2.1 SOT in clutter – Special case of MOT

→ only 1 object in env all times

Pros

1) No need to figure no. of objects.
2) No. of DA hypo <<

New challenges in SOT than KF:

1) missed detection
2) false detection – clutter detection
3) unknown DA (which is clutter which is object)

→ clutter is more random (no structure)

2.2 SOT motion, measurement models

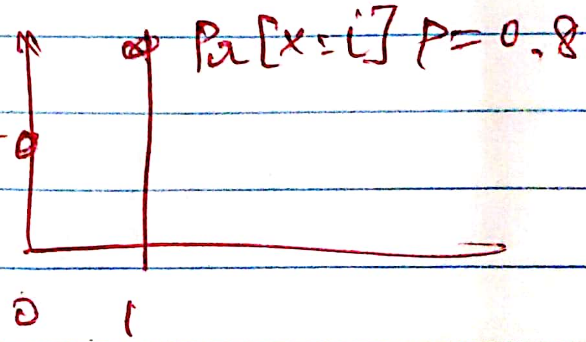
Bernoulli distn

Successfulness of experiment

Detection of object

$X \in$ Bernoulli distributed with $p \in [0,1]$

$$x = \begin{cases} 1 & \text{with prob } p \\ 0 & \text{with prob } 1-p \;\; 0.2 \end{cases}$$



$\boxed{\text{Pmf}}$

<u>motion model:</u> $P(x_k|x_{k-1}) = \pi_k(x_k|x_{k-1})$

motion model for SO

$x_k = f_{k-1}(x_{k-1}) + q_{k-1}$

$q_{k-1} = N(0, Q_{k-1})$

$$\boxed{\pi_k(x_k|x_{k-1}) = N(x_k; f_{k-1}(x_{k-1}), Q_{k-1})}$$

<u>Measurement model:</u>

object is detected with $\boxed{P_D}$

$P(o_k|x_k) = g_k(o_k|x_k)$

measurement for SO.
detection

$$\boxed{g_k(o_k|x_k) = N(o_k; h_k x_k, R_k)}$$

$o_k = h_k(x_k) + v_k$; $v_k \sim N(0, R_k)$

$o_k = \begin{cases} [\,] & \text{if undetected} \\ o_k & \text{if detected} \end{cases}$

$|o_k|$ = no. of column vectors in o_k.

$$\boxed{|o_k| = \begin{cases} 1 & \text{with } P_D(x_k) \\ 0 & \text{with } 1-P_D(x_k) \end{cases}}$$

Bernoulli distri

Generate measurements.

1) Initialise $O_k = [\,]$

2) if rand $<$ $P_D(x_k)$ then

3) $o_k \sim g_k(\cdot \mid x_k)$

4) $O_k = o_k$

5) end if

Update step:

$$p(O_k|x_k) = \begin{cases} 1 - P^D(x_k), & \text{if } O_k = [\,]. \\ P^D(x_k)\, g(O_k|x_k) & \text{if } O_k = o_k \end{cases}$$

posterior $\propto$ prior $\times$ likelihood.

$$P(x_k|O_{1:k}) \propto P(x_k|O_{1:k-1})\, p(O_k|x_k)$$

$$= \begin{cases} \underbrace{P(x_k|O_{1:k-1})}_{\text{prior}} \times (1 - P^D(x_k)) & \text{if } O_k = [\,] \\ \quad '' \times P^D(x_k)\, g(O_k|x_k) & \text{if } O_k = o_k \end{cases}$$

~~PD~~ $P^D(x_k)$ generally constant but can be informative sometimes

Assume $P^D(x_k) = \begin{cases} 1 & \text{if } x_k \geq 0 \\ 0 & \text{if } x_k < 0 \end{cases}$

Assume O_k is independ of x_k

if object is undetected, x_k is $-ve$
else $+ve$

if P^D = constant. Standard update if object is detected

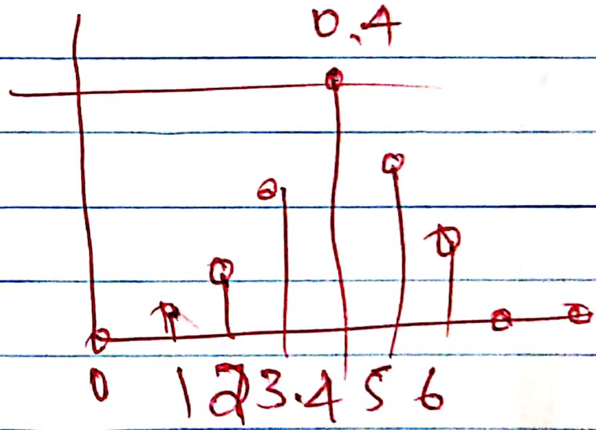
(2.4) clutter model: - environment, sensor factors.

