# GENERALIZED NON-PARAMETRIC FUNCTION APPROXIMATION

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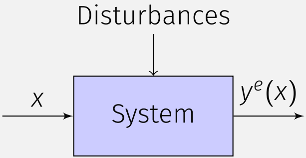
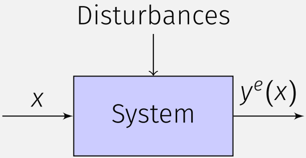
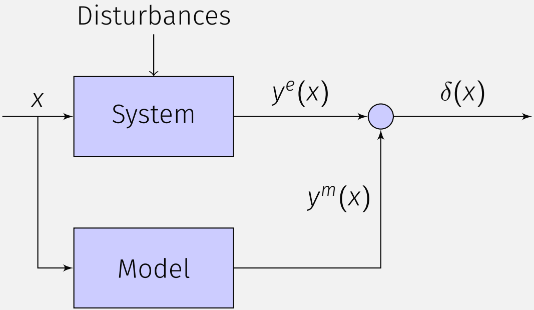
https://github.com/surenkum/uq_gaussian_processes

# INTRODUCTION


Disturbances
$x$
System
$y^e(x)$

# INTRODUCTION



## Bias Correction

$$y^e(x) = y^m(x) + \delta(x) + \epsilon$$

where $\epsilon$ is the experimental uncertainty.

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## Parameter Calibration

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Parameter Calibration

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Bias Correction and Parameter Calibration

$$y^e(x) = y^m(x, \theta^*) + \delta(x) + \epsilon$$

# NEED FOR FUNCTION APPROXIMATION

- Lack of simulation results $y^m(x, \theta)$
- Parameterizing discrepancy function $\delta(x)$

## Why Bayesian

- Integration of prior knowledge

Bayesian calibration of computer models, Marc C. Kennedy, Anthony O'Hagan in Journal of the Royal Statistical Society, 2001

# GAUSSIAN PROCESSES

## Function Approximation

$f : x \mapsto y$

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$f : x \mapsto y$

### Parametric Models: Pros

Easy to interpret

### Parametric Models: Cons

- Simpler models lack expressive power
- Complex models require lots of data
- Predictions are dependent on the model
- Representation of uncertainty in the input space

$$y = x + 0.005x^2 + \epsilon, x \in [1, 10] \cup [21, 25], \epsilon \sim \mathcal{N}(0, 10)$$

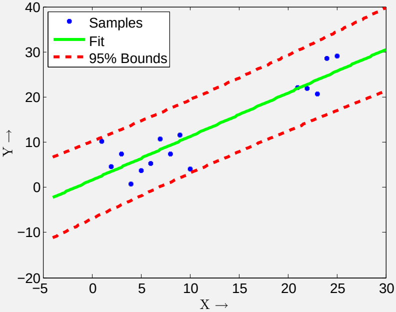


**Figure 1:** Linear Regression yields the model $y = mx + c$ with $m = 0.9644$ and $c = 1.5891$

$$y = x + 0.005x^2 + \epsilon, x \in [1, 10] \cup [21, 25], \epsilon \sim \mathcal{N}(0, 10)$$

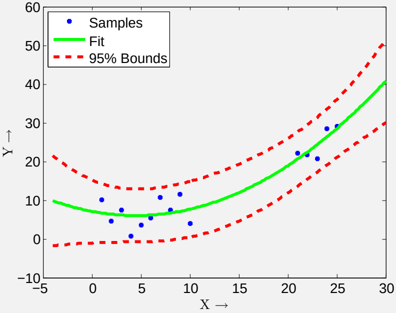


Figure 2: Quadratic model yields the model $y = ax^2 + bx + c$ with $a = 0.0534$, $b = -0.04781$ and $c = 7.1231$

$$y = x + 0.005x^2 + \epsilon, x \in [1, 10] \cup [21, 25], \epsilon \sim \mathcal{N}(0, 10)$$

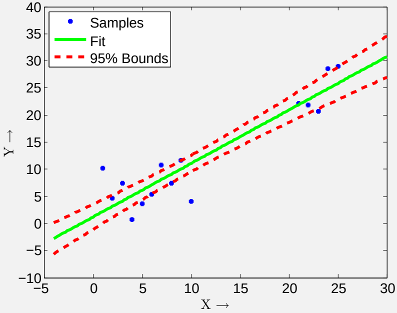


Figure 3: Bayesian Regression fit uses a zero mean Gaussian observation noise with variance 10 and a zero mean Gaussian with diagonal variance 5 prior on slope and intercept. The estimation process results in a mean parameter estimate of 0.9867 slope and 1.1797 intercept.

