

Lecture 04

Morphology and Segmentation

2024-02-29

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MÉXICO

1. Introduction

2. Mathematical Morphology

3. Image Segmentation

4. Analyze segmented image

Previous lecture:

convolution: $f(x, y), g(x, y), \underline{w}: \mathbb{N} \rightarrow \mathbb{R}$

where $w = \underline{\text{filter kernel}}$

→ (mostly) linear operators

Today:

morphology: $f(x, y), g(x, y), \underline{b}: \mathbb{N} \rightarrow \{0, 1\}$

where $b = \underline{\text{structuring element}}$

→ non-linear operators

→ concerned with connectivity and shape (close to *set theory*)

segmentation:

→ labeling image pixels to partition an image into regions

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2. Mathematical Morphology

1. Basic concepts
2. Primitive Morphological Operations
3. Composite Morphological Operations

3. Image Segmentation

4. Analyze segmented image

2.1. Basic concepts

- Initially proposed for binary images (*Matheron and Serra, 1964*)
→ later extended to gray-scale images, and later color images
- Binary images produced by simple thresholding are imperfect due to image noise, etc.
⇒ morphological image processing attempts to remove these imperfections
- Main applications:
 - Image pre-processing (noise filtering, shape simplification)
 - Enhancing object structure (skeletonizing, convex hull, ...)
 - Segmentation
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Morphological filtering mechanics are similar to spatial filtering using convolutions:

1) a kernel called a **structuring element** is used to determine filtering operation:

- the size is determined by the matrix dimensions
- the shape is determined by the pattern of 1 and 0 in the matrix
- the origin is usually the matrix center, although it can also off-centered or even outside it

NB: like convolution kernels, it is common to have structuring elements of odd dimensions with the center as the origin.

NB: the shape, size, and orientation of the structuring element depends on application

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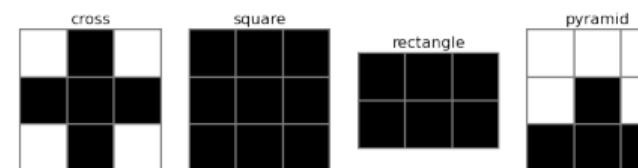
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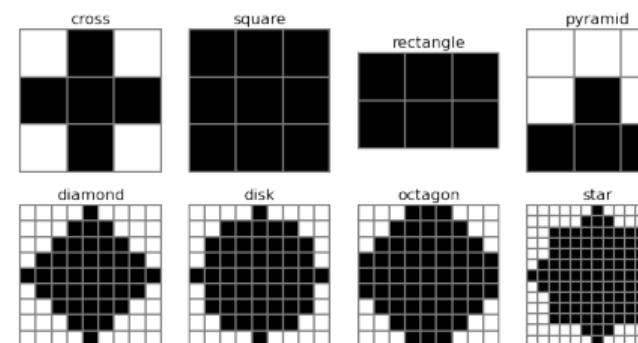
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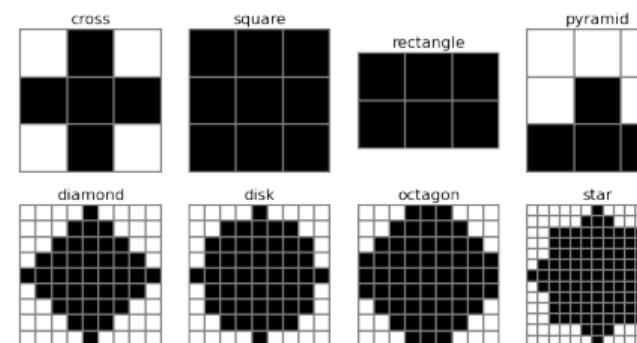
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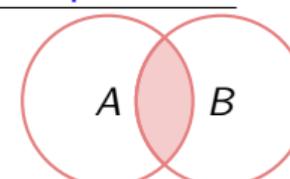
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2) the image is first **padded**, and the structuring element than **slides** across it

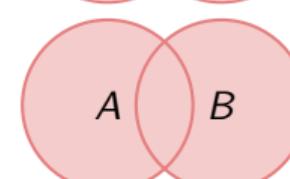
Morphological filters are essentially **set operations**

Intersection (AND)



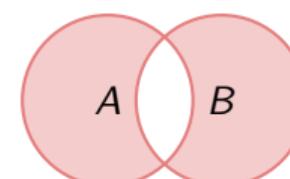
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Union (OR)



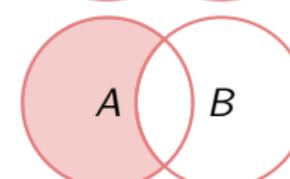
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Symmetric difference (XOR)



$$\overline{A \cap B}$$

Difference



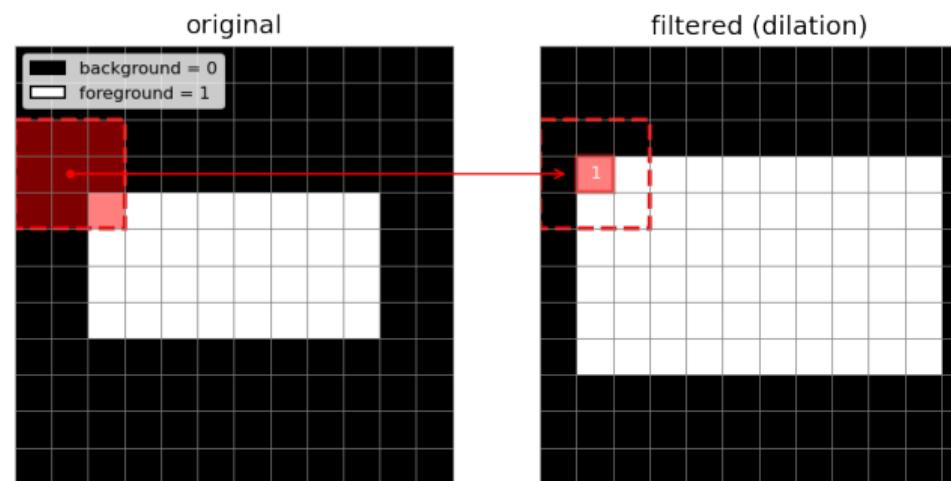
$$A - B$$

2.2. Primitive Morphological Operations

- Primary Morphological Operations are: dilation and erosion
- Concatenation of dilation and erosion result in higher level operations
 - ⇒ Composite Morphological Operations: closing and opening

1. **Dilation**: the dilation of a set F with a structuring element b is defined as:

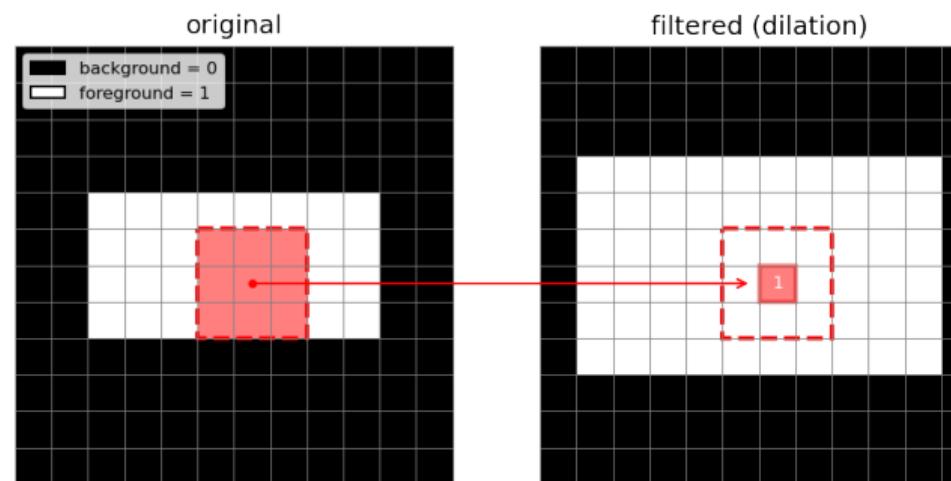
$$G = F \oplus b = \{x : (\hat{b})_x \cap F \neq \emptyset\}$$



if ≥ 1 pixel within the mask = "1", the result is "1", otherwise "0"

