

## Lecture 02

# Digital Image Basics

2024-02-01

Sébastien Valade



UNIVERSIDAD NACIONAL  
AUTÓNOMA DE  
MÉXICO

## 1. What is a digital image?

1. image acquisition
2. sampling and quantization
3. 3D projection on 2D plane
4. color image
5. color spaces
6. image histogram

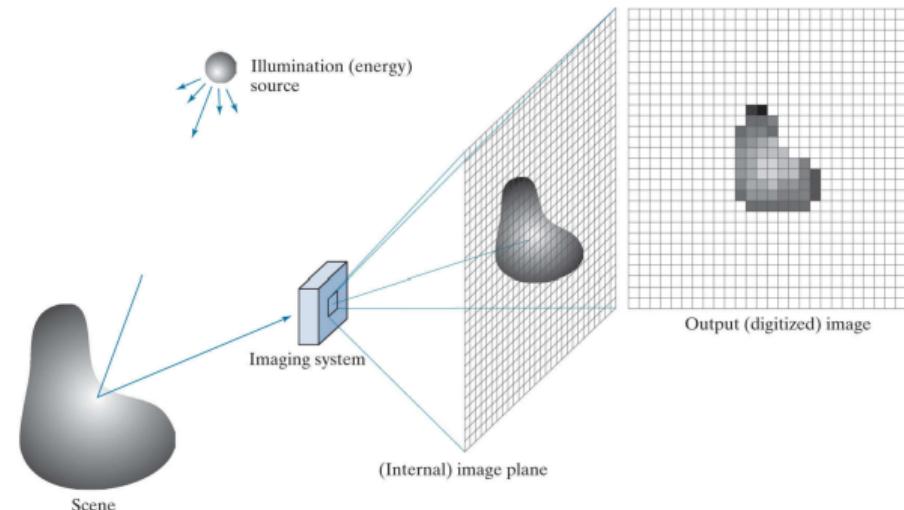
2. Point operations

3. Image processing levels

4. Image manipulation with Python

## 1.1. image acquisition

1. energy from an **illumination source** is reflected from a **scene**
2. the **imaging system** collects the incoming energy and focuses it onto an **image plane**  
NB: light-sensing instruments typically use 2-D arrays of photosensors to record incoming light  
intensity  $I(x)$ : the CCD (*Charge-Coupled Device*)
3. the image plane is sampled and quantized to produce a **digital image**



