GENERALIZED NON-PARAMETRIC FUNCTION APPROXIMATION

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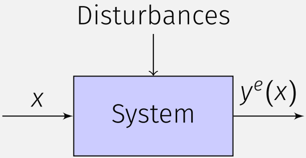
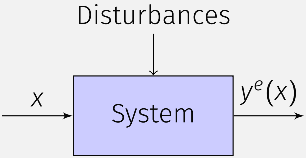
https://github.com/surenkum/uq_gaussian_processes

INTRODUCTION


Disturbances
x
System
$y^e(x)$


Disturbances
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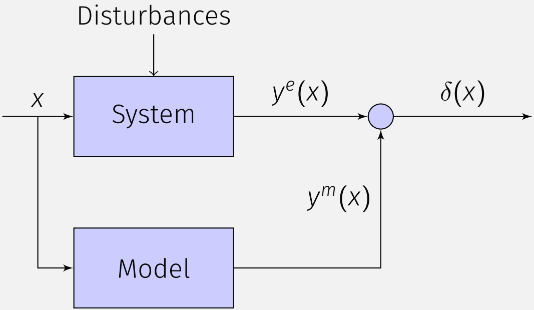
Disturbances
x
System
$y^e(x)$
$\delta(x)$
$y^m(x)$
Model

Bias Correction

$$y^e(x) = y^m(x) + \delta(x) + \epsilon$$

where ϵ is the experimental uncertainty.

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Parameter Calibration

$$y^e(x) = y^m(x, \theta^*) + \epsilon$$

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Bias Correction and Parameter Calibration

$$y^e(x) = y^m(x, \theta^*) + \delta(x) + \epsilon$$

- Lack of simulation results $y^m(x, \theta)$
- Parameterizing discrepancy function $\delta(x)$

Why Bayesian

- Integration of prior knowledge

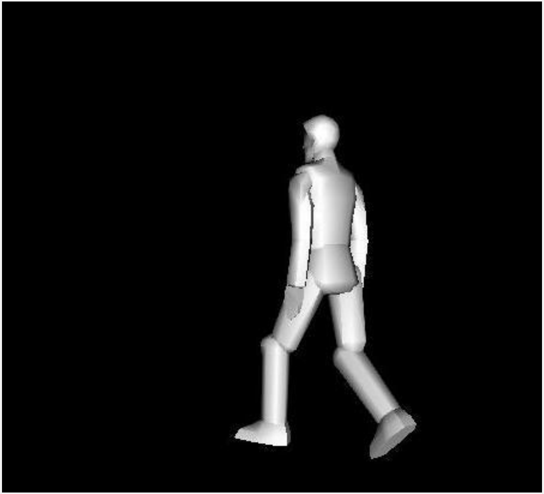
Bayesian calibration of computer models, Marc C. Kennedy, Anthony O'Hagan in Journal of the Royal Statistical Society, 2001

GAUSSIAN PROCESSES

System

POSE ESTIMATION

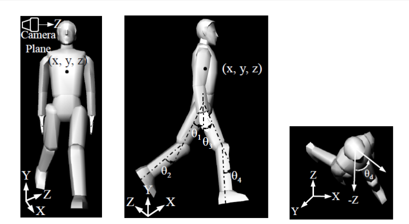
PROBLEM STATEMENT



PARTS OF POSE ESTIMATOR

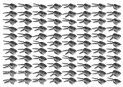
- Motion Model : Second Order motion continuity
- Observation Model : Mapping features to pose using Gaussian Processes
- Estimation : Kalman Filter

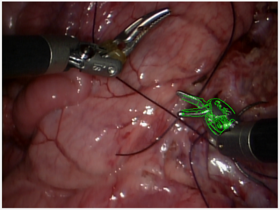
Model Based Optimization [Agarwal et. al, RSS 2012]:



- Computationally expensive
- Model Required: Articulation, CAD
- Often requires background subtraction

Template Based Methods [Reiter et. al, CARS 2012]