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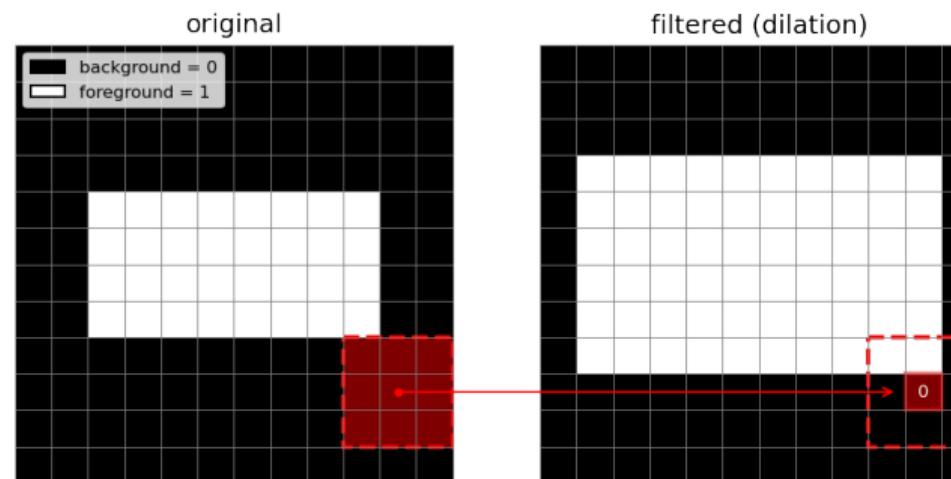
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- ⇒ Foreground objects get larger
- ⇒ Background objects get smaller
- ⇒ Small gaps are closed

original



background = 0
 foreground = 1

dilation ($b=3\times 3$)



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dilation ($b=7 \times 7$)



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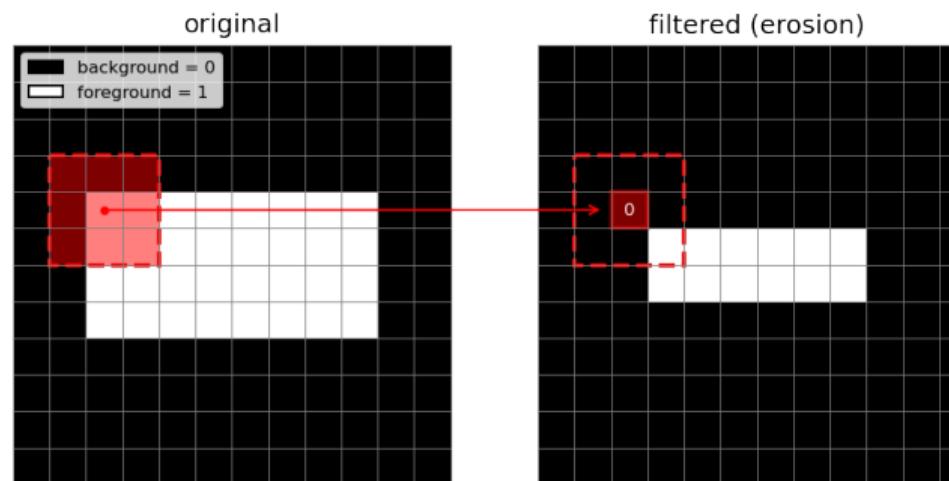
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dilation ($b=11\times 11$)



2. **Erosion**: the erosion of a set F with a structuring element b is defined as:

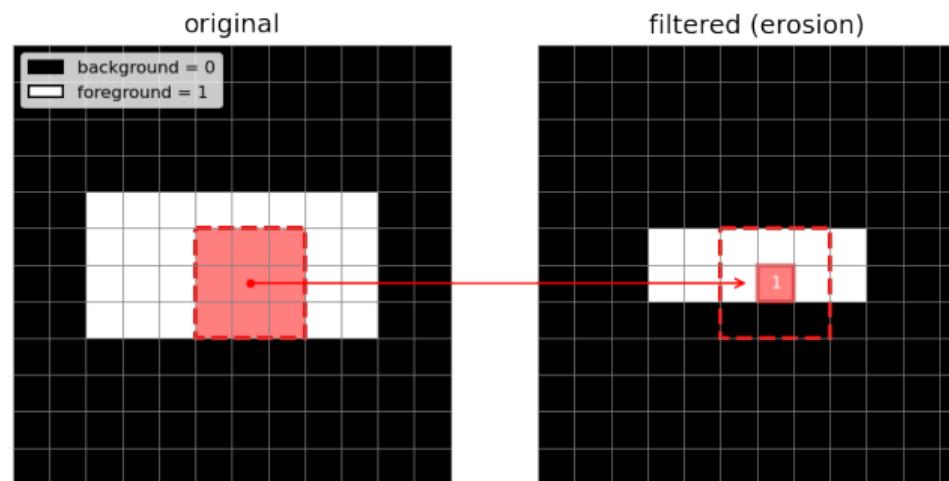
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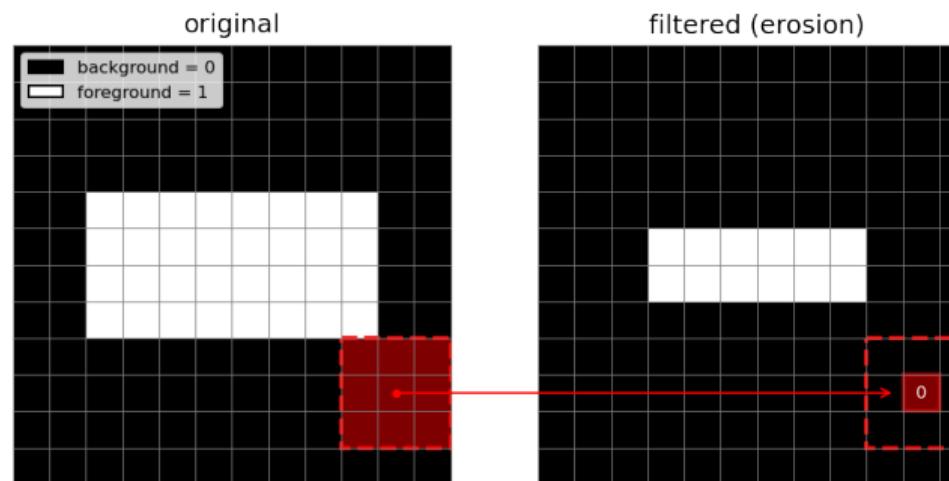
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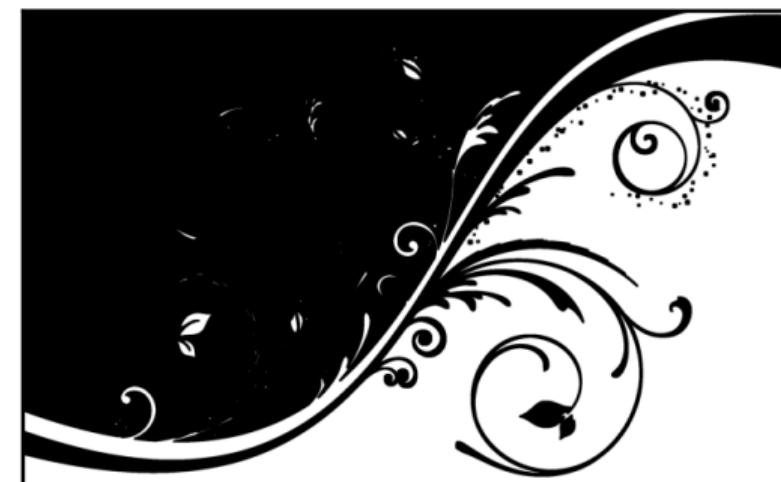
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Concatenation of **dilation** and **erosion** result in higher level operations: **closing**, **opening**

