Paper discusses the challenges that we are offering a solution to for the 0/1 designs. This paper was written in 2009.

The SIMP-SRV Method for Stiffness Topology Optimization of Continuum Structures

Xiangyang Zhou* **Liping Chen** **Zhengdong Huang**
CAD Center, School of Mechanical Science & Engineering, HuaZhong University of Science and Technology, Wuhan, Hubei, 430074, P.R. China

Abstract

In density-based topology optimization, 0/1 solutions are sought. Discrete topological problems are often relaxed with continuous design variables so that they can be solved using continuous mathematical programming. Although the relaxed methods are practical, grey areas appear in the optimum topologies. SIMP (Solid Isotropic Microstructures with Penalization) employs penalty schemes to suppress the intermediate densities. SRV (the Sum of the Reciprocal Variables) drives the solution to a 0/1 layout with the SRV constraint. However, both methods cannot effectively remove all the grey areas. SRV has some numerical aspects. In this work, a new scheme SIMP-SRV is proposed by combining SIMP and SRV approaches, where SIMP is employed to generate an intermediate solution to initialize the design variables and SRV is then adopted to produce the final design. The new method turned out to be very effective in conjunction with the method of moving asymptotes (MMA) when using for the stiffness topology optimization of continuum structures for minimum compliance. The numerical examples show that the hybrid technique can effectively remove all grey areas and generate stiffer optimal designs characterized with a sharper boundary in contrast to SIMP and SRV.

Key Words: Topology optimization, Discrete variable, SIMP method, SRV constraint

1. Introduction

Structural topology optimization has been becoming an interesting research area in the structural optimization community, which has been applied to many engineering areas successfully [1]. Topology optimization can be formulated as material distribution problem that optimally distributes solid and void material over a fixed design domain [2]. Each design variable defines the existence or non-existence of material at a particular location. Therefore, topology optimization problem is essentially an integer programming with 0 and 1 discrete design variables. Unfortunately, the design problems posed in this way usually ill-posed and cannot be directly solved by using many continuum-type optimization technologies, because the topology optimization problem formulated in this way usually tends to the so-called "combinatorial explosion". As a result, some alternative methods were developed to solve the optimization problems meaningfully.

∗ Corresponding author:
Tel: +86-(0)27-6711-9693
Fax: +86-(0)27-8725-1947
Email: zxy_paper @126.com

One common practice is to relax 0/1 design variables using the homogenization method [2] so that the material distribution problem can be solved using most mathematical programming with continuous design variables. However, the homogenization method need to compute the effective elastic modulus of the porous material and each element has several design variables. In addition, the optimal topologies generated by this way are difficult to manufacture since many regions with perforated material will be involved. As an alternative method, the power-law approach [3], which is also called the solid isotropic microstructure with penalization (SIMP), has got a general acceptance in recent years due to its computational efficiency and conceptual simplicity.

In the SIMP method, the dependence of material properties with design variables is expressed in terms of the material density using a simple 'power-law' interpolation, which means the intermediate values are suppressed by penalizing the bulk densities. In other words, the SIMP approach is to replace the integer variables with continuous variables, and then introduce some form of penalty that steers the solution to discrete 0/1 values. Once the original problem is relaxed in this way, grey regions with intermediate densities between 0 and 1 may occur in the optimum topologies. To ensure an easy interpretation of distinct 0/1 optimal topologies, penalty schemes are often employed to suppress the intermediate densities. In fact, the SIMP method does indeed have a tendency to remove grey areas, thus producing 0/1 results. SIMP is so easy to implement that it, in enhancing 0/1 solutions, has achieved prominent status in the topology optimization community. However, SIMP does not directly resolve the non-existence of solutions [1] and thus numerical instabilities may occur [4]. Consequently, many numerical techniques are incorporated to make the optimization problem well-posed [5, 6, 7, 8]. Unfortunately, SIMP can not absolutely preclude grey areas with intermediate densities around structural boundary in the final results.

Fuchs [9] proposes a possible alternative to SIMP for generating 0/1 structures. The design variables are still the densities of the finite elements but Young's modulus is a linear function of these densities, in some sense, a SIMP material without penalty. To drive the solution to a 0/1 layout, a new constraint labeled the sum of the reciprocal variables (SRV) is introduced into the optimization problem. The constraint stipulates that the SRV must be larger or equal to its value at a discrete design for a specified amount of material. This method has turned out to be very effective in conjunction with the method of moving asymptotes (MMA) [10]. 0/1 results can be obtained in the optimal design of piezoelectric (PZT) patches for reducing the noise of vibrating surfaces [9]. When applied to compliance topology optimizations, it is expected that SRV can produce stiffer and sharper 0/1 structures than SIMP for the same amount of material. But there are only preliminary results and some numerical aspects of the method are not addressed in Fuchs's paper. In addition, there are some unresolved instabilities in the program.