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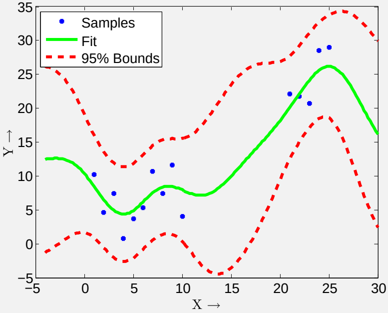
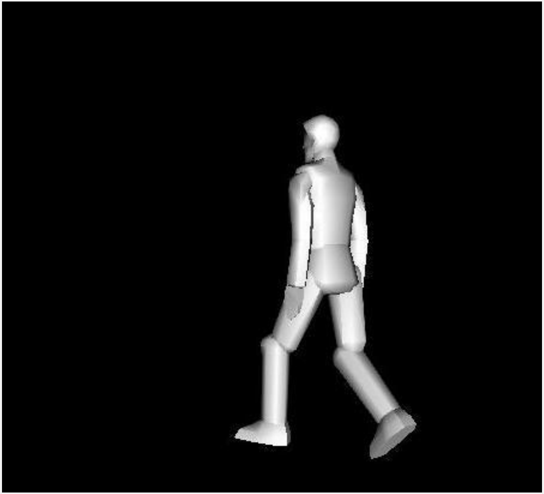



**Figure 4:** GPR with constant mean function, Gaussian likelihood and Squared Exponential covariance function

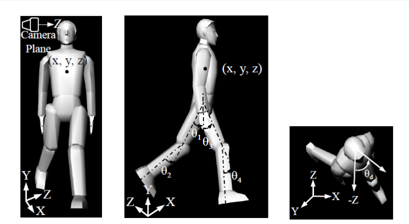
# POSE ESTIMATION



# PARTS OF POSE ESTIMATOR

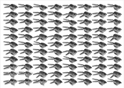
- Motion Model : Second Order motion continuity
- Observation Model : Mapping features to pose using Gaussian Processes
- Estimation : Kalman Filter

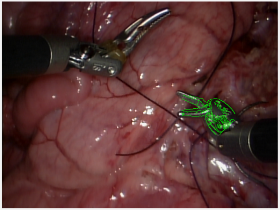
Model Based Optimization [Agarwal et. al, RSS 2012]:




- Computationally expensive
- Model Required: Articulation, CAD
- Often requires background subtraction

Template Based Methods [Reiter et. al, CARS 2012]



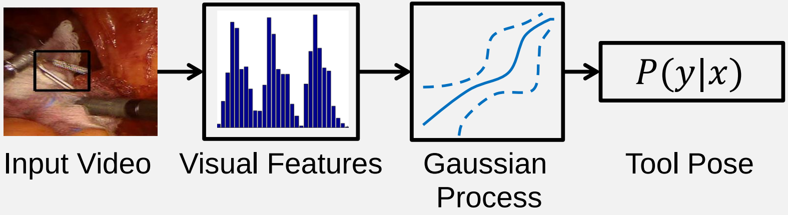


- Curse of Dimensionality
- Model Required: Articulation, CAD, Visual

# MODEL-LESS POSE ESTIMATION

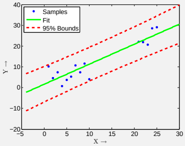
## Guiding Principles

- Real-time performance
- Measure of confidence in estimates

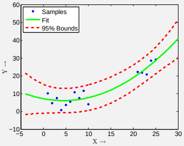



- Model Required: Articulation
- Map generic visual features $x$ to tool end-effector pose estimate $y$
- Tool Pose $y$ represents orientation, end-effector opening angle