1. Opening:

Problem: erosion causes deletion of small objects, BUT other objects shrink

Solution: after *erosion*, apply *dilation* with the same structuring element \Rightarrow **opening**

$$G = F \circ b = (F \ominus b) \oplus b$$

Usage example: removing small isolated “bright spots” (EX: volcanic *SO₂* detection from Sentinel-5P as foreground (mask=1))

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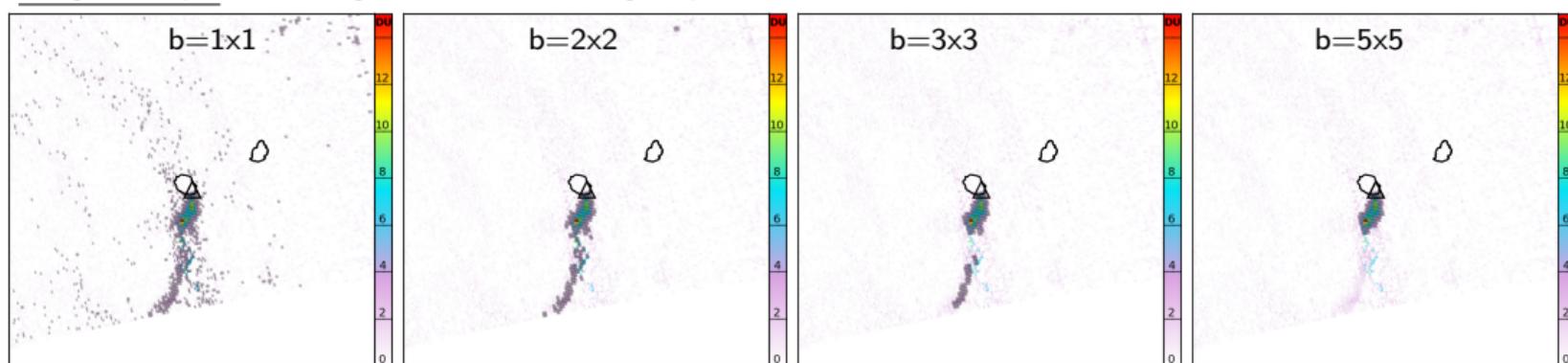
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Concatenation of dilation and erosion result in higher level operations: closing, opening

2. Closing:

Problem: dilation closes small holes and fractions, BUT objects get enlarged

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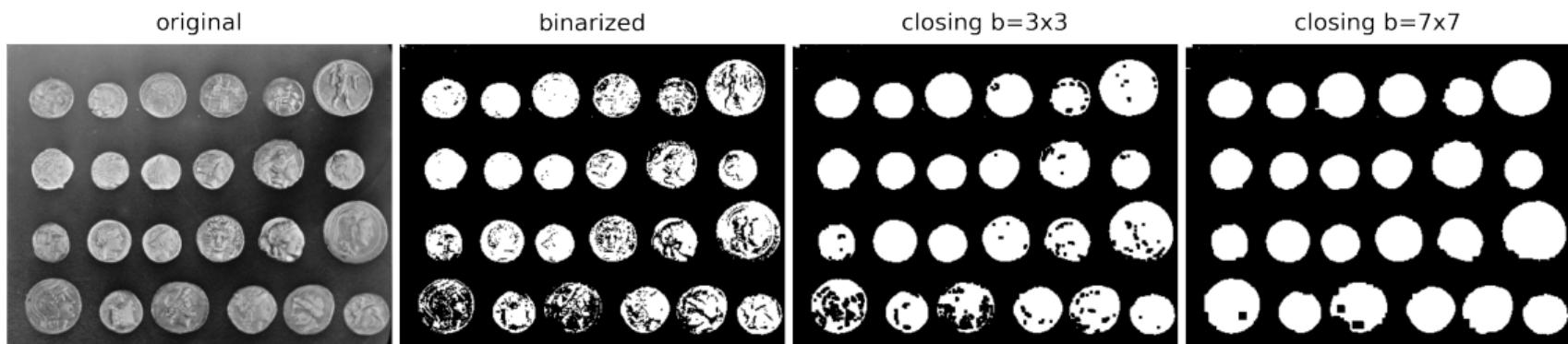
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1. Introduction

2. Mathematical Morphology

3. Image Segmentation

1. histogram-based segmentation
2. edge-based segmentation
3. region-based segmentation

4. Analyze segmented image

3.0. Composite Morphological Operations

Image segmentation = labeling image pixels to partition an image into regions

- Histogram-based segmentation
⇒ based on thresholding of pixel values
 - ex: manual thresholding
 - ex: automatic thresholding (e.g., Otsu)
 - ex: k-means clustering
- Edge-based segmentation
⇒ based on local contrast → uses gradients rather than the grey values
- Region-based segmentation
⇒ based on image gradients and region properties
 - ex: Watershed transform
 - ex: Random Walker
 - ex: Flood Fill
- Many other!
ex: Graph-cuts
ex: Active Contours, Region Growing, Weighted Pyramid Linking, Mean-Shift, etc.

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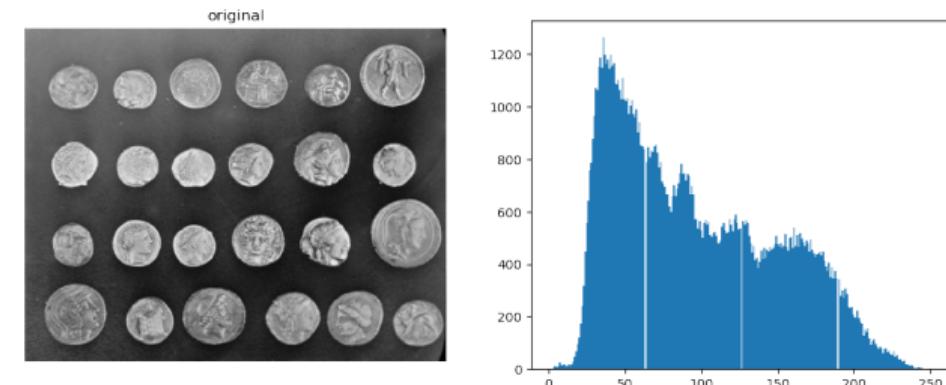
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Histogram-based segmentation

