(6. DZ)

**1**

a) $k(x, x') = \phi(x)^T \phi(x')$

Jezgrena fja mjeri sličnost dvaju vektora u nekom prostoru značajki. Jezgrenu fju ~~[illegible]~~ zamjenjujemo za obični skalarni produkt $\phi(x)^T \phi(x')$ (inače samo $x^T x'$), kada imamo dualnu prezentaciju SVM-a.

Da bi jezgreni trik funkcionirao, jezgre moraju biti Mercerove.

RBF su Mercerove jezgre koje ovise ~~samo~~ o udaljenosti primjera

$k(x, x') = k(\|x - x'\|)$

~~[illegible]~~ Primjer RBF-a je Gaussova jezgra:

$$k(x, x') = \exp\left\{-\frac{\|x - x'\|^2}{2\sigma^2}\right\} = \exp\left\{-\gamma \|x - x'\|^2\right\}$$

$\gamma = \frac{1}{2\sigma^2}$ je preciznost

Što su primjeri sličniji, k teži 1. Što su različitiji, teži

b) Mahalanobisova udaljenost:

$$k(x, x') = \exp\left(-\frac{1}{2}(x-x')^T \Sigma^{-1} (x-x')\right)$$

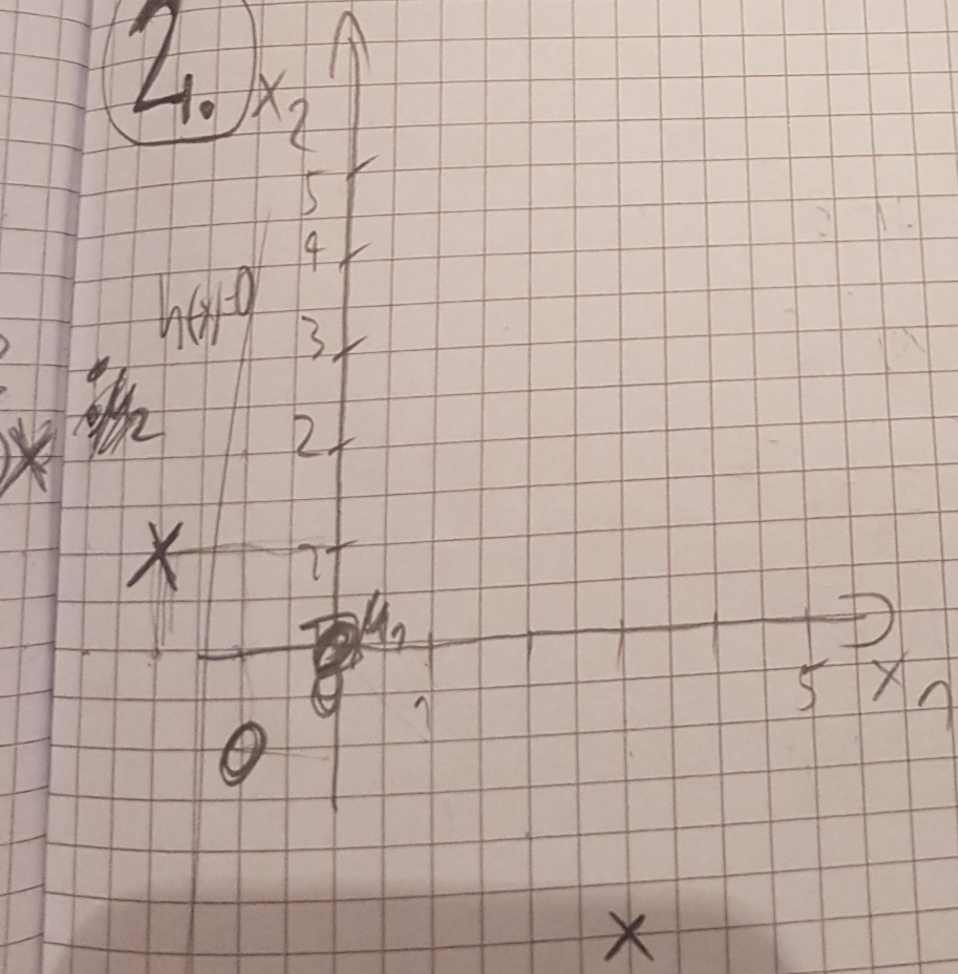
c)

$$\phi(x) = \left(k(x, \mu_1) \; . \; . \; . \; k(x, \mu_m)\right)$$

$\mu_j \in X$ su centroidi

Jezgreni stroj je PLM s takvim preslikavanjem

2.



