

Topological combined machine learning for consonant recognition



Yifei Zhu

Southern University of Science and Technology

2023.11.26

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In this talk, we mirror the question across senses and address instead:

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In this talk, we mirror the question across senses and address instead:

Can we see the sound of a human speech?

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Periodic phenomena: a motivating example

Let $\mathbb{T}^2 = (\mathbb{R}/\mathbb{Z})^2$ be the 2D torus. Consider the dynamical system given by

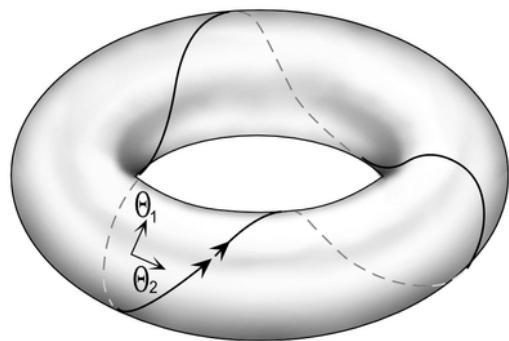
$$\begin{aligned}\Phi_\sigma: \mathbb{T}^2 \times \mathbb{R} &\rightarrow \mathbb{T}^2 \\ ((a, b), t) &\mapsto (a + t, b + \sigma t)\end{aligned}$$

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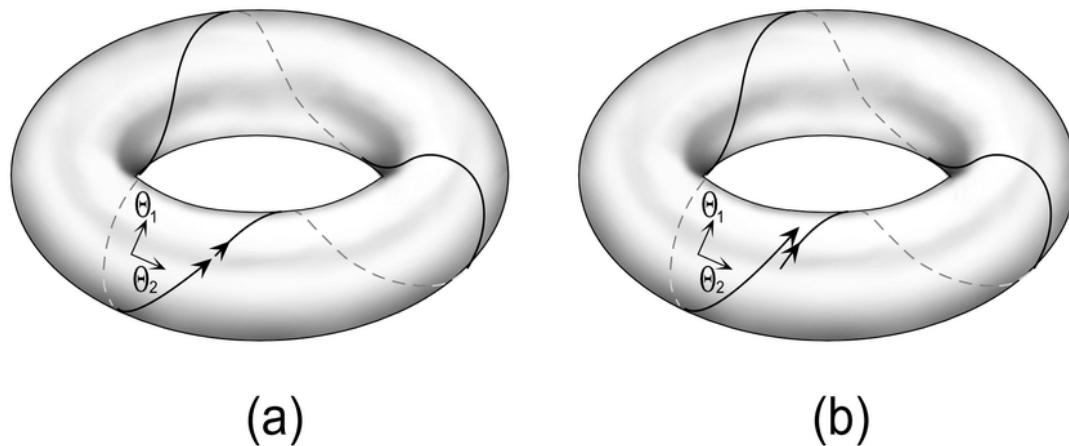


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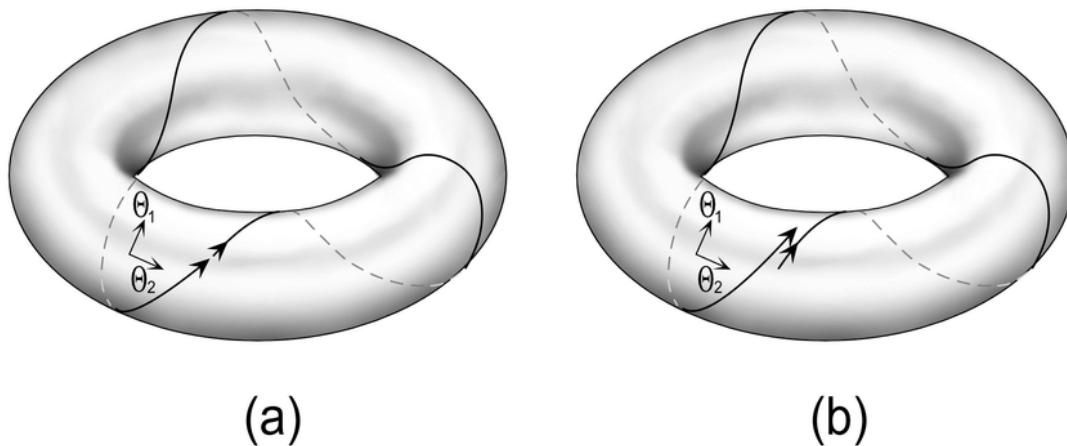


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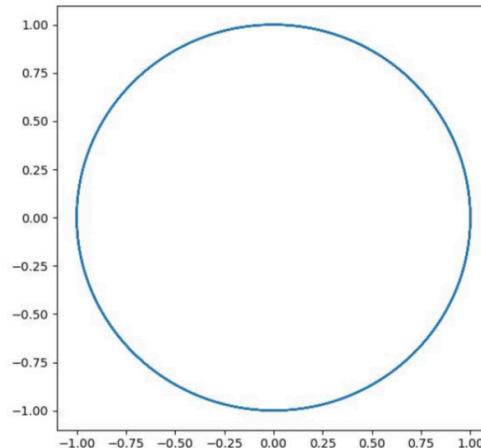
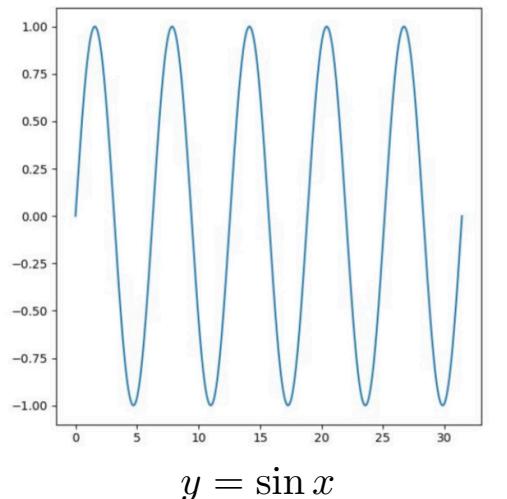


From time series to topological shapes

Most periodic time series can be realized by a **topological circle S^1** embedded in a Euclidean space of higher dimension.

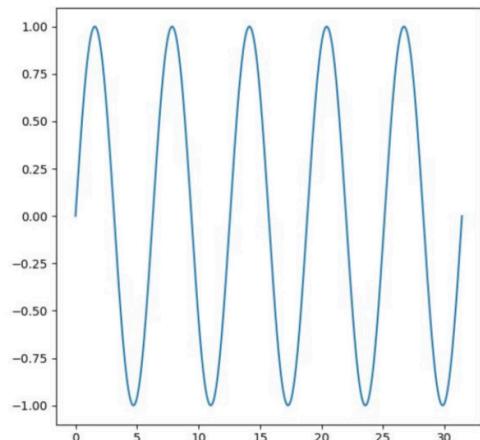
Ideas of topological data analysis (TDA)

The topological type (more precisely, homotopy type) is **robust** against perturbations.

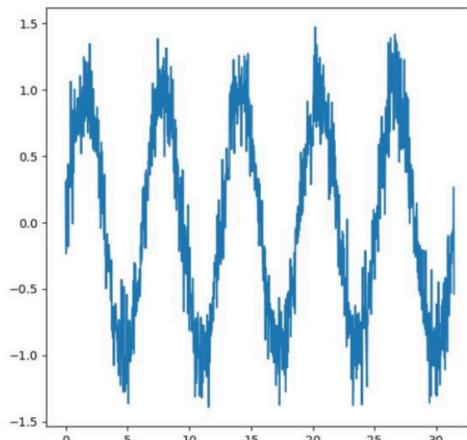
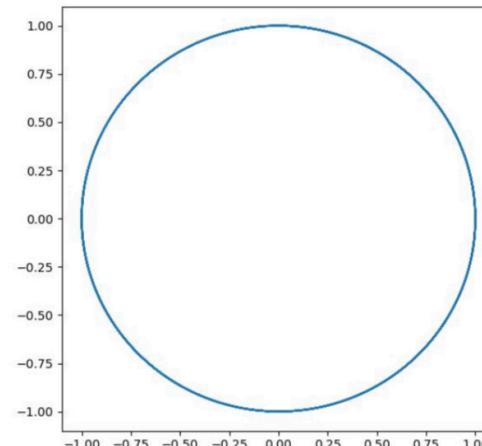


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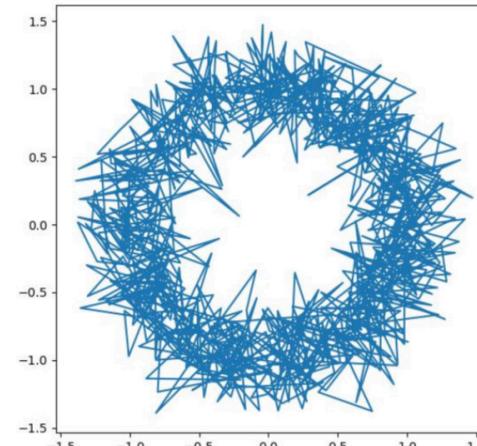
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$$y = \sin x$$



$$y = \sin x + \epsilon(x)$$



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Features of topological shapes, such as the number of holes, can be captured by algebraic invariants that are **computable**.

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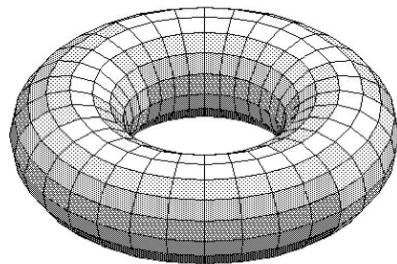
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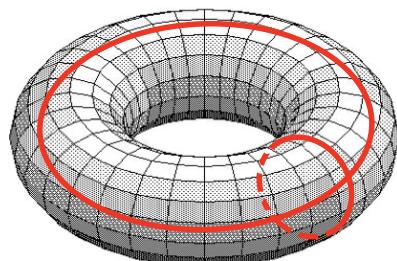


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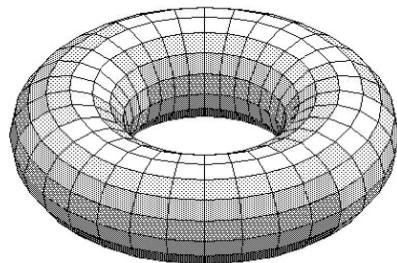


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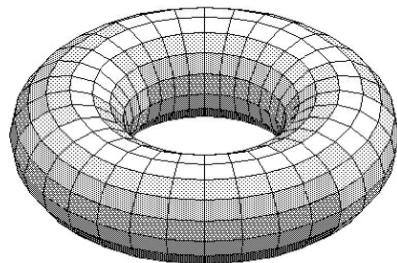


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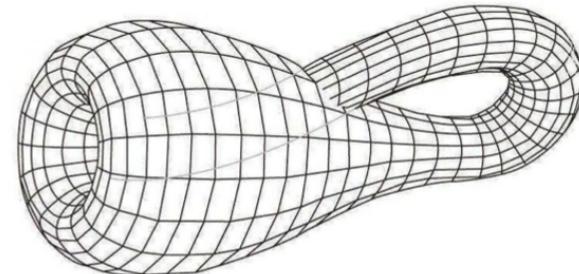
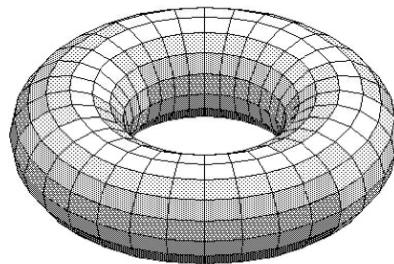


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$$H_k(\text{Klein bottle}) = \begin{cases} \mathbb{Z} & k = 0 \\ \mathbb{Z} \oplus \mathbb{Z}/2 & k = 1 \\ 0 & k = 2 \\ 0 & k > 2 \end{cases}$$

Topological time series analysis

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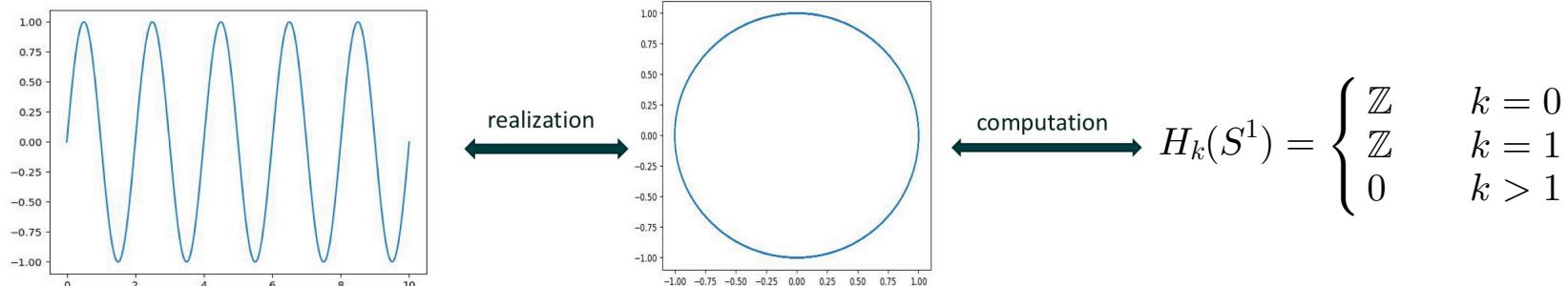
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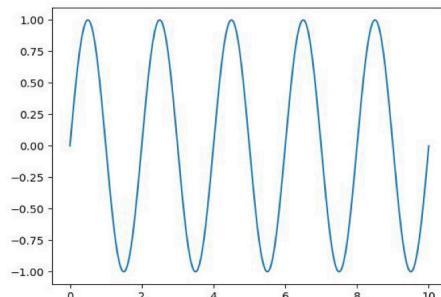