⇒ based on thresholding pixel values

- global thresholding
 - manual
 - automatic (e.g. [Otsu's method](#))

(threshold calculated to separate pixels into two classes,
minimizing intra-class intensity variance)

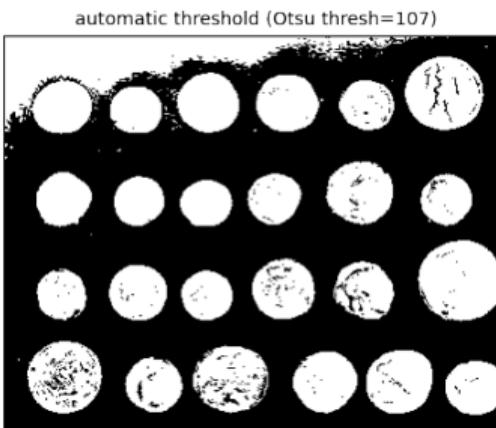
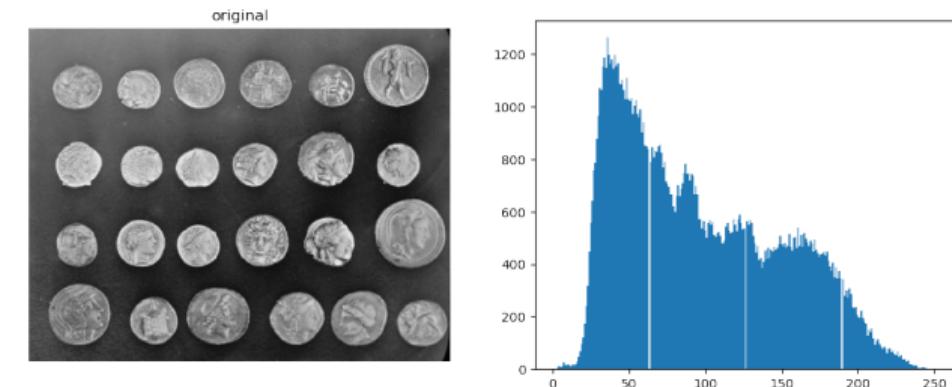


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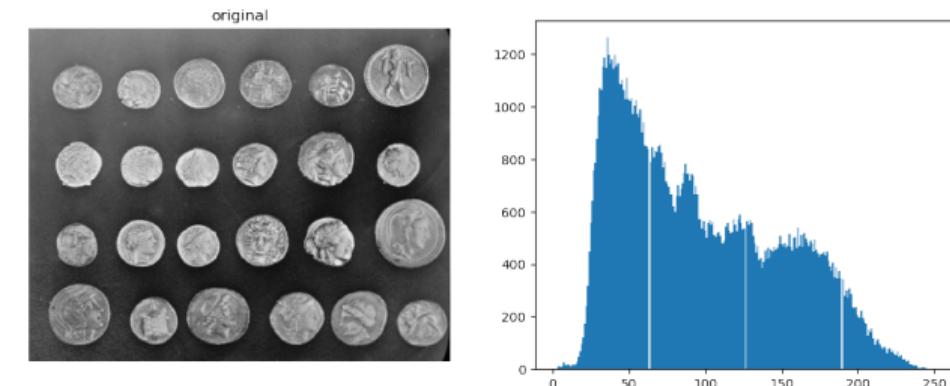


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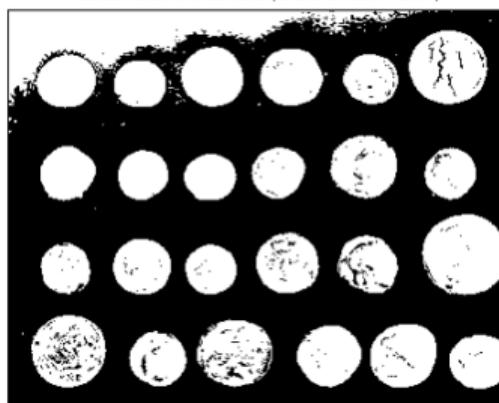
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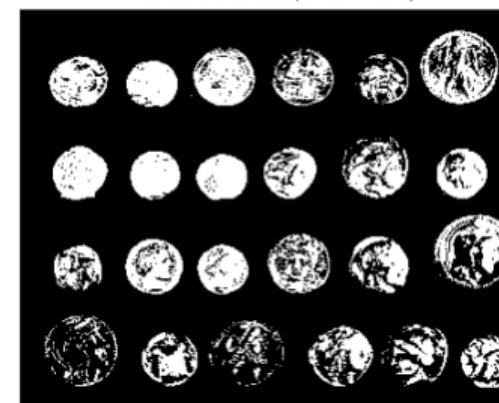
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automatic threshold (Otsu thresh=107)



manual threshold (thresh=150)

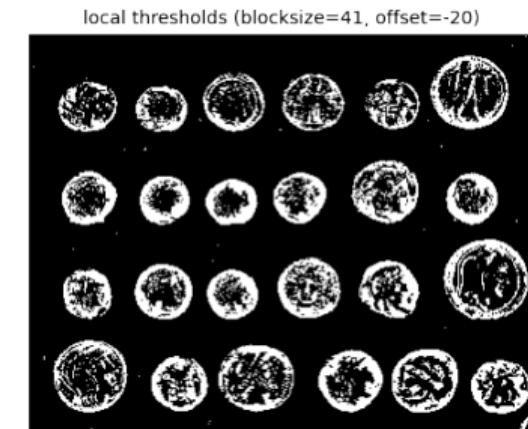
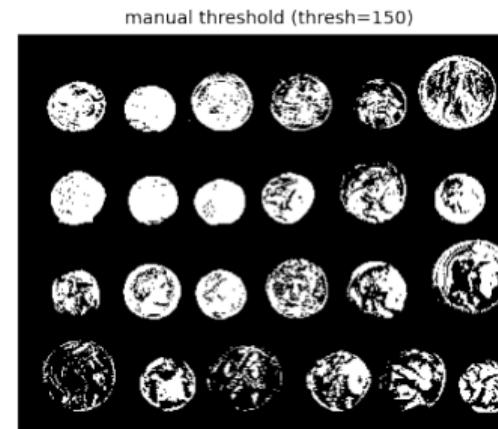
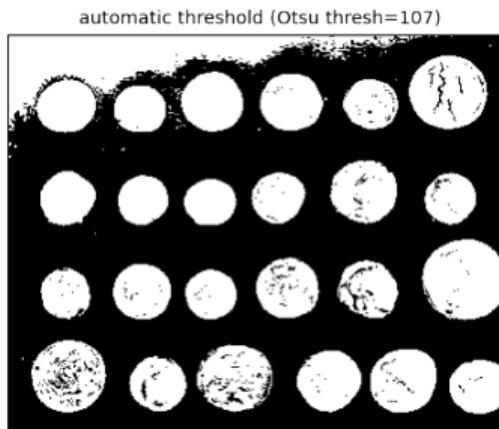
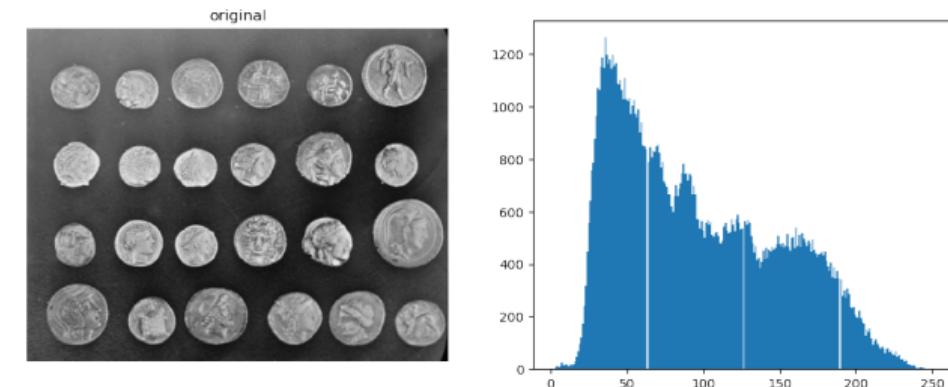