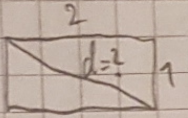
$e^{-1} = 0{,}36$

b)

$\mu_1 = (0,0)$

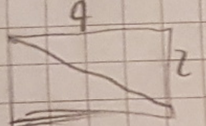
$\mu_2 = (-3,3)$

$\sigma_1 = \sigma_2 = 1$




$d = \sqrt{2^2 + 1^2}$

$= \sqrt{5}$




$d = \sqrt{16+4} = \sqrt{20}$

$\boxed{\phi_1}$:

$\phi_1(x_1) = \exp\left(-\frac{\sqrt{2}^2}{2}\right) = \exp\left(-\frac{2}{2}\right) = \exp(-1)$

$= e^{-1} = \frac{1}{e} = e^{-1} = \frac{1}{e}$

$= ?$

$\phi_1(x_2) = \exp\left(-\frac{0}{2}\right) = e^0 = \boxed{1}$

$\phi_1(x_3) = \exp\left(-\frac{(3\sqrt{2})^2}{2}\right)$

$= \exp(-9) = e^{-9}$

$\phi_1(x_4) = \exp\left(-\frac{5}{2}\right) = e^{-\frac{5}{2}}$

$\phi_1(x_5) = \exp\left(-\frac{\sqrt{20}^2}{2}\right) = e^{-10}$

$\boxed{\phi_2}$:

$\phi_2(x_1) = \exp\left(-\frac{\sqrt{20}^2}{2}\right) = e^{-10}$

$\phi_2(x_2) = \exp\left(-\frac{(3\sqrt{2})^2}{2}\right) = \exp(-9) = e^{-9}$

$\phi_2(x_3) = \exp\left(-\frac{(6\sqrt{2})^2}{2}\right) = \exp(-36) = e^{-36}$

$\phi_2(x_4) = \exp\left(-\frac{\sqrt{5}^2}{2}\right) = \exp\left(-\frac{5}{2}\right) = e^{-\frac{5}{2}}$

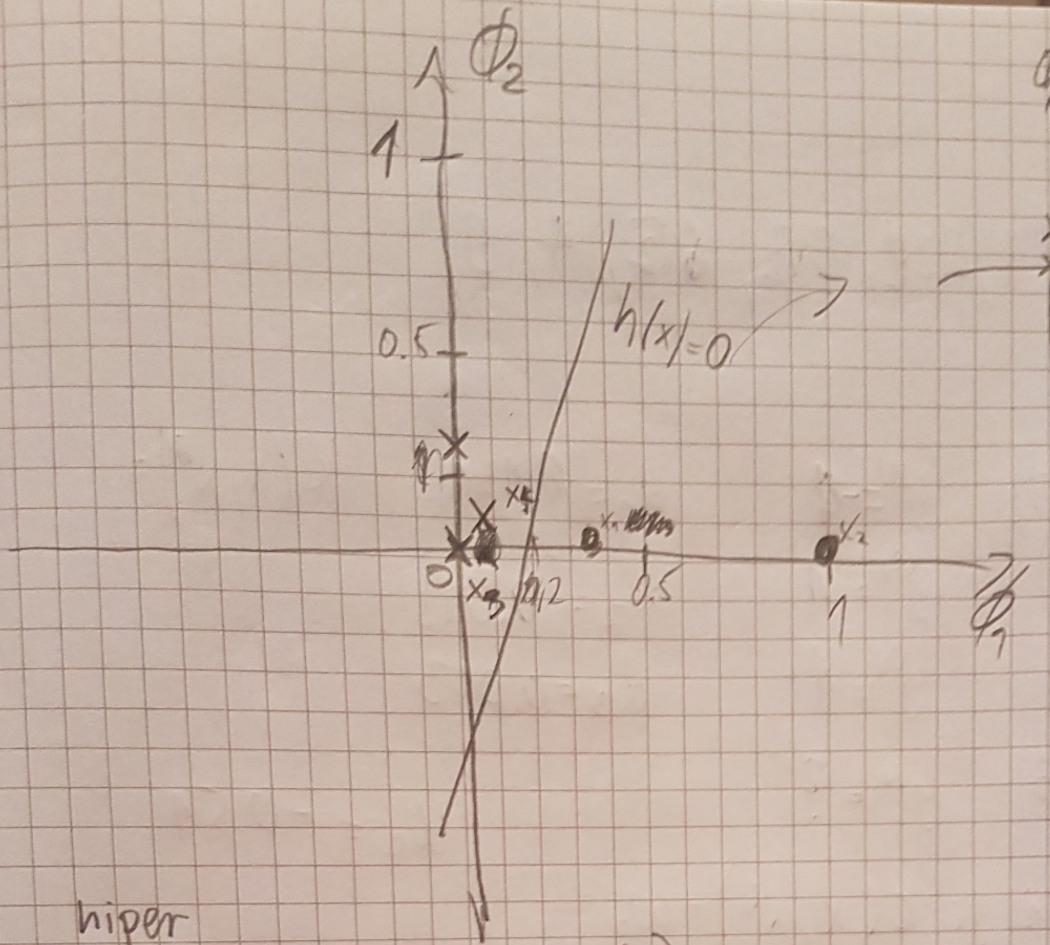
$\phi_2(x_5) = \exp\left(-\frac{\sqrt{2}^2}{2}\right) = \exp(-1) = e^{-1}$