In this paper, the authors attempted to propose a combined approach based on SIMP and SRV that employs the optimized design variables of SIMP to initialize the design variables of SRV, and we therefore call this approach SIMP-SRV. When the new method is used for classical stiffness topology optimization of structures for minimum compliance, there are no grey areas with intermediate densities and the obtained structures are stiffer than either the SIMP results or the SRV results.

This paper is organized as follows: In section 2, the SIMP method is briefly reviewed. The SRV constraint is briefly introduced in section 3. SIMP-SRV method is motivated in section 4. While in section 5, the SIMP-SRV method is presented. Some numerical results are presented in Section 6. Finally, in Section 7, the paper concludes with a summary and a succinct discussion.

2. The SIMP Method

The design region is meshed into a fixed grid of n finite elements. All elements carry densities that constitute the design variables. The objective is to find an optimal material distribution in the design domain that subjected to some given constraints, leading to minimizing a specified objective function, more often than not the compliance of the structure. The standard approach is to let the design variables represent the relative densities of the material in related elements, where the density can vary from zero to one. To avoid the singularity of the matrix, the density variables are given a lower limit. Topology optimization problem to minimize the compliance of the structure while it is subjected to a limited amount of material in the design domain can be written as

$$\begin{cases} \underset{X=(x_1,x_2,\ldots,x_n)^T}{\underline{\text{Minimize}}} : C(X) = \{F\}^T \{U\} \\ \text{Subject} \quad \text{to}: \begin{cases} V^T X \le V^*, \\ 0 < x_{\min} \le x_i \le 1 \ (i = 1 \ldots n), \\ \{F\} = [K]\{U\}. \end{cases} \end{cases} \tag{1}$$

where X is the design variable, C is the compliance of the structure, V^T is a vector containing the volume of the elements, V^* is the volume constraint, F is the load vector, U is the displacement vector, K is the stiffness matrix.

This formulation will yield solutions with intermediate densities (grey elements). In order to suppress these grey elements, the SIMP method defines the material elasticity modulus in element i as

$$E^p(x_i) = x_i^{\,p} E^0 \tag{2}$$

where E^0 is the modulus of the bulk material and p is the penalty exponent. The stiffness matrix is given by:

$$K = K(X) = \sum_i^n E K_i = \sum_i^n x_i^{\,p} E^0 K_i \tag{3}$$

Here, K_i is the element stiffness matrix. Bendsøe and Sigmund [3] prove that the power-law approach is perfectly valid when p is sufficiently big (in order to obtain true ‘0/1’ designs, $p \geq 3$ is usually required). The reason is that, for such a choice, intermediate densities are penalized when the volume constraint is active; volume is proportional to x_i, but stiffness is much less than proportional to x_i.

3. The SRV Constraint

The nomenclature SRV stands for the Sum of the Reciprocal Variables, which is defined as [9]

$$SRV = \sum_i \frac{1}{x_i}; x_{\min} \leq x_i \leq 1; i = 1 \ldots n \tag{4}$$

where topology optimization problem is the same as SIMP but the design domain is discretized into n equal (square) finite elements. Obviously, the denominator cannot be zero therefore the design variables have a small but finite lower limit $x_{\min}$, from which to 1 x_i can vary. It is reasonable since the element densities always have a minimum gage when using a fixed grid, as mentioned earlier. Every binary instance of the n-vector X in which there are m components with value 1 and $(n-m)$ components with value $x_{\min}$ is defined as

$$SRV_{\text{discrete}} = \frac{m}{1} + \frac{n-m}{x_{\min}} = m + \frac{n-m}{x_{\min}} \tag{5}$$

Fuchs [9] proves that for any vector X that is not discrete

$$SRV(X) < SRV_{\text{discrete}} \tag{6}$$

If the optimum design vector is posed to be discrete, $SRV_{\text{opt}} = SRV_{\text{discrete}}$, there will be $SRV(X) < SRV_{opt}$. In other words, for the solution to be discrete, a necessary condition is that SRV be a local maximum [9]. Now, the SRV constraint is defined as

$$\sum_i \frac{1}{x_i} \geq m + \frac{n-m}{x_{\min}} \quad \text{(the SRV constraint)} \tag{7}$$

Distinctly, since the maximum value of the sum of the inverses is $m + (n-m)/x_{\min}$, the only way to comply with the SRV constraint is to satisfy the equality, that is, to be at a discrete design. The SRV constrain could have been presented in the equality form but minimization algorithms usually perform better with inequalities. For similar numerical reasons the SRV constraint is usually relaxed to the form

$$\sum_i \frac{1}{x_i} \geq \eta\left(m + \frac{n-m}{x_{\min}}\right) \quad \text{(the relaxed SRV constraint)} \tag{8}$$

where the coefficient η was typically recommended by Fuchs [9] to be 0.95.