Binomial distribution.

$\Downarrow$

if x is $\Downarrow$, with j independent trials each with pr(success) $= p \in [0,1]$

$p = 0.8$, $j = 5$



$\Downarrow$

Total no. of experi is Binom. distributed

~~Pr[x = i] =~~

$$Pr[x = i] = \binom{j}{i} p^{i} (1-p)^{j-i}$$

$\Downarrow$

$$\frac{j!}{i!\,(j-i)!}$$

$$\boxed{E(x) = pj}$$

$Pr(x=4) = 0.4$

I have 0.4 chance of

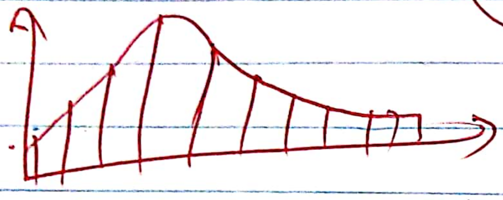
Poisson distributed

$\lambda = E[X]$: parameter $\boxed{\lambda > 0}$

$$\Pr[X=i] = Po[i;\lambda) = \frac{\lambda^{i}\exp(-\lambda)}{i!}$$

$\boxed{\lambda = 4}$

$$\boxed{E[X] = Var(X) = \lambda}$$



U can't increase mean w/o increasing its variance.

Observed measurement matrix

$$\boxed{Z_K = \Pi(O_K, C_K)}$$

C_K → clutter

Π → random colmn shuffle operator.

We need to model C_K

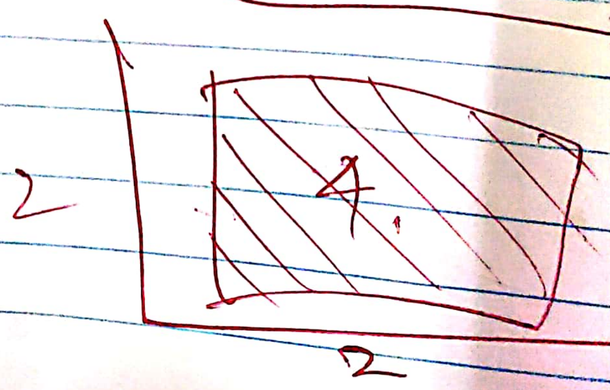
Stochastic model that covers

1) number of detections $|C_K|$
2) vectors in C_K

Assume $\mathbb{R}^{n_z}$ (FOV) of sensor Volume V.

$\boxed{V = 4, \ \lambda = 0.8}$

λ = expected number of clutter detections per unit volume.

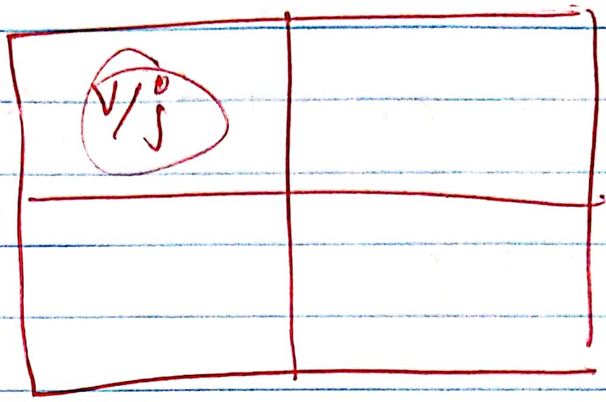


clutter Limited resolution:

Objects (nearby) generate atmost 1 detection.

clutter model

Split volume into 'j' cells



$C_k = \Pi(C_k^{(1)}, C_k^{(2)}, \ldots C_k^{(j)})$

↓

clutter detection in each cell

$C_k^{(1)}, C_k^{(2)}$ are independent

$|C_k^{(i)}|$ = Bernoulli distrib with $\Pr \boxed{\lambda\left(\frac{V}{j}\right)}$ → volume of 1 cell

probab of clutter in 1 cell

↳ means $|C_k|$ is binomial with j experiments each with $\Pr(\text{success}) = \lambda\left(\frac{V}{j}\right)$

Unlimited resolution:

As $j\uparrow$ → cells become smaller

→ ~~cells~~ possible to get more detections

→ $\Pr(\text{detection})$ in 1 cell $\left(\frac{\lambda V}{j}\right)\downarrow$.