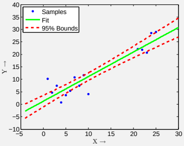
$$y = x + 0.005x^2 + \epsilon,\, x \in [1, 10] \cup [21, 25],\, \epsilon \sim \mathcal{N}(0, 10)$$



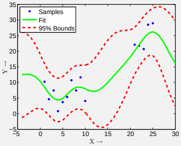

(a) *Linear Regression*




(b) *Quadratic Regression*




(c) *Bayesian Linear Regression*




(d) *GPR*

## Problem with Regression Models

- Parametric Models
- Prediction is dependent on the chosen model

## Gaussian Process Regression

$f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$

- Inference takes place in space of functions
- Makes minimal assumptions on the underlying data distribution
- High variance in regions with sparse ground truth data

$f(x) : x \mapsto y.$

- Input: Ground truth data from $(\mathbf{X}, \mathbf{y})$ from $n$ observations
- Output: Pose $y*$ for a new image with associated feature vector $x^*$, $p(y^*|x^*)$.
- Measurement Process: $y = f(x) + \epsilon$, $\epsilon \sim \mathcal{N}(0, \sigma^2)$

Inference:

- $f(x) \sim \mathbb{GP}(m(x), k(x, x^1))$
- $\mathrm{cov}(f(x_p), f(x_q)) = k(x_p, x_q) = \exp^{-\frac{1}{2l}(x_p - x_q)^2}$
- For two observation, $y_p, y_q$, we get, $\mathrm{cov}(y_p, y_q) = k(x_p, x_q) + \sigma^2 \delta_{pq}$

Prediction: Marginalize training data,

$$\mathbf{f}^*|\mathbf{X}, \mathbf{y}, X^* \sim \mathcal{N}(\mathbf{f}^*, \mathrm{cov}(\mathbf{f}^*)) \tag{1}$$

$$\mathbf{f}^* = K(X^*, X)[K(X, X) + \sigma^2 I]^{-1}\mathbf{y}$$

$$\mathrm{cov}(\mathbf{f}^*) = K(X^*, X^*) - K(X^*, X)[K(X, X) + \sigma^2 I]^{-1} K(X, X^*)$$

- Regression process predicts pose solely using one image
- Smoothness in surgical actions

## Motion Continuity

Consider a single $k^{th}$ element of the pose state $y(k)$,

$$\begin{bmatrix} y(k)_t \\ \dot{y}(k)_t \end{bmatrix} = \begin{bmatrix} 1 & \delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y(k)_{t-1} \\ \dot{y}(k)_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\delta t^2 \\ \delta t \end{bmatrix} \ddot{y}(k)_{t-1} \tag{2}$$

Acceleration is modeled as zero mean Gaussian white noise $\ddot{y}(k)_t \sim \mathcal{N}(0, \sigma_a^2) \forall t$.

The observation model is the Gaussian process regression framework which can be represented by

$$z_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y(k)_t \\ \dot{y}(k)_t \end{bmatrix} + v_t \tag{3}$$

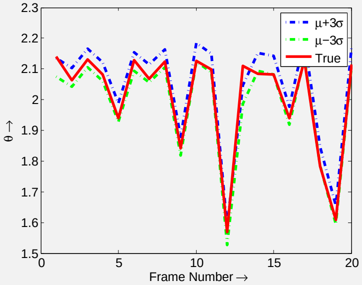



**Figure 5:** Gaussian Process regression with 3 sigma bounds plotted with true value of tool opening angle

### Ideal Features

Unique, Invariant, Computationally Efficient

Histograms of Oriented Gradients (HOG) and Local Binary Patterns (LBP)

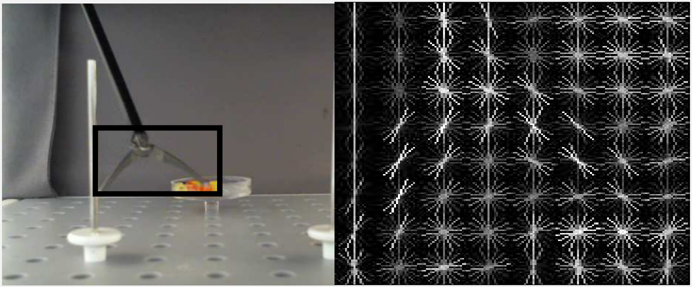


Figure 6: An example image with tool and corresponding HOG feature

- Both moving and fixed part of a tool are tracked at 50 fps
- Endoscopic camera : $640 \times 480$ pixels at 15 fps
- Entire dataset has 4346 different tool poses
- Sensing Noise: Motion blur, partial occlusions, lighting variation