So, the topological optimization problem to minimize the compliance of the structure subjected to a volume of material constraint with equal finite elements can be described as

$$\begin{cases} \underset{X=(x_1,x_2,\ldots,x_n)^T}{\underline{\text{Minimize}}} : C(X) = \{F\}^T \{U\} \\ \text{Subject to}: \begin{cases} \sum_i x_i = m + (n-m)x_{\min} \\ (n > m) \\ \sum_i \frac{1}{x_i} \geq \eta\left(m + \frac{n-m}{x_{\min}}\right) \\ 0 < x_{\min} \leq x_i \leq 1 \ (i = 1 \cdots n) \\ \{F\} = [K]\{U\} \end{cases} \end{cases} \tag{9}$$

Above is the Fuchs's SRV constraint method. It is successful in application of the layout design of PZT patches for minimizing the acoustical noise emanating from vibrating surfaces. When SRV is applied in classical topological design of structures for minimum compliance, sharp 0/1 results are obtained with smaller values of the objective functions than those of SIMP [9].

4. Motivation for SIMP-SRV Method

As Fuchs [9] mentioned, SRV has some objective advantages over SIMP. It seems that there is nothing to prove that SIMP can generate 0/1 solutions. And the proposed SRV method is a technique that can be substantiated analytically; it must converge to a 0/1 design, which appears in the formulation. But there are only preliminary results and some numerical aspects of the method are not addressed in Fuchs's paper. In addition, there are some unresolved instabilities in the program: the success of the implementation of SRV with MMA hinges on the coefficient η and the convergence of the algorithm is rather sensitive to the parameter. The authors try to resolve the problem.

4.1 An alternative of SRV method

Since minimization algorithms usually perform better with inequalities, we change the equation constraint in (9) into two inequalities. Analyzing the equations (4) and (5), there is a necessary condition for $SRV(X) < SRV_{discrete}$, which is the equality constraint on the material amount in (9). In other words, the equality constraint must be satisfied strictly. Otherwise, when $\sum_i x_i \neq m+(n-m)x_{\min}$, $SRV(X) < SRV_{discrete}$ is not always valid. Consequently, the equality constraint on the amount of material is replaced by two inequality constraints in Eq. (10) and is not relaxed as Fuchs uses $\eta = 0.95$ for the lower bound constraint.

$$\begin{cases} \sum_i x_i \leq m+(n-m)x_{\min} \\ m+(n-m)x_{\min} \leq \sum_i x_i \end{cases} \tag{10}$$

So, the model (9) is replaced by (11) as follows

$$\begin{cases} \underset{X=(x_1,x_2,\ldots,x_n)^T}{\underline{\text{Minimize}}} : C(X) = \{F\}^T \{U\} \\ \text{Subject to}: \begin{cases} \sum_i x_i \leq m+(n-m)x_{\min} \\ m+(n-m)x_{\min} \leq \sum_i x_i \\ (n>m) \\ \sum_i \dfrac{1}{x_i} \geq \eta(m+\dfrac{n-m}{x_{\min}}) \\ 0 < x_{\min} \leq x_i \leq 1 \;(i=1 \text{L} \;\; n) \\ \{F\} = [K]\{U\} \end{cases} \end{cases} \tag{11}$$

4.2 Numerical results of SRV method

The authors tested Fuchs's [9] method with model (11) for a cantilever example from Sigmund [11]. But the solution is not what is expected; the obtained structure is not like a cantilever and its compliance is much higher than both Fuchs's and Sigmund's. That's to say, the test failed. In order to find the reasons, the authors change some parameters. And the numerical results are given in following accordingly.

4.2.1 The lower limit of the design variables

While parameter $x_{\min}$ is originally set to 0.001 in Fuchs's [9] paper, some different values were observed in this work. When it is set to 0.01, the algorithm converges so fast that the structure is not shaped. But, when it is set to 0.0001, the algorithm converges very slowly and the structure is different from Fuchs's. Analyzing the SRV constraint (7), we can understand easily that the procedure is sensitive to $x_{\min}$ because the lower limit of density variables is small as a denominator and its slight variation will have a great impact on the right value of inequality (7).

4.2.2 The weighting factor η of the SRV constraint

Fuchs [9] emphasizes the parameter η has great influence on the success of the numerical procedure and the convergence of the algorithm. In fact, the solution varies with different η in this work. Even if η is set to 0.95 as Fuchs assigned in his paper [9], the obtained results are not what is expected. But, as will been seen, once initialized the design variables with the solution of SIMP, SRV with MMA can implement successfully with being immune from dependence on the coefficient η and the convergence of the algorithm is also not strongly sensitive to the parameter. We will discuss how η impacts on the results for using SIMP-SRV in the following section.