→ $E[|C_k|] = \cancel{j} \boxed{V\lambda}$

As $j \to \infty$

$|C_k|$ is Poisson distributed

C_k is Poisson point process

↓

Std. model for clutter

2.5 Poisson point process

$C_K = [c_K^1 \; c_K^2 \ldots c_K^{m_K^c}]$

m_k^c = number of clutter $\sim Po(\lambda V)$

vectors c_k^i are i.i.d. $\boxed{c_k^i \sim \text{unif}(V)}$

Sampling PPP

1) $C_K = [\,]$
2) generate $m_{k_c} \sim Po(\lambda V)$
3. for $i = 1$ to m_{k_c} do
4) Generate $c_k^i \sim \text{unif}(V)$
5) $C_K = [C_K, c_k^i]$
6) end for

Parametrize using either clutter

a) intensity fn

$\lambda_c(c) \geq 0$

region wise

eg: more in 1 area

b) combination of

expected number of detections

$\bar{\lambda}_c = \int \lambda_c(c) \cdot dc$ rate

$f_c(c)$ Spatial pdf.

$= \frac{\lambda_c(c)}{\bar{\lambda}_c}$

Intensity function is

$$\lambda_c(c) = \begin{cases} \lambda & \text{if } c \in V \\ 0 & \text{otherwise} \end{cases}$$

$\bar{\lambda}_c = \lambda V$

$$f_c = \begin{cases} \frac{1}{V} & \text{if } c \in V \\ 0 & \text{otherwise} \end{cases}$$

1) $C_k = [\,]$

2) generate $m_k^c \sim Po(\bar{\lambda}_c)$

3) for $i=1$ to m_k^c do

4) generate $c_k^i = f_c(\cdot)$

5) $C_k = [C_k \; c_k^i]$

) endfor

need not be uniform

$$P(C_k) = p(C_k, m_k^c)$$

$$= p(m_k^c)\, p(C_k | m_k^c)$$

$$= Po(m_k^c ; \bar{\lambda}_c) \prod_{i=1}^{m_k^c} f_c(c_k^i)$$

$$= \frac{\exp(-\bar{\lambda}_c)\cancel{\bar{\lambda}_c^{m_k^c}}}{m_k^c!} \prod_{i=1}^{m_k^c} \frac{\lambda_c(c_k^i)}{\cancel{\bar{\lambda}_c}}$$

con

prob. of clutter in that particular area.

$$= \frac{\exp(-\bar{\lambda}_c)}{m_k^c!} \prod_{i=1}^{m_k^c} \lambda_c(c_k^i)$$

piecewise intensity fn

2.b Complete measurement model

→ object detection model + clutter ~~dat~~ model

$\underbrace{\hspace{6em}}_{\text{DA hypothesis}}$

DA hypothesis

Obj

$P(Z_k | x_k)$

$$\boxed{Z_k = \Pi(O_k, C_k)}$$

challenges · z_k width is random
which column of z_k is object detection

We need to know which column of z_k is object measurement and perform $g_k(o_k|x_k)$ on that measurement & clutter model on other colns
But not possible

Notation $\theta = \begin{cases} 0, & \text{if undetected} \\ i>0, & \text{if } z^i \text{ is object detection} \end{cases}$

m = width

$\sim p(c_k, m_k^i)$ step

$$P(Z|x) = p(Z, m|x)$$

$$= \sum_{\theta=0}^{m} p(Z, m, \theta | x) \longrightarrow \text{law of total probability}$$

$$= \sum_{\theta=0}^{m} p(Z|m, \theta, x)\, p(\theta, m|x)$$

Using two variables m, θ to express complicated distribut. to simpler terms.

