

## Lecture 04

# Morphology and Segmentation

2024-02-29

Sébastien Valade



UNIVERSIDAD NACIONAL  
AUTÓNOMA DE  
MÉXICO

1. Introduction

2. Mathematical Morphology

3. Image Segmentation

4. Analyze segmented image

Previous lecture:

**convolution:**  $f(x, y), g(x, y), \underline{w}: \mathbb{N} \rightarrow \mathbb{R}$

where  $w = \underline{\text{filter kernel}}$

→ (mostly) linear operators

Today:

**morphology:**  $f(x, y), g(x, y), \underline{b}: \mathbb{N} \rightarrow \{0, 1\}$

where  $b = \underline{\text{structuring element}}$

→ non-linear operators

→ concerned with connectivity and shape (close to *set theory*)

**segmentation:**

→ labeling image pixels to partition an image into regions

Previous lecture:

**convolution:**  $f(x, y), g(x, y), \underline{w}: \mathbb{N} \rightarrow \mathbb{R}$

where  $w = \underline{\text{filter kernel}}$

→ (mostly) linear operators

Today:

**morphology:**  $f(x, y), g(x, y), \underline{b}: \mathbb{N} \rightarrow \{0, 1\}$

where  $b = \underline{\text{structuring element}}$

→ non-linear operators

→ concerned with connectivity and shape (close to **set theory**)

**segmentation:**

→ labeling image pixels to partition an image into regions

Previous lecture:

**convolution:**  $f(x, y), g(x, y), \underline{w}: \mathbb{N} \rightarrow \mathbb{R}$

where  $w = \underline{\text{filter kernel}}$

→ (mostly) linear operators

Today:

**morphology:**  $f(x, y), g(x, y), \underline{b}: \mathbb{N} \rightarrow \{0, 1\}$

where  $b = \underline{\text{structuring element}}$

→ non-linear operators

→ concerned with connectivity and shape (close to [set theory](#))

**segmentation:**

→ labeling image pixels to partition an image into regions

1. Introduction

## 2. Mathematical Morphology

1. Basic concepts
2. Primitive Morphological Operations
3. Composite Morphological Operations

3. Image Segmentation

4. Analyze segmented image

## 2.1. Basic concepts

- Initially proposed for binary images (*Matheron and Serra, 1964*)  
→ later extended to gray-scale images, and later color images
- Binary images produced by simple thresholding are imperfect due to image noise, etc.  
⇒ morphological image processing attempts to remove these imperfections
- Main applications:
  - Image pre-processing (noise filtering, shape simplification)
  - Enhancing object structure (skeletonizing, convex hull, ...)
  - Segmentation
  - Quantitative description of objects (area, perimeter, ...)

## 2.1. Basic concepts

- Initially proposed for binary images (*Matheron and Serra, 1964*)  
→ later extended to gray-scale images, and later color images
- Binary images produced by simple thresholding are imperfect due to image noise, etc.  
⇒ **morphological image processing** attempts to remove these imperfections
- Main applications:
  - Image pre-processing (noise filtering, shape simplification)
  - Enhancing object structure (skeletonizing, convex hull, ...)
  - Segmentation
  - Quantitative description of objects (area, perimeter, ...)

## 2.1. Basic concepts

- Initially proposed for binary images (*Matheron and Serra, 1964*)  
→ later extended to gray-scale images, and later color images
- Binary images produced by simple thresholding are imperfect due to image noise, etc.  
⇒ **morphological image processing** attempts to remove these imperfections
- Main applications:
  - Image pre-processing (noise filtering, shape simplification)
  - Enhancing object structure (skeletonizing, convex hull, ...)
  - Segmentation
  - Quantitative description of objects (area, perimeter, ...)

## 2.1. Basic concepts

Morphological filtering mechanics are similar to spatial filtering using convolutions:

1) a kernel called a **structuring element** is used to determine filtering operation:

- the size is determined by the matrix dimensions
- the shape is determined by the pattern of 1 and 0 in the matrix
- the origin is usually the matrix center, although it can also off-centered or even outside it

NB: like convolution kernels, it is common to have structuring elements of odd dimensions with the center as the origin.

NB: the shape, size, and orientation of the structuring element depends on application

## 2.1. Basic concepts

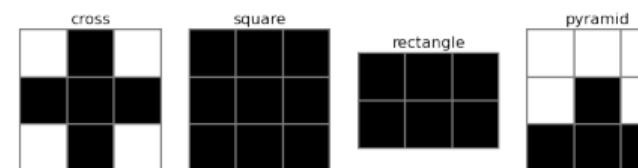
Morphological filtering mechanics are similar to spatial filtering using convolutions:

1) a kernel called a **structuring element** is used to determine filtering operation:

- the size is determined by the matrix dimensions
- the shape is determined by the pattern of 1 and 0 in the matrix
- the origin is usually the matrix center, although it can also off-centered or even outside it

NB: like convolution kernels, it is common to have structuring elements of odd dimensions with the center as the origin.

NB: the shape, size, and orientation of the structuring element depends on application



## 2.1. Basic concepts

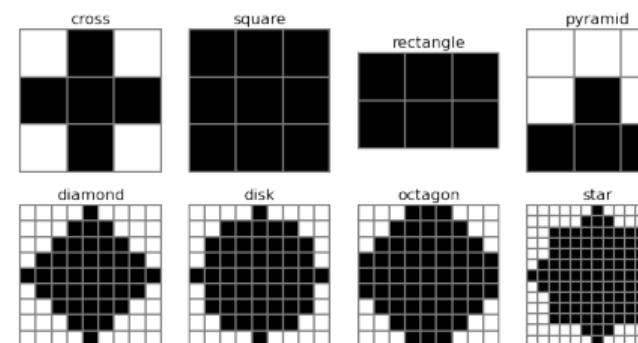
Morphological filtering mechanics are similar to spatial filtering using convolutions:

1) a kernel called a **structuring element** is used to determine filtering operation:

- the size is determined by the matrix dimensions
- the shape is determined by the pattern of 1 and 0 in the matrix
- the origin is usually the matrix center, although it can also off-centered or even outside it

NB: like convolution kernels, it is common to have structuring elements of odd dimensions with the center as the origin.

NB: the shape, size, and orientation of the structuring element depends on application



## 2.1. Basic concepts

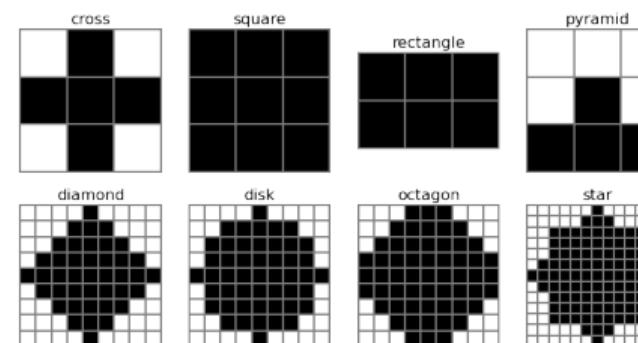
Morphological filtering mechanics are similar to spatial filtering using convolutions:

1) a kernel called a **structuring element** is used to determine filtering operation:

- the size is determined by the matrix dimensions
- the shape is determined by the pattern of 1 and 0 in the matrix
- the origin is usually the matrix center, although it can also off-centered or even outside it

NB: like convolution kernels, it is common to have structuring elements of odd dimensions with the center as the origin.

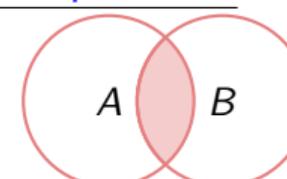
NB: the shape, size, and orientation of the structuring element depends on application



2) the image is first **padded**, and the structuring element than **slides** across it

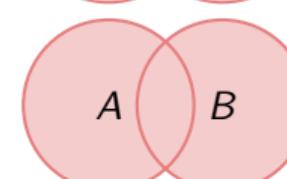
Morphological filters are essentially **set operations**

Intersection (AND)



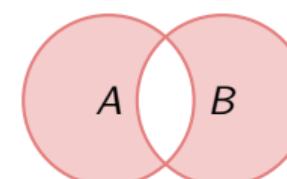
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Union (OR)



