GENERALIZED NON-PARAMETRIC FUNCTION APPROXIMATION

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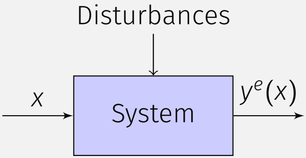
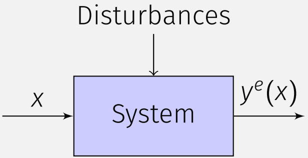
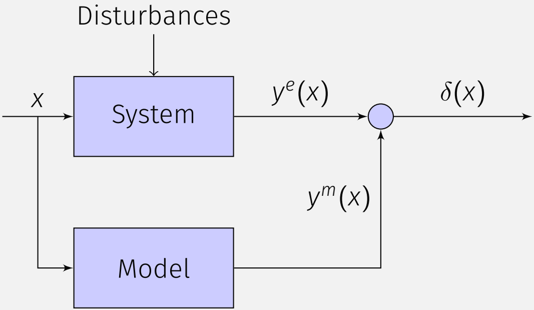
INTRODUCTION


Disturbances
x
System
$y^e(x)$


Disturbances
x
System
$y^e(x)$


Disturbances
x
System
$y^e(x)$
$\delta(x)$
$y^m(x)$
Model

Bias Correction

$$y^e(x) = y^m(x) + \delta(x) + \epsilon$$

where ϵ is the experimental uncertainty.

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Parameter Calibration

$$y^e(x) = y^m(x, \theta^*) + \epsilon$$

Bias Correction

$$y^e(x) = y^m(x) + \delta(x) + \epsilon$$

where ϵ is the experimental uncertainty.

Parameter Calibration

$$y^e(x) = y^m(x, \theta^*) + \epsilon$$

Bias Correction and Parameter Calibration

$$y^e(x) = y^m(x, \theta^*) + \delta(x) + \epsilon$$

NEED FOR FUNCTION APPROXIMATION

- Lack of simulation results $y^m(x, \theta)$
- Parameterizing discrepancy function $\delta(x)$

Why Bayesian

- Integration of prior knowledge

Bayesian calibration of computer models, Marc C. Kennedy, Anthony O'Hagan in Journal of the Royal Statistical Society, 2001

GAUSSIAN PROCESSES

Function Approximation

$f : x \mapsto y$

Function Approximation

$\mathrm{f} : x \mapsto y$

Parametric Models: Pros

Easy to interpret

Parametric Models: Cons

- Simpler models lack expressive power
- Complex models require lots of data
- Predictions are dependent on the model
- Representation of uncertainty in the input space

$$y = x + 0.005x^2 + \epsilon, x \in [1, 10] \cup [21, 25], \epsilon \sim \mathcal{N}(0, 10)$$

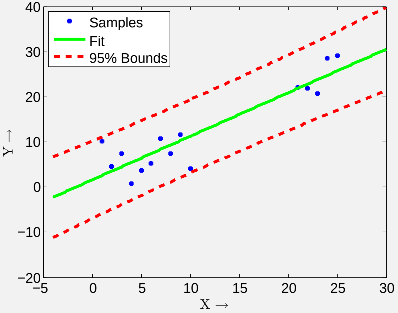


Figure 1: Linear Regression yields the model $y = mx + c$ with $m = 0.9644$ and $c = 1.5891$

$$y = x + 0.005x^2 + \epsilon,\ x \in [1, 10] \cup [21, 25],\ \epsilon \sim \mathcal{N}(0, 10)$$

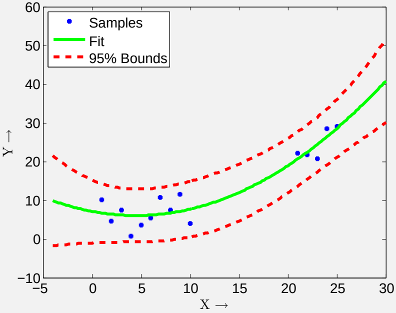


Figure 2: Quadratic model yields the model $y = ax^2 + bx + c$ with $a = 0.0534$, $b = -0.04781$ and $c = 7.1231$

$$y = x + 0.005x^2 + \epsilon, x \in [1, 10] \cup [21, 25], \epsilon \sim \mathcal{N}(0, 10)$$