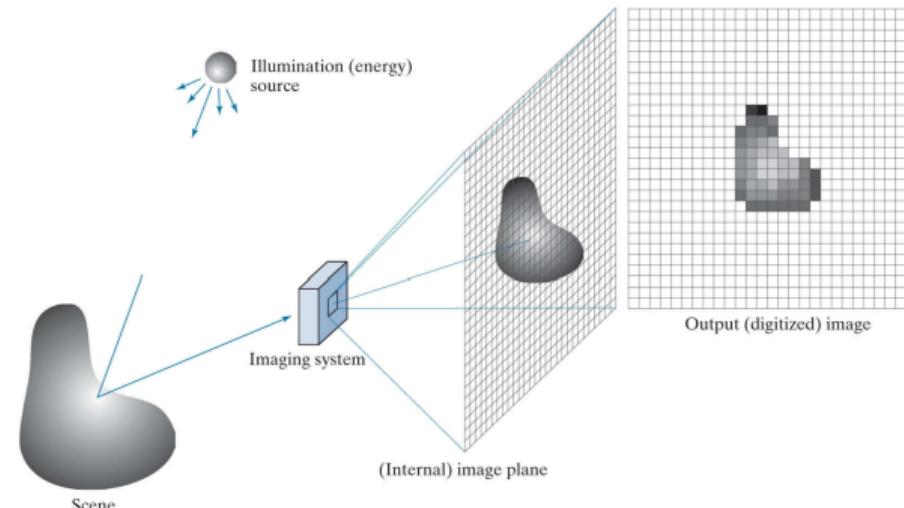
Credit: Gonzalez & Woods 2018

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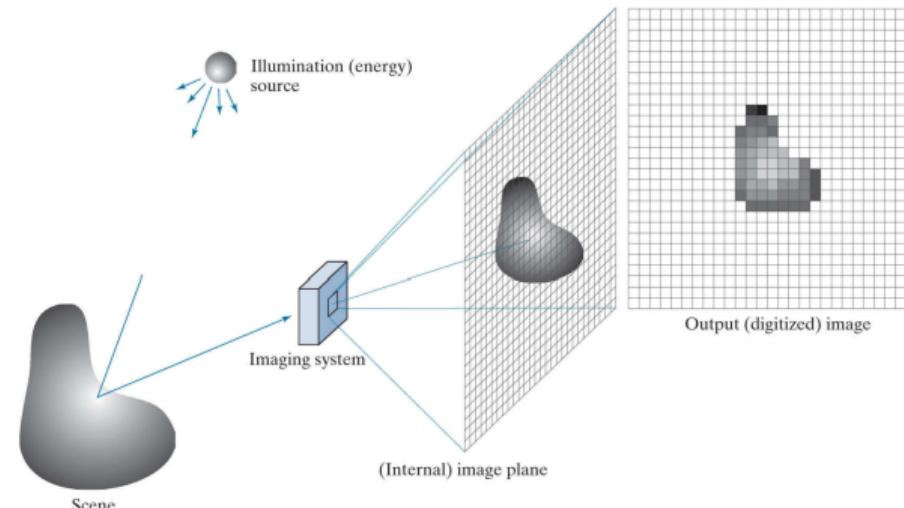
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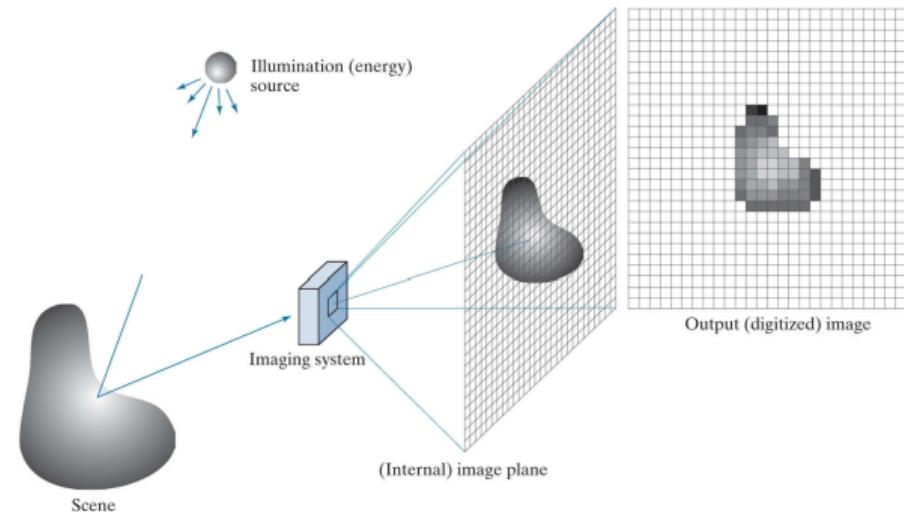
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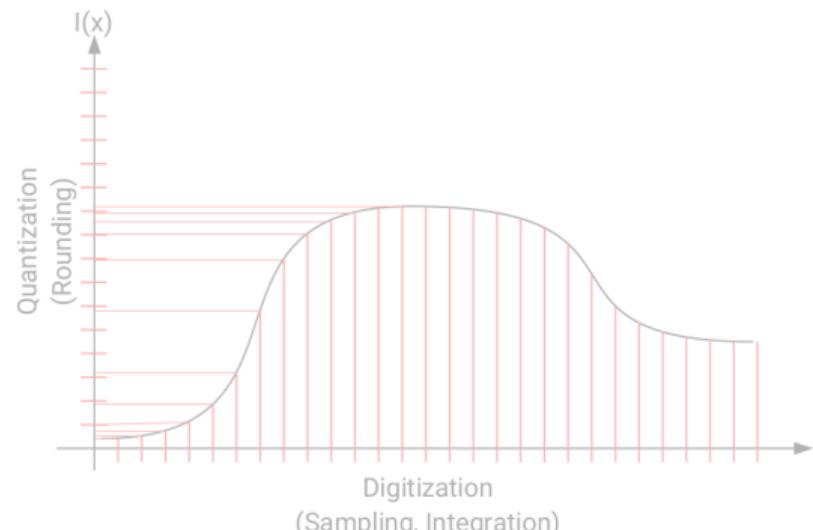
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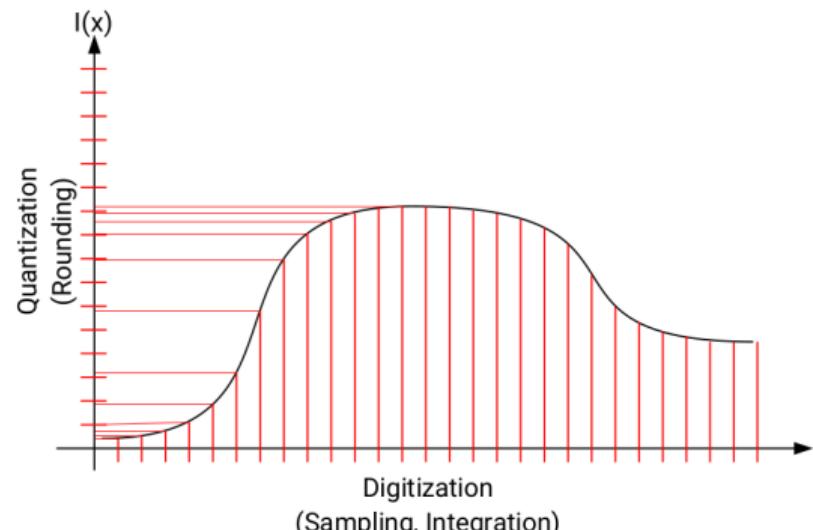
## 1.2. sampling and quantization

