Computing Input-Referred RSSI and Angle Estimates on the S.U.R.F.E.R. Platform

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***Abstract*—Computing RSSI and angle estimates of the received RFID backscatter waveform require some care due to the unique nature of the resource-saving algorithms used in the S.U.R.F.E.R. reader. This document shows how the reader or iDevice app software may be modified to use the Main/Alternate Path signal amplitude calculations to estimate the RSSI referred to the S.U.R.F.E.R. reader antenna port and to estimate the angle of the received signal. Also computed are the standard deviations of the error in these estimates due to limited SNR.**

I. Actual Amplitude Computations In Hardware

A. *Description of Hardware*

The "Main" and "Alternate" signal amplitudes reported to the iDevice are computed in the data recovery circuit of the S.U.R.F.E.R. reader FPGA. This block is rather large, and is shown in Fig. 1, and was originally reported in [1]. The amplitude calculations reported to the iDevice ultimately begin with the "INPUTS". Consider the "I DATA" and "Q DATA". We represent these two signals as $a_I[n]$ and $a_Q[n]$, respectively. In "PSEUDO-MATCHED FILTER INTEGRATION", these signals are assigned to either the "Main" integrator path or the "Alternate" integrator path based on the "set_use_i()" and "set_use_q()" functions controlled in the MCU firmware (in the figure, represented by the signal "EVAL_I"). For the purposes of this paper, we will assume that "I DATA" is assigned to the "Main" integrator path and that "Q DATA" is assigned to the "Alternate" integrator path and refer to these signals henceforth as I and Q data (and paths), with no loss of generality.

The 4-input multiplexers that assign I and Q data to the main and alternate paths also perform a square wave multiplication, with the square wave being generated by the "SQUARE WAVE" block. The square wave is synchronized to the clock recovery circuit output. Assuming that successful packet reception has occurred, it is safe to assume that the clock recovery circuit output is a faithful reproduction of the backscatter link frequency clock and that the edges of said clock are roughly aligned with the zero crossings of $a_I[n]$ and $a_Q[n]$. Multiplication of $a_I[n]$ and $a_Q[n]$ and subsequent integration in the "PSEUDO-MATCHED FILTER INTEGRATION" generates output waveforms similar to those shown in Fig. 2.

At each half-symbol boundary, when the magnitudes of the two bit decision integrators are at their peaks, the integrator outputs are flipped together such that the I PATH (Main) output is always positive and then subsequently integrated on the "I/Q

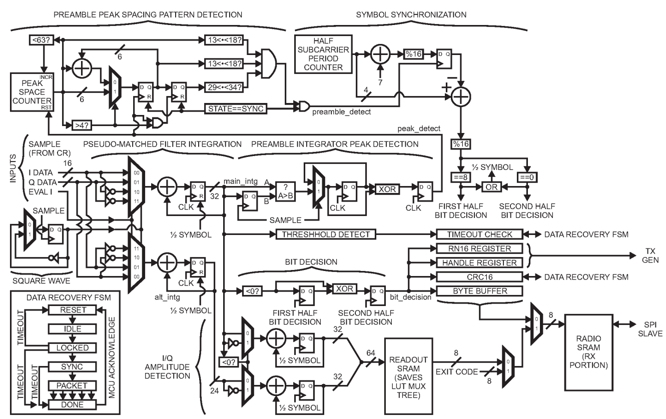

Fig. 1: Data Recovery Circuit.

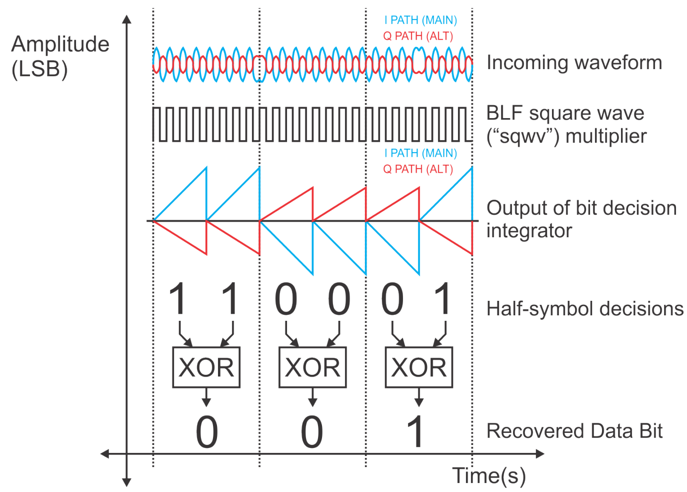

Fig. 2: Depiction of Half-Bit and Bit Decisions in Data Recovery Circuit.