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(thresholds calculated based on pixel local neighborhood)



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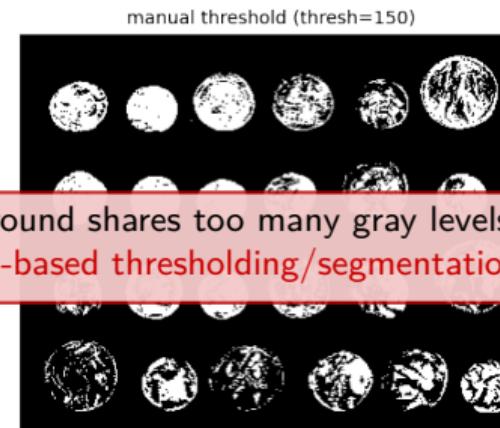
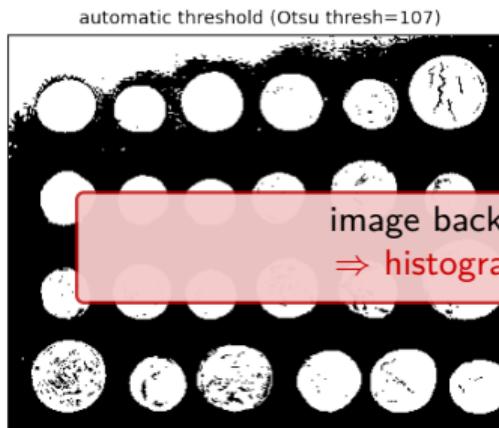
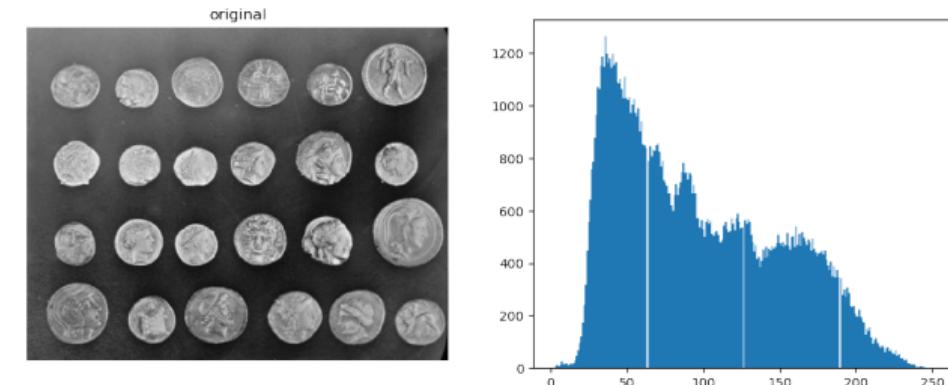
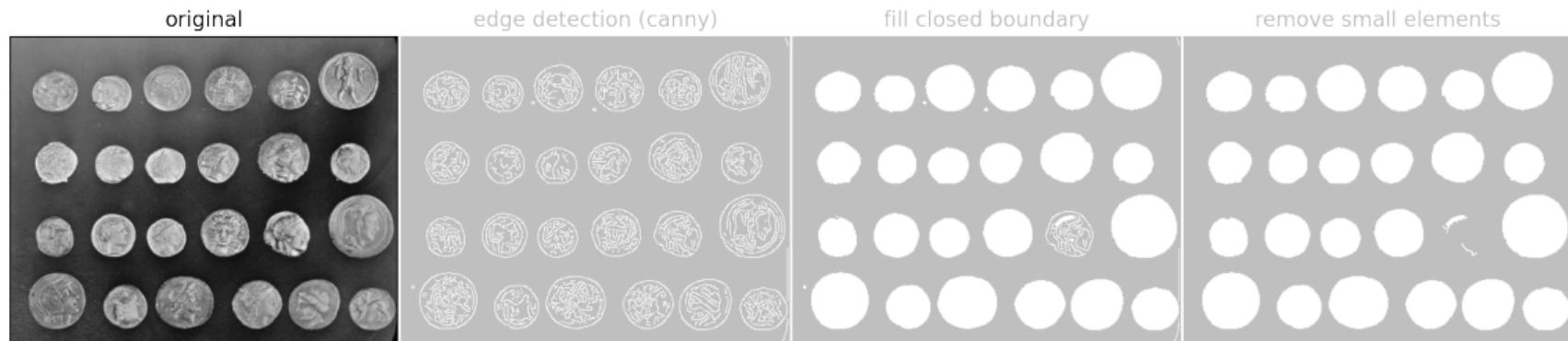


image background shares too many gray levels with the coins
⇒ histogram-based thresholding/segmentation is insufficient