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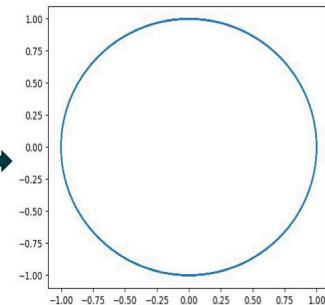
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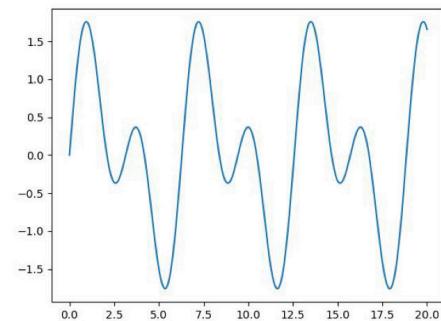


realization

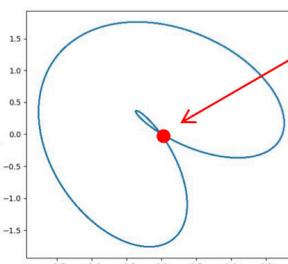


computation

$$H_k(S^1) = \begin{cases} \mathbb{Z} & k = 0 \\ \mathbb{Z} & k = 1 \\ 0 & k > 1 \end{cases}$$



not an embedding



2D

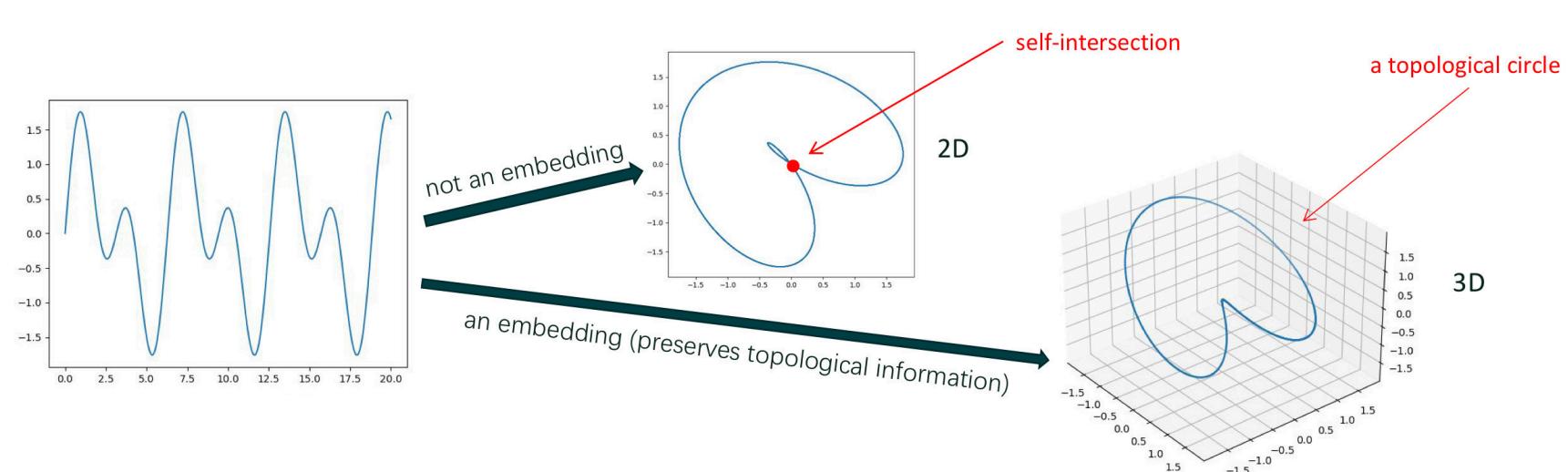
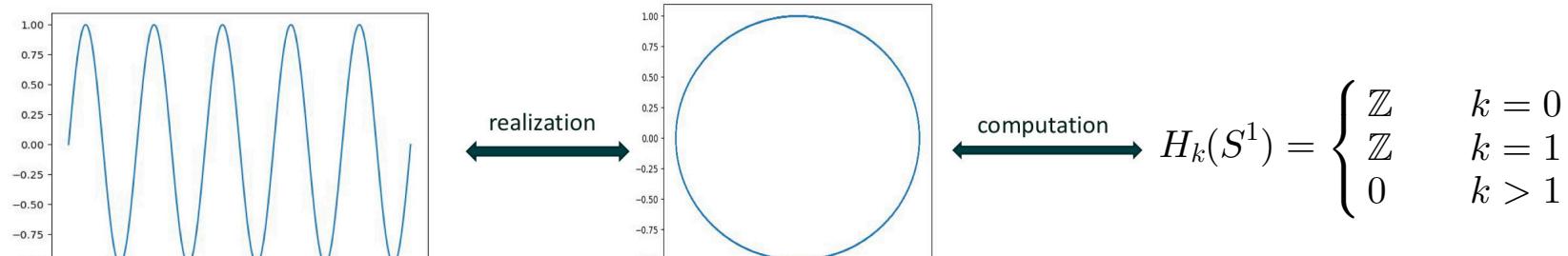
self-intersection

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An application: detection of wheeze in medical science (pulmonology)

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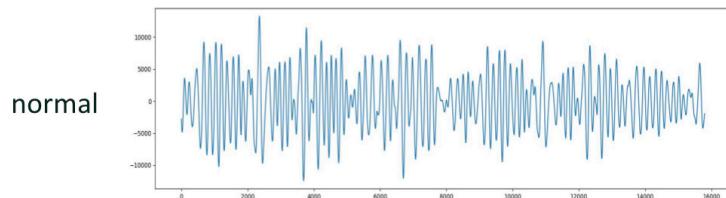
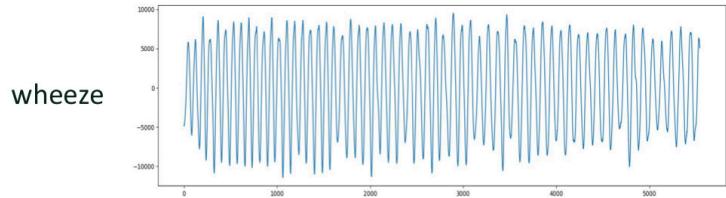


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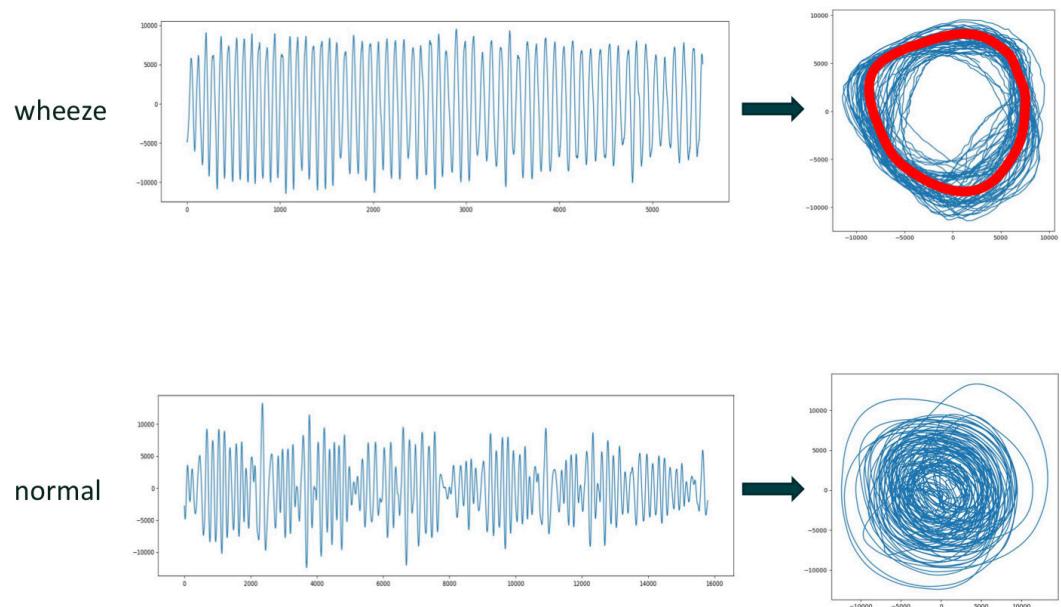
As a warm-up, our research group (Siheng Yi) has reproduced their results using the original data and open-source TDA programming package.

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Original sound signals

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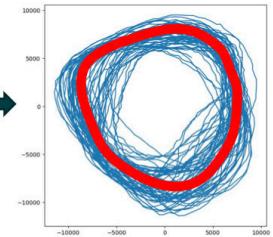
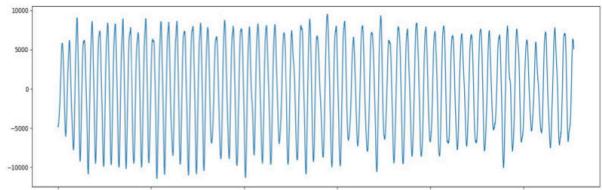


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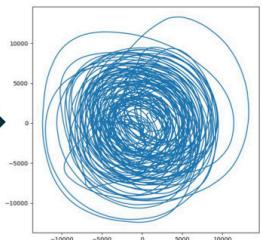
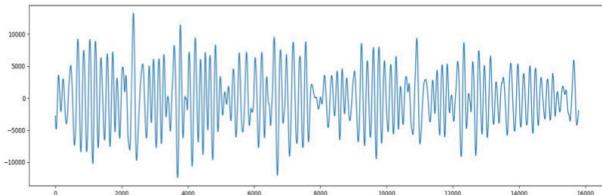
Realized topological
shapes embedded in
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An application: detection of wheeze in medical science (pulmonology)

wheeze



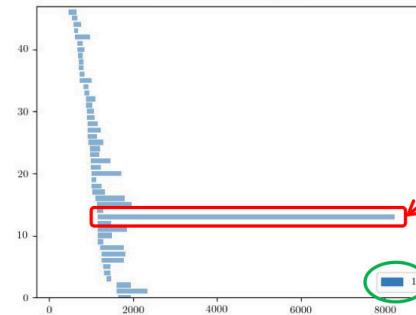
normal



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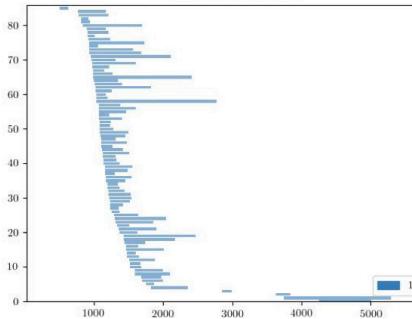
Persistence barcode



A long barcode indicates an essential one-dimensional hole.

Dimension of homology group

Persistence barcode

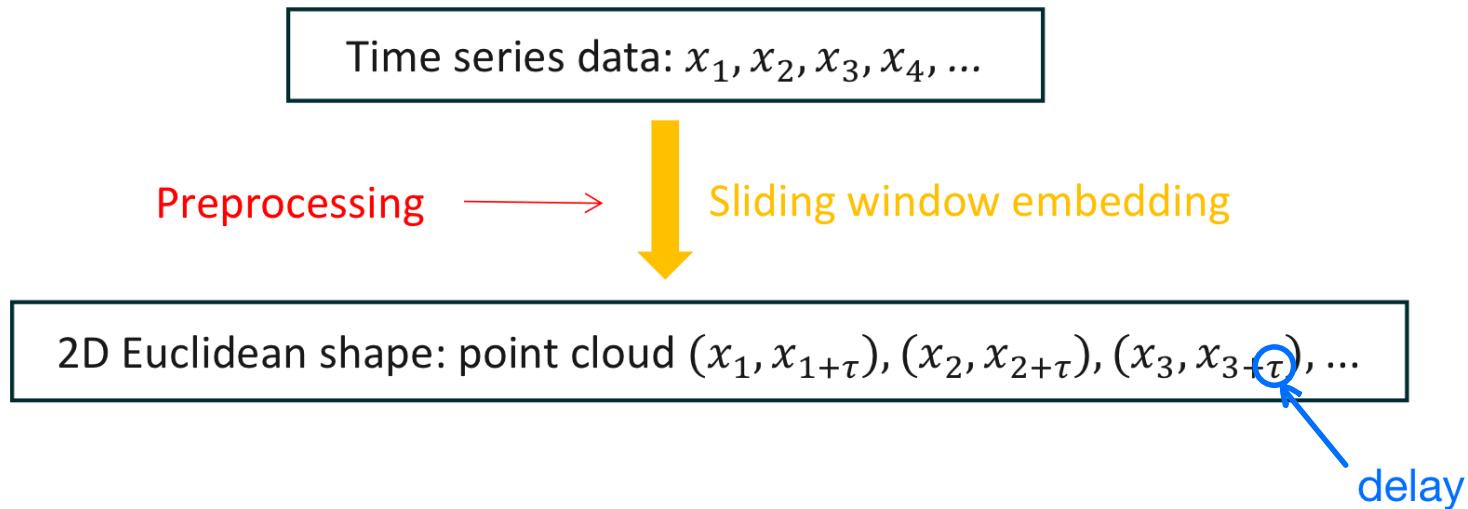


“Persistence barcodes” as representations of the algebraic invariant (1D homology group)