AMPLITUDE DETECTION" integrators at the center bottom of Fig. 1. For example, if the polarity of the I PATH and Q PATH signals are the same, the polarity of the Q PATH signal integrated on the amplitude integrator will be positive. If the polarity of the I PATH and Q PATH signals are different, the polarity of the Q PATH signal integrated on the amplitude integrator will be negative. Assuming that the angle of the returned tag backscatter signal is roughly constant over the response interval, the sign of the Q PATH signal integrated on the amplitude integrator will be constant. If the angle of the returned backscatter signal is close to 0°, the sign of the Q PATH signal integrated on the amplitude integrator will vary, but will average out to zero, the correct result.

B. Expression of Reported Main and Alternate Signal Amplitudes

In addition to the desired input signals $a_I[n]$ and $a_Q[n]$, there is thermal noise on each of the I and Q inputs of the data recovery circuit. The resulting inputs can therefore be expressed as: $a_I[n] + e_I[n]$ and $a_Q[n] + e_Q[n]$. The amplitude output of the I PATH "I/Q AMPLITUDE DETECTION" integrator, which is ultimately provided to the iDevice, can be expressed as:

$$AMP_I = \sum_{p=0}^{2B} \mathrm{sgn}(b_I[p]) b_I[p] \tag{1}$$

where B is the number of bits in the payload of the tag response (B=128 for an ACK relaying a full PCEPC) and

$$b_I[p] = \sum_{n=0}^{M\cdot OSR/2} \pm\,\mathrm{sgn}(a_I[n + M\cdot\frac{OSR}{2}\cdot p]) c_I[n,p] \tag{2}$$

$$c_I[n,p] = a_I[n + M\cdot\frac{OSR}{2}\cdot p] + e_I[n + M\cdot\frac{OSR}{2}\cdot p] \tag{3}$$

where M is the Miller Modulation index (in this case, fixed at 8) and OSR is 4.5MHz/BLF. While BLF can vary, lab observations have shown that it doesn't vary nearly as much as it is allowed to, and so for the time being we will assume it is fixed at 187.5kHz and that OSR=24. In future revisions of the FPGA design, we can send the clock recovery integrator output to the data recovery RAM to obtain the exact value of the BLF for greater accuracy if necessary. The summation in Eqn. 1 represents the integration at the "I/Q AMPLITUDE DETECTION". The summation in Eqn. 2 represents the correlation and integrate-and-dump output, depicted in Fig. 1 at the "PSEUDO-MATCHED FILTER INTEGRATION". The signum function in Eqn. 2 reflects the fact that for each half-bit period in a successful received waveform, the sign of the square wave multiplying the input signal is always the same or opposite sign as the desired input signal. While we may not know the sign of the ultimate result (this uncertainty is represented by the $\pm$), we will see later that the lack of this information is not an issue.

Similarly, the output of the Q PATH "I/Q AMPLITUDE DETECTION" integrator can be expressed as:

$$AMP_Q = \sum_{p=0}^{2B} \mathrm{sgn}(b_I[p]) b_Q[p] \tag{4}$$

where $b_I[p]$ is as in Eqn. 2 and where:

$$b_Q[p] = \sum_{n=0}^{M\cdot OSR/2} \pm\,\mathrm{sgn}(a_Q[n + M\cdot\frac{OSR}{2}\cdot p]) c_Q[n,p] \tag{5}$$

$$c_Q[n,p] = a_Q[n + M\cdot\frac{OSR}{2}\cdot p] + e_Q[n + M\cdot\frac{OSR}{2}\cdot p] \tag{6}$$

An alternative expression of $b_Q[p]$ that proves useful when computing the angle estimate is:

$$b_Q[p] = \sum_{n=0}^{M\cdot OSR/2} \pm\,\mathrm{sgn}(a_I[n + M\cdot\frac{OSR}{2}\cdot p]) c_Q[n,p] \tag{7}$$

This expression is similarly true, since the relation of $\mathrm{sgn}(a_I[n])$ and $\mathrm{sgn}(a_Q[n])$ can be assumed to remain constant over the tag backscatter response. As mentioned above, for reasonable tag movement speeds, if the tag backscatter reponse is such that the relation of $\mathrm{sgn}(a_I[n])$ and $\mathrm{sgn}(a_Q[n])$ does in fact change over the course of the tag backscatter response, either AMP_I or AMP_Q is close to 0 and what we have for the sign in Eqn. 5 won't alter the angle estimate substantially.