Q.2

Case 1: $\theta = 0$, $m = \begin{cases} \text{object is not detected} \\ m \text{ clutter detections} \end{cases}$

Case 2: ~~θ~~ $\theta = i > 0$, $m = \begin{cases} \text{object is detected} \\ m-1 \text{ clutter detections} \\ \text{object detection is @ } i \text{ index} \end{cases}$

Case 1:

$\theta = 0$, m clutter detections

$$p(Z \mid m, \theta) = \prod_{i=1}^{m} f_c(z^i)$$

$$p(0, m \mid x) = (1 - P^D(x)) \, Po(m; \bar{\lambda}_c)$$

$$\therefore p(z, m, \theta \mid x) = \prod_{i=1}^{m} f_c(z^i) \, (1 - P^D(x)) \, Po(m; \bar{\lambda}_c)$$

inserting definitions

$$\Rightarrow (1 - P^D(x)) \, \frac{\exp(-\bar{\lambda}_c)}{m!} \prod_{i=1}^{m} \lambda_c(z^i)$$

Pro for c_k being z_k.

Case 2: $\theta = 1, \dots, m$.

$$p(z|m, \theta, x) = \cancel{g_K(z^\theta|x)} \; g_K(z^\theta|x) \prod_{\substack{i=1 \\ i \neq \theta}}^{N} f_c(z^i)$$

except $i = \theta$
all clutter detecti

$$\boxed{p(\theta, m|x) = P^D(x)\, Po(m-1; \lambda_c) \left(\frac{1}{m}\right)}$$

uniform distri from 1 to m.

$$\therefore p(z|x) = P^D(x) \cancel{Po(m-1; \lambda_c)\left(\frac{1}{m}\right)} \; \frac{g_K(z^\theta|x)}{\lambda_c(z^\theta)} \frac{\exp(-\bar{\lambda}_c)}{m!}$$

$\therefore$ Total measurement model

$$\boxed{p(z|x) = \text{case 1} + \text{Case 2}}$$

Visualisation

$P^D(x) = 0.85$ $\quad g(0|x) = N(0; x; 0.2)$

$$\lambda_c = \begin{cases} 0.3 & \text{if } |x| \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

$z = [-1.6, 1] \Rightarrow 2$ measurements

$\theta = 0 \rightarrow$ flat line $\quad (1 - P_D(x)) \frac{\exp(-\lambda_c)}{m!} \prod_{i=1}^{m} \lambda_c(z^i)$

height is indicated by P_D

As $P_D \uparrow$, height $\downarrow$

2.13 Visualising SOT Recursion.

- Exact solns for few iteration $p(x_k | Z_{1:k})$
- Intractable for $n > t$ (exp increase in DA hypo)

$$P(x_k | Z_{1:k}) = \sum_{\theta_{1:k}} \underbrace{p(x_k | Z_{1:k}, \theta_{1:k})}_{\text{given a DA hypo perform update}} \underbrace{\Pr[\theta_{1:k} | Z_k]}_{\text{Prob of DA hyp given measurements}}$$

$$P(x_{k+1} | Z_{1:k}) = \sum_{\theta = 1:k} \underbrace{p(x_{k+1}}_{\text{motion model}} | Z_{1:k}, \theta_{1:k}) \Pr[\theta_{1:k} | Z_{1:k}]$$

2.13 visualisation

$(m_1 + 1) \times (m_2 + 1)$

hypothesis.

$\#10$

$64 \times 64 \times 64 \times 64$

MOT 2.9

Normalising posterior of mixture of Density

$$P(z_k \mid x_k) = \sum_{\theta_k=0}^{m} p(z_k, m_k, \theta_k \mid x_k)$$

measurement likelihood

Posterior $p(x/z) \propto p(z/x)\,p(x)$

$$= \sum_{\theta=0}^{m} p(x)\,p(z, m, \theta \mid x)$$

Desired form $p(x \mid z) = \sum_{m=0}^{\theta} w_\theta\, p_\theta(x)$

where w_θ = pmf $\Pr([\theta \mid z])$

$p_\theta(x)$ = pdf $p(x \mid \theta, z)$

weight for data association

Distribution assuming z, θ, known.

