PRISMS PhaseField
Coupled Cahn-Hilliard and Mechanics formulation

We only present the weak formulation of the governing equations here for the coupled model. Refer to the formulations for Cahn-Hilliard dynamics and Mechanics for the full derivation of the respective governing equations.

1 Weak formulation

In the weak formulation, considering an arbitrary variation w, the governing equations for the coupled problem are given by:

$$\int_\Omega w\mu^{n+1}\ dV = \int_\Omega w\ \underbrace{f^n_{,c}}_{r_{mu}} + \boldsymbol{\nabla} w \cdot \underbrace{\kappa \boldsymbol{\nabla} c^n}_{r_{mux}}\ dV \tag{1}$$

$$\int_\Omega w c^{n+1}\ dV = \int_\Omega w\ \underbrace{c^n}_{r_c} + \boldsymbol{\nabla} w\ \underbrace{(-\Delta t M)\ \cdot (\boldsymbol{\nabla}\mu^{n+1})}_{r_{cx}}\ dV \tag{2}$$

$$\int_\Omega \boldsymbol{\nabla} w : \sigma\ dV - \int_{\partial\Omega} w \cdot t\ dS = 0 \tag{3}$$

where $\sigma = C : (\varepsilon - \varepsilon^c)$ and $\varepsilon^c_{ij} = 0.01\ \delta_{ij}$ is the chemical strain.

The above values of r_μ, $r_{\mu x}$, r_c and r_{cx} are used to define the residuals in the following parameters file: *applications/coupledCahnHilliardMechanics/parameters.h*.