C. Simplification of Expressions

1) $b_I[p]$&$b_Q[p]$: The multiplication within the summation of $b_I[p]$ can be carried out to show:

$$\begin{aligned} &b_I[p] = \\ &\sum_{n=0}^{M\cdot OSR/2} \pm|a_I[n + M\cdot\frac{OSR}{2}\cdot p]| + \\ &\sum_{n=0}^{M\cdot OSR/2} e'_I[n + 2\cdot M\cdot\frac{OSR}{2}\cdot p] \end{aligned} \tag{8}$$

where

$$e'_I[n] = \mathrm{sgn}(a_I[n]) e_I[n] \tag{9}$$

We can see that since the argument of the signum function is statistically independent of $e_I[n]$, the statistics of $e'_I[n]$ should be the same as $e_I[n]$ if $e_I[n]$ is an i.i.d. random variable. In reality, $e_I[n]$ is only bandpass white noise around the backscatter link frequency due to the digital bandpass IIR filters and multiplying $e_I[n]$ by $\mathrm{sgn}(a_I[n])$ merely downconverts it to baseband, where it ($e'_I[n]$) appears as lowpass white noise around DC. However, due to the lowpass nature of the subsequent integration, the result is equivalent to if $e_I[n]$, and hence $e'_I[n]$, had been broadband white noise all along.

Furthermore, the $\pm$ ambiguity in the first term of Eqn. 8 can be moved outside of the summation, since we know that over a half-bit interval the phase relationship between neighboring $a_I[n]$ is constant (although this phase relationship in general changes at the boundaries of the half-bit intervals. Therefore, we have:

$$b_I[p] = \pm \sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]| + \sum_{n=0}^{M \cdot OSR/2} e'_I[n + 2 \cdot M \cdot \frac{OSR}{2} \cdot p] \quad (10)$$

Similarly, it can be shown for the first expression of $b_Q[p]$ above in Eqn. 5 :

$$b_Q[p] = \pm \sum_{n=0}^{M \cdot OSR/2} |a_Q[n + M \cdot \frac{OSR}{2} \cdot p]| + \sum_{n=0}^{M \cdot OSR/2} e'_Q[n + 2 \cdot M \cdot \frac{OSR}{2} \cdot p] \quad (11)$$

For the second expression of $b_Q[p]$ in Eqn. 7, noting that $a_Q[n] = a_I[n]\tan(\angle[n])$:

$$b_Q[p] = \pm \sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]| \tan(\angle[n + M \cdot \frac{OSR}{2} \cdot p]) + \sum_{n=0}^{M \cdot OSR/2} e'_Q[n + 2 \cdot M \cdot \frac{OSR}{2} \cdot p] \quad (12)$$

2) $\mathrm{sgn}(b_I[p])$: Sensitivity in the S.U.R.F.E.R. reader is dictated by the performance of the clock recovery circuit as mentioned in [1]. Sensitivity is reached when $SNR = \sqrt{E[a_I^2[n] + a_Q^2[n]]/E[e_I^2[n] + e_Q^2[n]]}$ is about a factor of 2 as also mentioned in [1]. For a large enough integration interval, in the case described above where $M \cdot OSR = 192$ samples, the $a_I[n]$ term will integrate to about $\pm M \cdot OSR \cdot E[|a_I[n]|]$ while the error $e'_I[n]$ term will average out to zero with a standard deviation of:

$$\sigma_e = \sqrt{E\left[\left(\sum_{n=0}^{M \cdot OSR/2} e'_I[n]\right)^2\right] - E\left[\left(\sum_{n=0}^{M \cdot OSR/2} e'_I[n]\right)\right]^2} \quad (13)$$

Assuming $e'_I[n]$ is a zero-mean i.i.d. process, we have:

$$\sigma_e = \sqrt{\sum_{n=0}^{M \cdot OSR/2} E[e'^2_I[n]]} \quad (14)$$

$$\sigma_e \approx \sqrt{M \cdot \frac{OSR}{2} \cdot \frac{1}{4} E[a_I^2[n]]} \quad (15)$$

The resulting ratio of desired signal to the noise standard deviation at the end of the half-bit integration interval is therefore approximately (assuming $E[|a_I[n]|] \approx E[a_I^2[n]]$ for now):

$$2\sqrt{M \cdot \frac{OSR}{2}} \quad (16)$$

For $M \cdot \frac{OSR}{2} = 96$, this means that the signal amplitude is nearly 20x that of the noise standard deviation at the end of the integration interval. Therefore, only very very rarely will the integrated noise dictate the output of the signum function and we can write:

$$\mathrm{sgn}(b_I[p]) = \mathrm{sgn}\left(\pm \sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]|\right) \quad (17)$$