3.2. edge-based segmentation

Edge-based segmentation

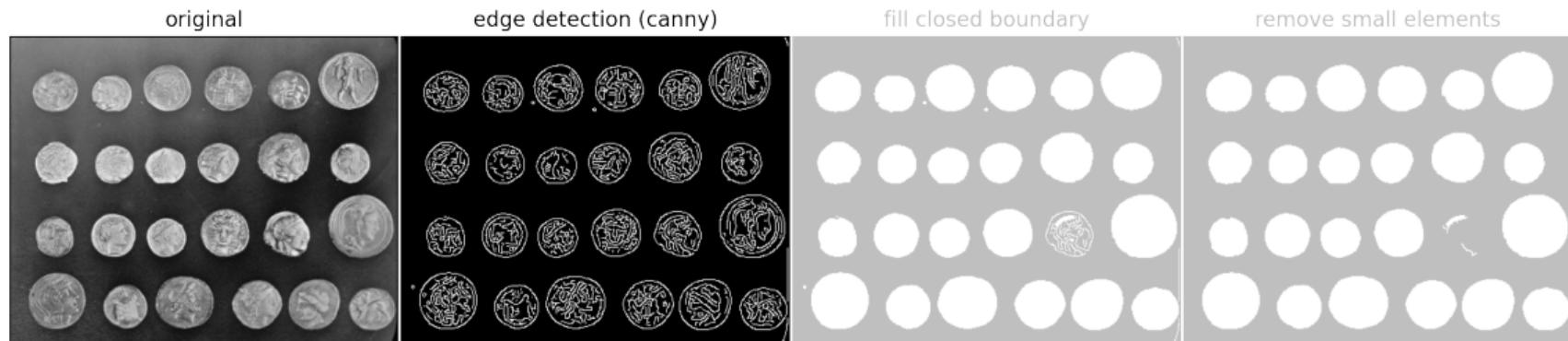
⇒ based on image gradients



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Edge-based segmentation

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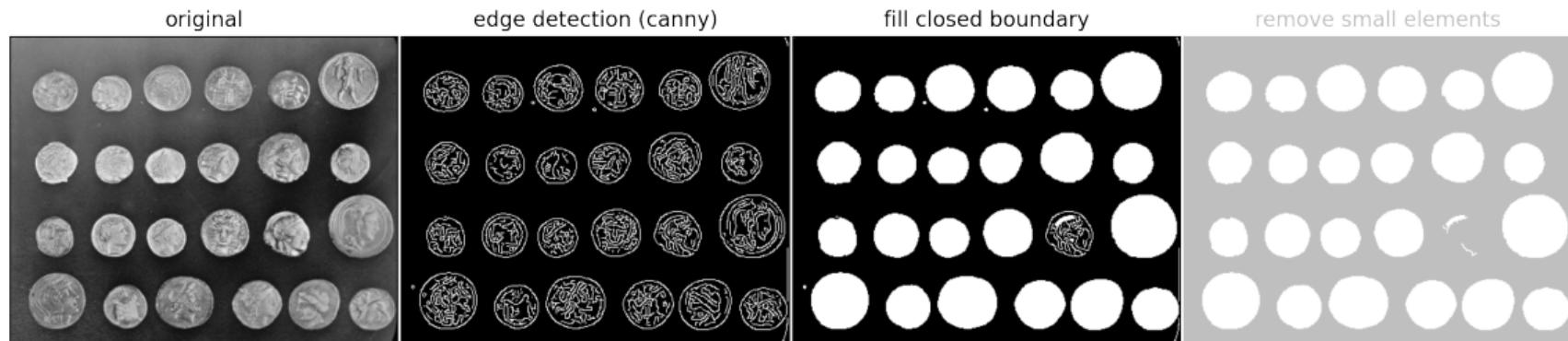


1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)

3.2. edge-based segmentation

Edge-based segmentation

⇒ based on image gradients

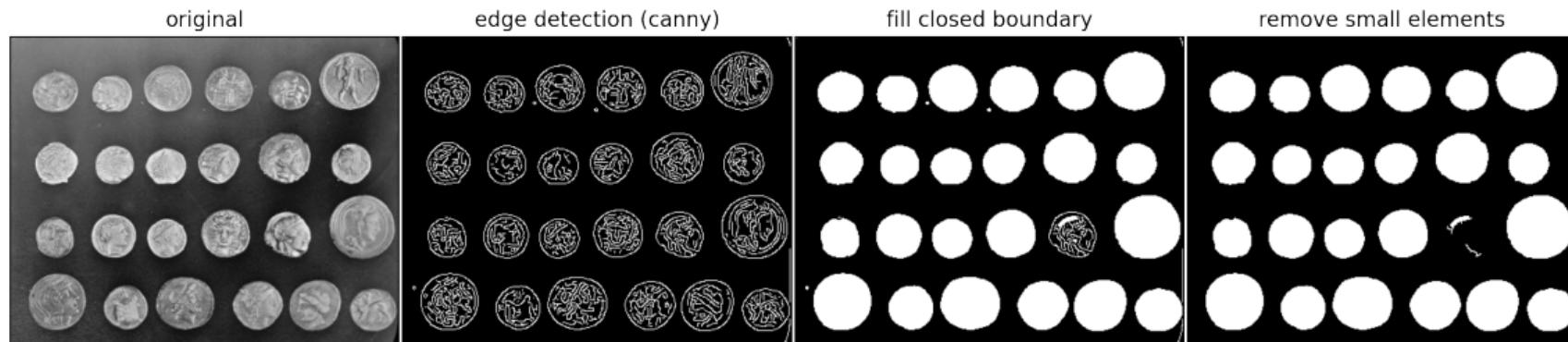


1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins

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Edge-based segmentation

⇒ based on image gradients

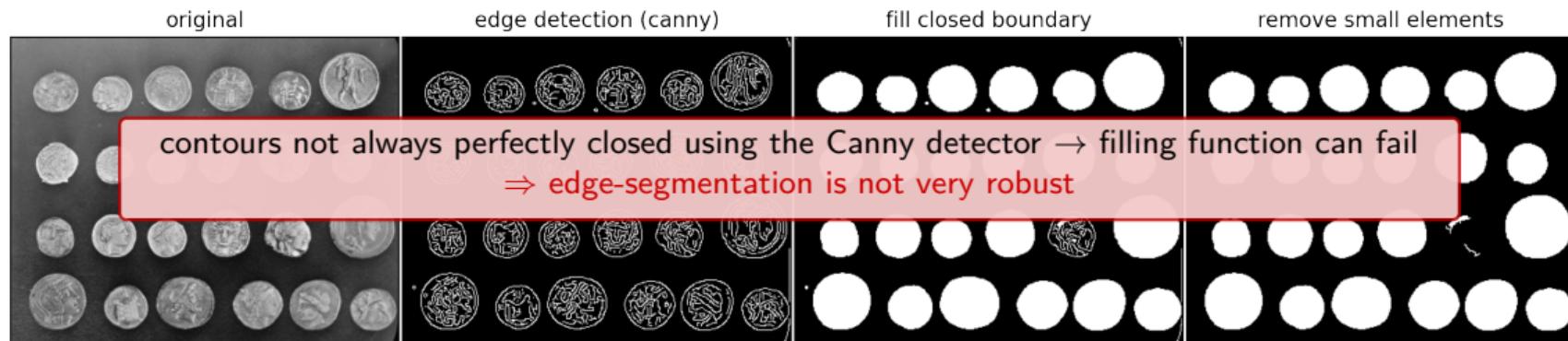


1. apply [Canny](#) edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins
3. remove objects smaller than a threshold

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Edge-based segmentation

⇒ based on image gradients



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3.3. region-based segmentation

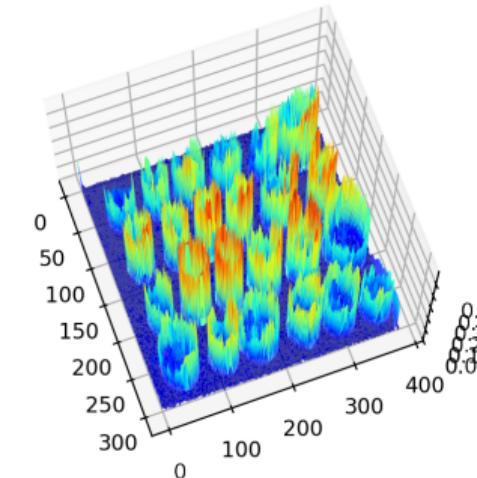
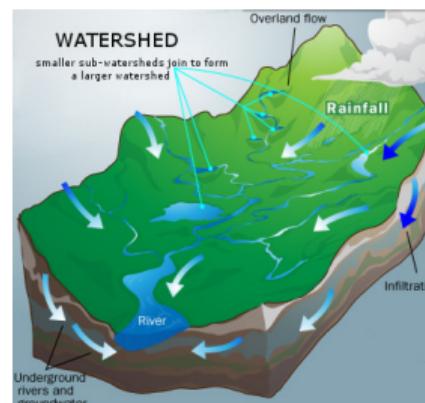
Region-based segmentation: *watershed transform*