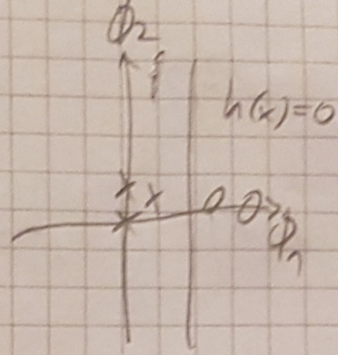
~~x(x_1 = (e^{-1},~~

$x_1 = (e^{-1}, e^{-10})$ $\quad x_4 = (e^{-\frac{5}{2}}, e^{-\frac{5}{2}})$

$x_2 = (1, e^{-9})$ $\quad x_5 = (e^{-10}, e^{-1})$

$x_3 = (e^{-9}, e^{-36})$




c) 2 hiper parametra: $\mu$ i $\sigma$ (?)

Centroide odredimo u točki u kojoj se grupira klasa.

Ako su dobro odabrani, smanjuju složenost. (?)

3 a) $K(x,z) = (x^T z + 1)^2 = (x^T z) + 2x^T z + 1$

$n=2$. $= x_1^2 z_1^2 + \sqrt{2}\, x_1 x_2 \sqrt{2} z_1 z_2 + x_2^2 z_2^2 + 2x_1 z_1 + 2x_2 z_2 + 1$

$$\boxed{\phi(x) = (1, \sqrt{2}x_1, \sqrt{2}x_2, \sqrt{2}x_1 x_2, x_1^2, x_2^2)}$$

K je moguće rastaviti na $K(x,z) = \phi(x)^T \phi(z)$, pa je jezgra Mercerova. To je bitan uvjet da bi jezgreni trik funkcionirao.

b) $x = (2,3)$

$$\phi(x) = (1, 2\sqrt{2}, 3\sqrt{2}, 6\sqrt{2}, 4, 9)$$

c)

$x^{(1)} = (-2,3,5)$ $y^{(1)} = -1$ $\alpha_1 = 0.131$ $K(x, x^{1}) =$

$x^{(2)} = (6,4,3)$ $y^{(2)} = 1$ $\alpha_2 = 0.048$

$x^{(3)} = (8,8,2)$ $y^{(3)} = 1$ $\alpha_3 = 0.013$

$x^{(4)} = (1,2,3)$ $y^{(4)} = ?$ $w_0 = -0.51$

$$h(x) = \sum_{i=1}^{3} \alpha_i y_i K(x, x^{(i)}) + w_0$$

$$K(x^4, x^1) = \left( [1\ 2\ 3] \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix} + 1 \right)^3 = (-2+6+15+1)^3 = 20^3$$

$$K(x^4, x^2) = \left( [1\ 2\ 3] \begin{bmatrix} 6 \\ 4 \\ 3 \end{bmatrix} + 1 \right)^3 = (6+8+9+1)^3 = 24^3$$

$$K(x^4, x^3) = \left( [1\ 2\ 3] \begin{bmatrix} 8 \\ 8 \\ 2 \end{bmatrix} + 1 \right)^3 = (8+16+6+1)^3 = 25^3$$

$$h(x^4) = -0.131 \cdot 20^3 + 0.048 \cdot 24^3 + 0.013 \cdot 25^3 + 0.51$$

$$= \underline{-181.833}$$ $x^4$ je negativan primjer.

4.

a) RBF: $k(x,z) = \phi(\|x-z\|)$

Gauss: $k(x,z) = \exp\left(-\frac{\|x-z\|^2}{2\sigma^2}\right) = \exp\left(-\gamma\|x-z\|^2\right)$

1. $\phi(\|x-z\|)$
2. $\phi(\|x-z\|^2)$ / Množenje 2 konc. jezgre
3. $\phi(-\gamma\|x-z\|^2)$ / Množenje s konst. $-\gamma$
4. $\cancel{\phi}\ \exp(-\gamma\|x-z\|^2)$ / eksponenciranje

$\| \ \ \nabla$

Gauss! QED

b) $\gamma$ i $C$ su u međuovisnosti ( $\gamma\uparrow \Leftrightarrow C\downarrow$ )

c) Za zadani $x$ možemo znati $\phi(x)$ ako imamo zadanu bazu fju, no mi imamo zadan kernel.

d) 1) ne 2) ne 3) da 4) da (?)