4.2.3 The initial values of design variable

The authors tested the SRV method with different initial values of design variable. After the design variable's initial value, which is marked as X_0, is changed, the results vary too. This implies that the success of the numerical procedure seems to hinge on the starting solution.

Some theoretical explanations may be presented. Analyzing the SRV constraint in Eq. (6), it is understandable easily that the constraint is non-convex highly and as such local optima may be abundant. The non-convexity typically means that one can find several different local minima (which is what the gradient based algorithms locate) and one can obtain different solutions to the same discretized problem when choosing different starting solutions and different parameters of the algorithms. In addition, the constraint conditions in SRV are stronger than those in SIMP. Consequently, various minima may encounter and as such the results are various.

5. SIMP-SRV Approach

From the above discussion, it is natural to combine SIMP with SRV by using the optimal X which obtained from SIMP for the initial X of SRV. In other words, SIMP-SRV is a two-pass method based on SIMP and SRV approaches: firstly, the standard SIMP is employed to generate an intermediate optimal design solution X^{int}; secondly, SRV approach is adopted to produce the final solution X^*, initializing the design variables X with X^{int}. This new method is called as SIMP-SRV. The main idea of SIMP-SRV is using the optimal solutions of SIMP for the starting solutions of SRV to obtain the final optimized solutions.

The algorithm and models are taken from Sigmund [11]. But the heuristic minimization is avoided and instead MMA is called as a subroutine because the constraints are more than one. The SIMP model is Eq. (1) in which p =3; the SRV model is Eq. (11) in which p =1, and the other relevant parameters are the same as Sigmund's [11]. The algorithm flowchart of SIMP-SRV is shown in Fig.1.

In topology design (as in structural problems in the large), most problems are not convex. Moreover, many problems have multiple optima, i.e. non-unique solutions. An example of the latter is the design of a structure in uniaxial tension. Non-convexity will produces some numerical problems such as local minima and non-uniqueness. Most global optimization methods seem to be unable to handle problems of the size of a typical topology optimization problem. Based on experience, it seems that continuation methods must be applied to ensure some sort of stable convergence towards reliably good designs [1]. The idea of continuation methods is to gradually change the optimization problem from an (artificial) convex (or quasi-convex) problem to the original (non-convex) design problem in a number of steps. In each step a gradient-based optimization algorithm is used until convergence.