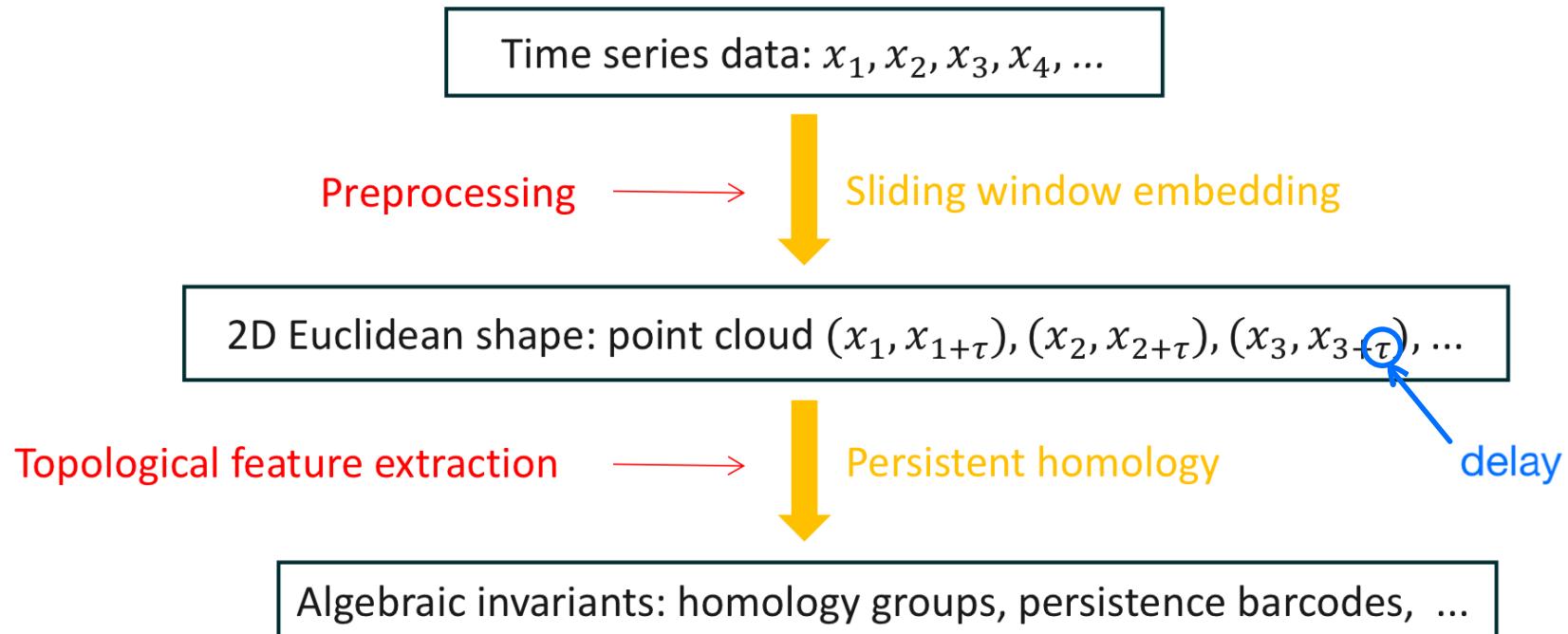
A pipeline for topological time series analysis

Time series data: $x_1, x_2, x_3, x_4, \dots$

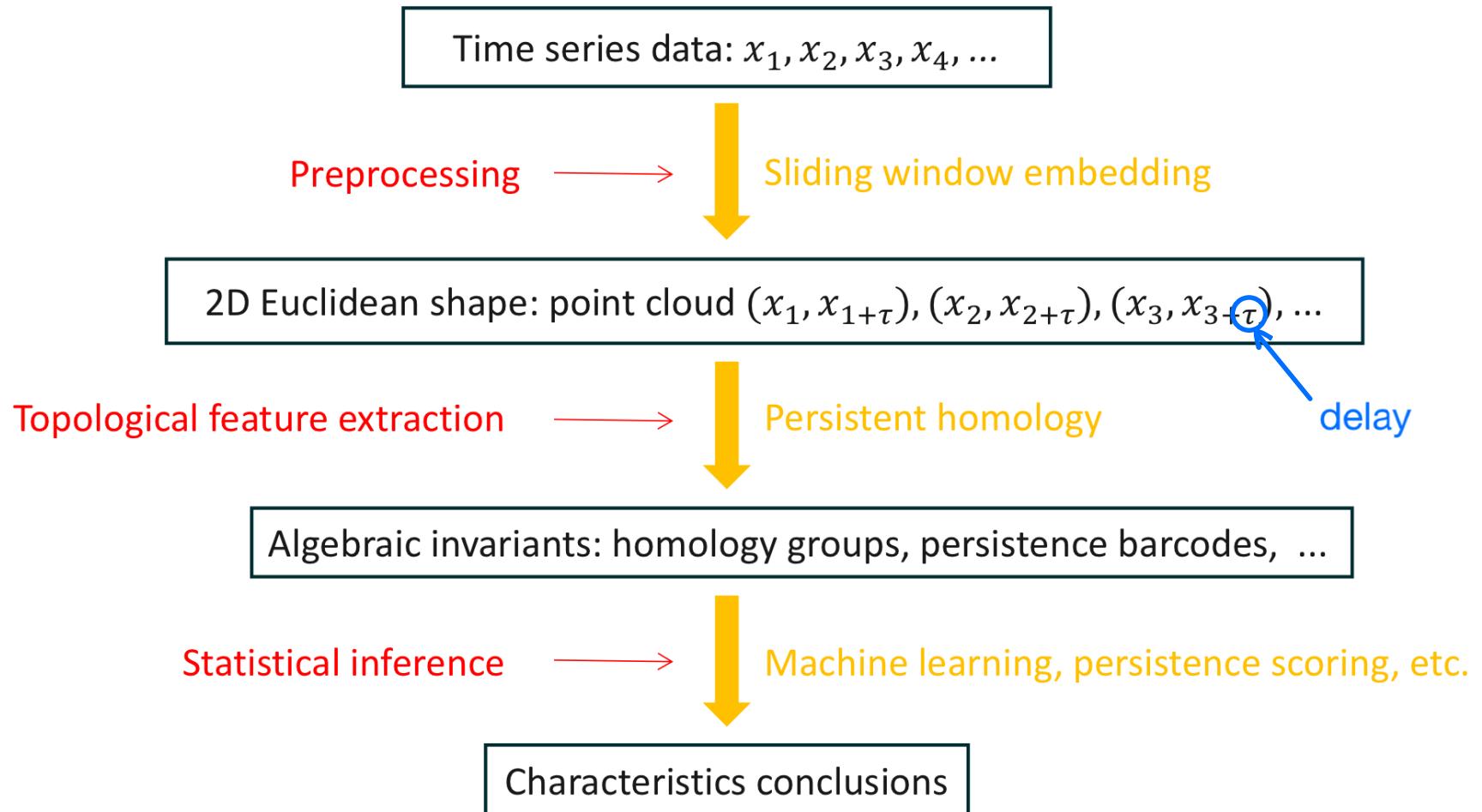
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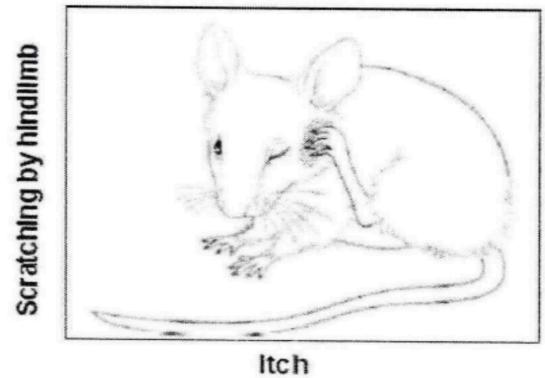


A pipeline for topological time series analysis



Application I: detection of mouse scratching behavior

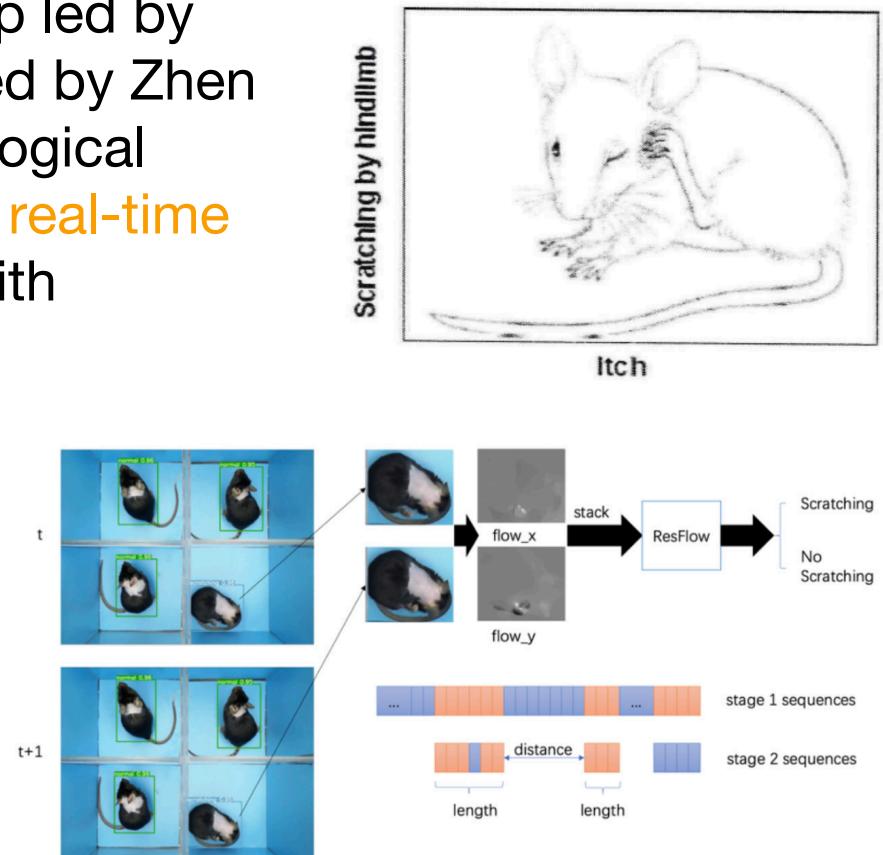
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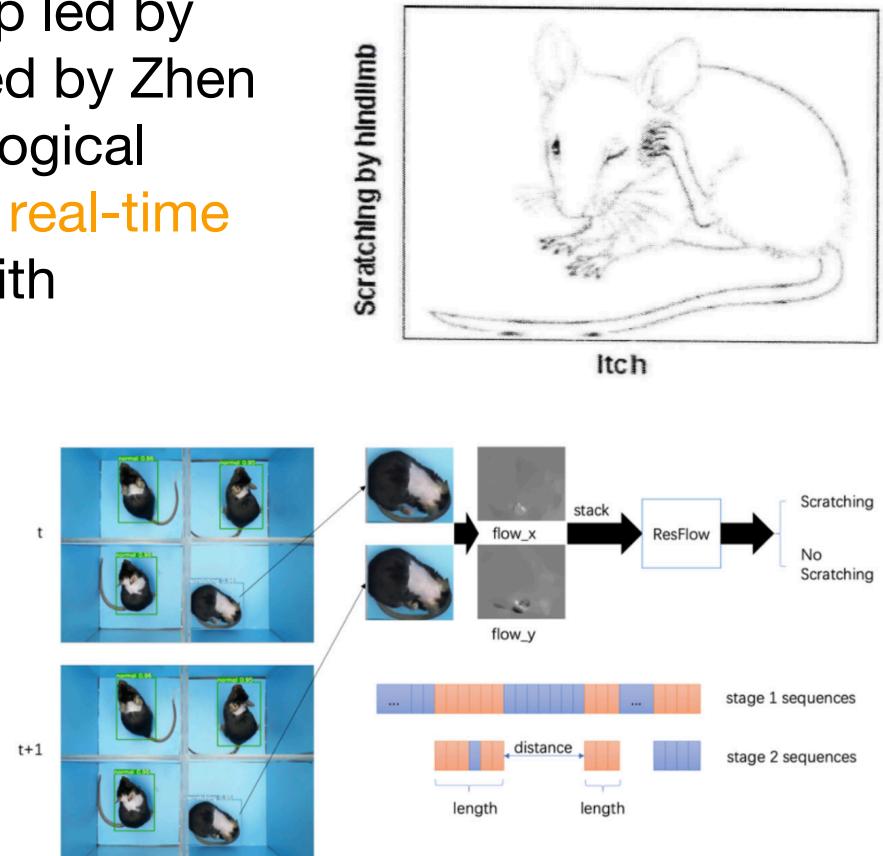
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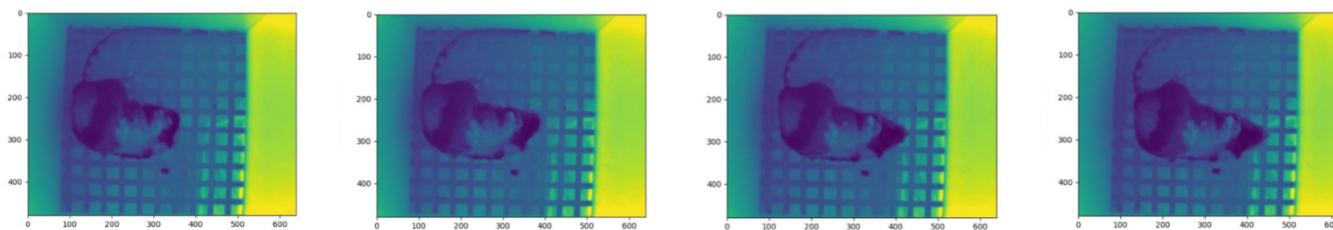
However, the learning process was **time consuming**, which is impractical for time-sensitive purposes and lab efficiency.

Application I: detection of mouse scratching behavior

We observed that the scratching behavior exhibits periodicity.

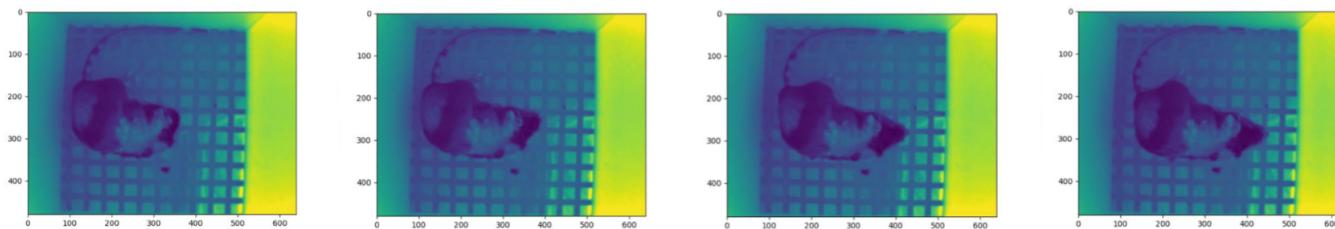
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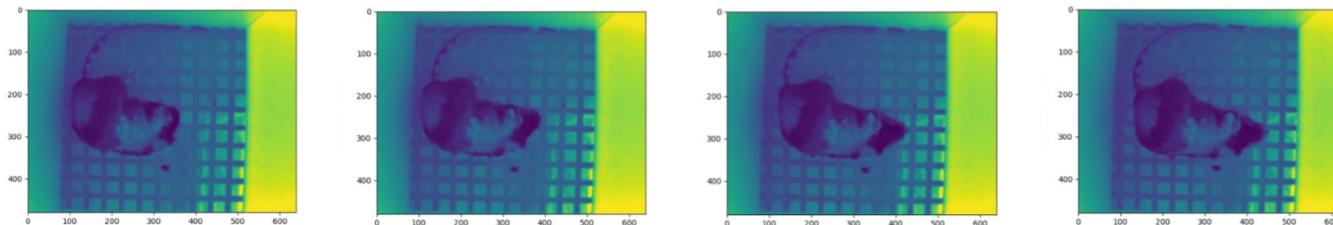


To resolve this issue, we adopted the following approaches:

Approach 1 **Sum up** all 460×640 pixels to extract a series of **1D data** which ignores differences caused by global movements. Too coarse?