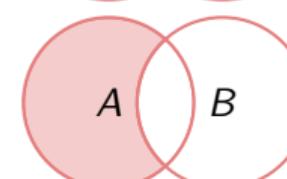
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Symmetric difference (XOR)



$$\overline{A \cap B}$$

Difference



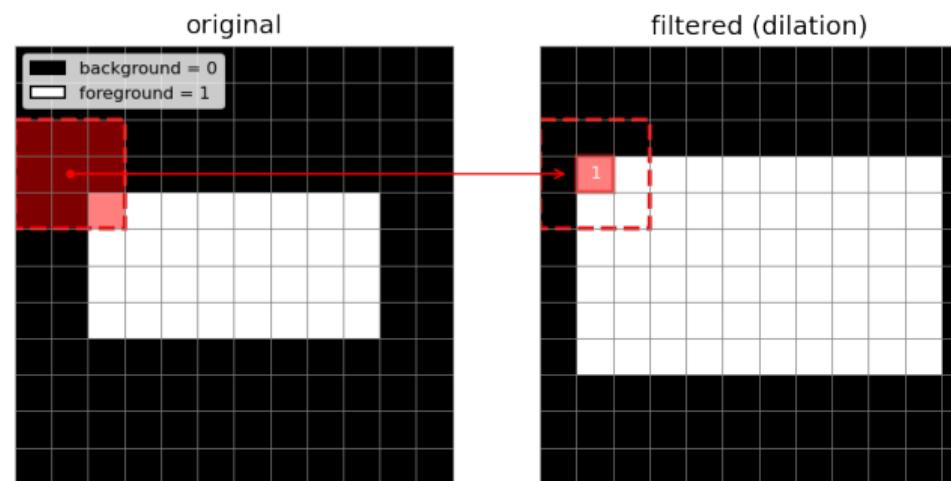
$$A - B$$

## 2.2. Primitive Morphological Operations

- Primary Morphological Operations are: dilation and erosion
- Concatenation of dilation and erosion result in higher level operations
  - ⇒ Composite Morphological Operations: closing and opening

1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

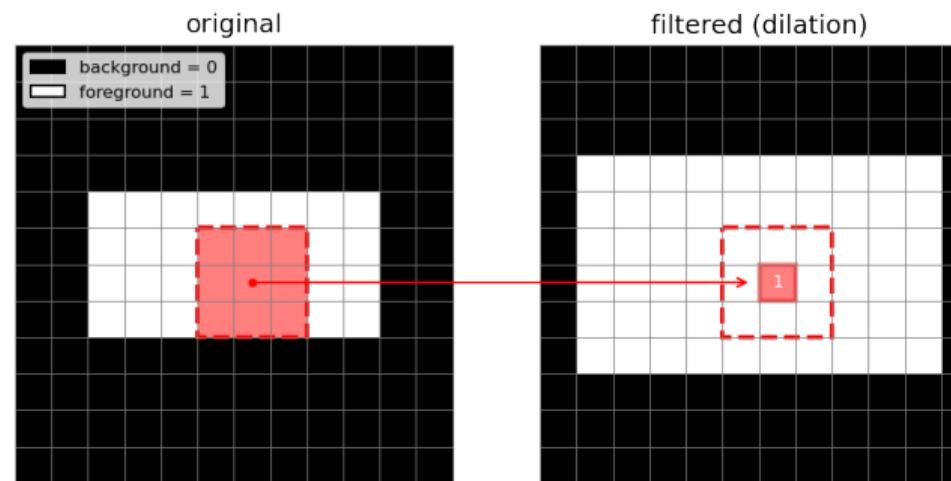
$$G = F \oplus b = \{x : (\hat{b})_x \cap F \neq \emptyset\}$$



if  $\geq 1$  pixel within the mask = "1", the result is "1", otherwise "0"

1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

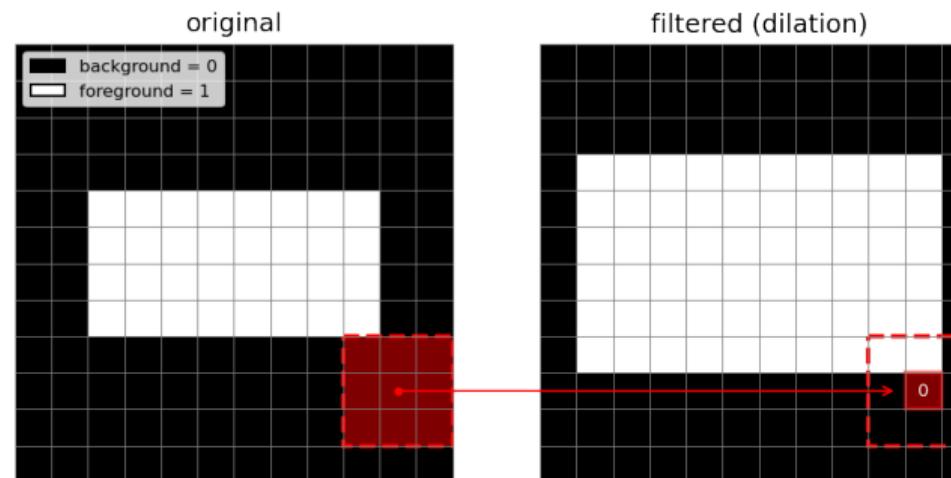
$$G = F \oplus b = \{x : (\hat{b})_x \cap F \neq \emptyset\}$$



if  $\geq 1$  pixel within the mask = "1", the result is "1", otherwise "0"

1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

$$G = F \oplus b = \{x : (\hat{b})_x \cap F \neq \emptyset\}$$



if  $\geq 1$  pixel within the mask = "1", the result is "1", otherwise "0"

1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get larger
- ⇒ Background objects get smaller
- ⇒ Small gaps are closed

original



■	background = 0
■	foreground = 1

dilation ( $b=3\times 3$ )



1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get larger
- ⇒ Background objects get smaller
- ⇒ Small gaps are closed

original



background = 0  
 foreground = 1

dilation ( $b=7 \times 7$ )



1. **Dilation**: the dilation of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get larger
- ⇒ Background objects get smaller
- ⇒ Small gaps are closed

original



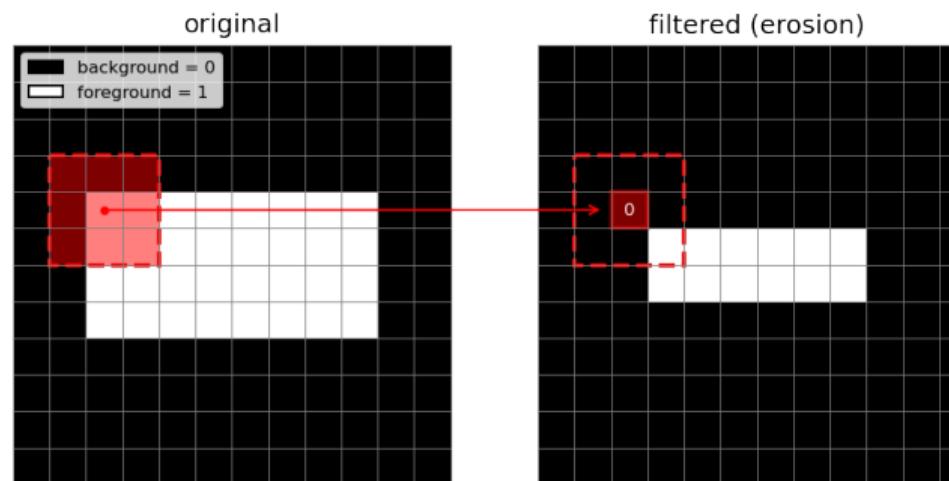
background = 0  
 foreground = 1

dilation ( $b=11\times 11$ )



2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

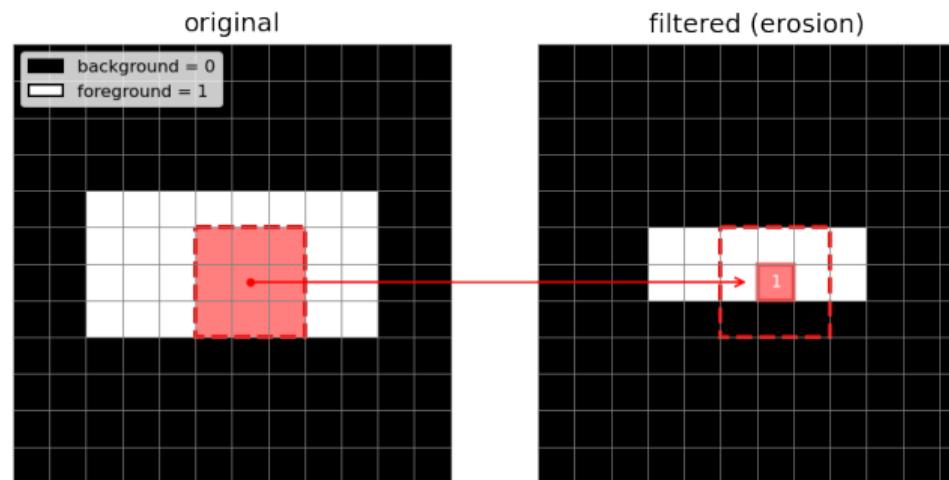
$$G = F \ominus b = \{x : (b)_x \subseteq F\}$$



if all pixel within the mask = "1" => the result is "1", otherwise "0"