⇒ region-growing approach that fills “basins” in the image

3.3. region-based segmentation

Region-based segmentation: *watershed transform*

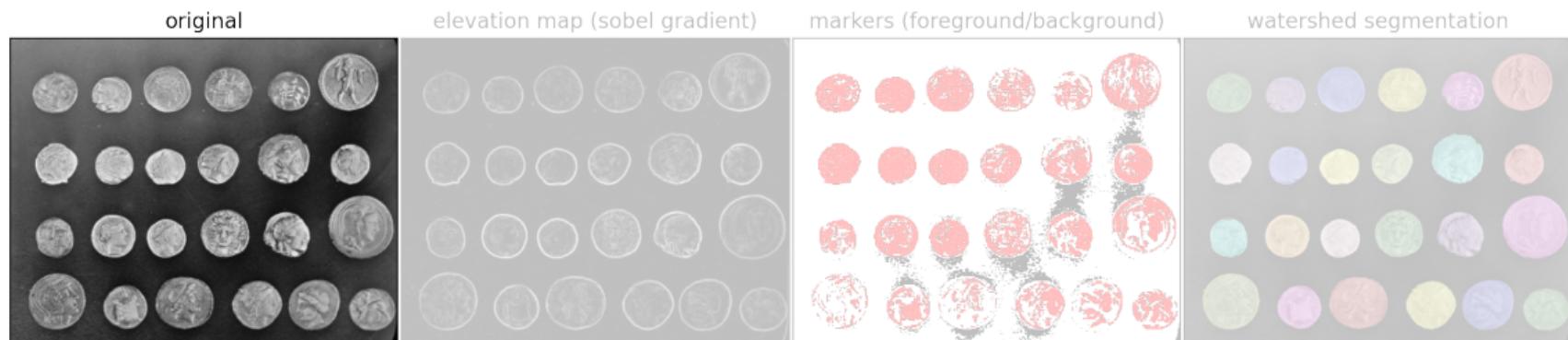
- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
 - the *watershed transform* “floods” a “topographic” representation of the image
 - flooding starts from “markers”, in order to determine the catchment basins of these markers



3.3. region-based segmentation

Region-based segmentation: *watershed transform*

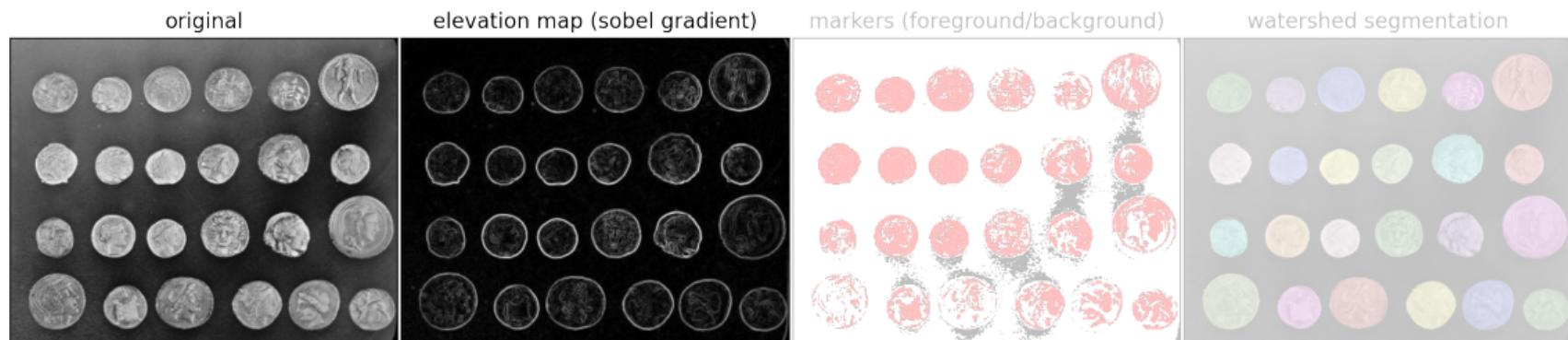
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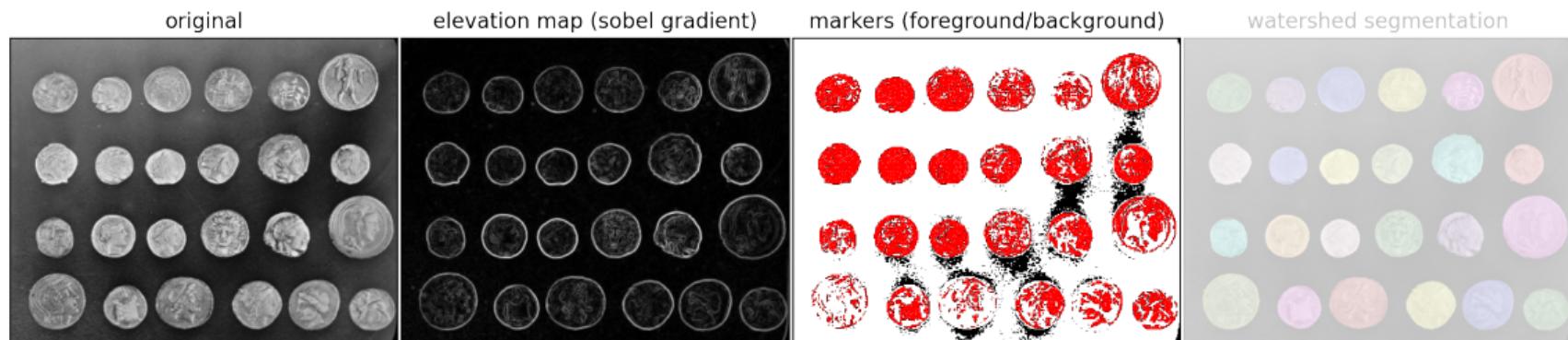


1. build “elevation map” from image gradient amplitude (using the Sobel operator)

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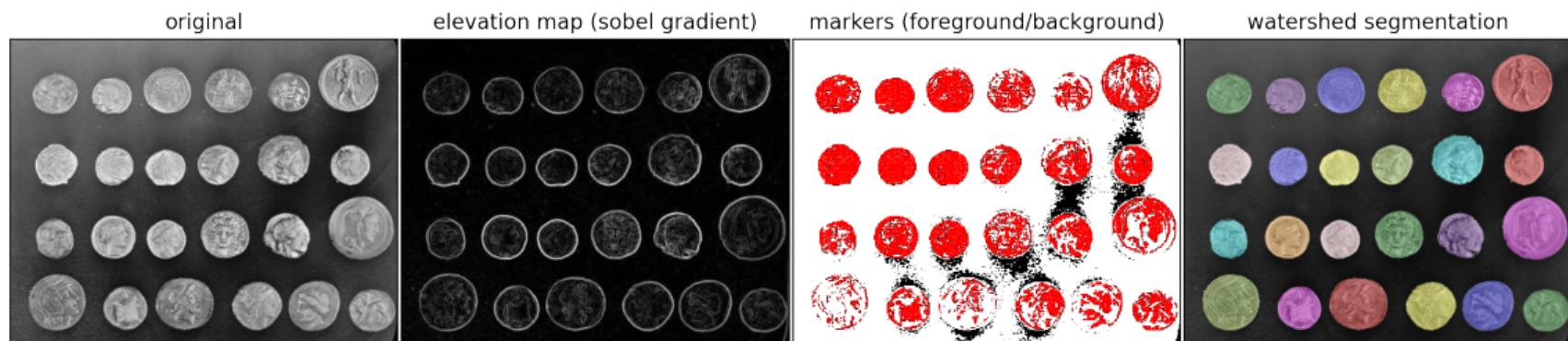


1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)