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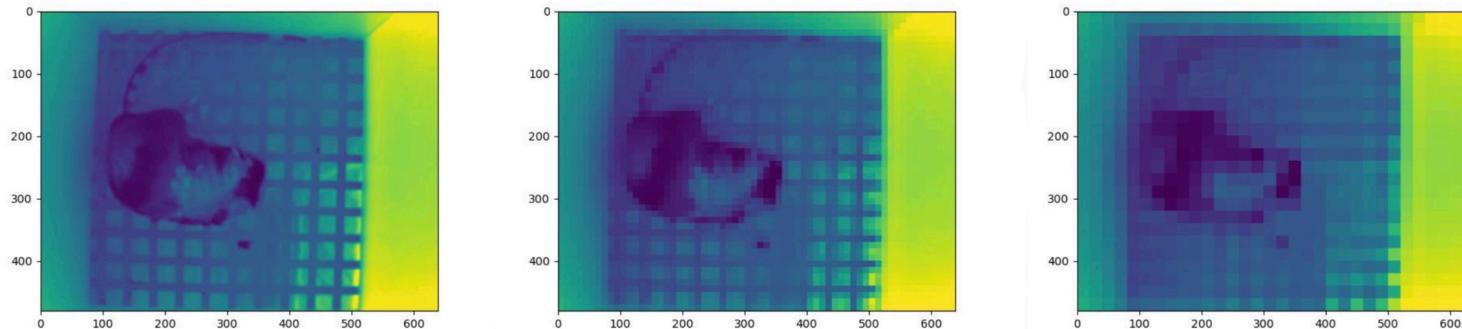
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Approach 2 Blur the images by **pooling**, and feed the topological pipeline with reduced **100-dimensional data**. Still too refined?

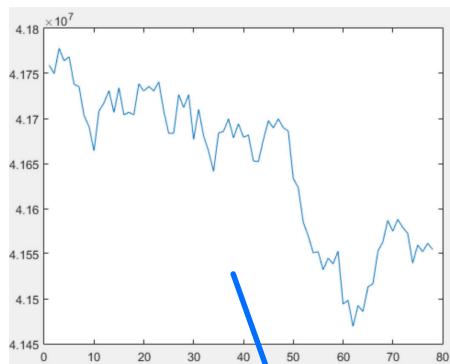


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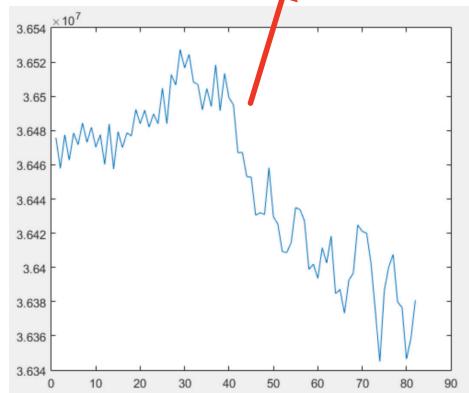
Approach 1 (1D data, Qingrui Qu), combined with carefully designed **filtration** for wave signals + suitably chosen **geometric statistics**, yielded a close-to-real-time, decently accurate detection performance.

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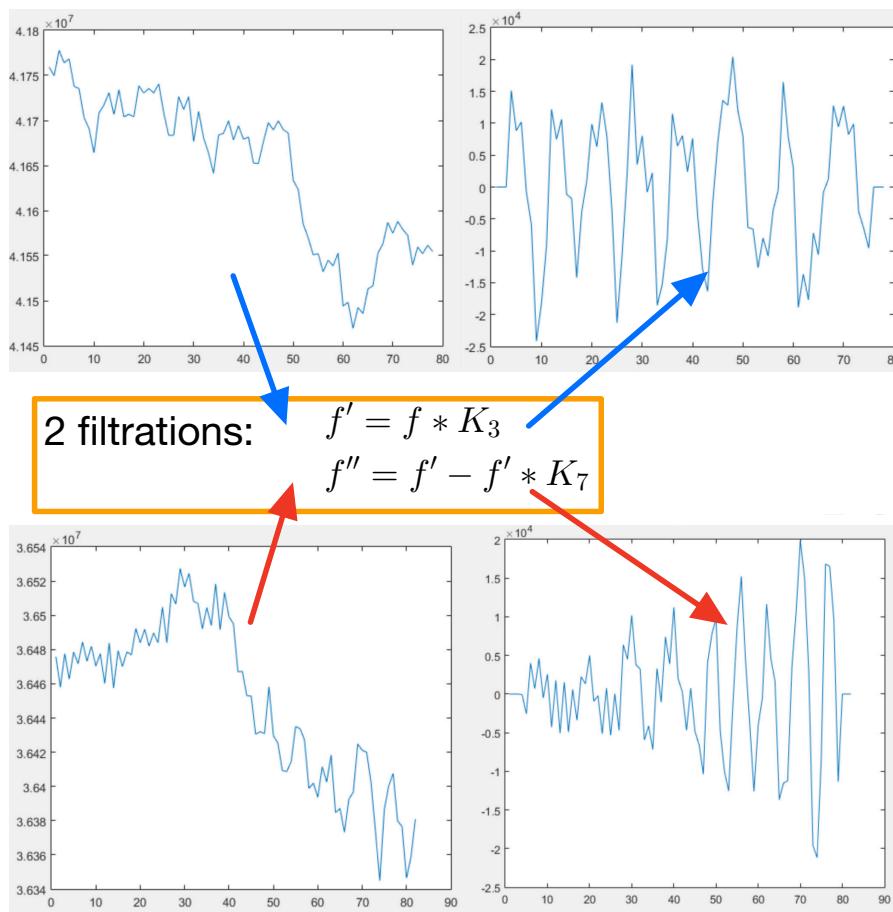


2 filtrations:

$$f' = f * K_3$$
$$f'' = f' - f' * K_7$$


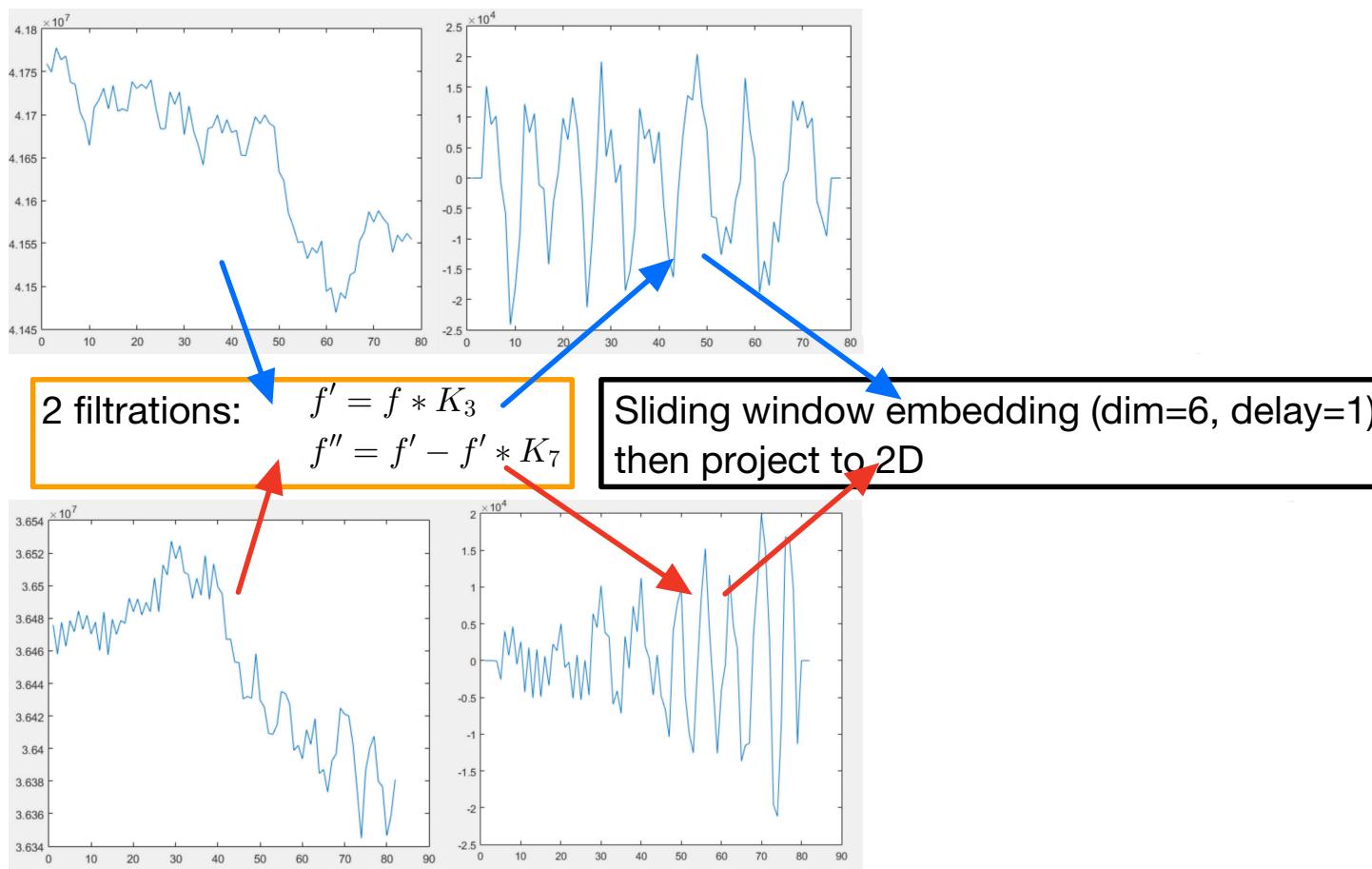
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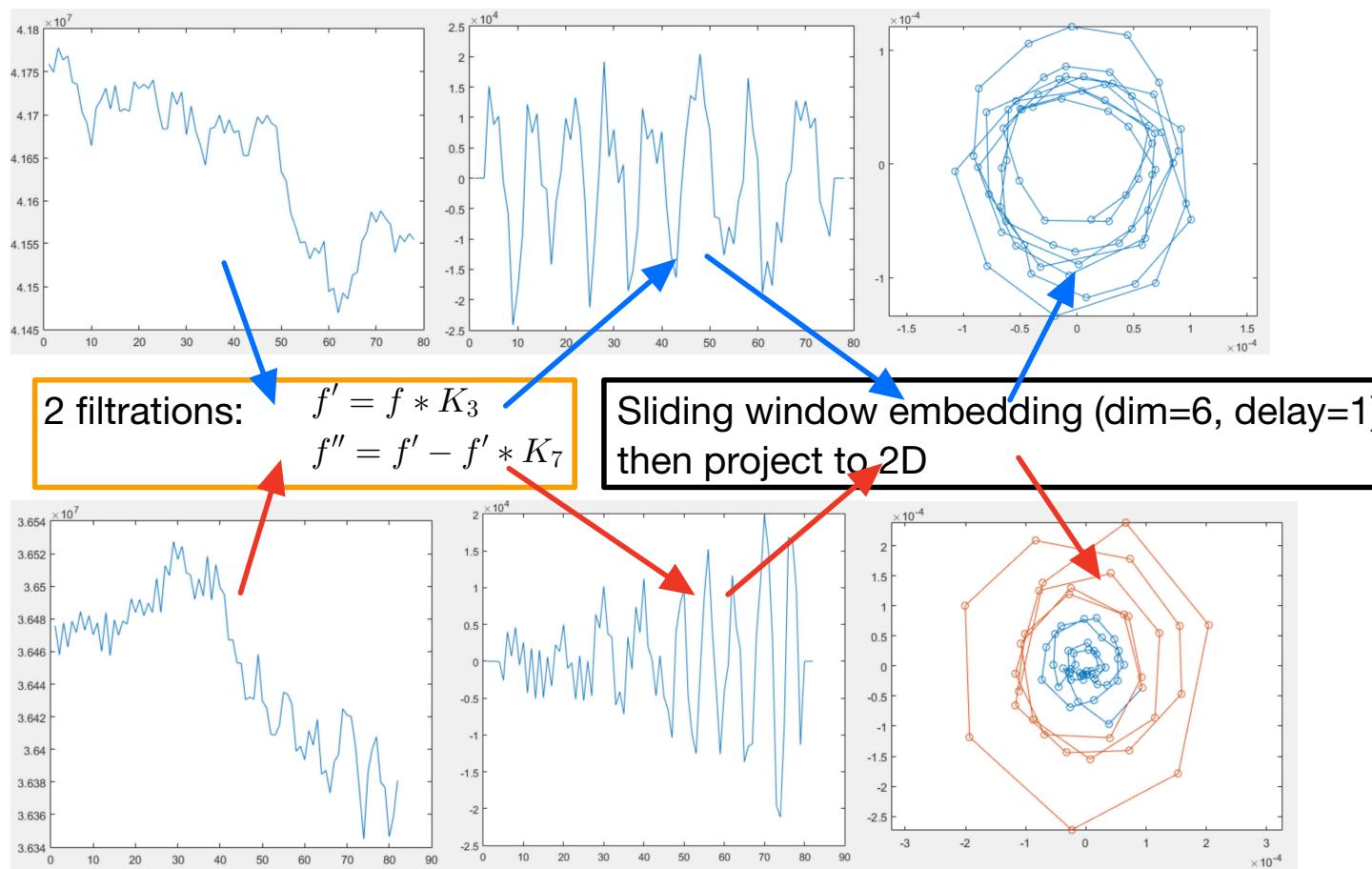
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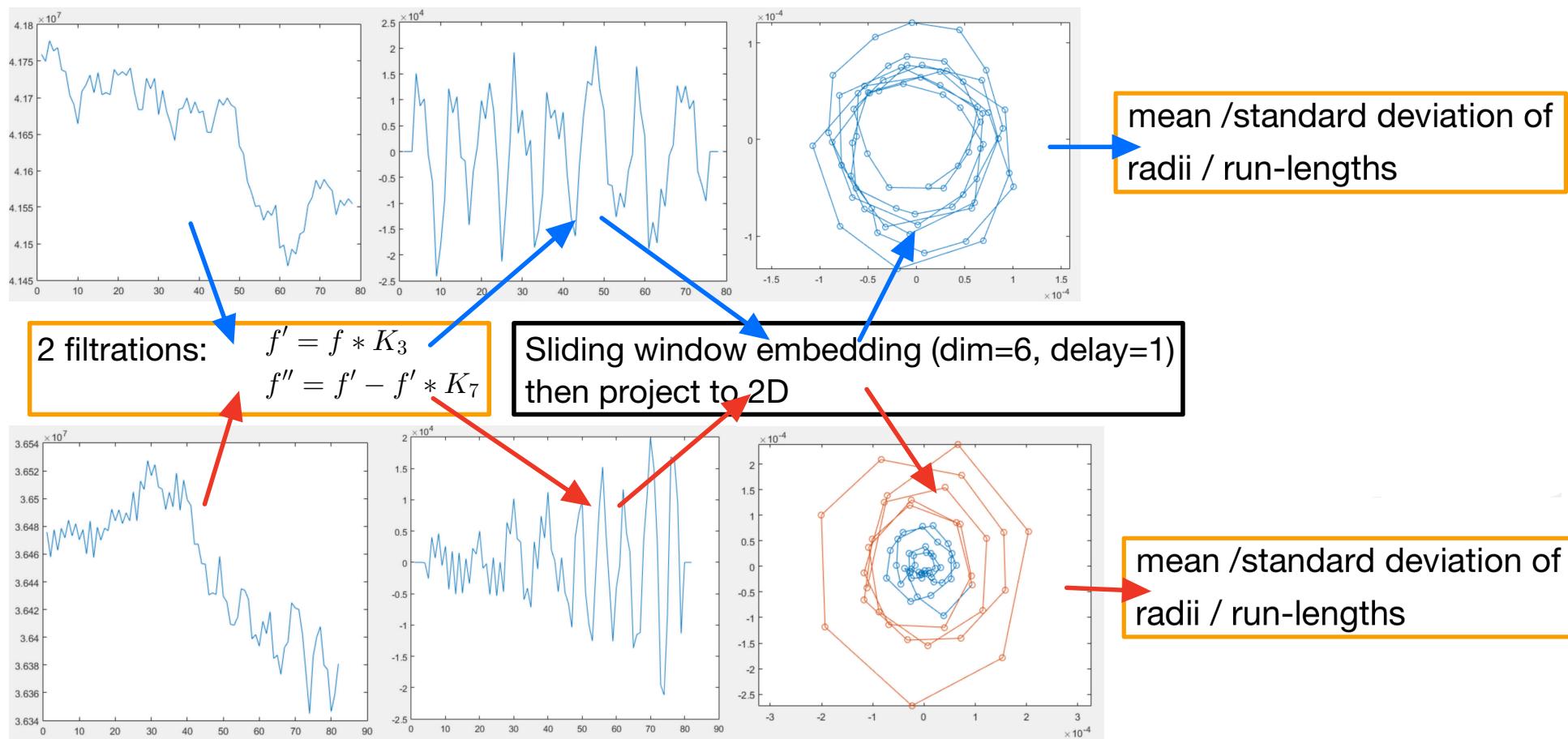
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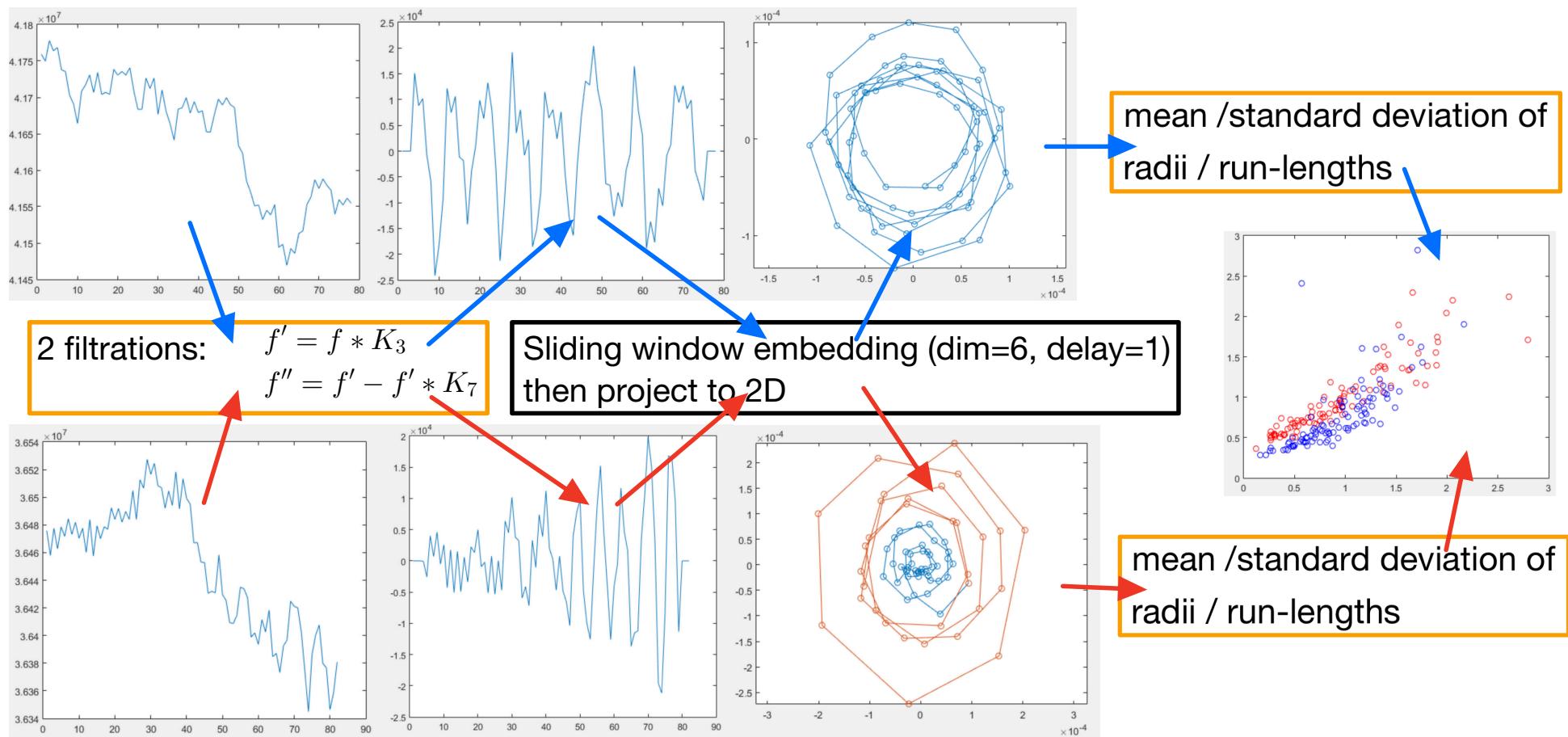
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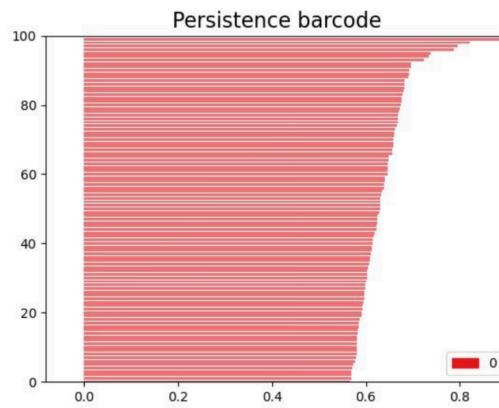
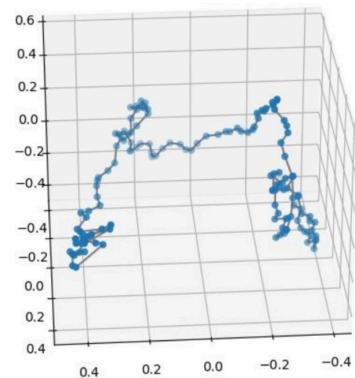
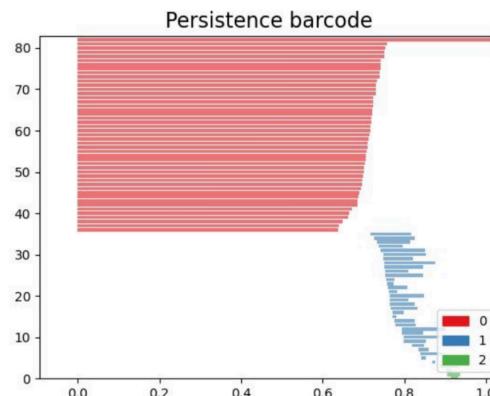
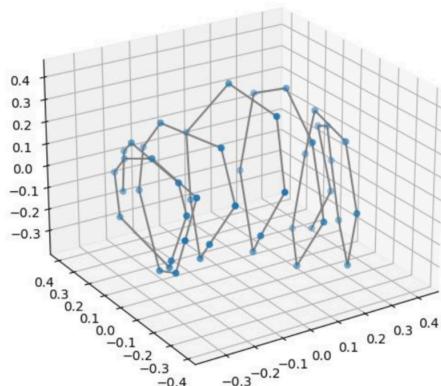
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Application I: detection of mouse scratching behavior

Approach 2 (multi-dimensional data, Siheng Yi), combined with **persistent homology** and its representations, yielded recognizable characteristics but required considerable computational time.



Application II: classification of speech signals

Joint with Meng Yu of Tencent AI Lab, we applied topological methods to classify **voiced/voiceless** and **vowel/consonant speech** data, with motivations from industrial applications.

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We were inspired by Carlsson et al.'s discovery of the **Klein-bottle** distribution of local natural **images**, as well as their subsequent recent work of **topological convolutional neural networks** learning **video** data.

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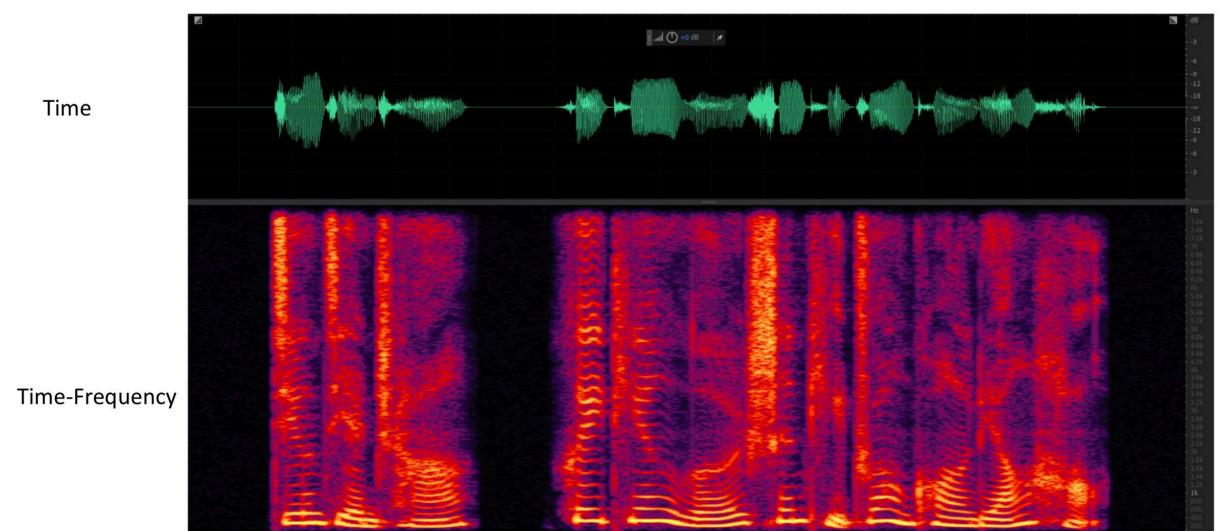
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Display of speech signals

There are speech signal processing softwares for professional use.