$$\boxed{P(x) \propto g(x) = \sum_{\theta=0}^{m} g_\theta(x)}$$

$g_0 \ldots g_m(x)$ are +ve integrals

g_0, g_1 must be normalised gaussian,

$$p(x) \propto g(x) = c\,g(x).$$

Given $p(x) = \text{pdf}$ $\therefore$ $1 = \int p(x)\,dx$

$$1 = \int c \cdot g(x) \cdot dx \qquad c = \frac{1}{\int g(x) \cdot dx} \approx \frac{1}{\text{area of curve}}.$$

$$\boxed{\therefore p(x) = \frac{g(x)}{\int g(x) \cdot dx}}$$

$\Rightarrow$ for each hypo v scale by area under the curve.

factorizing $\tilde{g}_\theta(x)$

$$\tilde{w}_\theta = \int g_\theta(x) \cdot dx$$

$$p_\theta(x) = \frac{g_\theta(x)}{\tilde{w}_\theta}, \qquad \boxed{g_\theta(x) = \tilde{w}_\theta\, p_\theta(x)}$$

$p_\theta(x) = \text{pdf}$

$$\boxed{p(x) \propto g(x) = \sum_{\theta=0}^{m} \tilde{w}_\theta\, p_\theta(x)}$$

$$\boxed{g(x) = 3p_0(x) + 2p_1(x)}$$

w_θ = normalised weights. $\quad w_\theta = \dfrac{\tilde{w}_\theta}{\sum_{i=0}^{m} \tilde{w}_i}$

where $\boxed{\tilde{w}_\theta = \int g_\theta(x) \cdot dx}$

$g(x) = 3p_0(x) + 2p_1(x)$

$p(x) = 0.6\, p_0(x) + 0.4\, p_1(x)$

$p(x) = \sum_{\theta=0}^{m} w_\theta p_\theta(x)$ where $p_\theta(x) = \dfrac{g_\theta(x)}{\tilde{w}_\theta}$

2.10 Interpretation of weights & Densities

$$p(x|z) \propto p(x)\, p(z|x)$$
$$= \sum_{\theta=0}^{m} p(x)\, p(z, m, \theta | x)$$

Final expression : $p(x|z) = \sum_{\theta=0}^{m} w_\theta p_\theta(x)$.

w_θ = pmf $\Pr[\theta|z] \rightarrow$ chance of θ being correct.

$p_\theta(x)$ = pdf $p(x|\theta, z)$

as $p_\theta(x) = P(x, \theta | Z)$.

Integrating over x $w_\theta = \Pr(\theta | Z) \rightarrow$ DA possibilities

$$p_\theta(x) = \frac{P(x, \theta | Z)}{w_\theta} \Rightarrow \text{conditional posterior pdfs.}$$

2.4 General update eqn:

$$P(x|Z) = \sum_{\theta=0}^{m} w_\theta \, p_\theta(x)$$

$$P(Z|x) = \left[(1 - P^D(x)) + P^D(x) \sum_{\theta=1}^{m} \frac{g(z^\theta | x)}{\lambda_c(z^\theta)} \right] \underbrace{\frac{\exp(-\bar{\lambda}_c)}{m!} \prod_{i=1}^{m} \lambda_c(z^i)}_{\text{constant.}}$$

$$P(x|Z) \propto p(x) \, p(Z|x).$$

$$\propto p(x) \left[1 - P^D + P^D \sum_{\theta=1}^{m} \frac{g}{\lambda_c} \right]$$

But $P(x|Z) \cancel{=} \sum_{\theta=0}^{m} w_\theta \, p_\theta(x)$

$\theta = 0$

object is undetected

$$\tilde{w}_0 = \int p(x)\,(1 - P^D(x))\,dx$$

$$p_0(x) = \frac{p(x)\,(1 - P^D(x))}{\int p(x')\,(1 - P^D(x'))\,dx'}$$

$\theta \in \{1, 2, \dots m\}$

z^θ = object detection

$$\tilde{w}_\theta = \frac{1}{\lambda_c(z^\theta)} \int p_x\, P^D(x)\, g(z^\theta|x)\, dx,$$

$$p_\theta(x) =$$

$$w_\theta \propto \tilde{w}_\theta'$$

object measurement model given θ

$$P(O|x) = \begin{cases} 1 - P^D(x) & \text{if } O = [\,] \\ P^D(x)\, g(O|x) & \text{if } O = o \end{cases}$$

$$p(x|O) \propto \begin{cases} p(x)\,(1 - P^D(x)) & \text{if } O = [\,] \\ p(x)\, P^D(x)\, g(o|x) & \text{if } O = o \end{cases}$$