3) AMP_I: Based on the work done so far, we can also considerably simplify the expression for AMP_I.

$$AMP_I = \sum_{p=0}^{2B} \mathrm{sgn}(b_I[p]) b_I[p] \quad (18)$$

$$AMP_I = \sum_{p=0}^{2B} \mathrm{sgn}\left(\pm \sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]|\right) \cdot \left(\pm \sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]| + \sum_{n=0}^{M \cdot OSR/2} e'_I[n + 2 \cdot M \cdot \frac{OSR}{2} \cdot p]\right) \quad (19)$$

Multiplying through, we have:

$$AMP_I = \sum_{p=0}^{2B} \left(\left|\sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]|\right| + \sum_{n=0}^{M \cdot OSR/2} e''_I[n + 2 \cdot M \cdot \frac{OSR}{2} \cdot p]\right) \quad (20)$$

where

$$e''_I[n] = \pm e'_I[n] \quad (21)$$

The relation in Eqn. 21 simply refers to the fact that some of the $e''_I[n]$ will change sign relative to $e'_I[n]$, in blocks of length p as a result of being multiplied by $\mathrm{sgn}\left(\pm \sum_{n=0}^{M \cdot OSR/2} |a_I[n + M \cdot \frac{OSR}{2} \cdot p]|\right)$. However, since we assumed $e_I[n]$, and hence $e'_I[n]$, were both i.i.d. zero-mean processes, then so will be $e''_I[n]$. Furthermore, it should be apparent that the standard deviation of $e''_I[n]$ should be the same as that of $e'_I[n]$ and $e_I[n]$. We can simplify the expression for AMP_I further by noting that, for the first term of Eqn. 20, a summation of the absolute value of a summation of absolute values is just a summation of absolute values. For the second term of Eqn. 20, the summation of a summation of i.i.d. zero-mean noise terms is just a summation of i.i.d. zero-mean noise terms. As such, we can write:

$$AMP_I = \sum_{n=0}^{B\cdot M\cdot OSR} \left(|a_I[n]| + e''_I[n]\right) \tag{22}$$

4) AMP_Q: Simplifying AMP_Q is similar to simplifying AMP_I with the added complication of mixed I and Q terms. The simplification is made easier by noting that over the tag backscatter response (about 6.5ms at most), the angular relationship between $a_I[n]$ and $a_Q[n]$ remains roughly constant. Certainly, this is even more true over a p interval. More specifically, for each p, the product of $\mathrm{sgn}\left(\sum_{n=0}^{M\cdot OSR/2}|a_I[n + M\cdot \frac{OSR}{2}\cdot p]|\right)$ and $\mathrm{sgn}\left(\sum_{n=0}^{M\cdot OSR/2}|a_Q[n + M\cdot \frac{OSR}{2}\cdot p]|\right)$ will be the same (except for perhaps one p at a quadrant crossing).

Using the assumption discussed above, we can write:

$$AMP_I = \sum_{n=0}^{B\cdot M\cdot OSR} \left(\pm|a_Q[n]| + e''_Q[n]\right) \tag{23}$$

While ambiguity on the sign of the $|a_Q[n]|$ term persists, we will see that when squaring AMP_Q to obtain an RSSI estimate, the ambiguity goes away. For the AMP_Q expression related to Eqn. 12, we can write:

$$AMP_Q = \sum_{n=0}^{B\cdot M\cdot OSR} \left(|a_I[n]|\tan(\angle[n]) + e''_Q[n]\right) \tag{24}$$

This latter expression proves useful when obtaining an angle estimate.

II. RSSI Estimate

A. Computing Input-Referred Power with $|a_I[n]|$ and $|a_Q[n]|$

A correct expression for RSSI will include the quantity $E[(a_I[n]^2 + a_Q[n]^2)] = E[|a_I[n]|^2] + E[|a_Q[n]|^2]$ plus a scaling and error term. However, the closest quantity that we will have available is: $E[|a_I[n]|]^2 + E[|a_Q[n]|]^2$. Is it possible for us to convert between the two quantities, via a simple scaling factor if possible? An easy way to do this is to note that the incoming signal in Fig. 2 is to good approximation a sine wave over a half-bit interval, after which the first correlation operation smears out any peculiarities of the phase transitions that occur at half-bit boundaries. So, assuming that the incoming signal is indeed a sine wave, and that its phase angle is 0°, with no loss of generality, we can write:

