

PRISMS PhaseField

Classical Diffusion: Fick's law

1 Kinetics

The Parabolic PDE for diffusion is given by:

$$\frac{\partial c}{\partial t} = -\nabla \cdot (-D \nabla c) + f \quad (1)$$

where D is the diffusion constant and f is a source term

2 Time discretization

Considering forward Euler explicit time stepping, we have the time discretized kinetics equation:

$$c^{n+1} = c^n + (\Delta t D) \Delta c^n + \Delta t f^n \quad (2)$$

3 Weak formulation

In the weak formulation, considering an arbitrary variation w , the above equation can be expressed as a residual equation:

$$\int_{\Omega} w c^{n+1} dV = \int_{\Omega} w c^n + w(\Delta t D) \Delta c^n + w \Delta t f^n dV \quad (3)$$

$$= \int_{\Omega} w(c^n + \Delta t f^n) - \nabla w \cdot (\Delta t D) \nabla c^n dV + \int_{\partial\Omega} w(\Delta t D) \nabla c^n \cdot n dS \quad (4)$$

$$= \int_{\Omega} w(c^n + \Delta t f^n) - \nabla w \cdot (\Delta t D) \nabla c^n dV + \int_{\partial\Omega} w(\Delta t D) j^n dS \quad (5)$$

$$= \int_{\Omega} w \underbrace{(c^n + \Delta t f^n)}_{r_c} + \nabla w \cdot \underbrace{(-\Delta t D) \nabla c^n}_{r_{cx}} dV \quad [\text{assuming flux } j = 0] \quad (6)$$

The above values of r_c and r_{cx} are used to define the residuals in the following parameters file:
applications/fickianFlux/parameters.h