3.3. region-based segmentation

Region-based segmentation: *watershed transform*

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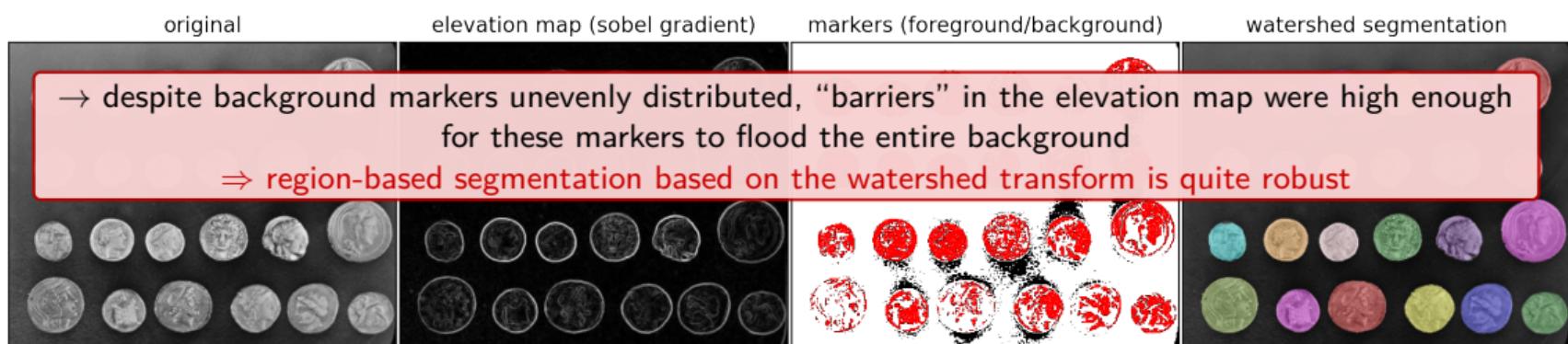


1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)
3. apply **watershed transform** (and colorize segmented elements)

3.3. region-based segmentation

Region-based segmentation: *watershed transform*

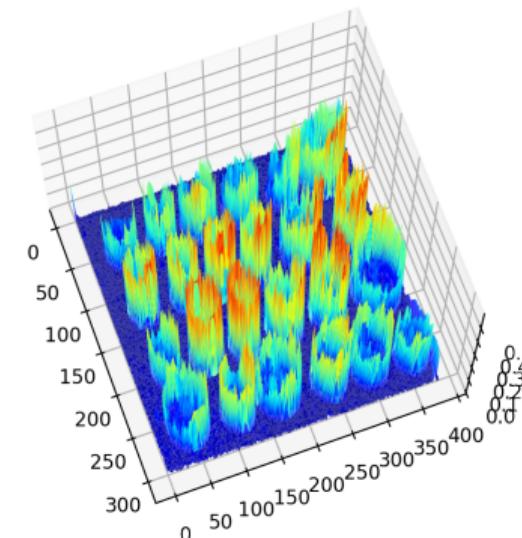
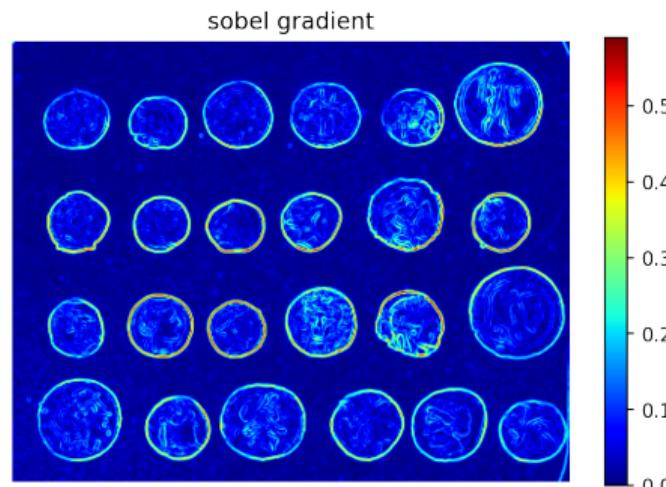
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3.3. region-based segmentation

Region-based segmentation: *watershed transform*



1. Start with lowest “altitude” (Gradient amplitude)
2. Increase the “water level” each time by 1
3. Merge all connected pixel with same/less level

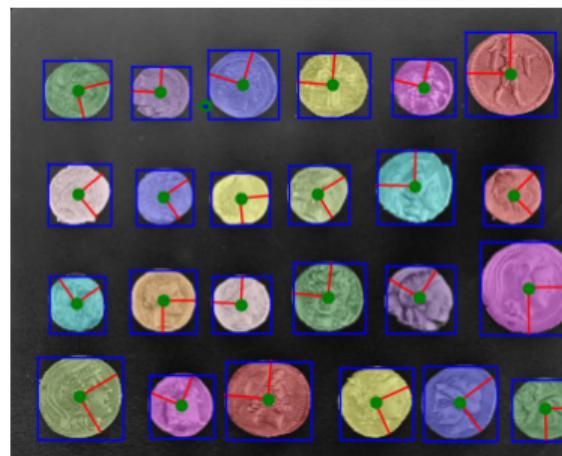
1. Introduction
2. Mathematical Morphology
3. Image Segmentation
4. Analyze segmented image

Analyze segmented image

The segmented elements can be analysed individually to:

→ provide statistics on their shape, distribution, orientation, etc.

(e.g. fields in a satellite image, crystal/bubble shape distribution in a rock sample, etc.)

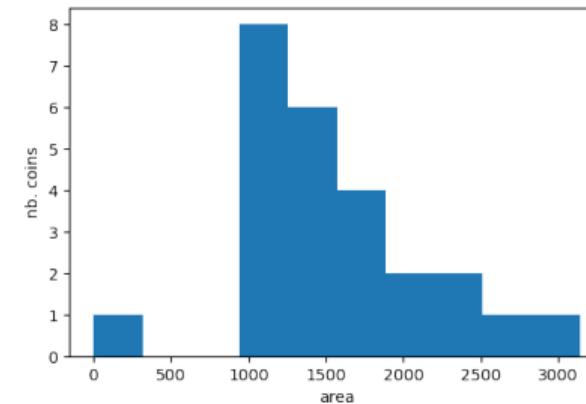
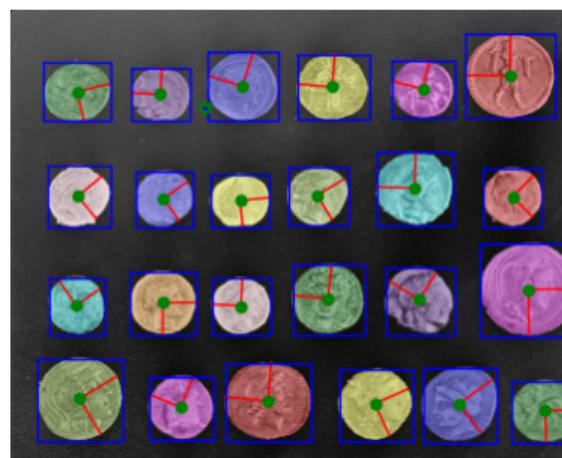


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The segmented elements can be analysed individually to:

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Analyze segmented image

Exercises:

1. Exercise 1:

⇒ histogram-based segmentation of Popocatépetl

2. Exercise 2:

⇒ analyze a thermal infrared image of a lava lake

→ segment the crustal plates from the incandescent cracks and analyze