2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

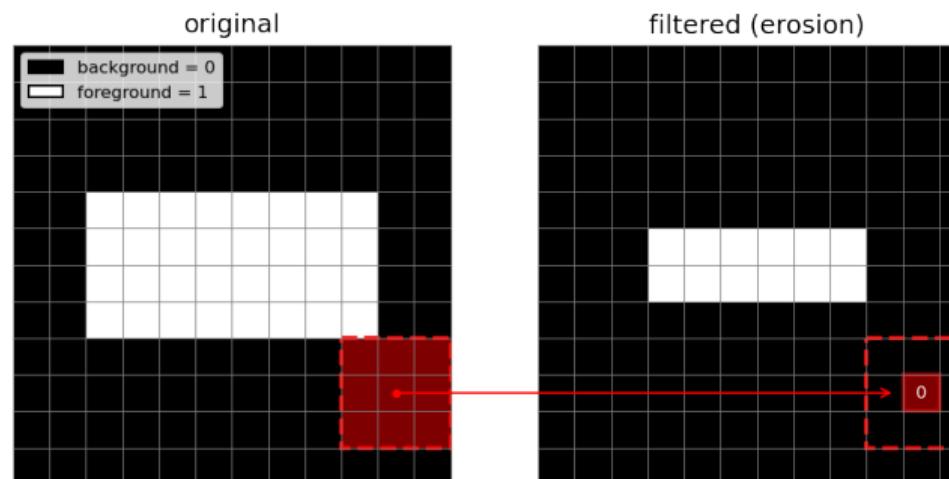
$$G = F \ominus b = \{x : (b)_x \subseteq F\}$$



if all pixel within the mask = "1" => the result is "1", otherwise "0"

2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

$$G = F \ominus b = \{x : (b)_x \subseteq F\}$$



if all pixel within the mask = "1" => the result is "1", otherwise "0"

2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get smaller, small objects disappear
- ⇒ Background objects get larger
- ⇒ Objects get separated

original



erosion ( $b=3 \times 3$ )



■	background = 0
■	foreground = 1

2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get smaller, small objects disappear
- ⇒ Background objects get larger
- ⇒ Objects get separated

original



erosion ( $b=7 \times 7$ )



■	background = 0
■	foreground = 1

2. **Erosion**: the erosion of a set  $F$  with a structuring element  $b$  is defined as:

- ⇒ Foreground objects get smaller, small objects disappear
- ⇒ Background objects get larger
- ⇒ Objects get separated

original



erosion ( $b=11 \times 11$ )



■	background = 0
■	foreground = 1

Concatenation of **dilation** and **erosion** result in higher level operations: **closing**, **opening**

### 1. Opening:

Problem: erosion causes deletion of small objects, BUT other objects shrink

Solution: after *erosion*, apply *dilation* with the same structuring element  $\Rightarrow$  **opening**

$$G = F \circ b = (F \ominus b) \oplus b$$

Usage example: removing small isolated “bright spots” (EX: volcanic SO<sub>2</sub> detection from Sentinel-5P as foreground (mask=1))

Concatenation of dilation and erosion result in higher level operations: closing, opening

## 1. Opening:

Problem: erosion causes deletion of small objects, BUT other objects shrink

Solution: after *erosion*, apply *dilation* with the same structuring element  $\Rightarrow$  opening

$$G = F \circ b = (F \ominus b) \oplus b$$

Usage example: removing small isolated “bright spots” (EX: volcanic SO<sub>2</sub> detection from Sentinel-5P as foreground (mask=1))

Concatenation of dilation and erosion result in higher level operations: closing, opening

## 1. Opening:

Problem: erosion causes deletion of small objects, BUT other objects shrink

Solution: after *erosion*, apply *dilation* with the same structuring element  $\Rightarrow$  opening

$$G = F \circ b = (F \ominus b) \oplus b$$

Usage example: removing small isolated “bright spots” (EX: volcanic SO<sub>2</sub> detection from Sentinel-5P as foreground (mask=1))

Concatenation of dilation and erosion result in higher level operations: **closing**, **opening**

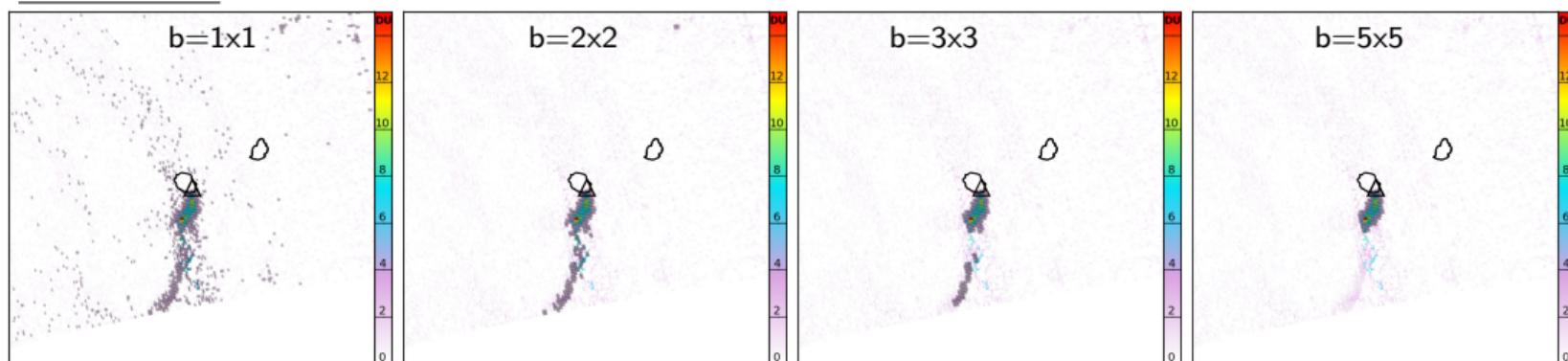
## 1. **Opening:**

Problem: erosion causes deletion of small objects, BUT other objects shrink

Solution: after *erosion*, apply *dilation* with the same structuring element  $\Rightarrow$  **opening**

$$G = F \circ b = (F \ominus b) \oplus b$$

Usage example: removing small isolated “bright spots” (EX: volcanic SO<sub>2</sub> detection from Sentinel-5P as foreground (mask=1))



Concatenation of dilation and erosion result in higher level operations: closing, opening

## 2. Closing:

Problem: dilation closes small holes and fractions, BUT objects get enlarged

Solution: after *dilation*, apply *erosion* with the same structuring element ⇒ closing

$$G = F \bullet b = (F \oplus b) \ominus b$$

Usage example: removing small isolated “dark spots” (binary mask value = 0)

Concatenation of dilation and erosion result in higher level operations: closing, opening

## 2. Closing:

Problem: dilation closes small holes and fractions, BUT objects get enlarged

Solution: after *dilation*, apply *erosion* with the same structuring element ⇒ closing

$$G = F \bullet b = (F \oplus b) \ominus b$$

Usage example: removing small isolated “dark spots” (binary mask value = 0)

Concatenation of dilation and erosion result in higher level operations: closing, opening

## 2. Closing:

Problem: dilation closes small holes and fractions, BUT objects get enlarged

Solution: after *dilation*, apply *erosion* with the same structuring element ⇒ closing

$$G = F \bullet b = (F \oplus b) \ominus b$$

Usage example: removing small isolated “dark spots” (binary mask value = 0)

Concatenation of dilation and erosion result in higher level operations: **closing**, **opening**

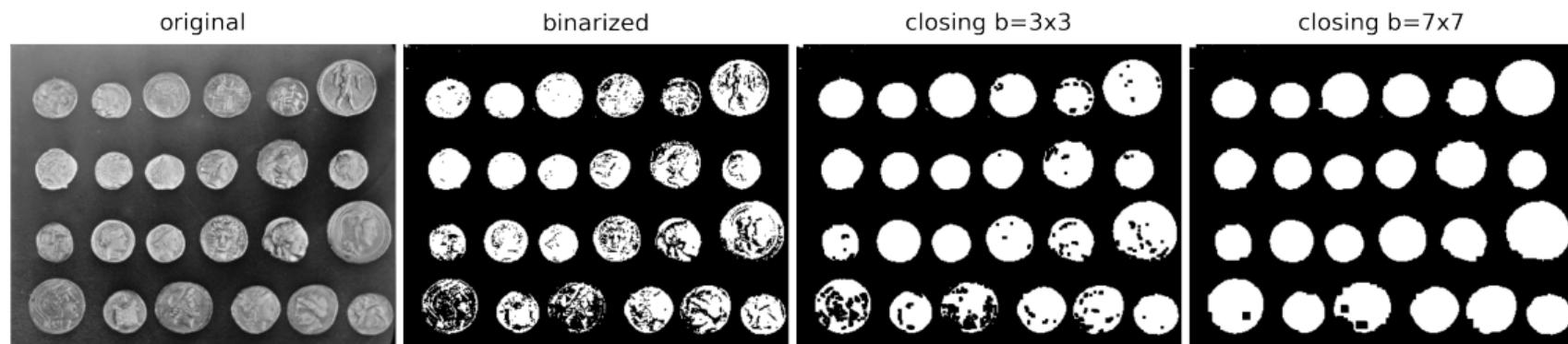
## 2. **Closing:**

Problem: dilation closes small holes and fractions, BUT objects get enlarged

Solution: after *dilation*, apply *erosion* with the same structuring element ⇒ **closing**

$$G = F \bullet b = (F \oplus b) \ominus b$$

Usage example: removing small isolated “dark spots” (binary mask value = 0)