$$\begin{aligned} E[|a_I[n]|^2] &= E[|A\sin(\omega t)^2|] = A^2E[\sin^2(\omega t)] \\ &= A^2\left[\frac{\frac{1}{2} + \frac{1}{2\omega}\sin(\omega t)\Big|_0^{\pi/\omega}}{\pi/\omega}\right] = A^2/2 \end{aligned} \tag{25}$$

and

$$\begin{aligned} E[|a_I[n]|]^2 &= E[|A\sin(\omega t)|]^2 = A^2\left[\frac{\int_0^{\frac{\pi}{\omega}}|\sin(\omega t)|\,dt}{\pi/\omega}\right]^2 \\ &= A^2\left[\frac{\frac{1}{\omega}\cos(\omega t)\Big|_0^{\pi/\omega}}{\pi/\omega}\right]^2 = 4A^2/\pi^2 \end{aligned} \tag{26}$$

In addition to the ratio between $E[|a_I[n]|^2]$ and $E[|a_I[n]|]^2$, another adjustment needs to be made to account for the fact that the tag backscatter signal is a square wave as described in [2] while the signal incident on the data recovery circuit is a sinusoid. It is expected that the actual waveform indeed is a square wave since the switch controlling the antenna reflection reflection coefficient should be able to switch into at least the GHz range in modern CMOS processes. Furthermore, the tag antenna bandwidth will be greater than 20MHz but the backscatter harmonics will be at frequency offset multiples of 187.5kHz, so there will be no filtering of these harmonics prior to analog baseband of the SX1257. However, suppression of these harmonics will be complete due to the digital bandpass channel filter.

Therefore, in order to get the true RSSI referred to the antenna port of the RFID reader, we need to multiply the input-referred RSSI estimate using our estimate of $E[|a_I[n]|^2]$ by the ratio of the power of the first harmonic of a $\pm A$V square wave divided by the total power in said $\pm A$V square wave (A^2). The first harmonic of such a square wave is widely known to be $\frac{4A}{\pi}\sin(\omega t)$.

$$\begin{aligned} &\frac{16A^2}{\pi^2}\frac{\int_0^{\pi/\omega}\sin^2(\omega t)dt}{A^2\pi/\omega} = \\ &\frac{16A^2}{\pi^2}\left[\frac{\left[\frac{1}{2} + \frac{1}{2\omega}\sin(\omega t)\right]\Big|_0^{\pi/\omega}}{A^2\pi/\omega}\right] = 8/\pi^2 \end{aligned} \tag{27}$$

Finally, in order to convert an RSSI estimate based on $E[|a_I[n]|]^2 + E[|a_Q[n]|]^2$ to the actual received power at the antenna port, we need to multiply said estimate by:

$$\frac{\pi^2}{8}\cdot\frac{A^2/2}{4A^2/\pi^2} = \frac{\pi^4}{64} = 1.522 \tag{28}$$

B. Gain from Reader Antenna Port to Data Recovery Circuit

Previously, the voltage gain around 915MHz from the reader antenna port to the data recovery circuit input was measured to be is 131.5dB. Part of the reason that this gain is so large is that in the digital domain, each LSB is counted as a "digital volt". In order to refer any RSSI estimate based on $E[|a_I[n]|]^2 + E[|a_Q[n]|]^2$, a "squared voltage" quantity, to the antenna port needs to therefore be divided by $10^{131.5/10}$. In order to convert this to a power value, the input-referred "squared voltage" quantity needs to be divided by the 50Ω characteristic impedance of the system.

C. A Proposed RSSI Estimate

We really don't have many options for estimating RSSI. Here, we propose the obvious and determine if we have chosen the correct estimator by substituting the expressions developed earlier in this paper. Assuming so, out of this exercise will also come any error terms, which we can evaluate for bias and standard deviation.

$$\widehat{RSSI(W)} = \frac{1}{PwrGain}\left(\frac{AMP_I^2}{N^2} + \frac{AMP_Q^2}{N^2}\right) \tag{29}$$

where $N = B{\cdot}M{\cdot}OSR$ and $PwrGain = 50{\cdot}10^{131.5(dB)/10}{\cdot}64/\pi^4$. Substituting in from Eqn. 22, we obtain:

$$\frac{AMP_I^2}{N^2} = \frac{1}{N^2}\left[\left(\sum_{n=0}^{N}|a_I[n]| + \sum_{n=0}^{N} e_I''[n]\right)\left(\sum_{m=0}^{N}|a_I[n]| + \sum_{m=0}^{N}|e_I''[n]|\right)\right] \tag{30}$$