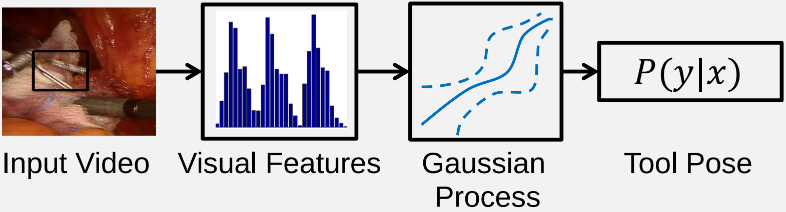




- Curse of Dimensionality
- Model Required: Articulation, CAD, Visual

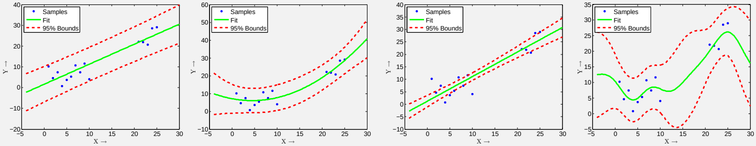
Guiding Principles

- Real-time performance
- Measure of confidence in estimates



- Model Required: Articulation
- Map generic visual features x to tool end-effector pose estimate y
- Tool Pose y represents orientation, end-effector opening angle

$$y = x + 0.005x^2 + \epsilon,\ x \in [1, 10] \cup [21, 25],\ \epsilon \sim \mathcal{N}(0, 10)$$



(a) *Linear Regression*

(b) *Quadratic Regression*

(c) *Bayesian Linear Regression*

(d) *GPR*

Problem with Regression Models

- Parametric Models
- Prediction is dependent on the chosen model

Gaussian Process Regression

$f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$

- Inference takes place in space of functions
- Makes minimal assumptions on the underlying data distribution
- High variance in regions with sparse ground truth data

GAUSSIAN PROCESS REGRESSION

$f(x) : x \mapsto y$.

- Input: Ground truth data from (X, y) from n observations
- Output: Pose $y*$ for a new image with associated feature vector x^*, $p(y^*|x^*)$.
- Measurement Process: $y = f(x) + \epsilon$, $\epsilon \sim \mathcal{N}(0, \sigma^2)$

Inference:

- $f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$
- $\mathrm{cov}(f(x_p), f(x_q)) = k(x_p, x_q) = \exp^{-\frac{1}{2l}(x_p - x_q)^2}$
- For two observation, y_p, y_q, we get, $\mathrm{cov}(y_p, y_q) = k(x_p, x_q) + \sigma^2 \delta_{pq}$

Prediction: Marginalize training data,

$$
\begin{aligned}
\mathbf{f}^* | \mathsf{X}, \mathbf{y}, X^* &\sim \mathcal{N}(\mathbf{f}^*, \mathrm{cov}(\mathbf{f}^*)) \qquad (1)\\
\mathbf{f}^* &= K(X^*, X)[K(X, X) + \sigma^2 I]^{-1} \mathbf{y} \\
\mathrm{cov}(\mathbf{f}^*) &= K(X^*, X^*) - K(X^*, X)[K(X, X) + \sigma^2 I]^{-1} K(X, X^*)
\end{aligned}
$$

- Regression process predicts pose solely using one image
- Smoothness in surgical actions

Motion Continuity

Consider a single k^{th} element of the pose state $y(k)$,

$$\begin{bmatrix} y(k)_t \\ \dot{y}(k)_t \end{bmatrix} = \begin{bmatrix} 1 & \delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y(k)_{t-1} \\ \dot{y}(k)_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\delta t^2 \\ \delta t \end{bmatrix} \ddot{y}(k)_{t-1} \tag{2}$$

Acceleration is modeled as zero mean Gaussian white noise $\ddot{y}(k)_t \sim \mathcal{N}(0, \sigma_a^2) \forall t$.

The observation model is the Gaussian process regression framework which can be represented by

$$z_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y(k)_t \\ \dot{y}(k)_t \end{bmatrix} + v_t \tag{3}$$

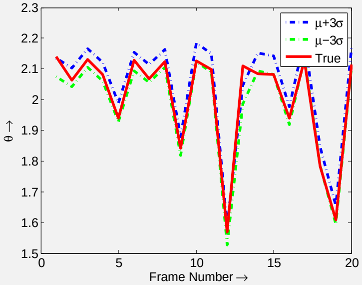


Figure 1: Gaussian Process regression with 3 sigma bounds plotted with true value of tool opening angle

Ideal Features

Unique, Invariant, Computationally Efficient

Histograms of Oriented Gradients (HOG) and Local Binary Patterns (LBP)