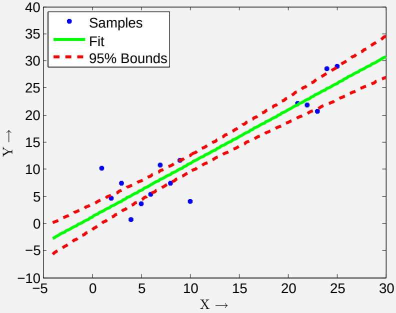


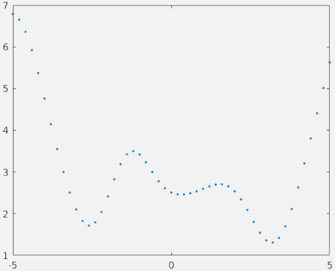
Figure 3: Bayesian Regression fit uses a zero mean Gaussian observation noise with variance 10 and a zero mean Gaussian with diagonal variance 5 prior on slope and intercept. The estimation process results in a mean parameter estimate of 0.9867 slope and 1.1797 intercept.

Definition

Gaussian Process is a collection of random variables, any finite number of which are joint Gaussian distribution.
$f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$

- Generalization from mean vector and covariance matrix
- Indexed by x
- At every value of x, we have an associated random variable

$f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$
$m(x) = \frac{1}{4}x^2$, and $k(x, x^1) = \exp\left(-\frac{1}{2}(x - x^1)^2\right)$
Let's sample at n different locations
$\mu(x_i) = \frac{1}{4}x_i^2$, and $\Sigma_{ij}(x_i, x_j) = \exp\left(-\frac{1}{2}(x_i - x_j)^2\right)$, $i, j \in [1, ..., n]$



POSTERIOR GAUSSIAN PROCESS

$f(x) : x \mapsto y$.

- Input: Ground truth data from $(\mathrm{x}, \mathrm{f}(\mathrm{x}))$ from n observations
- Output: Output f_* for a novel input x_*, $p(y_*|x_*)$.
- Measurement Process: $y = f(x) + \epsilon$, $\epsilon \sim \mathcal{N}(0, \sigma^2)$

Joint Distribution:

$$\begin{bmatrix} f \\ f_* \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu \\ \mu_* \end{bmatrix}, \begin{bmatrix} \Sigma & \Sigma_* \\ \Sigma_*^T & \Sigma_{**} \end{bmatrix}\right) \tag{1}$$

Marginalization:

$$f_*|f \sim \mathcal{N}(u_* + \Sigma_*^T \Sigma^{-1}(f - \mu), \Sigma_{**} - \Sigma_*^T \Sigma^{-1} \Sigma_*) \tag{2}$$

Posterior Process:

$$\begin{aligned} \mathrm{f}|\mathcal{D} \sim & \mathbb{GP}(m_D, k_D) \\ & m_D(x) = m(x) + \Sigma(X, x)^T \Sigma^{-1}(f - m) \\ & k_D(x, x^1) = k(x, x^1) - \Sigma(X, x)^T \Sigma^{-1} \Sigma(X, x^1) \end{aligned} \tag{3}$$

Observation Noise:

$$y \sim \mathbb{GP}(m, k + \sigma_n^2 \delta_{ii^1}) \tag{4}$$

Parameterizing GP Priors
$m(x) = ax^2 + bx + c$, and $k(x, x^1) = \sigma_y^2 \exp\left(-\frac{(x-x^1)^2}{2l^2}\right) + \sigma_n^2 \delta_{ii^1}$
Hyperparameters: $\theta = \{a, b, c, \sigma_y, \sigma_n, l\}$
Fitting:

$$\begin{aligned} L &= \log p(y|x, \theta) \\ &= -\frac{1}{2}\log|\Sigma| - \frac{1}{2}(y-\mu)^T \Sigma^{-1}(y-\mu) - \frac{n}{2}\log 2\pi \end{aligned} \tag{5}$$

Estimation:

$$\begin{aligned} \frac{\partial L}{\partial \theta_m} &= -(y-\mu)^T \Sigma^{-1} \frac{\partial m}{\partial \theta_m} \\ \frac{\partial L}{\partial \theta_k} &= \frac{1}{2} trace(\Sigma^{-1} \frac{\partial \Sigma}{\partial \theta_k}) + \frac{1}{2}(y-\mu)^T \frac{\partial \Sigma}{\partial \theta_k} \Sigma^{-1} \frac{\partial \Sigma}{\partial \theta_k}(y-\mu) \end{aligned} \tag{6}$$