$$\cancel{p_\theta}\ p_\theta(x) \propto \begin{cases} p(x)\,(1 - P^D(x)) & \text{if } \theta = 0 \\ p(x)\, P^D(x)\, g(z^\theta|x) & \text{if } \theta = 1, 2, \dots m \end{cases}$$

$$p_\theta(x) = p(x|O)$$

with O defined by θ, Z

Object measurement $g(o|x) = N(o; Hx, R)$

$\boxed{\text{clutter } \cancel{\text{densit}} \text{ inten } \lambda_c(c) \geq 0,}$ → $\boxed{\text{No assumptions needed}}$

$$p(x|z) = \sum_{\theta=0}^{m} w_\theta p_\theta(x)$$

~~When P~~

$$p_\theta(x) \propto \begin{cases} p(x)(1 - P_D(x)) & \text{if } \theta = 0 \\ p(x)\, P_D(x)\, g(z^\theta|x) & \text{if } \theta \in \{1, \dots, m\} \end{cases}$$

~~If~~ P_D = constant

KF eqns to compute posterior

$$S = HPH^T + R$$

$$KG = PH^T S^{-1}$$

$$\hat{x}_\theta = \mu + K(z^\theta - H\mu)$$

$$P_+ = P - KHP.$$

$$p^\theta(x) = N(x; \hat{x}_\theta, P_+)$$

If $\theta > 0$, then normal KF update with θ as object detected

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Computing weights

$$\tilde{w}_\theta(x) = \begin{cases} \int p(x)(1 - P^D(x))\,dx & \text{if } \theta = 0 \\ \left(\int \cancel{p(x)}\, P^D(x)\, g(z^\theta|x)\right) \cdot \left(\frac{1}{\lambda_c z^\theta}\right) & \text{if } \theta \in (1 \ldots m) \end{cases}$$

$\downarrow$

Data association probabilities.

$\left(\frac{1}{\lambda_c z^\theta}\right) \rightarrow$ clutter/object probability

if $\lambda_c \gg$ then its a clutter

if P_D = constant $\quad \boxed{\because \int p(x)\,dx = 1}$

$$\tilde{w}_\theta(x) = \begin{cases} 1 - P_D \\ \left(\frac{P_D}{\lambda_c z^\theta}\right) \int p(x) \cdot g(z^\theta|x)\,dx \end{cases}$$

$\Downarrow$

Linear and gaussian assumptions

$$\tilde{w}_\theta = \frac{P_D}{\lambda_c(z^\theta)} \int N(x, \mu, P)\, N(z^\theta, Hx, R)\,dx$$

$\int$ terms $\Rightarrow \mathcal{N}(z\theta; Hy, HPH^T + R)$

$\bar{z} = Hy$, $S = HPH^T + R$,

$$\tilde{w}_\theta = \frac{P_D \mathcal{N}(z^\theta; \bar{z}, S)}{\lambda c(z\theta)} \rightarrow \boxed{\text{predicted likelihood}}$$

Variance.

$P_D \propto c$ then, area under $P_\theta(x)$ for $\theta \in (1, 2, \ldots, m)$

St. line area $\uparrow$ for $P_\theta(x)$ $\boxed{\theta = 0}$

2.13 Solution pt1:

Do prediction step for each hypothesis

2.14. sum $\sum_{\theta_{1:k-1}} \sum_{\theta_{k}=1}^{m_k} \lambda_c(z_c^{\theta_k}) \; w_\theta \; p_k^{\theta_{1:k-1}} \; x_k P$

over time steps

over all hypothesis

There's a weight $w_\theta^{1:k-1}$ → for prediction & update terms

indicating influence of previous timestep weights

approximations needed as $\prod_{k=1}^{K} (m_k + 1)$

(exp) increase in hypothesis

2.15 SOT algorithms intractible t/o calculate accurate over all timesteps

Gaussian mixture setting.

