eXtreme Ordinary Differential Equation Solver

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This is the documentation of the eXtreme Ordinary Differential Equation Solver (XODES) as detailed in arxiv:1903.02624. This includes an overview of each component and the necessary steps to compile and execute the program. For any questions, contact Andrew Sullivan at andrew.sullivan4@montana.edu.

1 Folder Structure

The XODES folder is structured as such:

- Code
  - BVP_BICO.c: BICOnjugate Gradient Method
  - BVP_control.c: Main loop controller for Newton-Raphson method.
  - BVP_GMRES.c: GEneralized RESidual Method
  - BVP_gridsize.c: Grid adjustment and interpolation.
  - BVP_header.h: Main header file.
  - BVP_IC.c: Initial conditions.
  - BVP_LUdcmop.c: LU Decomposition Method.
  - BVP_out.c: Output generator. Output solution to directory 'Data/BVPout_sols/Sol_Funcs/'
  - BVP_physics.c: Extract physical observables. Outputs observable parameters to directory 'Data/BVPout_sols/Sol_Props/'.
  - BVP_sys.c: System residual and Jacobian evaluation.
  - BVP.c: Main file to initialize executable variables.
- Documentation
  - Documentation File
  - Linear Solvers.pdf: Documentation of the linear solver algorithms, LU decomposition, Biconjugate gradient stabilized, and generalized residual method.
- Execs
  - BVP.exe: Main executable
- Funcs
  - FEqEEPSIout.c: C file with procedurally generated function declarations of the scalar field equation and its partial derivatives that is #included in the C header file.
  - FEqEERRout.c: C file with procedurally generated function declarations of the r-r component of modified Einstein equation and its partial derivatives that is #included in the C header file.

  - FEqEETTout.c: C file with procedurally generated function declarations of the t-t component of modified Einstein equation and its partial derivatives that is #included in the C header file.
  - FEqISCOLRout.c: C file with procedurally generated function declarations of the differential equation that governs the location of the ISCO and light ring that is #included in the C header file.

- Gens
  - Maple_Export Folder
    * This folder contains the Maple field equation calculation and export. Filenames have the structure "FE_THEORY_export.mw". It creates the C-files in 'Funcs' folder.
- BVPcompile.sh: Bash compile file
- BVPbatch.sh: Bash executable file

1.1 Compilation Pipeline

The complete step-by-step process is:

1. Execute Maple worksheet of modified Einstein equations of interest. (Crtl+Shift+Enter executes entire Maple worksheet). This will calculate the modified Einstein equations and procedurally calculate and export each component and its partial derivatives. Then it will export the results to the 'FEqEETTout.c', 'FEqEERRout.c', 'FEqEEPSIout.c' C-files. Alternatively, one can simply copy-paste these already generated files for each coupling function located in the folders 'Funcs-LinCoupling' and 'Funcs-ExpCoupling' respectively.
2. Execute Maple worksheet of ISCO and light ring differential equation calculation. This will calculate and export the two differential equations to the 'FEqISCOLRout.c' C-file. Alternatively, one can simply use the already generated file located in the 'Funcs' folder.
3. The 'BVPcompile.sh' bash script will gcc compile the program to the 'BVP.exe' executable located in the 'Execs' folder.
4. Execute program with 'BVPbatch.sh' to determine inputs. This will also automatically remove and create the directories 'Data/BVPout_sols' where the solution and physical observables are located.

There are 5 inputs of the executable 'BVP.exe {1} {2} {3} {4} {5}':

1. {1}: Coupling parameter α, entered as $\alpha = 10^{-5}\{1\}$. For example, to enter $\alpha = 0.001$, $\{1\} = 100$.
2. {2}: Desired tolerance, entered as $\text{tol} = 10^{\{2\}}$. For example, to enter $\text{tol} = 10^{-5}$, $\{2\} = -5$.
3. {3}: Number of grid points, entered as $n = \{3\}$.
4. {4}: Newton polynomial order, $r = \{4\}$.
5. {5}: Horizon radius parameter, $r_{\text{H}} = \{5\}$.

As an example, to execute for a coupling value $\alpha = 0.001$, tolerance $\text{tol} = 10^{-5}$, $n = 101$ grid points, Newton polynomial order $r = 12$ and a horizon radius $r_{\text{H}} = 1.0$: enter 'BVP.exe 100 -5 101 12 1.0'