Non-parametric does not mean no parameters. Parametric models can through away data once parameters are tuned.

$$y = x + 0.005x^2 + \epsilon, x \in [1, 10] \cup [21, 25], \epsilon \sim \mathcal{N}(0, 10)$$

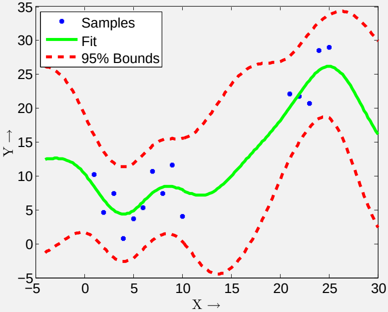
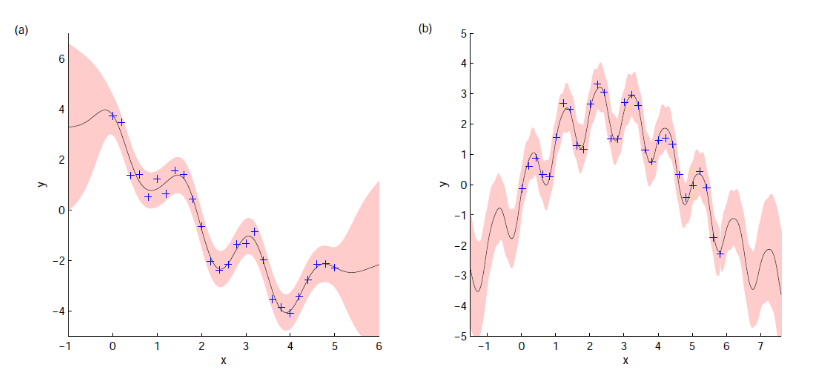


Figure 4: GPR with constant mean function, Gaussian likelihood and Squared Exponential covariance function

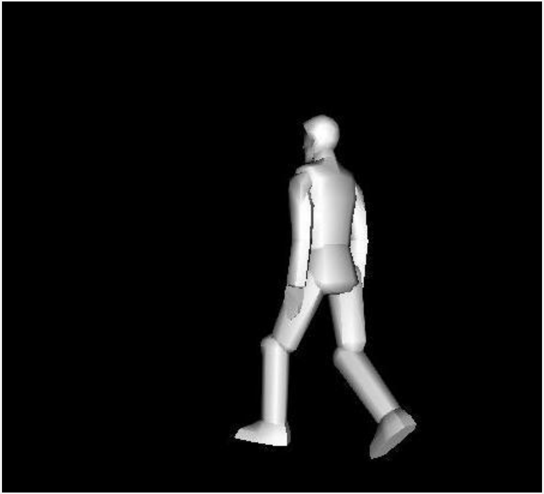
EFFECT OF COVARIANCE FUNCTION



- Long term and short term dependencies
- Periodic nature

POSE ESTIMATION

PROBLEM STATEMENT

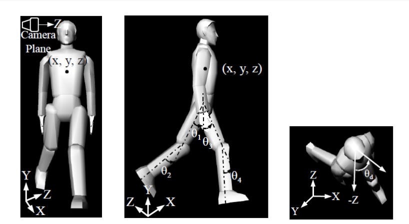


PARTS OF POSE ESTIMATOR

- Motion Model : Second Order motion continuity
- Observation Model : Mapping features to pose using Gaussian Processes
- Estimation : Kalman Filter

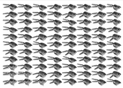
PREVIOUS WORK

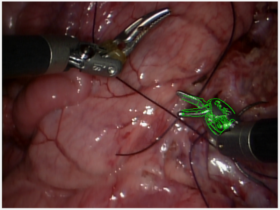
Model Based Optimization [Agarwal et. al, RSS 2012]:



- Computationally expensive
- Model Required: Articulation, CAD
- Often requires background subtraction

Template Based Methods [Reiter et. al, CARS 2012]