1. Introduction

2. Mathematical Morphology

### 3. Image Segmentation

1. histogram-based segmentation
2. edge-based segmentation
3. region-based segmentation

4. Analyze segmented image

## 3.0. Composite Morphological Operations

**Image segmentation** = labeling image pixels to partition an image into regions

- Histogram-based segmentation  
⇒ based on thresholding of pixel values
  - ex: manual thresholding
  - ex: automatic thresholding (e.g., Otsu)
  - ex: k-means clustering
- Edge-based segmentation  
⇒ based on local contrast → uses gradients rather than the grey values
- Region-based segmentation  
⇒ based on image gradients and region properties
  - ex: Watershed transform
  - ex: Random Walker
  - ex: Flood Fill
- Many other!  
ex: Graph-cuts  
ex: Active Contours, Region Growing, Weighted Pyramid Linking, Mean-Shift, etc.

## 3.0. Composite Morphological Operations

**Image segmentation** = labeling image pixels to partition an image into regions

- Histogram-based segmentation

⇒ based on thresholding of pixel values

ex: manual thresholding

ex: automatic thresholding (e.g., Otsu)

ex: k-means clustering

- Edge-based segmentation

⇒ based on local contrast → uses gradients rather than the grey values

- Region-based segmentation

⇒ based on image gradients and region properties

ex: Watershed transform

ex: Random Walker

ex: Flood Fill

- Many other!

ex: Graph-cuts

ex: Active Contours, Region Growing, Weighted Pyramid Linking, Mean-Shift, etc.

## 3.0. Composite Morphological Operations

**Image segmentation** = labeling image pixels to partition an image into regions

- Histogram-based segmentation

⇒ based on thresholding of pixel values

ex: manual thresholding

ex: automatic thresholding (e.g., Otsu)

ex: k-means clustering

- Edge-based segmentation

⇒ based on local contrast → uses gradients rather than the grey values

- Region-based segmentation

⇒ based on image gradients and region properties

ex: Watershed transform

ex: Random Walker

ex: Flood Fill

- Many other!

ex: Graph-cuts

ex: Active Contours, Region Growing, Weighted Pyramid Linking, Mean-Shift, etc.

## 3.0. Composite Morphological Operations

**Image segmentation** = labeling image pixels to partition an image into regions

- Histogram-based segmentation

⇒ based on thresholding of pixel values

ex: manual thresholding

ex: automatic thresholding (e.g., Otsu)

ex: k-means clustering

- Edge-based segmentation

⇒ based on local contrast → uses gradients rather than the grey values

- Region-based segmentation

⇒ based on image gradients and region properties

ex: Watershed transform

ex: Random Walker

ex: Flood Fill

- Many other!

ex: Graph-cuts

ex: Active Contours, Region Growing, Weighted Pyramid Linking, Mean-Shift, etc.

## 3.1. histogram-based segmentation

**Histogram-based segmentation**

⇒ based on thresholding pixel values

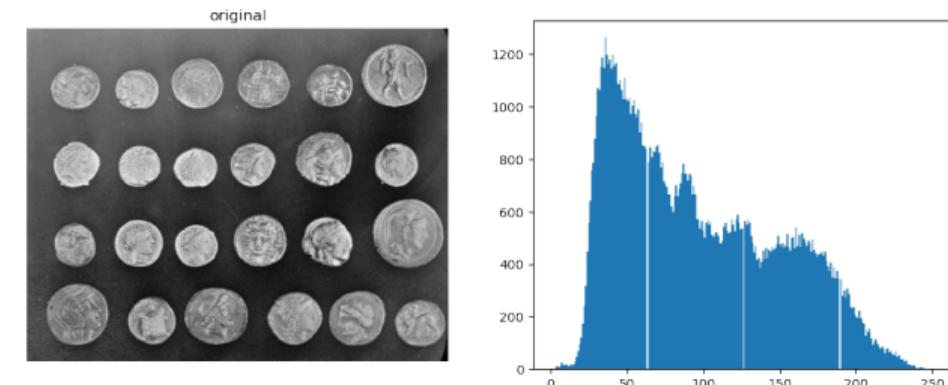
## 3.1. histogram-based segmentation

## Histogram-based segmentation

⇒ based on thresholding pixel values

- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))

(threshold calculated to separate pixels into two classes,  
minimizing intra-class intensity variance)

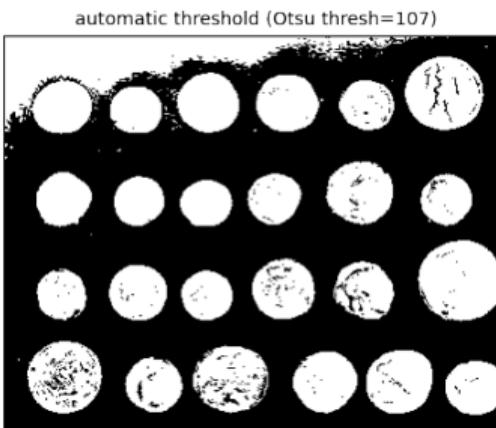
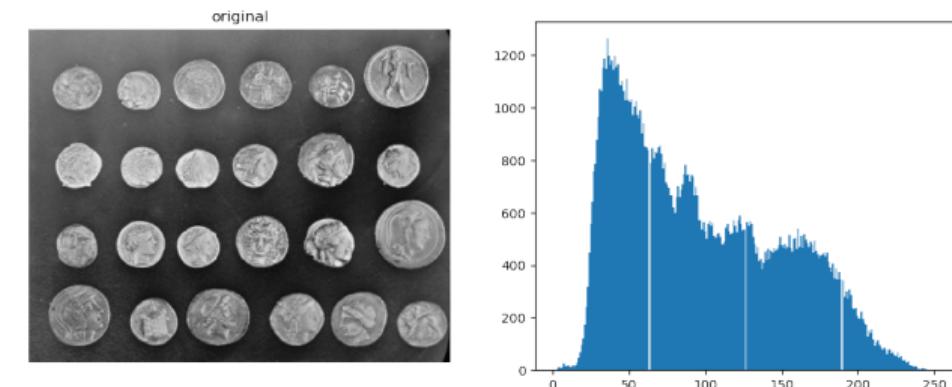


## 3.1. histogram-based segmentation

## Histogram-based segmentation

⇒ based on thresholding pixel values

- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)