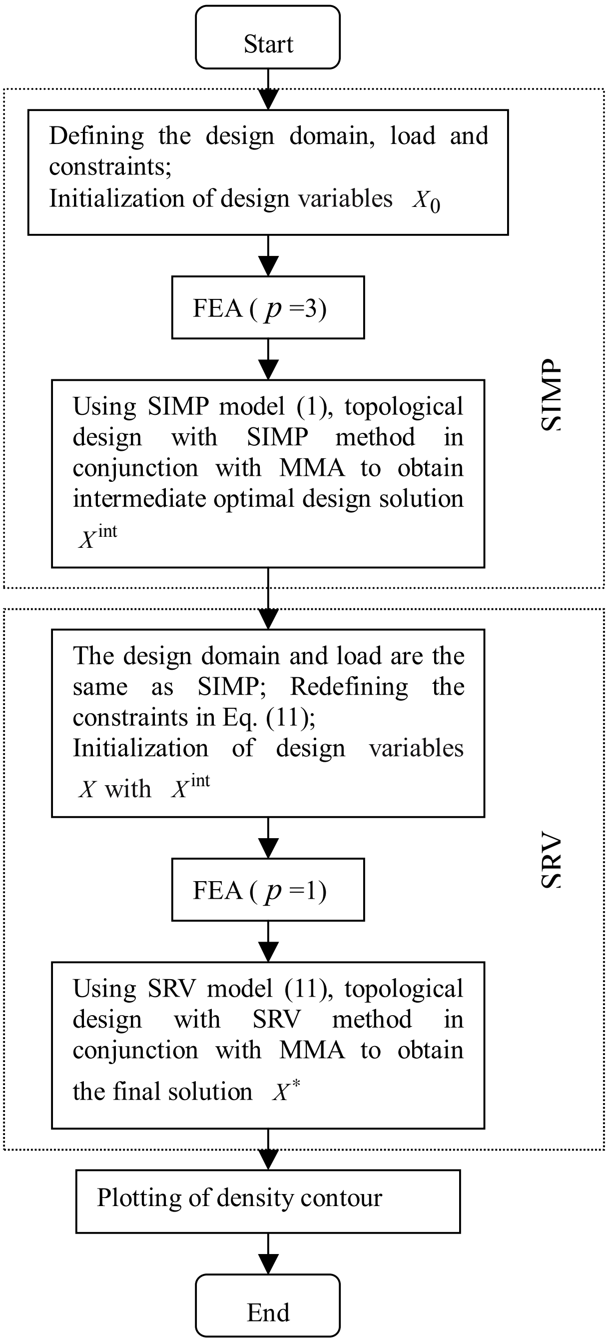


Fig.1 Flowchart for SIMP-SRV method

According to the continuation methods, for the mesh-independence filter it is normally recommended to start with a large value of the filter size $r_{\min}$ (which gives designs with blurry edges) and gradually decrease it, to end up with a well-defined 0/1design. So, a clear 0/1 design can also be obtained by simply stopping the filter when the optimization nears convergence in SIMP. But a key is when the filter should stop. The different stopping time of the filter will lead to different results, which will be a new numerical instability. From another point of view, we would like to advocate another method for achieving 0/1 solutions in density-based topological design, mainly because it is in the nature of research to tread unbeaten paths. SIMP-SRV can be an alternative to SIMP. It could turn out that each technique (SIMP or SIMP-SRV) has its particular niches where it performs best.

Incidentally, we employ SIMP but not SRV in the first stage because SIMP seems not to strongly depend on the initial values of design variables as SRV does. The authors have tested a group of different initial values of design variables for cantilever example under a tip load with the SIMP method. The results show that all the topological structures are similar to the cantilever beam in Fig.2 (a). As can be seen in Table 1, the differences of the compliances for different starting solutions are small.

Table 1 The compliance of cantilever example for different initial values of design variable with SIMP. The volume constraint is 40% (vol=0.4).

x_{0i}	0.01	0.25vol	0.50vol	0.75vol	1.00vol	1.25vol	1.50vol	1.75vol	2.00vol	0.90	1.00
C	57.3286	57.3142	57.3332	57.3327	57.3337	57.3466	57.3113	57.3465	57.3626	57.3520	57.3585

This is all the SIMP-SRV approach. The Matlab implementation of MMA was kindly provided by Svanberg and all the examples were run on that platform. Although it needs further study, some encouraging preliminary results, which will be presented in the following section, have been obtained. Of course, perhaps there are some other conjunction ways.

6. Numerical Examples

In this work, the SIMP-SRV approach was only implemented for achieving a minimum compliance in classical topological design of structures. The first example, in which a group of η were tested and its influence on the results for using SIMP-SRV was analyzed, was the cantilever under a tip load from Sigmund [11]. The other was a bridge under a group of loads which are distributed uniformly in the upper edge. In all instances we have compared SIMP-SRV with SIMP under similar conditions. And only the first is compared with SRV because there was no example for the second problem in Fuchs' [9] paper. We will see the advantages of the hybrid method over SIMP and SRV in terms of sharpness and stiffness of the 0/1 results.

6.1 Cantilever example

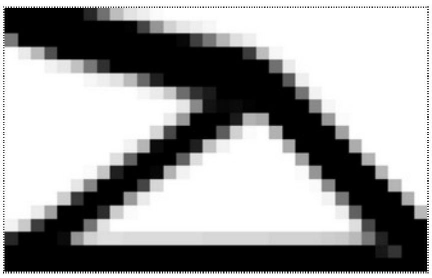
In this case, the relevant parameters in the SIMP stage are the same as Sigmund's [11] with $x_{\min} = 0.001$. In the SRV stage, where $p = 1$, the initial X is the optimal X of the SIMP stage and the other parameters are unchanged. In SRV design of Fuchs [9], the weighting factors η is set to 0.95; the equality constraint on the amount of material is replaced by two inequality constraints and a coefficient 0.95 is used for the lower bound. Here, there is no coefficient for the lower limit of the volume; in other words, the coefficient is set to 1. Fuchs [9] emphasizes the parameter η has great influence on the success of the numerical procedure and the convergence of the algorithm. Consequently, the author analyzed five cases with different η in SIMP-SRV design. The optimal topologies of cantilever beam under a down-load at the tip with different approaches are shown in Fig.2. The values of the objective function and the iterations can be seen in Table 2.

6.1.2 Comparison with numerical results

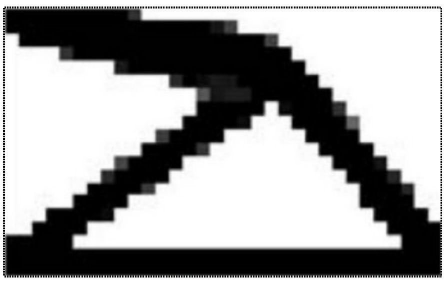
To validate the optimal structures found with the developed computer code, a comparison with the result found by SIMP in conjunction with MMA and the one in the paper by Fuchs [9] is given here. This subsection also investigates the influence of varied η.

