Collective communication costs in MPI

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https://surakuma.github.io/courses/daamtc.html

Cost model

- The links of the network are bidirectional
- Each process can send and receive at-most one message at the same time
- Time taken to send a message with n bytes between any two nodes – $T = \alpha + n\beta$
  - α: latency cost per message, β: transfer time per byte
- In case of reduction operation, γ: computation cost per byte
- P is the total number of processes

Allgather

p0	p1	p2	p3	p4	p5
a_0	a_1	a_2	a_3	a_4	a_5

Initial data

p0	p1	p2	p3	p4	p5
a_0	a_0	a_0	a_0	a_0	a_0
a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_2	a_2	a_2	a_2	a_2
a_3	a_3	a_3	a_3	a_3	a_3
a_4	a_4	a_4	a_4	a_4	a_4
a_5	a_5	a_5	a_5	a_5	a_5

Data after operation

Ring algorithm

- Takes total $P - 1$ steps
- In each step, process i sends its contribution to process $i + 1$ and receives the contribution from process $i - 1$
- $T_{ring} = (P - 1)\alpha + \left(\frac{P-1}{P}\right) n\beta$
- n is the total amount of data gathered on each processor

Recursive doubling algorithm for Allgather

p0	p1	p2	p3	p4	p5	p6	p7
a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7

Initial data

a_0	a_0	a_2	a_2	a_4	a_4	a_6	a_6
a_1	a_1	a_3	a_3	a_5	a_5	a_7	a_7

a_0	a_0	a_0	a_0	a_4	a_4	a_4	a_4
a_1	a_1	a_1	a_1	a_5	a_5	a_5	a_5
a_2	a_2	a_2	a_2	a_6	a_6	a_6	a_6
a_3	a_3	a_3	a_3	a_7	a_7	a_7	a_7

a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
a_3	a_3	a_3	a_3	a_3	a_3	a_3	a_3
a_4	a_4	a_4	a_4	a_4	a_4	a_4	a_4
a_5	a_5	a_5	a_5	a_5	a_5	a_5	a_5
a_6	a_6	a_6	a_6	a_6	a_6	a_6	a_6
a_7	a_7	a_7	a_7	a_7	a_7	a_7	a_7

- Assume P is a perfect power of 2
- In each step $k (0 \leq k < \lg P)$, processes that are at 2^k distance exchange their data
- $T_{rec\_dbl} = \lg P \alpha + \left(\frac{P-1}{P}\right) n\beta$
- Requires adaptation when P is not a-power-of-two

Bruck's algorithm for Allgather

p0	p1	p2	p3	p4	p5
a_0	a_1	a_2	a_3	a_4	a_5

Initial data

p0	p1	p2	p3	p4	p5
a_0	a_1	a_2	a_3	a_4	a_5
a_1	a_2	a_3	a_4	a_5	a_0

After step 0

p0	p1	p2	p3	p4	p5
a_0	a_1	a_2	a_3	a_4	a_5
a_1	a_2	a_3	a_4	a_5	a_0
a_2	a_3	a_4	a_5	a_0	a_1
a_3	a_4	a_5	a_0	a_1	a_2

After step 1

p0	p1	p2	p3	p4	p5
a_0	a_1	a_2	a_3	a_4	a_5
a_1	a_2	a_3	a_4	a_5	a_0
a_2	a_3	a_4	a_5	a_0	a_1
a_3	a_4	a_5	a_0	a_1	a_2
a_4	a_5	a_0	a_1	a_2	a_3
a_5	a_0	a_1	a_2	a_3	a_4

After step 2

p0	p1	p2	p3	p4	p5
a_0	a_0	a_0	a_0	a_0	a_0
a_1	a_1	a_1	a_1	a_1	a_1
a_2	a_2	a_2	a_2	a_2	a_2
a_3	a_3	a_3	a_3	a_3	a_3
a_4	a_4	a_4	a_4	a_4	a_4
a_5	a_5	a_5	a_5	a_5	a_5

After local shift

- $T_{bruck} = \lceil \lg P \rceil \alpha + \frac{P-1}{P} n\beta$
- In each step $k (0 \leq k < \lceil \lg P \rceil)$, process i sends data to process $(i - 2^k)\%P$ and receives data from process $(i + 2^k)\%P$