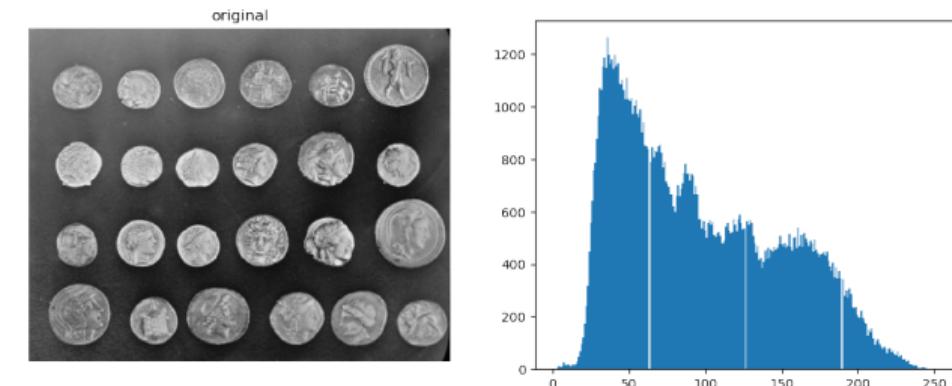


## 3.1. histogram-based segmentation

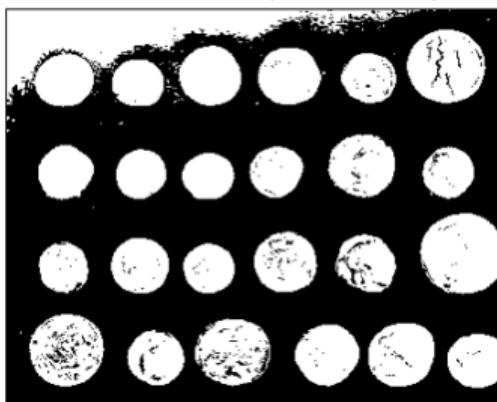
## Histogram-based segmentation

⇒ based on thresholding pixel values

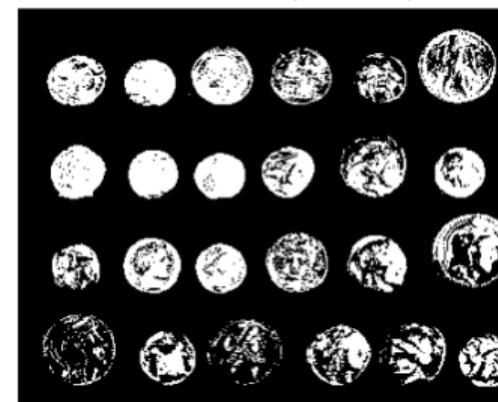
- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)



automatic threshold (Otsu thresh=107)



manual threshold (thresh=150)

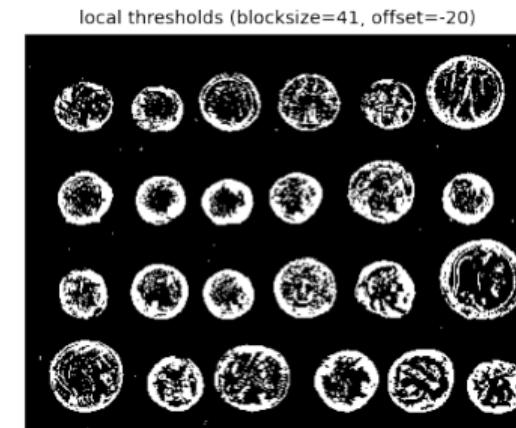
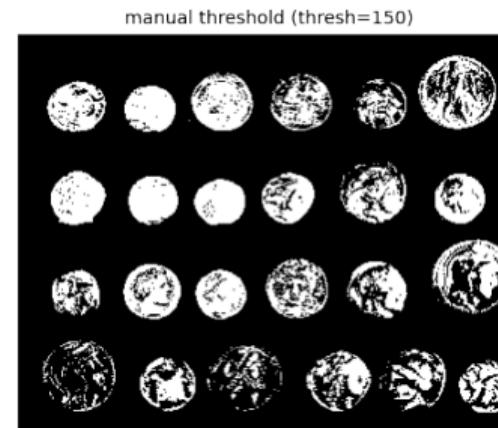
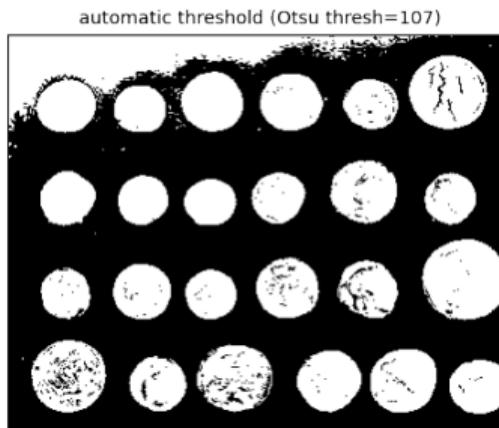
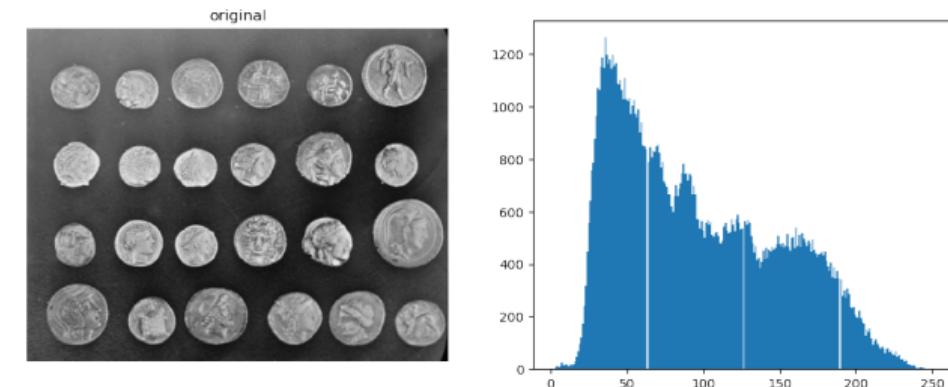


## 3.1. histogram-based segmentation

## Histogram-based segmentation

⇒ based on thresholding pixel values

- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)
- local thresholding (adaptive)  
(thresholds calculated based on pixel local neighborhood)



## 3.1. histogram-based segmentation

## Histogram-based segmentation

⇒ based on thresholding pixel values

- global thresholding
  - manual
  - automatic (e.g. [Otsu's method](#))  
(threshold calculated to separate pixels into two classes, minimizing intra-class intensity variance)
- local thresholding (adaptive)  
(thresholds calculated based on pixel local neighborhood)

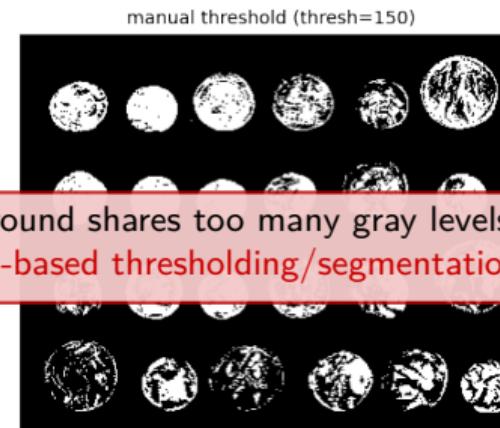
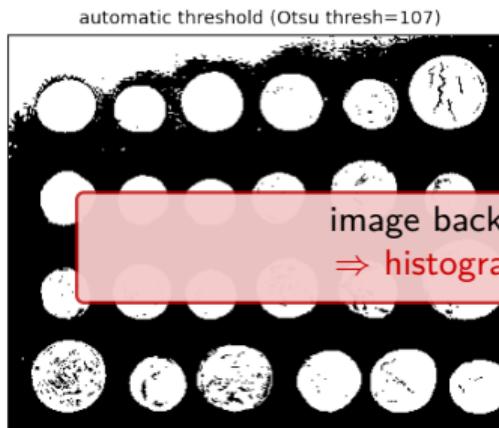
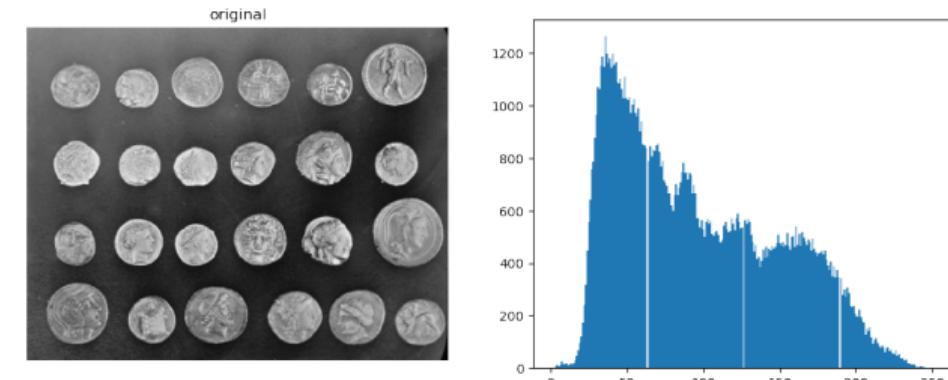
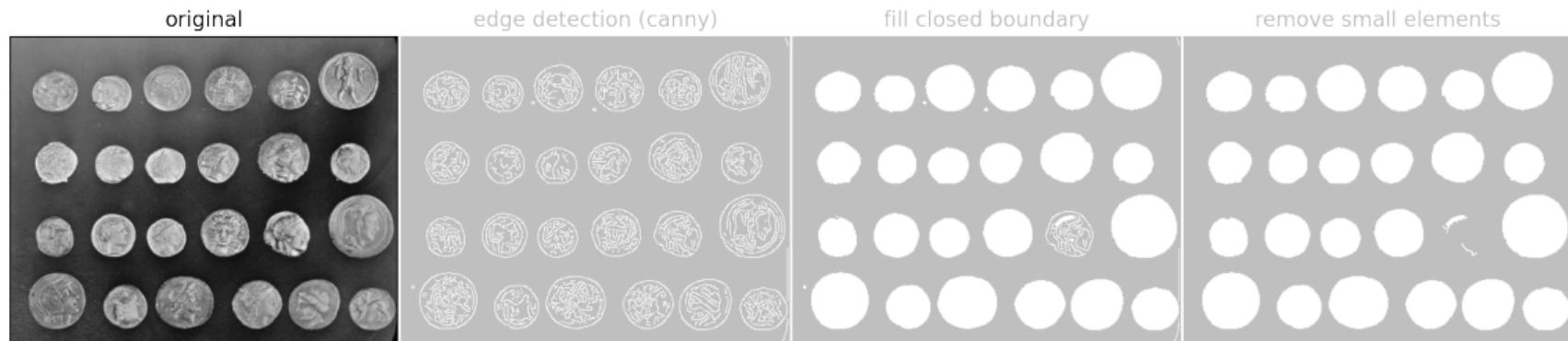


image background shares too many gray levels with the coins  
⇒ histogram-based thresholding/segmentation is insufficient