$\hat{p}(x_k | z_{1:k}) \approx p(x_k | z_{1:k})$

Limiting to Gaussian mixture with fewer components

pruning. merging

Small wts remove and renormalise

single density. depends on all components.

eg:- $p(x) = w_1 p_1(x) + p_2(x) w_2$

$w_1 = 0.07$

$w_2 = 0.93$

then set $\hat{p}(x) = p_2(x)$

NN filter (pruning)

PDA (probabilistic DA) filter (merging)

Gaussian sum filter (pruning/merging)

Assumed Density filtering

NN, PDA - Gaussian densities

GSF → Gaussian mixture densities

Gating (pruning) → disregard unreasonable predictions before update step

$$\check{p}^{NN}(x_k | Z_1 : k) = \sum_{\theta_k \neq 0}^{m_k} \omega_k^{\theta_k} p_{k|k}^{\theta_k}(x_k)$$

before approximatione

Gaussian mixture

prune everything other than most probable solution

Q.17

$z_k^{\theta_k^*}$ is nearest measurement to $\bar{z}_{k|k-1}$

$= \arg\max \tilde{w}_\theta^{\theta_k}$

$$= \arg\max_{\theta = \{1, 2, \dots m\}} \frac{P^D N(z_k^\theta, z_{k|k-1}, S_k)}{\lambda_c(z_k^\theta)}$$

$=$ assume samp for all

Ignores uncertainties in

$$\Rightarrow \arg\max_{\theta = \{1, 2 .. m\}} N(z_k^\theta, z_{k|k-1}, S_k)$$

$$\Rightarrow \arg\max_{\theta \in (1, 2, 3 .. m)} \exp\left(-\frac{1}{2}(z_k^\theta - z_k) S_k^{-1} (z_k^\theta - z_k)\right)$$

$$= \arg\min_{\theta \in (1, \dots m)} \underbrace{(z_k^\theta - z_{k|k-1})^T S_k^{-1} (z_k^\theta - z_{k|k-1})}_{\text{"norm" distance.}}$$

Compared against clutter measurements,

if θ_k < clutter ; its considered as clutter,

∴ If ~~its~~ loses track of 1 measurement in middle, later it can't come back ~~as~~ will regard as clutter

2.18 PDA filtering

merging

$$\hat{x}_{k|k}^{PDA} = E_{p^{PDA}(x_k|z_{1:k})}[x_k]$$

minimises Kullback Liebler divergence

$\sum w_k^{\theta_k} \hat{x}_k^{\theta_k}$ = weighted sum of mean

$$\sum w_k^{\theta_k} P_k^{\theta_k} + w_k^{\theta_k} (\hat{x}_k^{\theta_k} - \bar{x}_k^{\theta_k})(x_k - \hat{x}_k^{\theta_k})^T$$

PDA

1) Compute $w_k^{\theta_k}$, $\hat{x}_k^{\theta_k}$, $P_k^{\theta_k}$ $\theta = 0, 1, \ldots m$

2) set $\bar{x}_{k|k}^{PDA}$, $P_{k|k}^{PDA}$ $\cancel{\sum w_\theta^{\theta_k} \hat{x}_k^{\theta_k} + \sum}$

= weighted sum of $\hat{x}_k^{\theta_k}$

2.19: $\bar{z}_{k|k-1} = H_k \hat{x}_{k|k-1}^{PDA}$

PDA Large uncertainty than NN.

fast easy to implement

simple scenarios

Acknowledges uncertainties

– complicated scenarios

Posterior dominated by few components

2.20 Gaussian sum filtering,

$$p^{GSF} = \sum_{h_{k-1}=1}^{H_{k-1}} w_{k-1}^{h_{k-1}} \, p_{k-1|k-1}^{h_{k-1}}(x_{k-1})$$

(1) prune all component w_c < threshold

after pruning renormalise weights

(2) merging similar

Prediction & update for all H_{k-1} hypothesis

$(H_{k-1}) \times (m_{k+1})$ components

1) Input γ, w^i, p^i, $i = 1 \ldots H$.

~~2) Output $\tilde{w}^i \tilde{x}^i \tilde{p}^i$, $i = 1, \ldots H$~~

3. ind = find $[[w^1, w^2, \ldots w^H] > \gamma]$

4. $H' =$ length (ind) $\quad c = \sum_{i=1}^{H'} w^{ind(i)}$ ~~renormalised~~ new weights sum

5. for $i = 1$ to H do

Set $\tilde{w}^i = \dfrac{w^{ind(i)}}{c}$, $\tilde{x}^i = \hat{x}^{ind(i)}$

$\tilde{p}^i = p^{ind(i)}$

end for.

