+2\lambda-3)(\lambda^2-4\lambda+3)=(\lambda-1)^2(\lambda+3)(\lambda-3)$$

$\Rightarrow$ svojstvene vrijednosti od A su $\lambda_1=1,\ \lambda_2=-3,\ \lambda_3=3$

Tražimo pripadne svojstvene vektore:

1° $\lambda_1=1$

$$(I-A)\vec{x}=\vec{0}$$

$$\left[\begin{array}{cccc|c}4 & 12 & 0 & 0 & 0\\ -2 & -6 & 0 & 0 & 0\\ 0 & 0 & 6 & 3 & 0\\ 0 & 0 & -4 & -2 & 0\end{array}\right]\begin{array}{l}\uparrow^{+}\\ |\cdot 2\\ \uparrow^{+}\\ |\cdot\frac{2}{3}\end{array}\sim\left[\begin{array}{cccc|c}0 & 0 & 0 & 0 & 0\\ -2 & -6 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -4 & -2 & 0\end{array}\right]\begin{array}{l}\\ \Rightarrow x_1=-3x_2\\ \\ \Rightarrow x_4=-2x_3\end{array}$$

$$\Rightarrow\vec{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}-3x_2\\x_2\\x_3\\-2x_3\end{bmatrix}=x_2\begin{bmatrix}-3\\1\\0\\0\end{bmatrix}+x_3\begin{bmatrix}0\\0\\1\\-2\end{bmatrix},\quad x_{2,3}\in\mathbb{R}$$

$2^\circ \ \lambda_2 = -3$

$(-3I - A)\vec{x} = \vec{0}$

$$\left[\begin{array}{cccc|c} 0 & 12 & 0 & 0 & 0 \\ -2 & -10 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 \\ 0 & 0 & -4 & -6 & 0 \end{array}\right] \begin{array}{l} \Rightarrow x_2 = 0 \\ \Rightarrow x_1 = -5x_2 = 0 \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} \Rightarrow x_3 = -\frac{3}{2}x_4 \end{array}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{3}{2}x_4 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} 0 \\ 0 \\ -\frac{3}{2} \\ 1 \end{bmatrix}, \quad x_4 \in \mathbb{R}$$

$3^\circ \ \lambda_3 = 3$

$(3I - A)\vec{x} = \vec{0}$

$$\left[\begin{array}{cccc|c} 6 & 12 & 0 & 0 & 0 \\ -2 & -4 & 0 & 0 & 0 \\ 0 & 0 & 8 & 3 & 0 \\ 0 & 0 & -4 & 0 & 0 \end{array}\right] \begin{array}{l} \left.\begin{array}{l} \\ \\ \end{array}\right\} x_1 = -2x_2 \\ \Rightarrow x_4 = -\frac{8}{3}x_3 = 0 \\ \Rightarrow x_3 = 0 \end{array}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad x_2 \in \mathbb{R}$$