The resulting topologies of cantilever beam are showed in Fig.2. An important observation is that the grey elements of SIMP-SRV are much less than those of SIMP. With the increase of η, the grey elements can be removed gradually. According to the formulation of the SRV constraint, the topological design must be in clear 0/1 patterns if it is not relaxed. This is in fact obtained by setting η to 1. As seen in Fig.2 (g), there is no any grey element. Such turns out that the quantity of grey elements is sensitive to η.

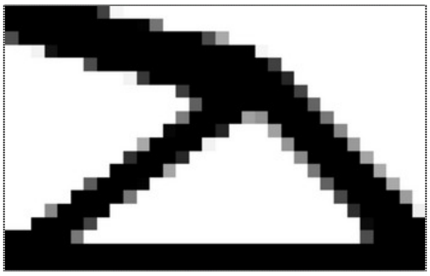
In Table 2, the resulting objectives and iterations are presented. As can be seen, the difference in the seven objectives is significant. It is obvious that the compliance of SIMP is the highest. The objective of the two-pass method can decrease about 10%. Under similar conditions with η=0.95, the optimal structure of SIMP-SRV is also stiffer than that of SRV. It can be seen from the numbers following the plus sign that the additional cost of computation is not much greater than SIMP.



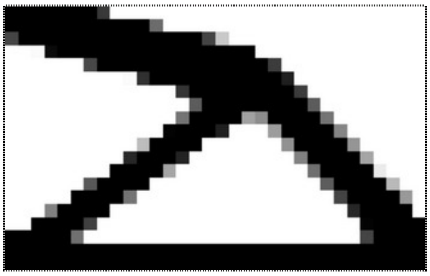
(a) SIMP design



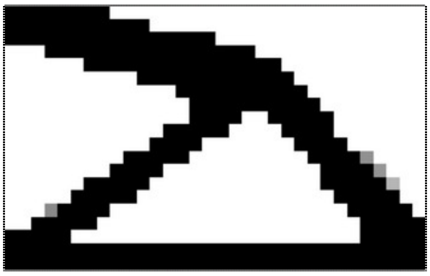
(b) SRV design (Fuchs, 2005)



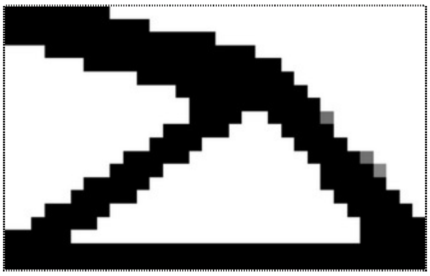
(c) SIMP-SRV design, $\eta = 0.90$



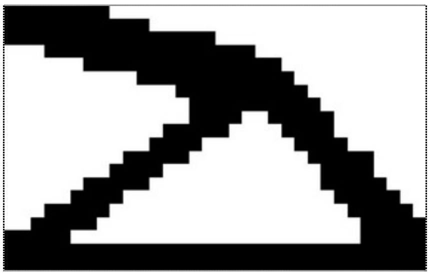
(d) SIMP-SRV design, $\eta = 0.95$



(e) SIMP-SRV design, $\eta = 0.97$



(f) SIMP-SRV design, $\eta = 0.99$



(g) SIMP-SRV design, $\eta = 1.0$

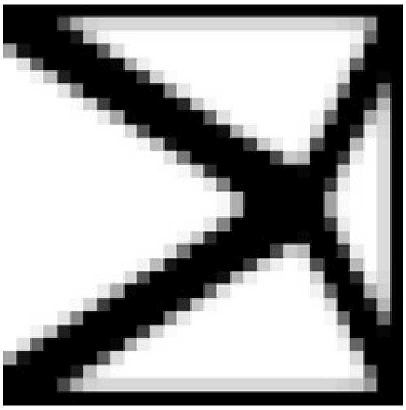
Fig.2 Optimal topology of cantilever beam under a down-load at the tip

Table 2 Different compliance values and iterations of cantilever. The numbers within parenthesis are the relative compliances at the minimum. 114 is the iteration in the SIMP stage and the numbers following the plus sign are the iterations in the SRV stage respectively.

	SIMP	SRV	SIMP-SRV				
		(Fuchs 2005)	$\eta = 0.90$	$\eta = 0.95$	$\eta = 0.97$	$\eta = 0.99$	$\eta = 1.0$
Compliance	57.3337	~	51.8361	52.0330	53.1416	53.2145	53.4508
(Percent)	(100%)	(92%)	(90.41%)	(90.75%)	(92.69%)	(92.82%)	(93.23%)
Iteration	114	~	114+6=120	114+19=133	114+13=127	114+17=131	114+23=137

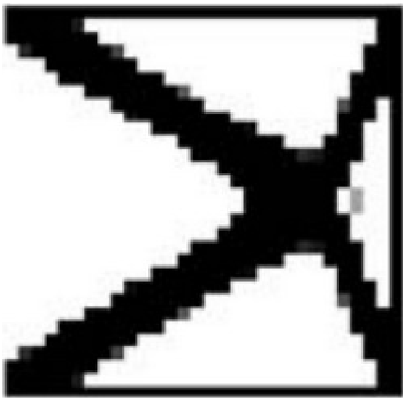
6.2 Cantilever with two loading example

We have also run the SIMP-SRV method without filtering (see Fig.3). In this case, the cantilever beam is under two loading conditions: a down-load and an up-load at the tip. The results are compared with SIMP and SRV.