## 3.2. edge-based segmentation

## Edge-based segmentation

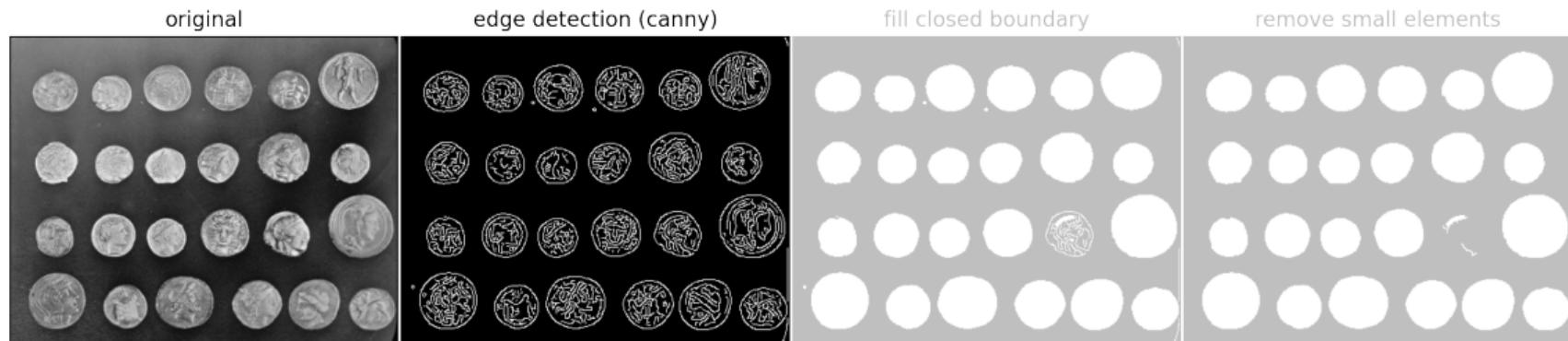
⇒ based on image gradients



## 3.2. edge-based segmentation

## Edge-based segmentation

⇒ based on image gradients

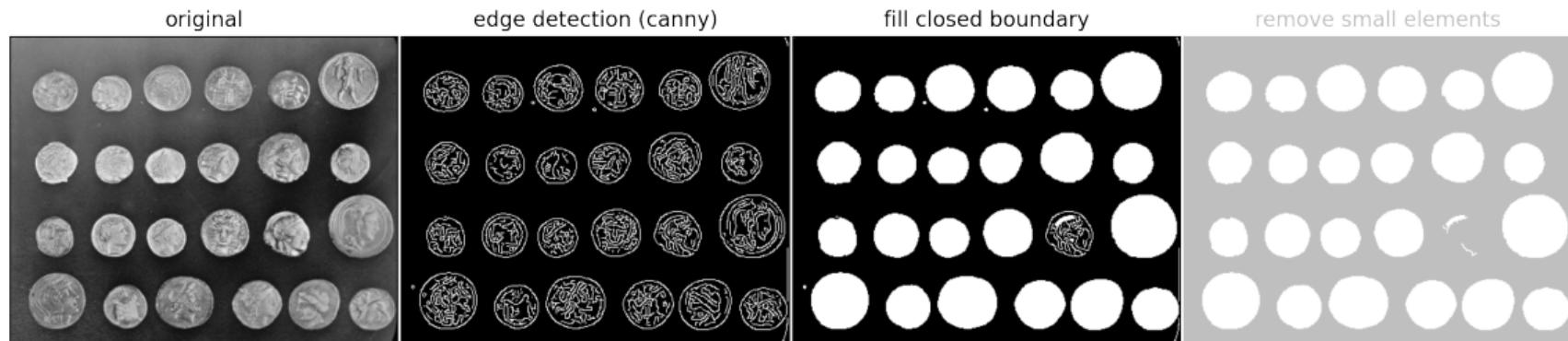


1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)

## 3.2. edge-based segmentation

## Edge-based segmentation

⇒ based on image gradients

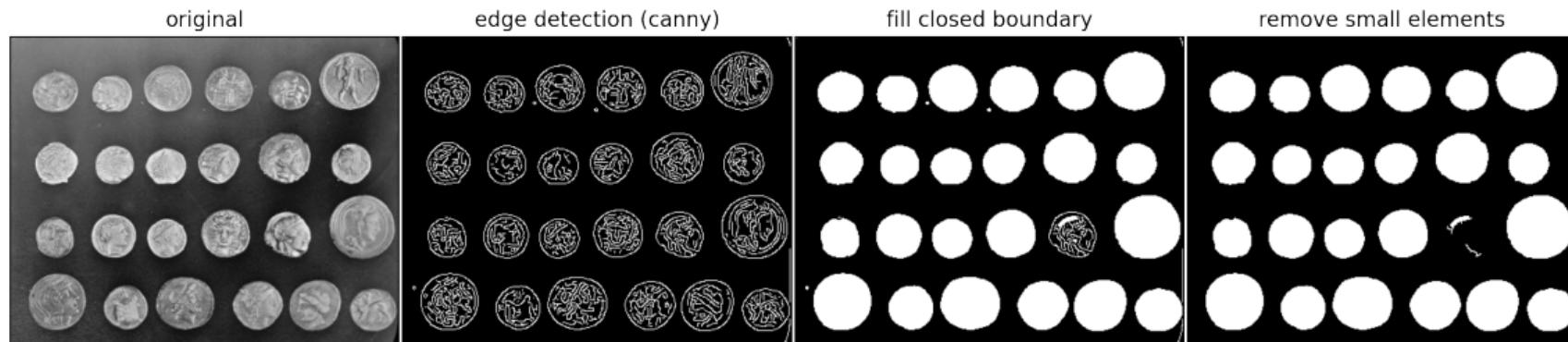


1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins

## 3.2. edge-based segmentation

## Edge-based segmentation

⇒ based on image gradients

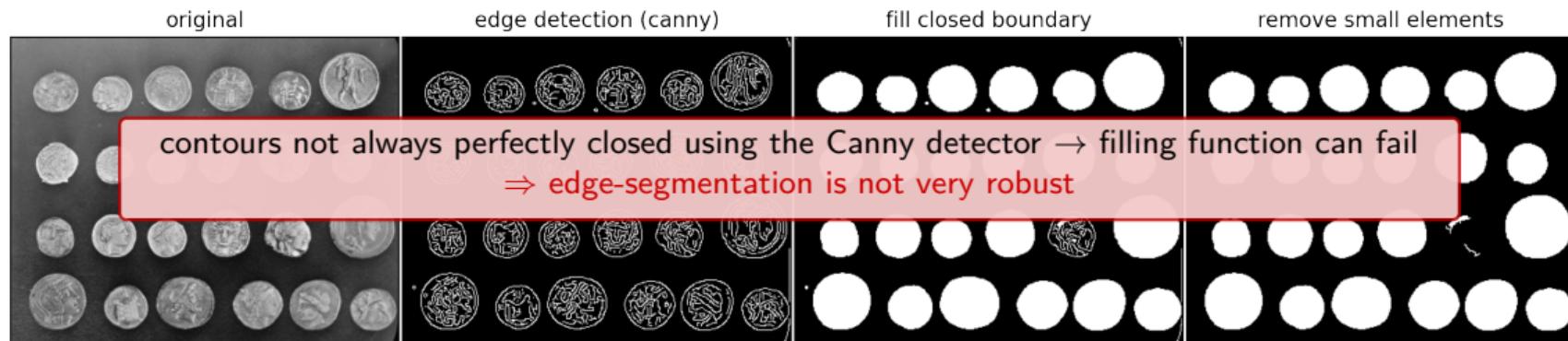


1. apply [Canny](#) edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins
3. remove objects smaller than a threshold