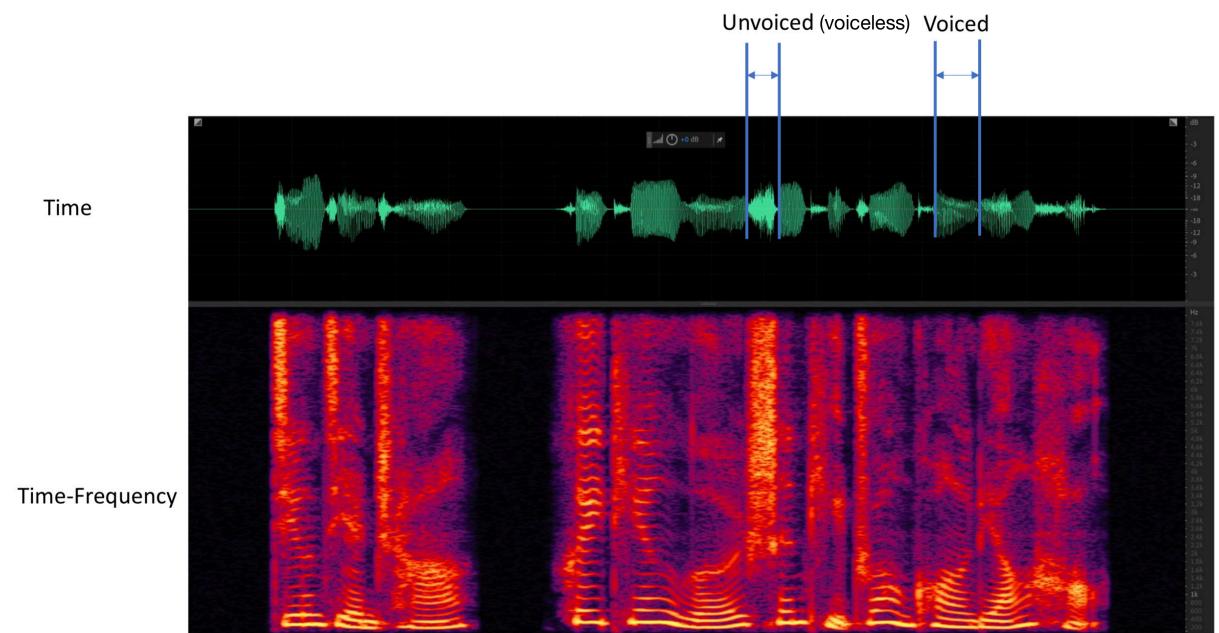
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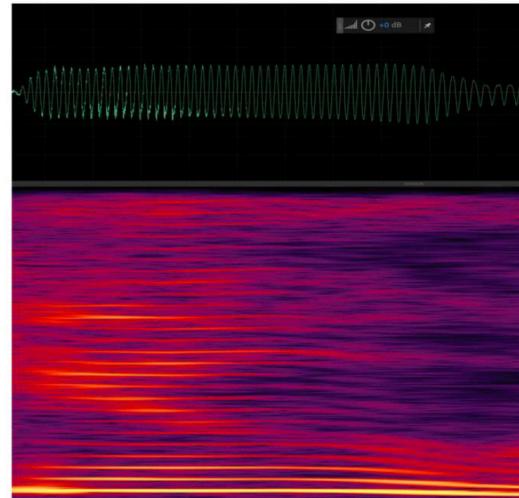


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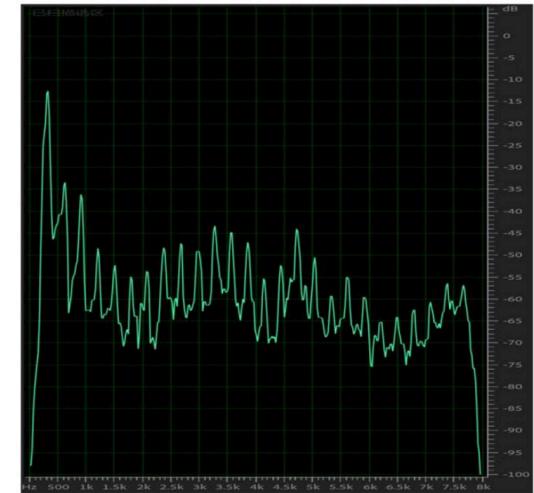
Voiced

Sinusoid in
time domain

Harmonics in
frequency
domain



Time and Time-
Frequency domain

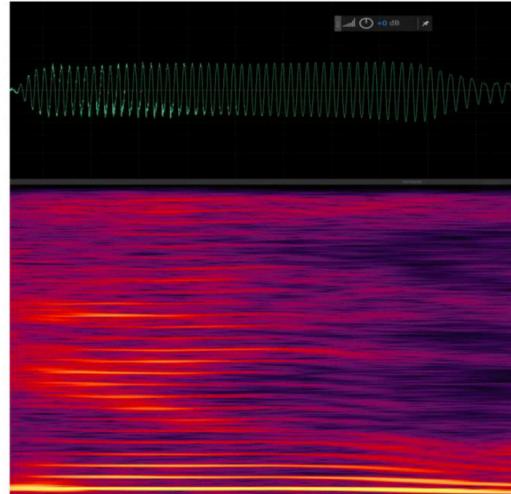


Frequency response

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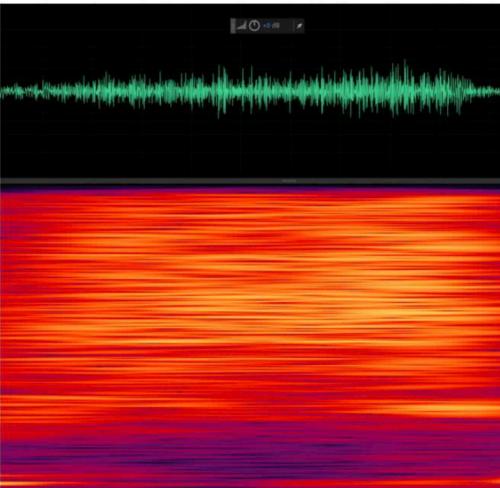
Voiced

Sinusoid in time domain

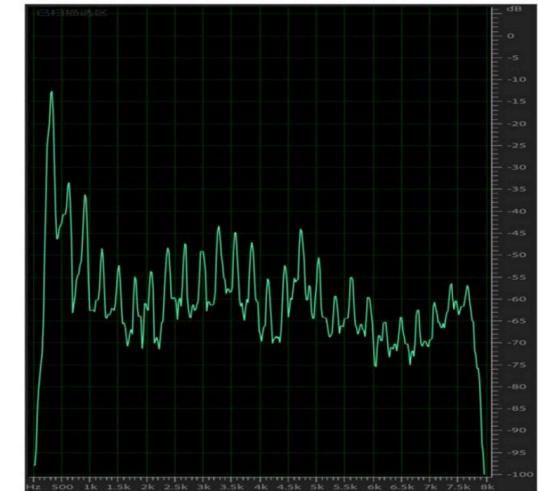


Time and Time-Frequency domain

Like a white noise



Voiceless

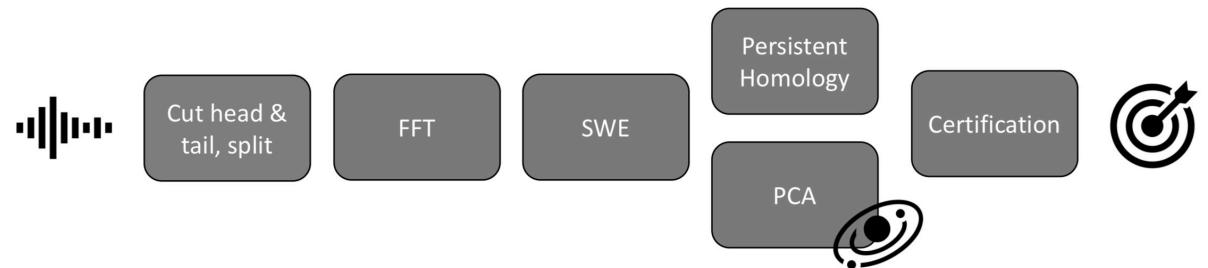


Frequency response



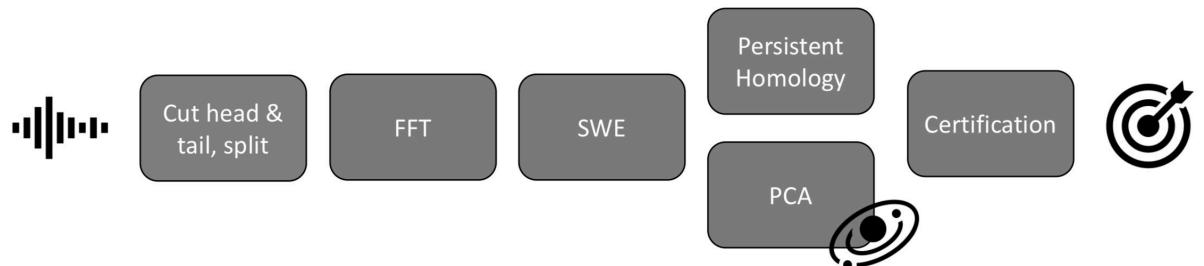
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Here is a flowchart for our topological approach:

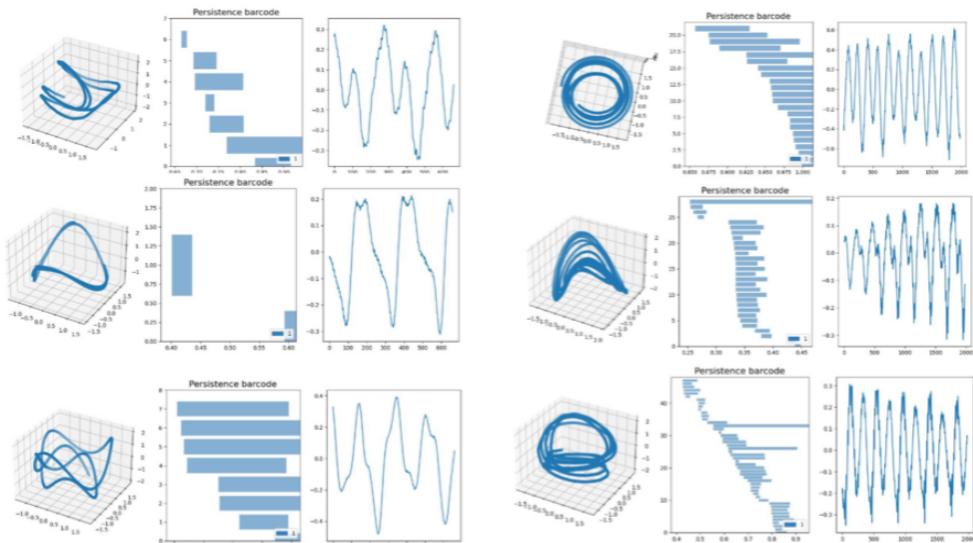


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Topological profiles for vowels and consonants (Pingyao Feng)

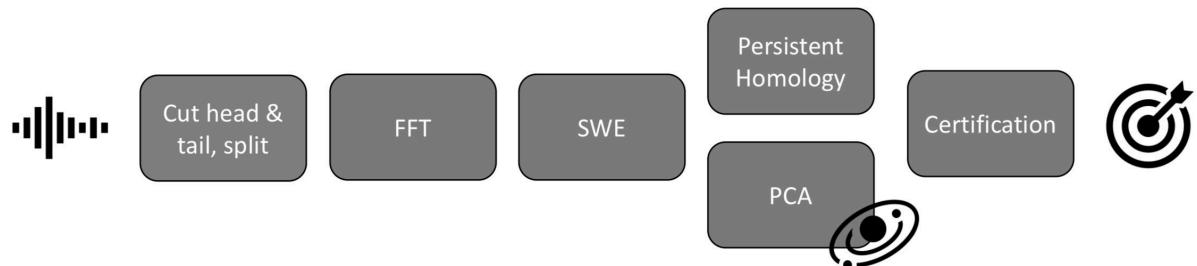


Features for vowels

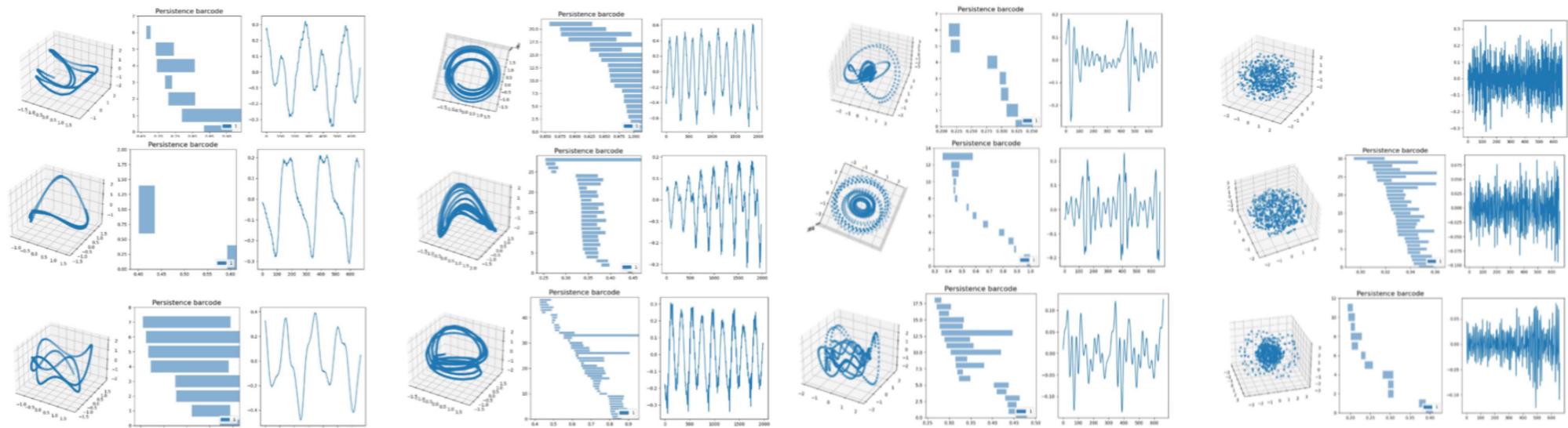
Left: frame size: 15ms, frame shift: 5ms; Right: frame size: 45ms, frame shift: 22.5ms

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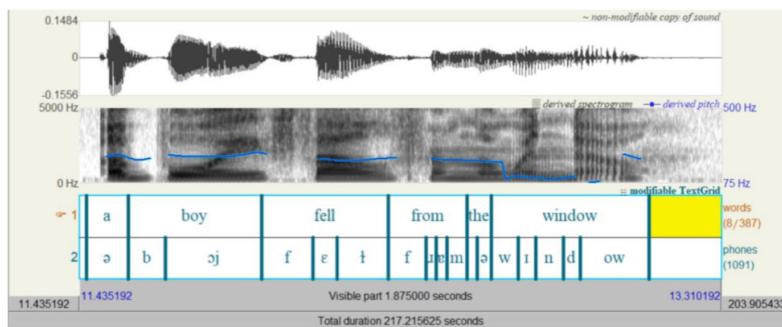
Left: pulmonic consonant; Right: non-pulmonic consonant

Application II: classification of speech signals

Using real-world speech data from the MFA aligner, our research group (Feng) further fed the topological features for machine learning, and obtained positive preliminary results for classification.