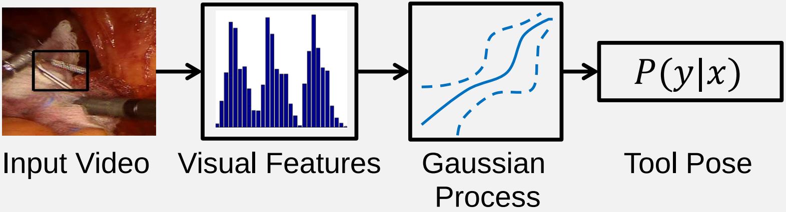


- Curse of Dimensionality
- Model Required: Articulation, CAD, Visual

MODEL-LESS POSE ESTIMATION

Guiding Principles

- Real-time performance
- Measure of confidence in estimates

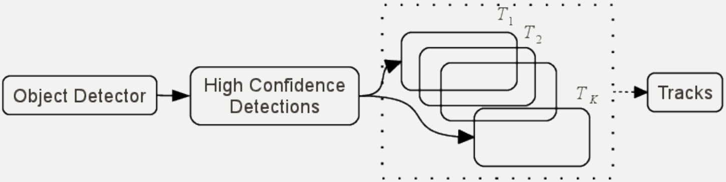


- Model Required: Articulation
- Map generic visual features x to tool end-effector pose estimate y
- Tool Pose y represents orientation, end-effector opening angle

OBJECT TRACKING

- Motion Model: Second Order Motion Continuity
- Observation Model: Multiple observers
- Interaction Model: Constant Velocity assumption
- Estimation: Depends on the observation model

PRODUCT OF TRACKING EXPERTS



Mixture Models:

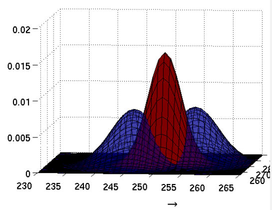
$$P(x) = \sum_{j=1}^{M} \pi_i p_j(x|\theta_j), \quad \sum_{j=1}^{M} \pi_i = 1$$

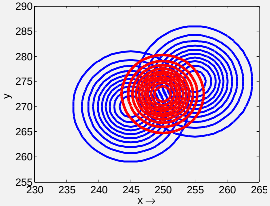
Product of Experts Model

$$P(x) = \frac{1}{Z} \prod_{j=1}^{M} f_j(x|\theta_j), \quad Z = \int \prod_{j=1}^{M} f_j(x|\theta_j) \mathrm{d}x$$

PRODUCT OF EXPERTS MODEL

- Expert 1: $\mu = [245, 270]^T$, $\Sigma = diag([16.66, 25])$
- Expert 2: $\mu = [255, 275]^T$, $\Sigma = diag([16.66, 25])$





Resulting Output: $\mu = [250, 272.5]^T$, $\Sigma = [8.33, 12.5]$

- Merges output of various trackers
- Can faithfully use information from detectors
- Allows to use complete densities for inference in tracking

$$\mu = [x_{CB}, y_{CB}]^T, \Sigma = \frac{1}{6}\begin{bmatrix} w_B & 0 \\ 0 & h_B \end{bmatrix}$$

PoTE Model with K Gaussian Experts ($\underline{\mu_k}, \Sigma_k$)

$$p(\underline{x}|\theta_{T_1}, \theta_{T_2}, ..., \theta_{T_K}) = \frac{\prod_{k=1}^{K} \frac{1}{2\pi|\Sigma_k|^{\frac{1}{2}}} \exp(-\frac{1}{2}[\underline{x} - \underline{\mu_k}]^T \Sigma_k^{-1} [\underline{x} - \underline{\mu_k}])}{\int \prod_{k=1}^{K} p_k(\underline{x}|\theta_k) \mathrm{d}\underline{x}}$$

$$p(\underline{x}|\theta_{T_1}, \theta_{T_2}, ..., \theta_{T_K}) \sim \mathcal{N}(\underline{\mu}, \Sigma), \text{ where}$$

$$\Sigma^{-1} = \sum_{k=1}^{K} \Sigma_k^{-1}, \underline{\mu} = \Sigma \left(\sum_{k=1}^{K} \Sigma_k^{-1} \underline{\mu_k} \right)$$