Multiplying through, we have:

$$\frac{AMP_I^2}{N^2} = \frac{1}{N^2}\cdot \left[\left(\sum_{n=0}^{N}|a_I[n]|\right)^2 + 2\left(\sum_{n=0}^{N}|a_I[n]|\right)\left(\sum_{m=0}^{N} e_I''[m]\right) + \dots\right] \tag{31}$$

Noting that the a_I terms are in the form of expected value estimators, we arrive at one of the quantities of interest plus an error term:

$$\frac{AMP_I^2}{N^2} = \left[\left(\widehat{E}|a_I[n]|\right)^2 + 2\left(\frac{1}{N}\widehat{E}[|a_I[n]|]\right)\left(\sum_{m=0}^{N} e_I''[m]\right) + \dots\right] \tag{32}$$

Continuing and substituting in from Eqn. 23, we have that:

$$\frac{AMP_Q^2}{N^2} = \left[\left(\pm\widehat{E}|a_Q[n]|\right)^2 \pm 2\left(\frac{1}{N}\widehat{E}[|a_Q[n]|]\right)\left(\sum_{m=0}^{N} e_Q''[m]\right) + \dots\right] \tag{33}$$

In the first term of Eqn. 33, the $\pm$ term will go away by squaring, while in the second term we can just define yet another error term $e_Q'''[m]$ with its sign potentially reversed. Again, changing the sign of the noise does not change its statistical properties. As a result, we have:

$$\frac{AMP_Q^2}{N^2} = \left[\left(\widehat{E}|a_Q[n]|\right)^2 + 2\left(\frac{1}{N}\widehat{E}[|a_Q[n]|]\right)\left(\sum_{m=0}^{N} e_Q'''[m]\right) + \dots\right] \tag{34}$$

Finally, we can put it all together to see that:

$$\widehat{RSSI(W)} = \frac{1}{PwrGain}\left(\left(\widehat{E}|a_I[n]|\right)^2 + \left(\widehat{E}|a_Q[n]|\right)^2 + \frac{2}{N}\left(\widehat{E}[|a_I[n]|]\left(\sum_{m=0}^{N} e_I''[m]\right) + \widehat{E}[|a_Q[n]|]\left(\sum_{m=0}^{N} e_Q'''[m]\right)\right)\right) \tag{35}$$

We can see that the first line of Eqn. 35 is the term of interest. The second line of Eqn. 35 is an error term, and we care to know its standard deviation.

D. Error Statistics in RSSI Estimate

Denoting

$$\epsilon(W) = \frac{2}{PwrGain \cdot N}\cdot \left(\widehat{E}[|a_I[n]|]\left(\sum_{m=0}^{N} e_I''[m]\right) + \widehat{E}[|a_Q[n]|]\left(\sum_{m=0}^{N} e_Q'''[m]\right)\right) \tag{36}$$

We note that we are looking for the standard deviation of the error $\sqrt{E[\epsilon(W)^2]}$. This can be confusing because we are squaring watts, when usually the "watts" term is the one that has been squared already. It can be shown that the desired quantity is equal to:

$$\sigma_{\epsilon(W)} = \frac{2}{PwrGain \cdot N}\cdot \sqrt{E[|a_I[n]|]^2\left(\sum_{m=0}^{N} E[e_I''^2[m]]\right) + E[|a_Q[n]|]^2\left(\sum_{m=0}^{N} E[e_Q'''^2[m]]\right)} \tag{37}$$

since the expected value of cross-terms of the squaring are zero due to independence of I and Q noise and since the noise processes are i.i.d. zero-mean. Continuing simplifications, assuming that I and Q noise statistics are equal:

$$\sigma_{\epsilon(W)} = \frac{2}{PwrGain \cdot \sqrt{N}} \cdot \sqrt{(E[|a_I[n]|]^2 + E[|a_Q[n]|]^2)\, E[e_I''^2[n]]} \tag{38}$$

Given this result, the best interpretation is to compute the rms percentage error at sensitivity. We begin with:

$$\frac{\sigma_{\epsilon(W)}}{\mu_{\widehat{RSSI}}} = \sqrt{\frac{2}{N}}\sqrt{\frac{E[e_I^2[n]] + E[e_Q^2[n]]}{E[|a_I[n]|]^2 + E[|a_Q[n]|]^2}} \tag{39}$$

$$\frac{\sigma_{\epsilon(W)}}{\mu_{\widehat{RSSI}}} = \sqrt{\frac{\pi^2}{4N}}\sqrt{\frac{E[e_I^2[n]] + E[e_Q^2[n]]}{E[a_I^2[n]] + E[a_Q^2[n]]}} \tag{40}$$

$$\frac{\sigma_{\epsilon(W)}}{\mu_{\widehat{RSSI}}} = \frac{\pi}{2\sqrt{N}}\frac{1}{SNR} \tag{41}$$

For $N = B \cdot M \cdot OSR$=24576 and SNR=2 at sensitivity, we see that the rms error is about 0.5%. For larger input magnitudes, SNR only increases, yielding a better measurement. One thing we can take away from this is that the RSSI estimator is not biased by noise, so averaging successive RSSI estimator measurements should improve the accuracy.