## 3.2. edge-based segmentation

## Edge-based segmentation

⇒ based on image gradients



1. apply **Canny** edge detection algorithm (involves gradient detection using e.g. Sobel operator)
2. apply mathematical morphology to fill inner part of the coins
3. remove objects smaller than a threshold

## 3.3. region-based segmentation

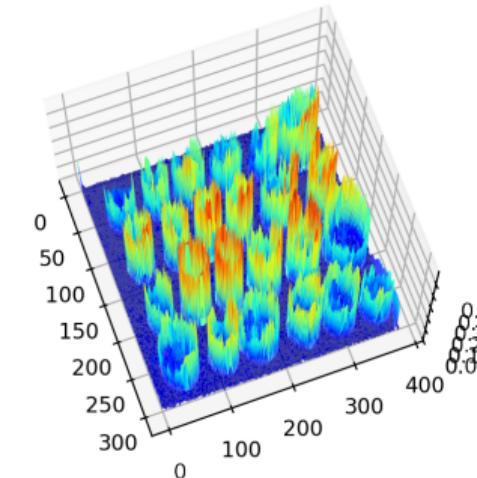
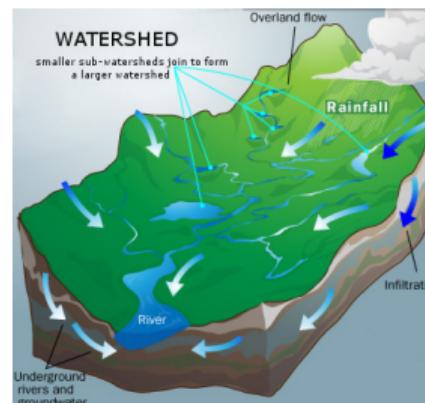
## Region-based segmentation: *watershed transform*

⇒ region-growing approach that fills “basins” in the image

## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*

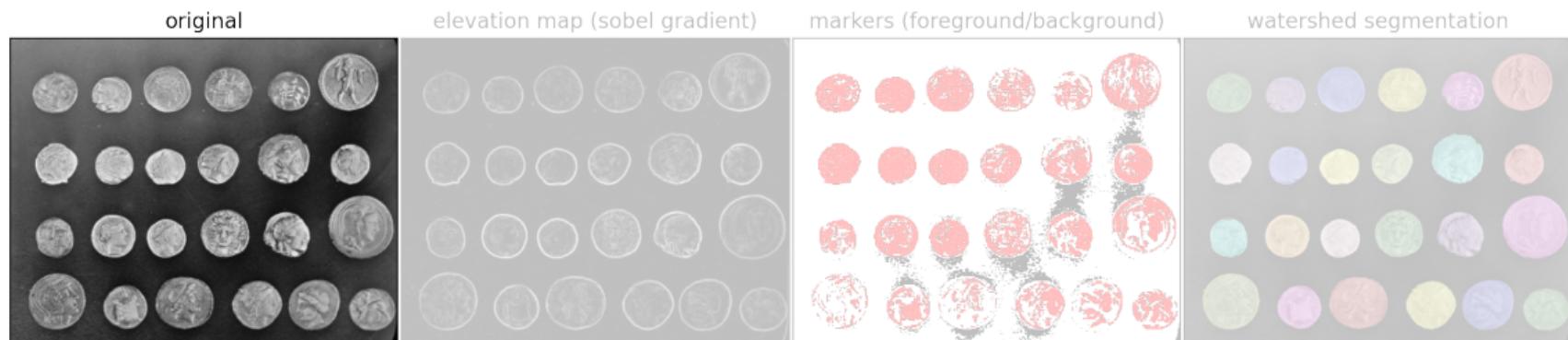
- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers



## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*

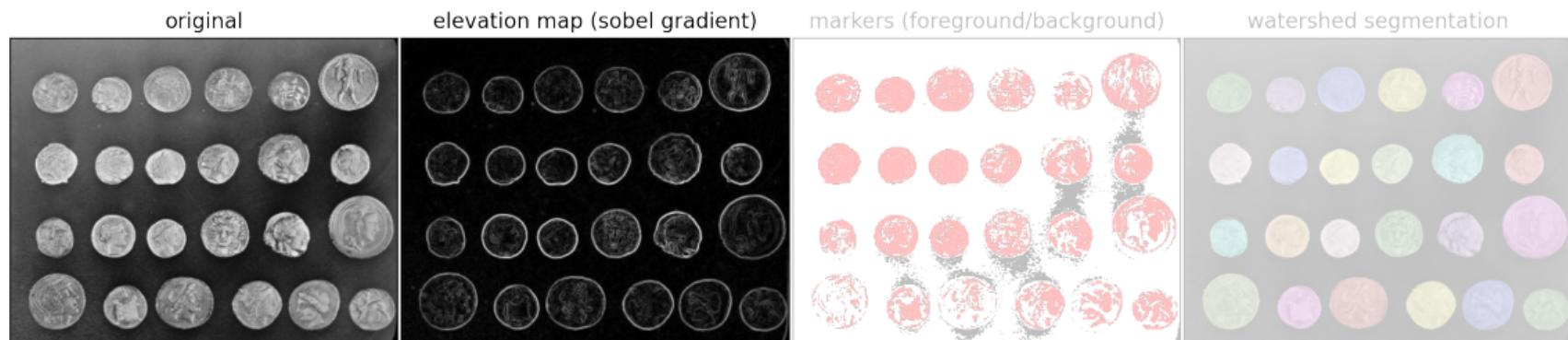
- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers



## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*

- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers

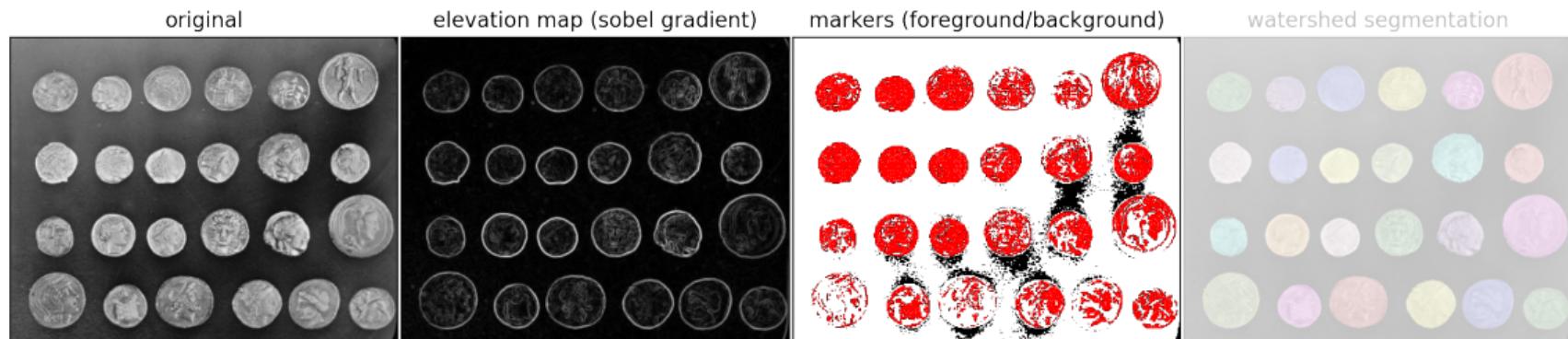


1. build “elevation map” from image gradient amplitude (using the Sobel operator)

## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*

- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers

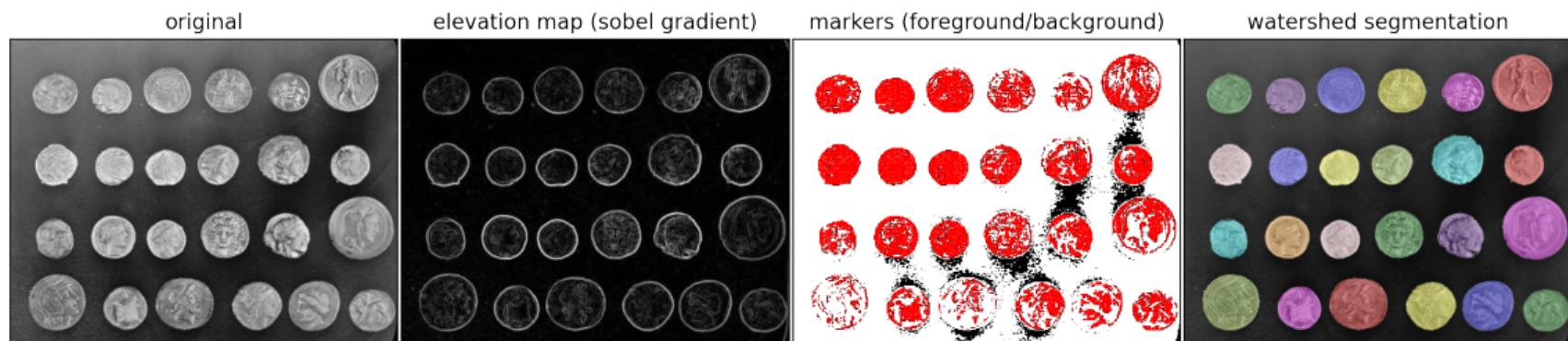


1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)

## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*

- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers

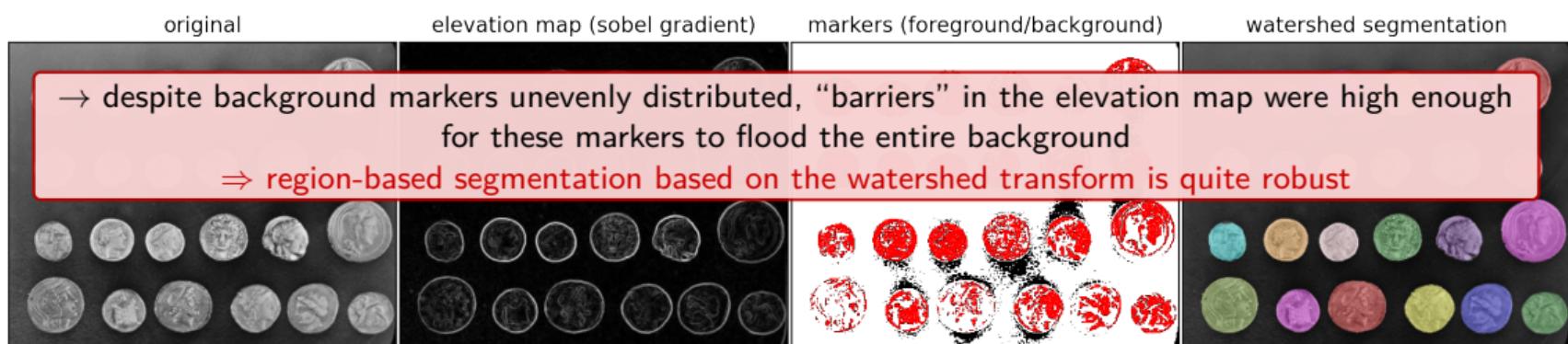


1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)
3. apply **watershed transform** (and colorize segmented elements)

## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*

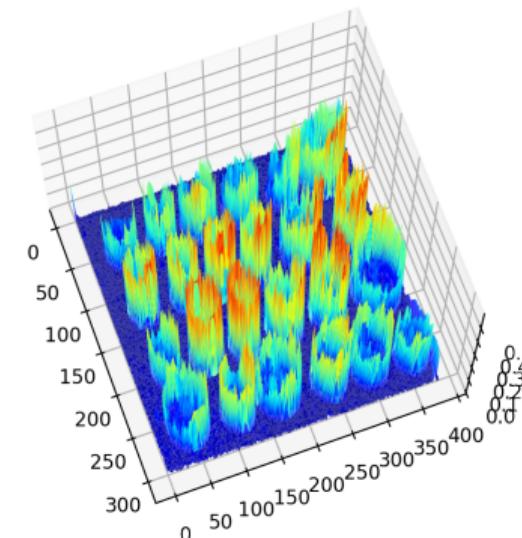
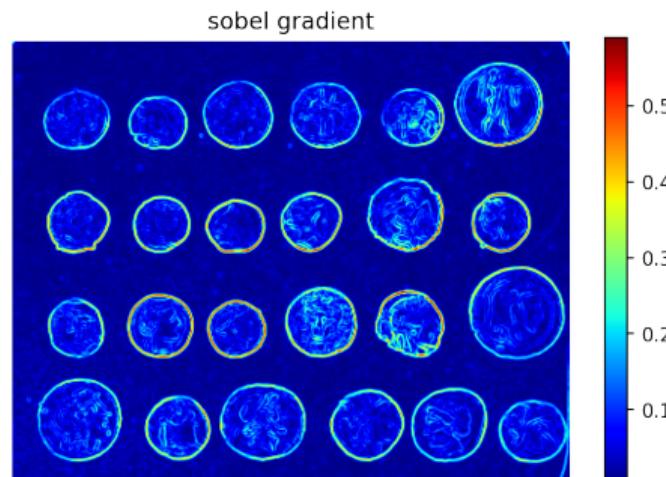
- ⇒ region-growing approach that fills “basins” in the image
- ⇒ the name “watershed” comes from an analogy with hydrology:
  - the *watershed transform* “floods” a “topographic” representation of the image
  - flooding starts from “markers”, in order to determine the catchment basins of these markers



1. build “elevation map” from image gradient amplitude (using the Sobel operator)
2. define markers for background / foreground (here based on the extreme parts of the histogram)
3. apply **watershed transform** (and colorize segmented elements)

## 3.3. region-based segmentation

## Region-based segmentation: *watershed transform*



1. Start with lowest “altitude” (Gradient amplitude)
2. Increase the “water level” each time by 1
3. Merge all connected pixel with same/less level

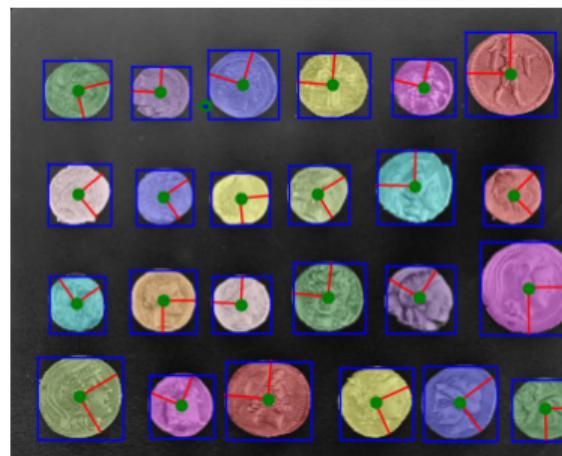
1. Introduction
2. Mathematical Morphology
3. Image Segmentation
4. Analyze segmented image

## Analyze segmented image

The segmented elements can be analysed individually to:

→ provide statistics on their shape, distribution, orientation, etc.

(e.g. fields in a satellite image, crystal/bubble shape distribution in a rock sample, etc.)

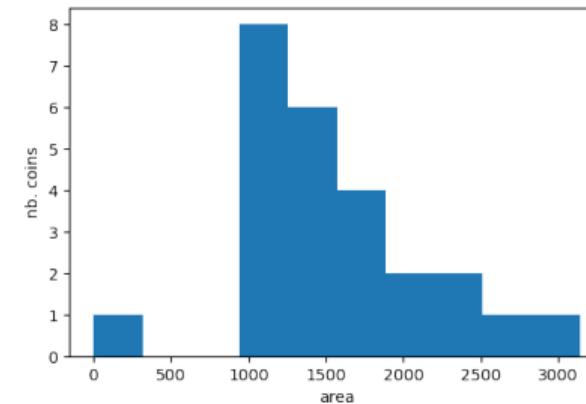
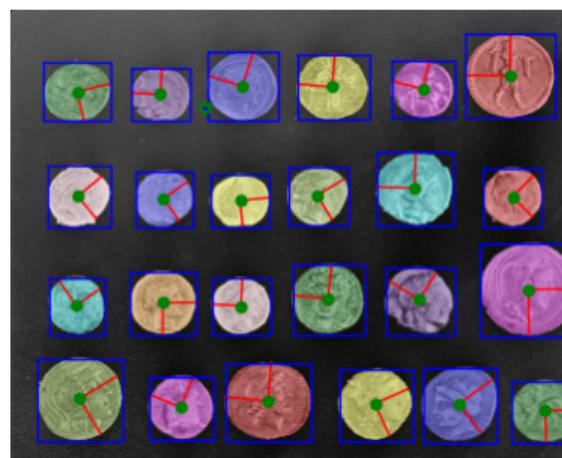


## Analyze segmented image

The segmented elements can be analysed individually to:

→ provide statistics on their shape, distribution, orientation, etc.

(e.g. fields in a satellite image, crystal/bubble shape distribution in a rock sample, etc.)



## Analyze segmented image

### Exercises:

#### 1. Exercise 1:

⇒ histogram-based segmentation of Popocatépetl

#### 2. Exercise 2:

⇒ analyze a thermal infrared image of a lava lake

→ segment the crustal plates from the incandescent cracks and analyze