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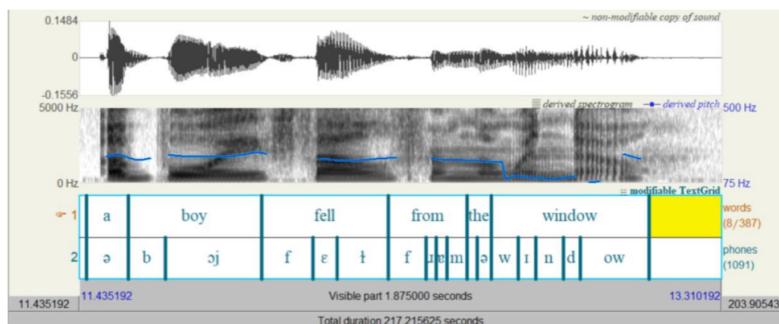
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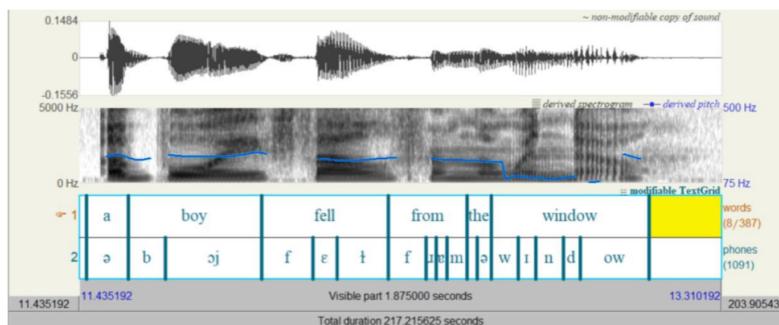
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Last change: Optimizable Tree	10/10 features
6 Ensemble	Accuracy (Validation): 77.1%
Last change: Optimizable Ensemble	10/10 features
1 Tree	Accuracy (Validation): 75.0%
Last change: Fine Tree	10/10 features
5 KNN	Accuracy (Validation): 75.0%
Last change: Optimizable KNN	10/10 features
8 Tree	Accuracy (Validation): 75.0%
Last change: Medium Tree	10/10 features
3 Optimizable Discr...	Accuracy (Validation): 72.9%
Last change: Optimizable Discriminant	10/10 features
4 SVM	Accuracy (Validation): 70.8%
Last change: Optimizable SVM	10/10 features
7 Neural Network	Accuracy (Validation): 70.8%
Last change: Optimizable Neural Network	10/10 features
9 KNN	Accuracy (Validation): 66.7%
Last change: Hyperparameter option(s)	10/10 features

32 vowels, 16 consonants.
10 features: 5 are barcodes
number of 5 diag, other 5
are number of barcodes that
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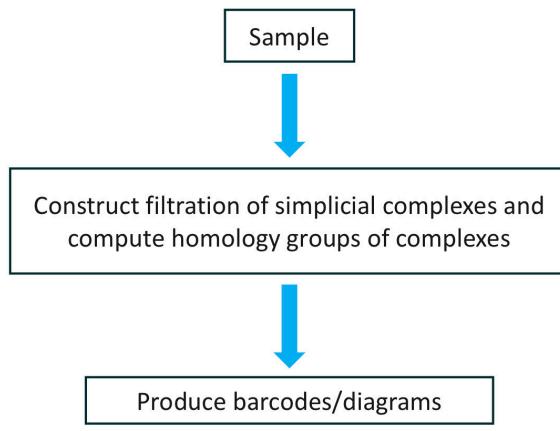
1 Tree	Accuracy (Validation): 81.5%
Last change: Fine Tree	4/4 features
2 Tree	Accuracy (Validation): 81.5%
Last change: Optimizable Tree	4/4 features
7 Tree	Accuracy (Validation): 81.5%
Last change: Medium Tree	4/4 features
4 Tree	Accuracy (Validation): 78.5%
Last change: Coarse Tree	4/4 features
3 KNN	Accuracy (Validation): 69.2%
Last change: Optimizable KNN	4/4 features
5 Neural Network	Accuracy (Validation): 46.2%
Last change: Hyperparameter option(s)	4/4 features
6 Neural Network	Accuracy (Validation): 46.2%
Last change: Narrow Neural Network	4/4 features

32 vowels, 33 consonants. 4
features: bottleneck distance
between neighborhood
barcode(currently the best
result)

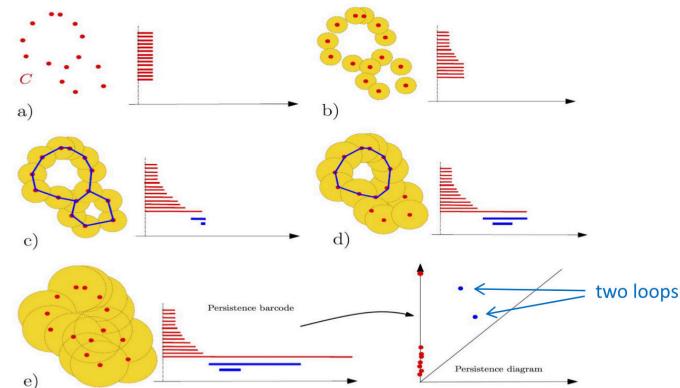
A formal recap of the topological methods applied

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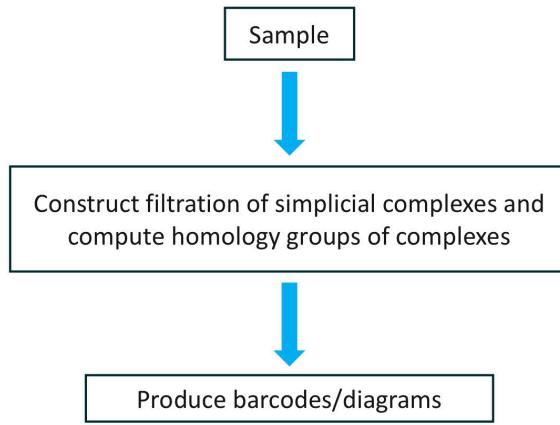


How filtration through varying distance measure reveals essential topological features

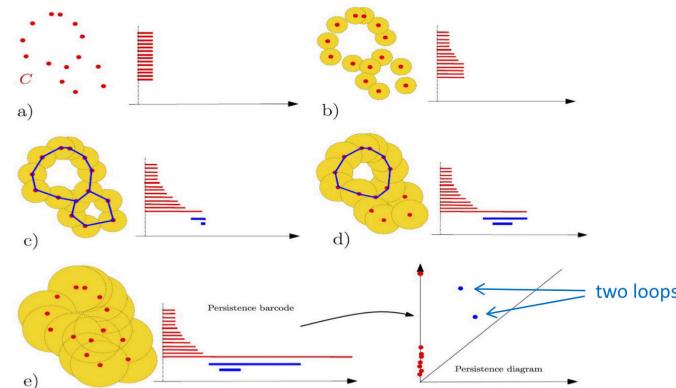


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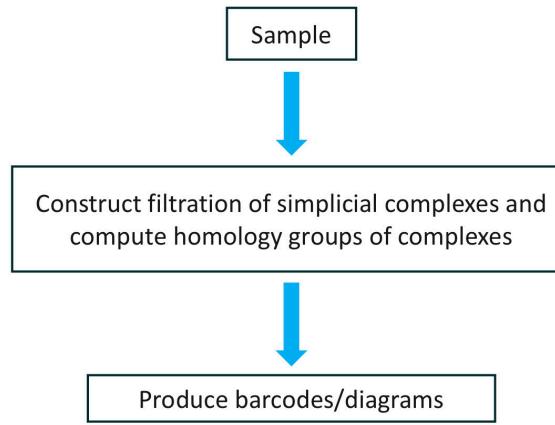


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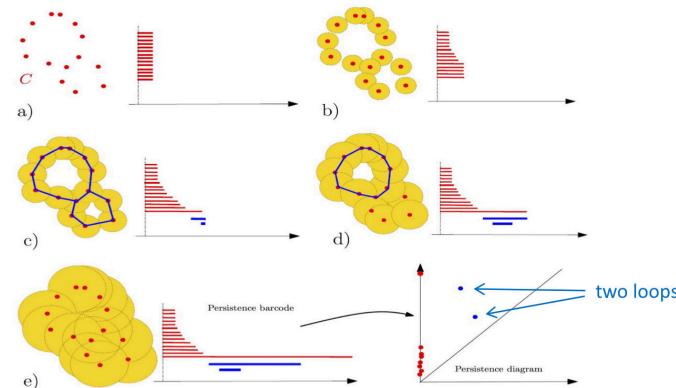
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Theorem (Takens 1981). Let M be a compact manifold of dimension n . Given pairs (ϕ, y) with $\phi: M \rightarrow M$ a smooth diffeomorphism and $y: M \rightarrow \mathbb{R}$ a smooth function, it is a generic property that the map $\Phi_{(\phi, y)}: M \rightarrow \mathbb{R}^{2n+1}$ defined by

$$\Phi_{(\phi, y)}(x) = (y(x), y(\varphi(x)), \dots, y(\varphi^{2n}(x)))$$

is an **embedding**.

From topological data analysis to topological deep learning

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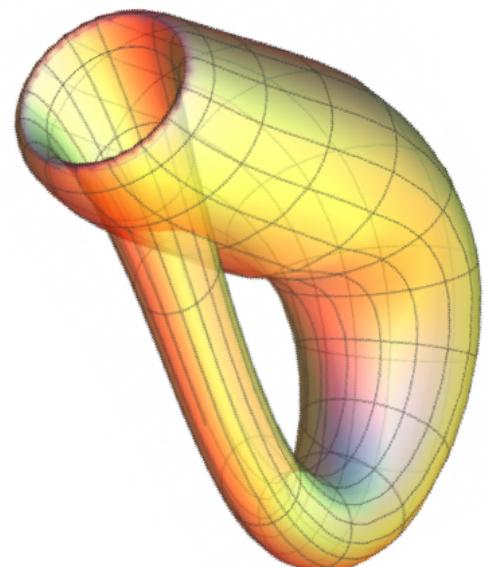
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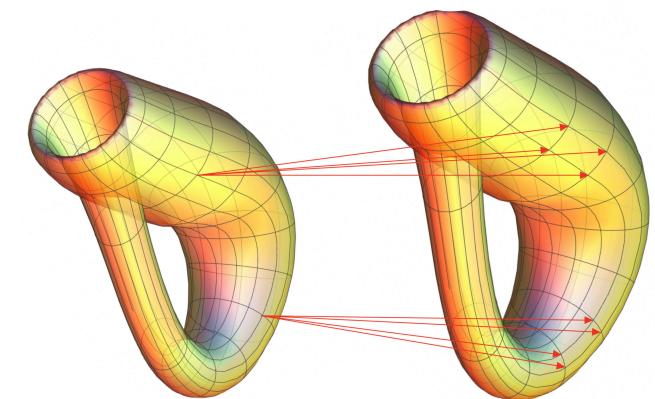
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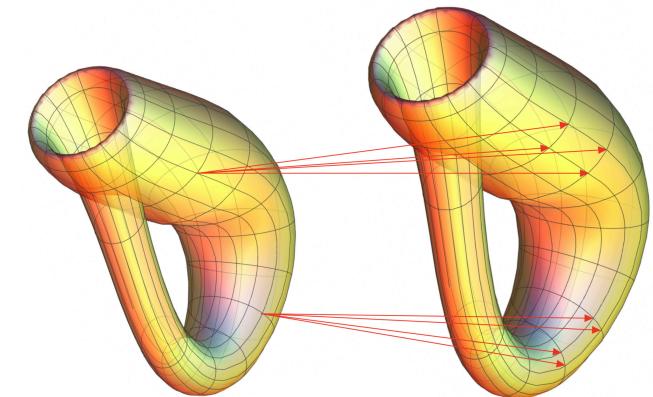
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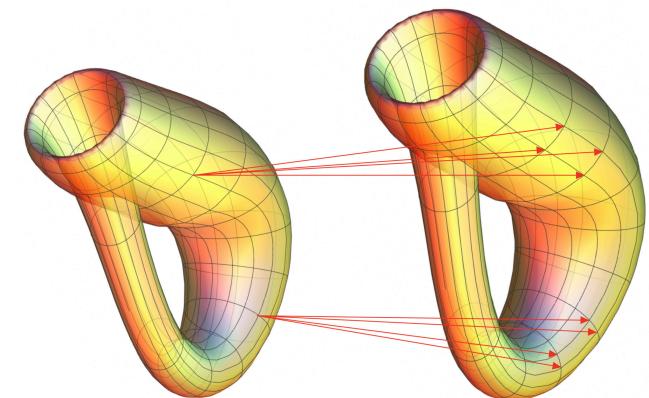
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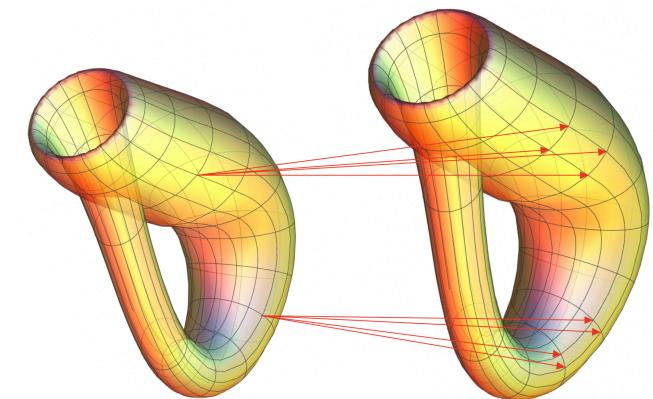


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As a second warm-up, our research group (Zhiwang Yu) have reproduced some of their results.