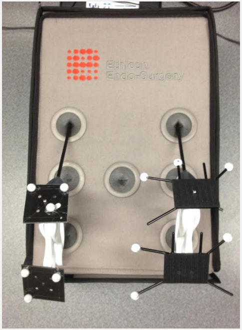
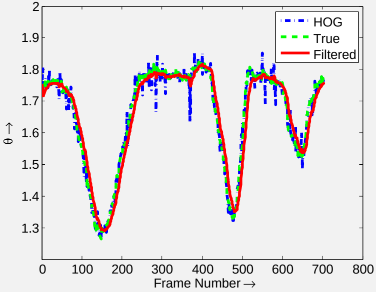
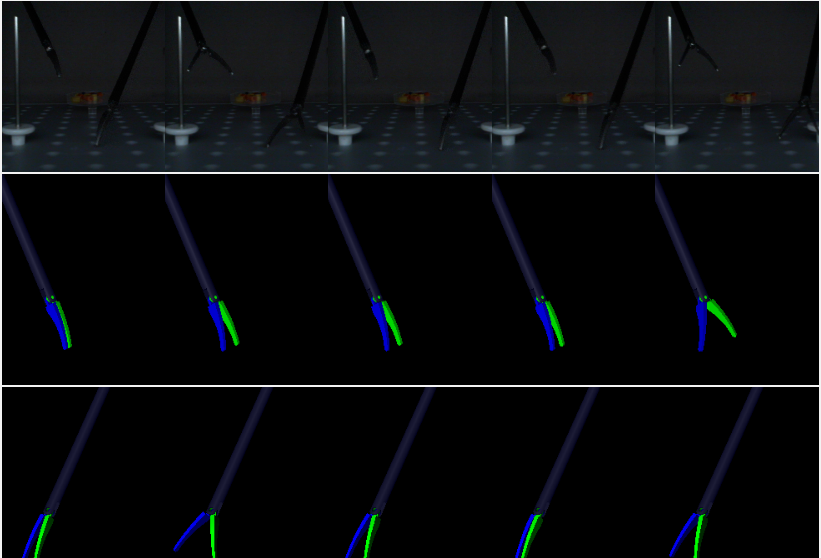


Figure 7: Customized Box Trainer Setup Retrofitted with Optical Reflective Markers

| Features | Orientation Error | Opening Angle |
| --- | --- | --- |
| HOG | 2.42 | 2.49 |
| LBP | 1.92 | 2.48 |

**Table 1:** Tool pose estimate angular accuracy in degrees using different visual features





- Real-time method to predict tool pose using generic visual features
- Robust variance estimates along with mean predictions
- Variance estimate is demonstrated to be useful for filtering
- Experimental results using a customized box trainer demonstrate good tool pose prediction

## Journal

1. **S. Kumar**, J. Sovizi, V. Krovi, "Error Propagation on SE(3) for Surgical Tool Pose Filtering" (In Preparation)
2. P. Agarwal, **S. Kumar**, J. Ryde, J. Corso, and V. Krovi, "Estimating Dynamics On-the-fly Using Monocular Video For Vision-Based Robotics", IEEE/ASME Transactions on Mechatronics, 2013.

## Conference

1. **S. Kumar**, J. Sovizi, M. S. Narayanan, V. Krovi, "Surgical Tool Pose Estimation from Monocular Endoscopic Videos", IEEE International Conference on Robotics and Automation (ICRA), 2015
2. P. Agarwal, **S. Kumar**, J. Ryde, J. Corso, and V. Krovi, "Estimating Human Dynamics On-the-fly Using Monocular Video for Pose Estimation", Robotics: Science and Systems VIII, 2013

Questions?