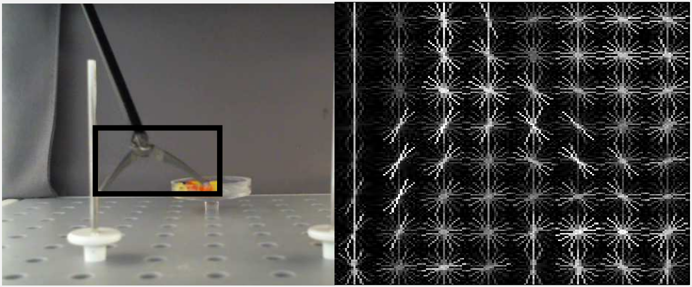


Figure 2: An example image with tool and corresponding HOG feature

EXPERIMENTAL DATA

- Both moving and fixed part of a tool are tracked at 50 fps
- Endoscopic camera : 640×480 pixels at 15 fps
- Entire dataset has 4346 different tool poses
- Sensing Noise: Motion blur, partial occlusions, lighting variation

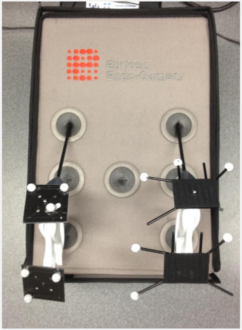
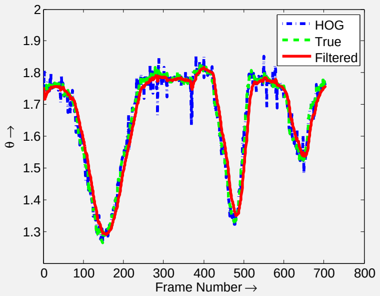


Figure 3: Customized Box Trainer Setup Retrofitted with Optical Reflective Markers

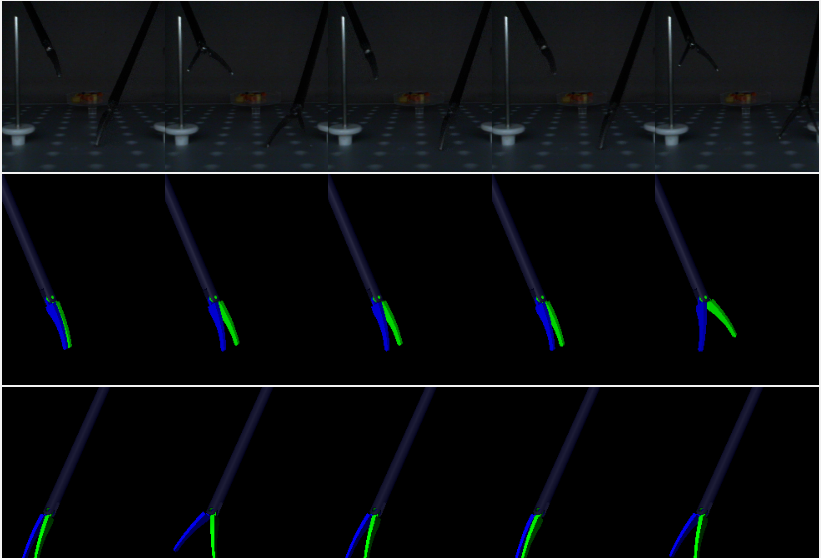
RESULTS

Features	Orientation Error	Opening Angle
HOG	2.42	2.49
LBP	1.92	2.48

Table 1: Tool pose estimate angular accuracy in degrees using different visual features



VISUAL RESULTS



CONCLUSION

- Real-time method to predict tool pose using generic visual features
- Robust variance estimates along with mean predictions
- Variance estimate is demonstrated to be useful for filtering
- Experimental results using a customized box trainer demonstrate good tool pose prediction

Journal

1. **S. Kumar**, J. Sovizi, V. Krovi, "Error Propagation on SE(3) for Surgical Tool Pose Filtering" (In Preparation)
2. P. Agarwal, **S. Kumar**, J. Ryde, J. Corso, and V. Krovi, "Estimating Dynamics On-the-fly Using Monocular Video For Vision-Based Robotics", IEEE/ASME Transactions on Mechatronics, 2013.

Conference

1. **S. Kumar**, J. Sovizi, M. S. Narayanan, V. Krovi, "Surgical Tool Pose Estimation from Monocular Endoscopic Videos", IEEE International Conference on Robotics and Automation (ICRA), 2015
2. P. Agarwal, **S. Kumar**, J. Ryde, J. Corso, and V. Krovi, "Estimating Human Dynamics On-the-fly Using Monocular Video for Pose Estimation", Robotics: Science and Systems VIII, 2013

Questions?