Broadcast

It broadcasts n words from the root to all processes.

p0	p1	p2	p3	p4	p5
a					

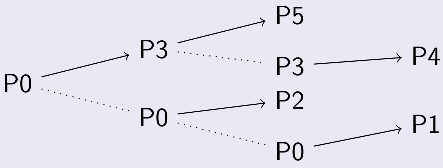
Initial setting

p0	p1	p2	p3	p4	p5
a	a	a	a	a	a

Data after operation

Bionomial tree algorithm

In the first step, the root sends data to process $(root + \frac{P}{2})$. This process and the root then act as new roots and recursively continue this algorithm.



- $T_{tree} = \lceil \lg P \rceil (\alpha + n\beta)$

Alternative approach: Scatter + Allgather

All-to-All

p0	p1	p2	p3	p4	p5
a_0	b_0	c_0	d_0	e_0	f_0
a_1	b_1	c_1	d_1	e_1	f_1
a_2	b_2	c_2	d_2	e_2	f_2
a_3	b_3	c_3	d_3	e_3	f_3
a_4	b_4	c_4	d_4	e_4	f_4
a_5	b_5	c_5	d_5	e_5	f_5

Initial settings

p0	p1	p2	p3	p4	p5
a_0	a_1	a_2	a_3	a_4	a_5
b_0	b_1	b_2	b_3	b_4	b_5
c_0	c_1	c_2	c_3	c_4	c_5
d_0	d_1	d_2	d_3	d_4	d_5
e_0	e_1	e_2	e_3	e_4	e_5
f_0	f_1	f_2	f_3	f_4	f_5

Data after operation

Algorithm by Thakur et al.

- In each step $k(1 \leq k < P)$, process i receives data from process $(i-k)\%P$ and send data to process $(i+k)\%P$
- $T = (P-1)\alpha + \frac{P-1}{P} n\beta$
- n is the total amount of data on any process in the beginning or end

Reduce-Scatter

p0	p1	p2	p3	p4	p5
a_0	b_0	c_0	d_0	e_0	f_0
a_1	b_1	c_1	d_1	e_1	f_1
a_2	b_2	c_2	d_2	e_2	f_2
a_3	b_3	c_3	d_3	e_3	f_3
a_4	b_4	c_4	d_4	e_4	f_4
a_5	b_5	c_5	d_5	e_5	f_5

Initial settings

p0	p1	p2	p3	p4	p5
x_0	x_1	x_2	x_3	x_4	x_5

Data after operation

$x_i = \text{Reduce}(a_i, b_i, c_i, d_i, e_i)$
Each process has n amount of data in the beginning.

Recursive halving algorithm (Assuming P is a perfect power of 2)

- Analogous to the recursive-doubling algorithm for Allgather
- In each step $k (1 \leq k \leq P)$, processes that are at $\frac{P}{2^k}$ distance exchange parts of their data
- Each process sends the data needed by all processes in the other half, receives the data needed by all processes in its own half
- $T_{rec\_half} = \lg P\alpha + \frac{P-1}{P} n\beta + \frac{P-1}{P} n\gamma$
- Requires adaptation when P is not a power-of-two

- With an adaptation of Bruck's algorithm:
  $T = \lceil \lg P \rceil \alpha + \frac{P-1}{P} n\beta + \frac{P-1}{P} n\gamma$

Reduce and Allreduce

Each process has n amount of data in the beginning.

Reduce

- With binomial tree algorithm, $T_{tree} = \lceil \lg P \rceil (\alpha + n\beta + n\gamma)$
- With Reduce-Scatter(Bruck's algorithm) + Gather(Binomial tree), $T = 2\lceil \lg P \rceil \alpha + 2\frac{P-1}{P} n\beta + \frac{P-1}{P} n\gamma$

Allreduce

- With Reduce-Scatter(Bruck's algorithm) + Allgather(Bruck's algorithm), $T = 2\lceil \lg P \rceil \alpha + 2\frac{P-1}{P} n\beta + \frac{P-1}{P} n\gamma$

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