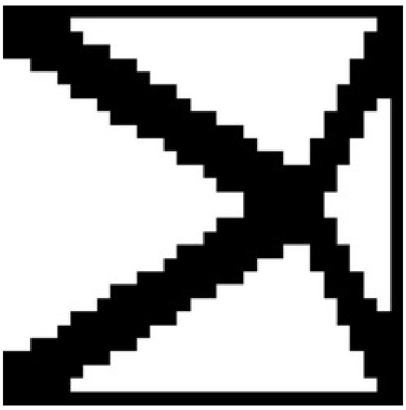


(a) SIMP design

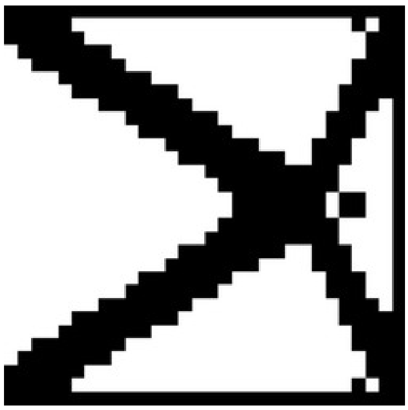
(b) SRV design including filter (Fuchs, 2005)



(c) SRV design without filter (Fuchs, 2005)



(d) SIMP-SRV design including filter($\eta = 1$)



(e) SIMP-SRV design without filter($\eta = 1$)

Fig.3 Optimal topology of cantilever beam under a down-load at the tip

The relative minimum compliances are: (a) 100% (61.5279); (b) 91.2%; (c) 91.6%; (d) 92.29% (56.7865); (e) 92.87% (57.1388), respectively. The results indicate that the grey areas would be removed even if there is no filter, which shows the 0/1 feature is inherent to the method.

6.3 Bridge example

The design problem has been shown in Fig. 4. The domain is meshed with 50×20 elements. The support areas are fixed at the right and the left edge and the middle of the bottom edge. The top edge is solid to endure a group of distributed uniformly roads. The volume constraint is 40% of the total volume.

The other relevant parameters are the same to the cantilever example. In this test case, we only compared the SIMP-SRV with SIMP and in the former design two cases are analyzed; in one $\eta = 0.95$ and in the other $\eta = 1.0$. The compliance at the minimum with SIMP is C= 3137.6281. And the compliance in the two cases with SIMP-SRV are C = 2862.8064 and C = 2967.5242, respectively. The optimal topological structures of bridge are shown in Fig.5. It is obvious that the objective of SIMP-SRV is better than that of SIMP. And there are fewer grey areas in the structures of SIMP-SRV. Once the SRV constraint is not relaxed, the grey areas disappear completely. The bridge example shows again that the SIMP-SRV method is superior to the SIMP method.

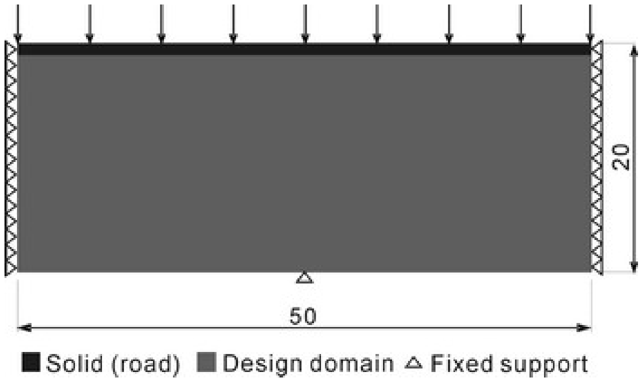
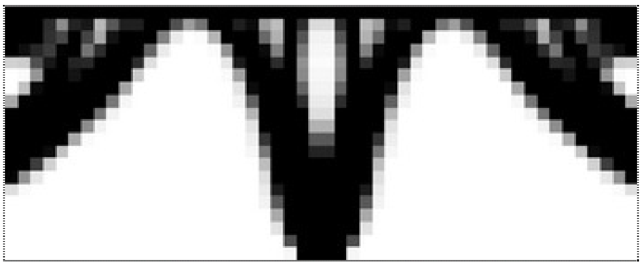
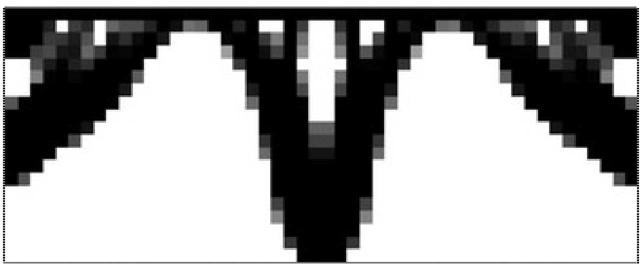