COMMONLY USED TRACKING EXPERTS

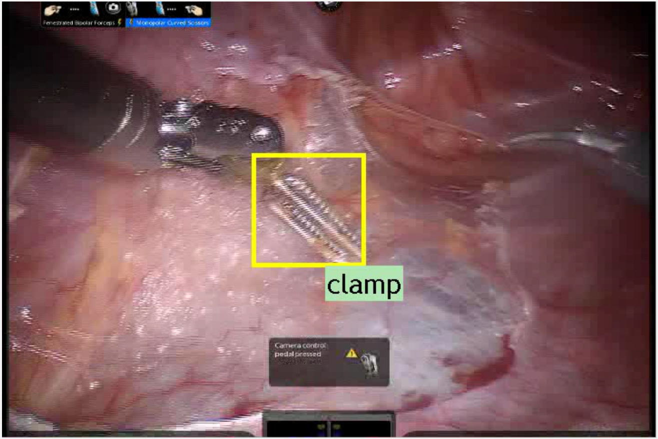
Kanade Lucas Tracking Point Feature Tracking for bounding box prediction

Background Subtraction Helpful for stationary camera

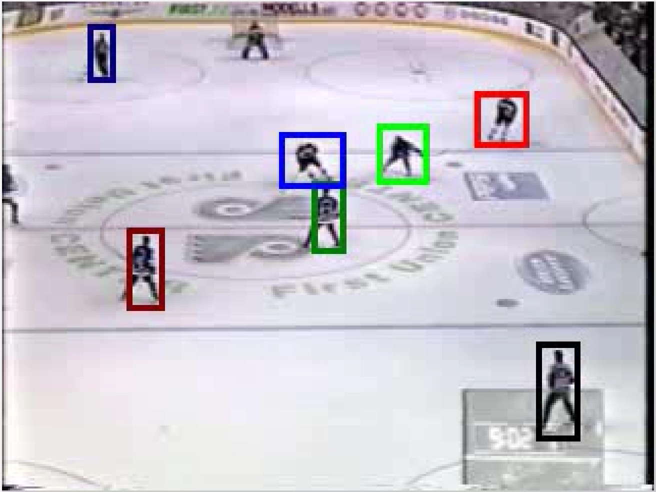
Motion Prediction Second order motion continuity

Object Detector High Confidence detection from a detector

Dense Optical Flow Texture-less objects


clamp
Camera control
pedal pressed

PERSON TRACKING RESULTS



GAUSSIAN PROCESS BASED POSE ESTIMATION

Problem with Regression Models

- Parametric Models
- Prediction is dependent on the chosen model

Gaussian Process Regression

$f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$

- Inference takes place in space of functions
- Makes minimal assumptions on the underlying data distribution
- High variance in regions with sparse ground truth data

- Regression process predicts pose solely using one image
- Smoothness in surgical actions

Motion Continuity

Consider a single k^{th} element of the pose state $y(k)$,

$$\begin{bmatrix} y(k)_t \\ \dot{y}(k)_t \end{bmatrix} = \begin{bmatrix} 1 & \delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y(k)_{t-1} \\ \dot{y}(k)_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\delta t^2 \\ \delta t \end{bmatrix} \ddot{y}(k)_{t-1} \tag{7}$$

Acceleration is modeled as zero mean Gaussian white noise $\ddot{y}(k)_t \sim \mathcal{N}(0, \sigma_a^2) \forall t$.

The observation model is the Gaussian process regression framework which can be represented by

$$z_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y(k)_t \\ \dot{y}(k)_t \end{bmatrix} + v_t \tag{8}$$

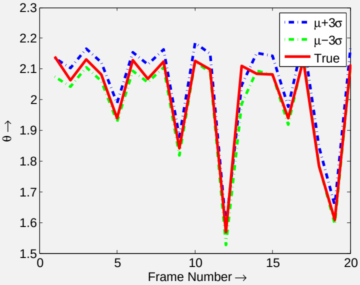


Figure 5: Gaussian Process regression with 3 sigma bounds plotted with true value of tool opening angle

Ideal Features

Unique, Invariant, Computationally Efficient

Histograms of Oriented Gradients (HOG) and Local Binary Patterns (LBP)

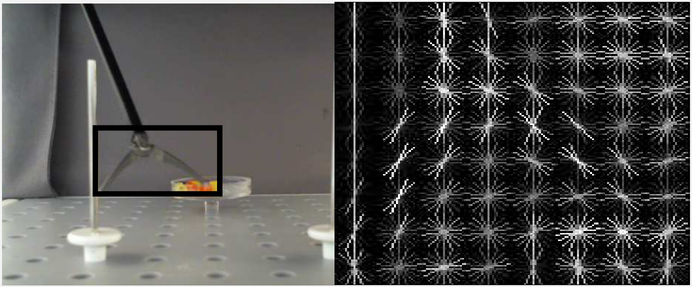


Figure 6: An example image with tool and corresponding HOG feature

- Both moving and fixed part of a tool are tracked at 50 fps
- Endoscopic camera : 640×480 pixels at 15 fps
- Entire dataset has 4346 different tool poses
- Sensing Noise: Motion blur, partial occlusions, lighting variation