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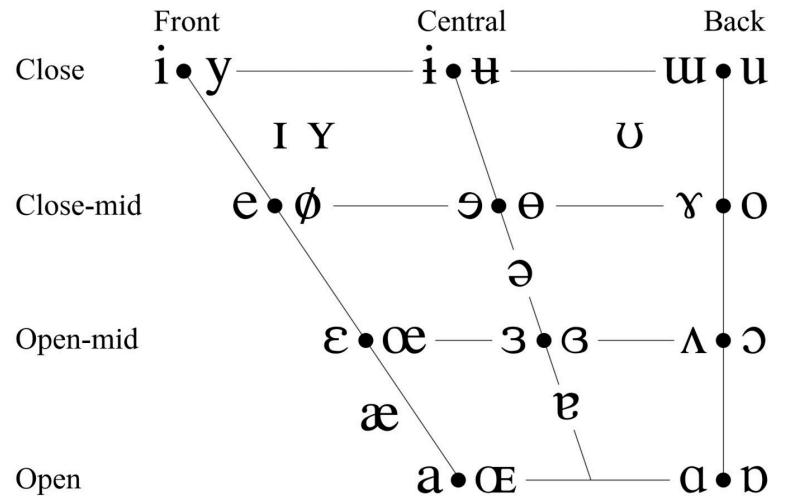
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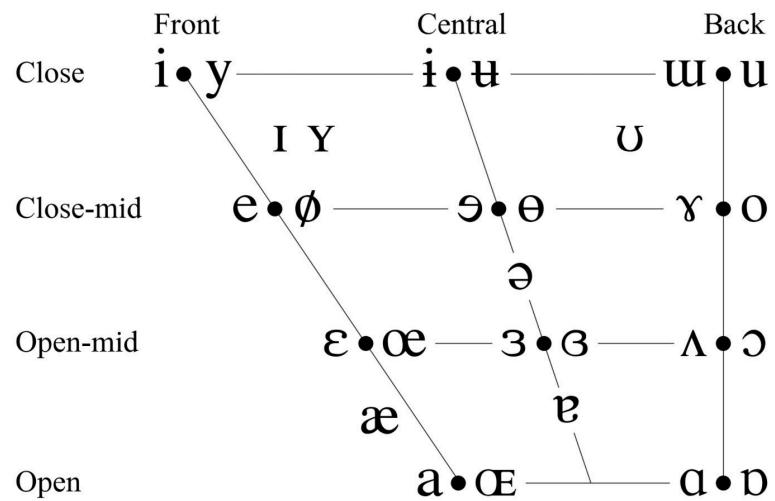


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The vertical axis of the chart denotes vowel height. Vowels pronounced with the tongue lowered are at the bottom and raised are at the top. The horizontal axis of the chart denotes vowel backness. Vowels with the tongue moved towards the front of the mouth are in the left of the chart, while those with the tongue moved to the back are placed in right. The last parameter is whether the lips are rounded. At each given spot, vowels on the right and left are rounded and unrounded, respectively.



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The image shows a screenshot of a scientific paper. At the top left, it says "1 of 19". The title is "Topology combined machine learning for consonant recognition". Below the title, the authors are listed as Pingyao Feng, Siheng Yi, Qingrui Qu, Zhiwang Yu & Yifei Zhu. The abstract begins with: "Abstract—In artificial-intelligence-aided signal processing, existing deep learning models often exhibit a black-box structure, and their validity and comprehensibility remain elusive. The integration of topological methods, despite its relatively nascent application, serves a dual purpose of making models more interpretable and as a means of extracting information from time-delayed data. Here, we provide a transparent and broadly applicable methodology, TopCap, to capture the most salient topological features inherent in time series for machine learning. Rooted in persistent homology, TopCap is capable of capturing features more relevant in contexts with high dimensionality. Applying time-delay embedding and persistent homology, we obtain descriptors which encapsulate information such as the vibration of a time series, in terms of its variability of frequency, amplitude, and average line, demonstrating the strength of persistent homology in capturing the shape of data. We then extract information from these descriptors and fed into multiple machine learning models such as k-nearest neighbours and support vector machine. Notably, in classifying voiced and voiceless consonants, TopCap achieves an accuracy exceeding 96% and is geared towards designing topological convolutional layers for deep learning of speech and audio signals." The abstract is followed by a section titled "1 INTRODUCTION" and a detailed explanation of TDA's applications in various fields like image recognition, time series forecasting, and speech processing.

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1 INTRODUCTION

In 1966, Mark Kac asked the famous question: “Can you hear the shape of a drum?” To hear the shape of a drum is to hear the sound of the drum. In the short article of the same name, he inquired about the shape of a drum from the sound it makes using mathematical theory. In this article, we mirror the question across senses and address instead: “Can we see the sound of a human speech?”

The artificial intelligence (AI) advancements have led to a widespread adoption of voice recognition technologies, encompassing applications such as speech-to-text conversion and music generation. The rise of topological data analysis (TDA) [1] has integrated topological methods into various fields, including image recognition [2], protein folding prediction [3], and speech recognition [4]. TDA methods are more interpretable and efficient, with a focus on structural information. In the field of voice recognition [4], [5], more specifically consonant recognition [6]–[10], prevalent methodologies frequently revolve around the analysis of energy and spectral information. While topological approaches are still rare in this area, we combine TDA and machine learning to obtain a classification for speech data, based on geometric patterns hidden within the speech signals. The method we propose, TopCap (referring to capturing topological structures of data), is not only applicable to audio data but also to general-purpose time series data that require extraction of structural information for machine learning algorithms. Initially, we endow phonetic time series with point-cloud structure in a high-dimensional Euclidean space via time-delay embedding (TDE, see Fig. 1a) with appropriate choices of parameters. Subsequently, 1-dimensional persistence diagrams are computed using persistent homology (see Section 3 of Supplementary Information for the explanation of the terminologies). We then conduct evaluations with nine machine learning algorithms to demonstrate the significant capabilities of TopCap in descripted classification.

Conceptually, TDA is an approach which facilitates the examination of data structure through the lens of topology. This discipline was originally formulated to investigate the “shape” of data, particularly point-cloud data in high-dimensional spaces [11]. Characterised by a unique measure of topological features, TDA has been applied under continuous deformation, and coordinate-free computation [1]. TDA has been combined with machine learning methods to uncover intricate and concealed information within datasets [3], [12]–[16]. In these contexts, topological methods have been employed to extract structural information from the dataset, thereby enhancing the efficiency of the original algorithms. Notably, TDA excels in identifying patterns in complex data, and its potential is estimated to have a broad range of applications in various fields [17]. As a nascent field of study, the majority of theoretical results pertaining to topological methods have yet found their optimal applications and benefited everyday life. Nevertheless, with its distinctive emphasis on the shape of data, TDA has led to novel applications in various far-reaching fields, as evidenced in the literature. These include image recognition [18]–[20], time series forecasting [21] and classification [22], speech recognition [23], protein folding prediction [24], and signal analysis [25], [26], speech recognition [27], signal processing [28], [29], neural networks [2], [30]–[32], among others. It is anticipated that with the further development of theoretical foundations and their applications, the promising future of TDA will pave a new direction to enhance numerous aspects of daily lives.

The task of extracting features that pertain to structural information is both interesting and formidable. This process is integral to a multitude of practical applications, as evidenced by various studies [33]–[36]. Scholars strive to identify the most effective representatives and descriptors of shape within a given dataset. Despite the fact that TDA

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The image shows a digital document page with a light gray background. At the top left, it says "1 of 19". In the center, the title "Topology combined machine learning for consonant recognition" is displayed in a black serif font. Below the title, the authors' names are listed: Pingyao Feng, Siheng Yi, Qingrui Qu, Zhiwang Yu & Yifei Zhu. A horizontal line follows this information. The main text begins with an abstract: "Abstract—In artificial-intelligence-aided signal processing, existing deep learning models often exhibit a black-box structure, and their validity and comprehensibility remain elusive. The integration of topological methods, despite its relatively nascent application, serves a dual purpose of making models more interpretable and as extracting structural information from time-delayed data. Here, we provide a transparent and broadly applicable methodology, TopCap, to capture the most salient topological features inherent in time series for machine learning. Rooted in persistent homology, the TopCap is capable of capturing features more relevant in contexts with high dimensionality. Applying time-delay embedding and persistent homology, we obtain descriptors which encapsulate information such as the vibration of a time series, in terms of its variability of frequency, amplitude, and average line, demonstrating how persistent diagram information can be extracted and fed into multiple machine learning models such as k-nearest neighbours and support vector machine. Notably, in classifying voiced and voiceless consonants, TopCap achieves an accuracy exceeding 96% and is geared towards designing topological convolutional layers for deep learning of speech and audio signals." The abstract is followed by a section titled "1 INTRODUCTION" and a detailed explanation of the methodology and its applications in speech recognition.

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Topology combined machine learning for consonant recognition

Pingyao Feng , Siheng Yi, Qingrui Qu, Zhiwang Yu & Yifei Zhu

Abstract—In artificial-intelligence-aided signal processing, existing deep learning models often exhibit a black-box structure, and their validity and comprehensibility remain elusive. The integration of topological methods, despite its relatively nascent application, serves a dual purpose of making models more interpretable and as an effective structural interpretation for time-delay embedded data. Here, we provide a transparent and broadly applicable methodology, TopCap, to capture the most salient topological features inherent in time series for machine learning. Rooted in persistent homology, the TopCap is capable of capturing features more relevant in contexts with high dimensionality. Applying time-delay embedding and persistent homology, we obtain descriptors which encapsulate information such as the vibration of a time series, in terms of its variability of frequency, amplitude, and average line, demonstrating that the shape of a time series is derived from the sound it makes using mathematical theory. In this article, we mirror the question across senses and address instead: “Can we see the sound of a human speech?”

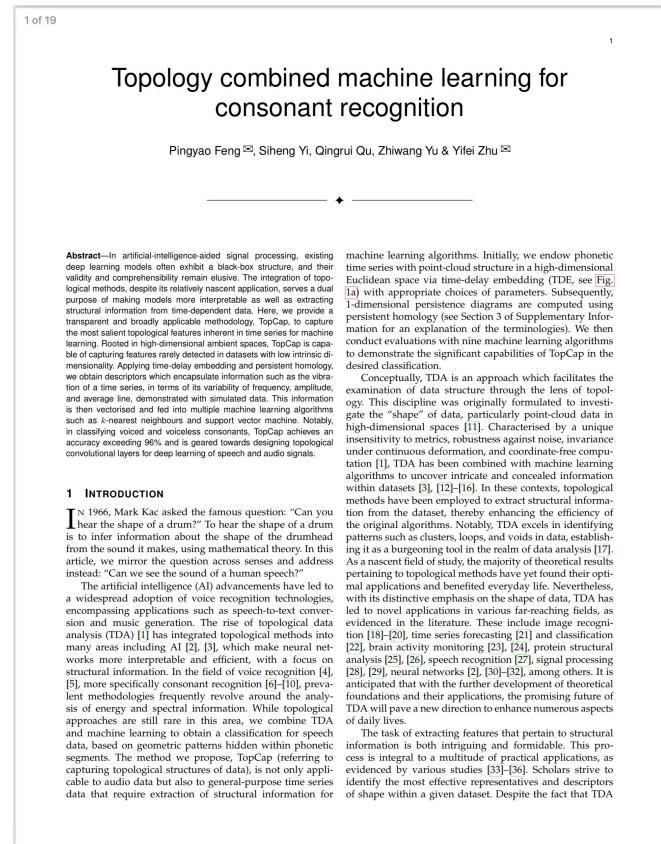
The abstract intelligence (AI) advancements have led to a widespread adoption of voice recognition technologies, encompassing applications such as speech-to-text conversion and music generation. The rise of topological data analysis (TDA) [1] has integrated topological methods into various fields, including image recognition [2], protein structural analysis [3], speech recognition [27], signal processing [28], [29], neural networks [2], [30]-[32], among others. It is anticipated that with the further development of theoretical foundations and their applications, the promising future of TDA will pave a new direction to enhance numerous aspects of daily lives.

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1 INTRODUCTION

In 1966, Mark Kac asked the famous question: “Can you hear the shape of a drum?” To hear the shape of a drum is to identify the shape of the drum from the sound it makes using mathematical theory. In this article, we mirror the question across senses and address instead: “Can we see the sound of a human speech?”

The artificial intelligence (AI) advancements have led to a widespread adoption of voice recognition technologies, encompassing applications such as speech-to-text conversion and music generation. The rise of topological data analysis (TDA) [1] has integrated topological methods into various fields, including image recognition [2], protein folding [3], network analysis [24], speech recognition [27], signal processing [28], [29], neural networks [2], [30]-[32], among others. It is anticipated that with the further development of theoretical foundations and their applications, the promising future of TDA will pave a new direction to enhance numerous aspects of daily lives.

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Reservoir networks and photonic circuits have been applied to vowel recognition, too.



Deep learning with coherent nanophotonic circuits

Yichen Shen^{1*}, Nicholas C. Harris^{1†}, Scott Skirlo¹, Mihika Prabhu¹, Tom Baehr-Jones², Michael Hochberg², Xin Sun³, Shijie Zhao⁴, Hugo Larochelle⁵, Dirk Englund¹ and Marin Soljačić¹

Artificial neural networks are computational network models inspired by signal processing in the brain. These models have dramatically improved performance for many machine-learning tasks, including speech and image recognition. However, today's computing hardware is inefficient at implementing neural networks, in large part because much of it was designed for von Neumann computing schemes. Significant effort has been made towards developing electronic architectures tuned to implement artificial neural networks that exhibit improved computational speed and accuracy. Here, we propose a new architecture for a fully optical neural network that, in principle, could offer an enhancement in computational speed and power efficiency over state-of-the-art electronics for conventional inference tasks. We experimentally demonstrate the essential part of the concept using a programmable nanophotonic processor featuring a cascaded array of 56 programmable Mach-Zehnder interferometers in a silicon photonic integrated circuit and show its utility for vowel recognition.

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It will be useful to design and fine-tune them topologically (joint with Huan Li of optical science and engineering at Zhejiang University and Xinxiang Niu of Huawei).

Thank you.