III. Angle Estimate

A. A Proposed Angle Estimate

We really don't have many options for estimating angle, either. Here, we propose the obvious and determine if we have chosen the correct estimator by substituting the expressions developed earlier in this paper. Assuming so, out of this exercise will also come any error terms, which we can evaluate for bias and standard deviation.

$$\widehat{\angle(rads)} = \tan^{-1}\left(\frac{AMP_Q}{AMP_I}\right) \tag{42}$$

$$\widehat{\angle(rads)} = \tan^{-1}\left(\frac{\sum_{n=0}^{N}\left(|a_I[n]|\tan(\angle[n]) + e_Q''[n]\right)}{\sum_{n=0}^{N}(|a_I[n]| + e_I''[n])}\right) \tag{43}$$

where again $N = B \cdot M \cdot OSR$. We can convert the summations into approximate expected value operations and then separate out the signal term from the angle term using their independence.

$$\widehat{\angle(rads)} = \tan^{-1}\left(\frac{\widehat{E}[|a_I[n]|\tan(\angle[n])] + \frac{1}{N}\sum_{n=0}^{N} e_Q''[n]}{\widehat{E}|a_I[n]| + \frac{1}{N}\sum_{n=0}^{N} e_I''[n]}\right) \tag{44}$$

$$\widehat{\angle(rads)} \approx \tan^{-1}\left(\frac{\widehat{E}[|a_I[n]|]\widehat{E}[\tan(\angle[n])] + \frac{1}{N}\sum_{n=0}^{N} e_Q''[n]}{\widehat{E}|a_I[n]| + \frac{1}{N}\sum_{n=0}^{N} e_I''[n]}\right) \tag{45}$$

$$\widehat{\angle(rads)} \approx \tan^{-1}\left(\widehat{E}[\tan(\angle[n])]\frac{1}{1 + \frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]}} + \frac{\frac{\frac{1}{N}\sum_{n=0}^{N} e_Q''[n]}{\widehat{E}[|a_I[n]|]}}{1 + \frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]}}\right) \tag{46}$$

Assuming that the error terms are small, and converting $\frac{1}{1+\epsilon}$ to $1-\epsilon$, we have:

$$\widehat{\angle(rads)} \approx \tan^{-1}\left(\widehat{E}[\tan(\angle[n])]\left(1 - \frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]}\right) + \frac{\frac{1}{N}\sum_{n=0}^{N} e_Q''[n]}{\widehat{E}[|a_I[n]|]}\left(1 - \frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]}\right)\right) \tag{47}$$

Discounting the double error term at the end, we can rearrange to give:

$$\widehat{\angle(rads)} \approx \tan^{-1}\left(\widehat{E}[\tan(\angle[n])] - \widehat{E}[\tan(\angle[n])]\frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]} + \frac{\frac{1}{N}\sum_{n=0}^{N} e_Q''[n]}{\widehat{E}[|a_I[n]|]}\right) \tag{48}$$

Now, noting that for small ϵ_1 and ϵ_2, we have $\tan^{-1}(x + \epsilon_1 + \epsilon_2) \approx \tan^{-1}(x) + (\epsilon_1 + \epsilon_2)\frac{1}{1+x^2}$. Applying this relation to Eqn. 48 yields:

$$\widehat{\angle(rads)} \approx \tan^{-1}\left(\widehat{E}[\tan(\angle[n])]\right) - \widehat{E}[\tan(\angle[n])]\frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]}\frac{1}{1 + \widehat{E}[\tan(\angle[n])]^2} + \frac{\frac{1}{N}\sum_{n=0}^{N} e_Q''[n]}{\widehat{E}[|a_I[n]|]}\frac{1}{1 + \widehat{E}[\tan(\angle[n])]^2} \tag{49}$$

It is tempting to assume that the first term of Eqn. 49 is equal to $\widehat{\angle[n]}$, but this is only true if $\angle[n]$ is not changing over the measurement interval. If $\angle[n]$ is changing, a biased estimate of $\angle[n]$ results. For example, consider a tag angle changing from 75° to 85° over the N-sample (≈5.5ms) interval. At a carrier frequency of 915MHz, this corresponds to a tag moving at about 1.66m/s in free space. (By comparison, the fastest 100m sprint record is 10.44m/s). In this case, $E[\angle[n]] = 80°$ but $\tan^{-1}(\tan(E[\angle[n]])) = 80.89°$.