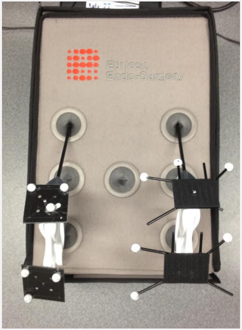
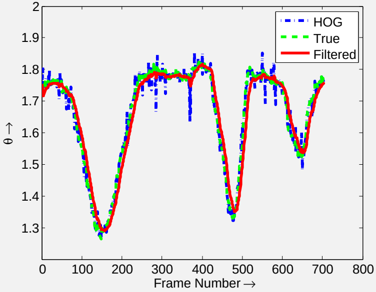


Figure 7: Customized Box Trainer Setup Retrofitted with Optical Reflective Markers

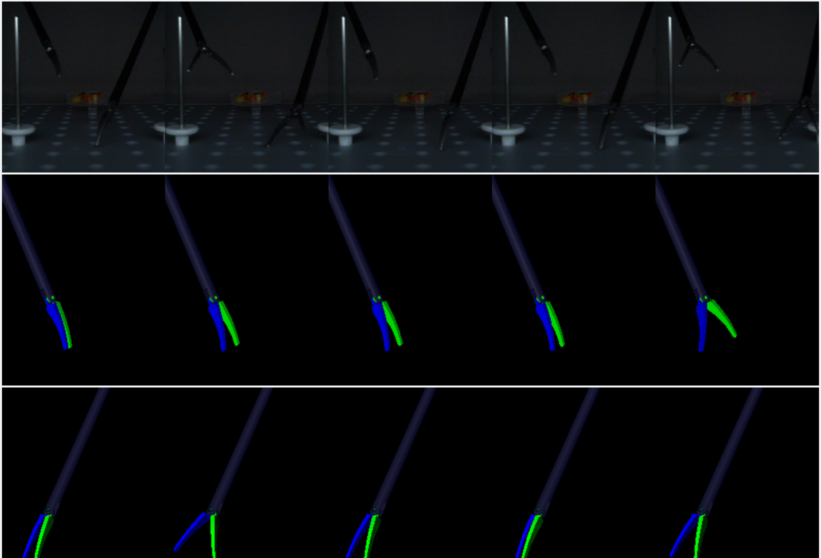
RESULTS

Features	Orientation Error	Opening Angle
HOG	2.42	2.49
LBP	1.92	2.48

Table 1: Tool pose estimate angular accuracy in degrees using different visual features



VISUAL RESULTS



- Real-time method to predict tool pose using generic visual features
- Robust variance estimates along with mean predictions
- Variance estimate is demonstrated to be useful for filtering
- Experimental results using a customized box trainer demonstrate good tool pose prediction

Journal

1. **S. Kumar**, J. Sovizi, V. Krovi, “Error Propagation on SE(3) for Surgical Tool Pose Filtering” (In Preparation)
2. P. Agarwal, **S. Kumar**, J. Ryde, J. Corso, and V. Krovi, “Estimating Dynamics On-the-fly Using Monocular Video For Vision-Based Robotics”, IEEE/ASME Transactions on Mechatronics, 2013.

Conference

1. **S. Kumar**, J. Sovizi, M. S. Narayanan, V. Krovi, “Surgical Tool Pose Estimation from Monocular Endoscopic Videos”, IEEE International Conference on Robotics and Automation (ICRA), 2015
2. P. Agarwal, **S. Kumar**, J. Ryde, J. Corso, and V. Krovi, “Estimating Human Dynamics On-the-fly Using Monocular Video for Pose Estimation”, Robotics: Science and Systems VIII, 2013

REFERENCES

- Gaussian Processes for Regression: A Quick Introduction, M. Ebden
- Gaussian Processes for Machine Learning, Carl Edward Rasmussen and Chris Williams, the MIT Press, 2006

Questions?