2.20 continuation

Merging 2 out of three components

suppose $p^1(x)$ and $p^2(x)$ are similar.

$$p(x) = w^1 p^1(x) + w^2 p^2(x) + w^3 p^3(x)$$

Using $\boxed{w^{12} = w^1 + w^2}$

$$p(x) = w^{12} \left(\frac{w^1 p^1(x)}{w_{12}} + \frac{w^2 p^2(x)}{w_{12}} \right) + w^3 p^3(x)$$

$\boxed{\approx p^{12}}$ - see PDA

$$\boxed{\approx w^{12} p^{12} + w^3 p^3}$$

Prune hypo until we have N_{max} hypo

if $< N_{max}$, no need to prune

2) a) prune $< \gamma$

2) merge similar components

3) cap to N_{max}

2.21

$$p^{GSF}(x_{k-1}|Z_{1:k-1}) = \sum_{h_{k-1}=1}^{H_{k-1}} w_{k-1}^{h_{k-1}} \, p_{k-1}^{h_{k-1}}(x_{k-1})$$

$$\Downarrow$$

predicted density $\Rightarrow$ $p_{k|k-1}^{h_{k-1}}(x_k)$

$$\boxed{\breve{H}_k = H_{k-1} \times (m_k+1)}$$

Update step: for every pair of hypothesise $h_{k-1} \in \{1, 2, \dots H_{k-1}\}$ and $\theta_k \in \{0, 1, \dots m_k\}$

$$\breve{w}_{k|k} \propto \begin{cases} w_{k-1}^{h_{k-1}} \int (1-P^D(x_k)) \, p_{k|k-1}(x_k) \, dx_k & \text{if } \theta_k = 0 \\ \dfrac{w_{k-1}^{h_{k-1}}}{\lambda_c(z_k^{\theta_k})} \int P^D(x_k) \, p_{k|k-1}(x_k) \, g_k(z_k^{\theta_k}|x_k) \cdot dx_k & \text{if } \theta_k = 1, \dots m \end{cases}$$

$$\breve{p}_{k|k} \propto \begin{cases} (1-P^D(x_k)) \, p_{k|k-1}^{h_{k-1}}(x_k) & \text{if } \theta = 0 \\ P^D(x_k) \, p_{k|k-1}(x_k) \, g_k(z_k^{\theta_k}|x_k) & \text{if } \theta = 1, \dots m \end{cases}$$

Indexing h_k based on h_{k-1}, θ_k

1) $h_k = h_{k-1} + H_{k-1} \theta_k$

2) $h_k = 1 + \theta_k + H_{k-1}(h_{k-1} - 1)$.

1-1 mapping b/w (θ_k, h_{k-1}) to h_k.

2.22 State estimation: (x_K)

posterior mean
MMSE

argmax $w^h_{K|K}$

++ accurate
++ complexity adjusted
-- complex
-- computation >> PDA NN

GSF >> PDA NN.

Difference is noticeable in medium-scale scenarios

2.23 Gating:

PDA with large m_K,

1) large P_D
2) small λ_c
3) High FOV

=> m_K may be large

Since

$$\boxed{\bar{x}_{K|K} = \sum_{\theta_K=0}^{m_K} w_K^{\theta_K} \hat{x}_K^{\theta_K}}$$

if weights are small can be ignored

reduce posterior estimation for certain components

Gating is general.

only consider measurements within a gate.

→ rectangular (simple)
→ ellipsoidal

$$\tilde{w}_K = \frac{P_D P(x_K) N(z^{\theta_K}; \bar{z}_K^{h_{K-1}}, S_{K, h_{K-1}})}{\lambda_c(z_K^{\theta})}$$

unnormalised weights.

$\tilde{w}_K$ is maximised as $N(z\theta_K)\uparrow$, $\boxed{\lambda_c \simeq \text{constant}}$ assumed

$\tilde{w}_K \downarrow$ if $d^2 = (z_K^{\theta_K} - z_{K|K}^{h_{K-1}})(\;)^T$ >> Threshold

Ellipsoidal: disregard measurement if $d^2_{h_{k-1} \_ \theta_k} > G$

$\boxed{G \approx \text{size of ellipsoid}}$

$$P_G = P_r\left[d^2_{h_{k-1} \_ \theta_k} > G \mid h_{k-1} \_ \theta\right]$$

$$\boxed{d^2_{h_{k-1}} \sim \chi^2(n_z)}$$

Kai-square distribution

$\boxed{P_G = 99.5\%}$ Cummulative distribution,