For now, we will accept that $\tan^{-1}\left(\widehat{E}[\tan(\angle[n])]\right)$ is an acceptable estimate for $E[\angle[n]]$ but the reader is advised to be aware of the implications of this choice. One potential solution to the problem of the angle measurement being smeared is to change M to 4 or 2. In this case, sensitivity may not be terribly affected, as it is dictated by the clock recovery circuit. This would also have the effect of speeding up the read rate.

B. Error Statistics in Angle Estimate

Manipulating the error term in Eqn. 49 yields, using various approximations and manipulations discussed above:

$$\epsilon_{\angle} = \left(\frac{\frac{1}{N}\sum_{n=0}^{N} e_I''[n]}{\widehat{E}[|a_I[n]|]} + \frac{\frac{1}{N}\sum_{n=0}^{N} e_Q'''[n]}{\widehat{E}[|a_Q[n]|]} \right) \cdot \frac{\widehat{E}[\tan(\angle[n])]}{1+\widehat{E}[\tan(\angle[n])]^2} \quad (50)$$

We can see here that the error has zero-mean; i.e. it is unbiased by the thermal noise at the data recovery circuit input. Taking the rms value of Eqn. 50 yields:

$$\sigma_{\epsilon_{\angle}} = \frac{1}{\sqrt{N}} \frac{\widehat{E}[\tan(\angle[n])]}{1+\widehat{E}[\tan(\angle[n])]^2} \sqrt{\frac{E[e_I^2[n]]}{\widehat{E}[|a_I[n]|]^2} + \frac{E[e_Q^2[n]]}{\widehat{E}[|a_Q[n]|]^2}} \quad (51)$$

As $\angle[n]$ approaches a quadrant boundary, the second term of Eqn. 51 goes to zero while the third term goes to infinity. It is likely that their product is a constant value. We note that $\widehat{E}[\tan(\angle[n])]$=$\widehat{E}[|a_Q[n]|]/\widehat{E}[|a_I[n]|]$. Substituting this relation and multiplying through yields:

$$\sigma_{\epsilon_{\angle}} = \frac{1}{\sqrt{N}} \sqrt{\frac{E[e_I^2[n]]\widehat{E}[|a_Q[n]|]^2 + E[e_Q^2[n]]\widehat{E}[|a_I[n]|]^2}{(\widehat{E}[|a_I[n]|]^2 + \widehat{E}[|a_Q[n]|]^2)^2}} \quad (52)$$

Noting that $E[e_I^2[n]$=$E[e_Q^2[n]$, we can write:

$$\sigma_{\epsilon_{\angle}} = \frac{1}{\sqrt{N}} \sqrt{\frac{E[e_I^2[n]]}{\widehat{E}[|a_I[n]|]^2 + \widehat{E}[|a_Q[n]|]^2}} \quad (53)$$

$$\sigma_{\epsilon_{\angle}} = \frac{1}{\sqrt{N}} \sqrt{\frac{\frac{1}{2}(E[e_I^2[n]] + E[e_Q^2[n]])}{\widehat{E}[|a_I[n]|]^2 + \widehat{E}[|a_Q[n]|]^2}} \quad (54)$$

$$\sigma_{\epsilon_{\angle}} = \frac{1}{\sqrt{N}} \sqrt{\frac{\frac{1}{2}\frac{\pi^2}{8}(E[e_I^2[n]] + E[e_Q^2[n]])}{\widehat{E}[a_I^2[n]] + \widehat{E}[a_Q^2[n]]}} \quad (55)$$

$$\sigma_{\epsilon_{\angle}} = \frac{\pi}{4\sqrt{N}} \frac{1}{SNR} \quad (56)$$

As described earlier, SNR is about 2 at sensitivity, while N=24576 for the existing S.U.R.F.E.R. system implementation. Therefore, the error standard deviation at sensitivity is about 0.0025 radians, or 0.14°.

IV. Conclusion

In this paper, the I and Q signal amplitude quantities directly available from the S.U.R.F.E.R. reader FPGA were mathematically analyzed and shown to be approximately equal to sums of absolute values of the I and Q input components plus noise terms. Conversion factors between sums of absolute values and input-referred rms signal power were then developed. Finally, RSSI and angle estimates were proposed and shown to be equivalent to the desired expressions. The rms error of these expressions was calculated, and the implication of a moving object on the angle estimator was discussed.

References

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