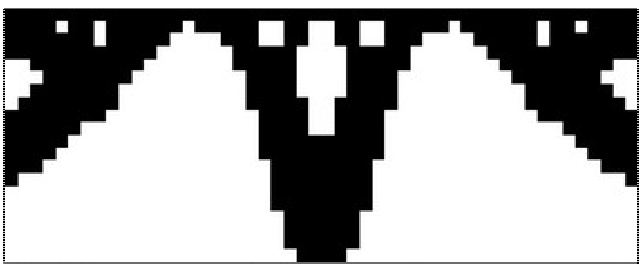
Fig.4 Design domain of bridge



(a) SIMP design



(b) SIMP-SRV design, $\eta = 0.95$



(c) SIMP-SRV design, $\eta = 1.0$

Fig.5 Optimal topology of bridge defined in Fig.4.

7. Summary and conclusions

In density-based topology optimization, 0/1 solutions are the objective sought by researchers. Recently, SIMP has been accepted in the field for its simplicity and efficiency. Lately, the SRV technique is presented by considering its simplicity and validity. SRV is in the initial stage and has some numerical issues that need to be further addressed, but the authors combine SIMP approach and SRV constraint for topology optimization: the SIMP-SRV method. In order to obtain better design, a two stages' algorithm is used in SIMP-SRV: the first was done by SIMP in conjunction with MMA; in the second, the solution of the previous stage is used for this stage's starting solution and then SRV accomplishes the following optimizing process to obtain the final optimal solution with the help of MMA. It can be turned out SIMP-SRV is more stable than SRV in the implementation of program. Cantilever and bridge examples were employed to test the potential of the SIMP-SRV method. Different relaxed coefficients of the SRV constraint were tested in order to study how it affects the solutions in cantilever example.

Although the SRV method seems not be more advantageous than the SIMP method because of the non-convexity of the SRV constraint, the examples of classical topology design for structural stiffness designs have shown the effectiveness of the proposed combined method SIMP-SRV. SRV tenders the results in the second stage, which is close to the binary set and even is a real binary set. Moreover, compared with the results of SIMP, the total compliance of SIMP-SRV decreases, which is of more important meaning. Of course, the main work was done by SIMP, but the additional cost of computation is not great in the SRV stage. In a word, the SIMP-SRV method has some obvious advantages and deserves a close attention.

8. Acknowledgements

This work is supported by the National Natural Science Foundation of China (60574053). The authors show their acknowledgments to Prof. Krister Svanberg (Royal Institute of Technology, Sweden) for his supplying of MMA codes with Matlab.

References

1. Bendsøe, M.P., Sigmund, O. (2003), *Topology optimization: Theory, Methods, and Applications*, Springer, Berlin Heidelberg, New York.
2. Bendsøe, M.P. (1989), Optimal shape design as a material distribution problem, *Struct. Opt.* 1,193-202.
3. Bendsøe, M.P., Sigmund, O. (1999), Material interpolation schemes in topology optimization, *Archive of*

Applied Mechanics 69, 635-654.

4. Sigmund, O., Petersson, J. (1998), Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima, *Structural and Multidisciplinary Optimization* 16, 68-75.
5. Haber, R.B., Bendsøe, M.P., Jog, C.S. (1996), A new approach to variable-topology design using a constraint on the perimeter, *Structural and Multidisciplinary Optimization* 11, 1-12.
6. Bourdin, B. (2001), Filters in topology optimization, *International Journal for Numerical Methods in Engineering* 50, 2143-2158.
7. Sigmund, O. (2001), Design of Multiphysics Actuator using Topology Optimization-PartⅠ: One Material Structure, *Computer Methods in Applied Mechanics and Engineering* 190(49-50), 6577-6604.
8. Wang, S.Y., Wang, M.Y. (2006), Structural shape and topology optimization using an implicit free boundary parameterization method, *Computer Modeling in Engineering and Science* 13(2), 119-147.
9. Fuchs, M.B., Jiny, S., Peleg, N. (2005), The SRV constraint for 0/1 topological design, *Structural and Multidisciplinary Optimization* 30, 320-326.
10. Svanberg, K. (2002), A class of globally convergent optimization methods based on conservative con-vex separable approximations, *SIAM Journal of Optimization* 12(2), 555-573.
11. Sigmund, O. (2001), A 99 line topology optimization code written in MATLAB, *Structural and Multidisciplinary Optimization* 31(6), 419-429.