- each photosensor records incident light
- digitalization of an analog signal involves two operations
  - spatial sampling (= discretization of space domain)
  - intensity quantization (= discretization of incoming light signal)



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  - **intensity quantization** (= discretization of incoming light signal)



## 1.2. sampling and quantization

**spatial sampling** (= discretization of space domain)

⇒ smallest element resulting from the discretization of the space is called a pixel (=picture element)

(512, 512)



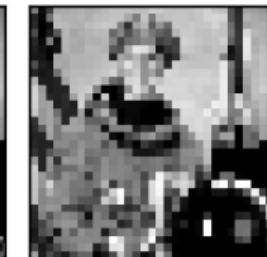
(128, 128)



(64, 64)



(32, 32)

**intensity quantization** (= discretization of light intensity signal)

⇒ typically, 256 levels (8 bits/pixel =  $2^8$  values) suffices to represent the intensity

8-bit resolution

$$2^8 = 256 \text{ gray levels}$$



3-bit resolution

$$2^3 = 8 \text{ gray levels}$$



2-bit resolution

$$2^2 = 4 \text{ gray levels}$$



1-bit resolution

$$2^1 = 2 \text{ gray levels}$$



## 1.2. sampling and quantization

**spatial sampling** (= discretization of space domain)

⇒ smallest element resulting from the discretization of the space is called a pixel (=picture element)

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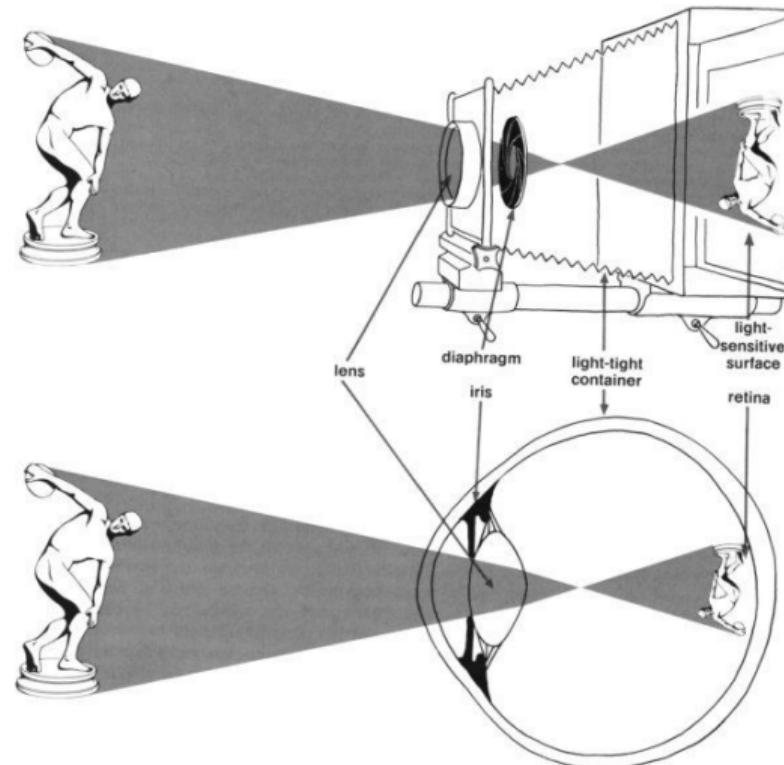
8-bit resolution $2^8 = 256$ gray levels	3-bit resolution $2^3 = 8$ gray levels	2-bit resolution $2^2 = 4$ gray levels	1-bit resolution $2^1 = 2$ gray levels
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## 1.3. 3D projection on 2D plane

But how is the 3D world projected on a 2D plane?

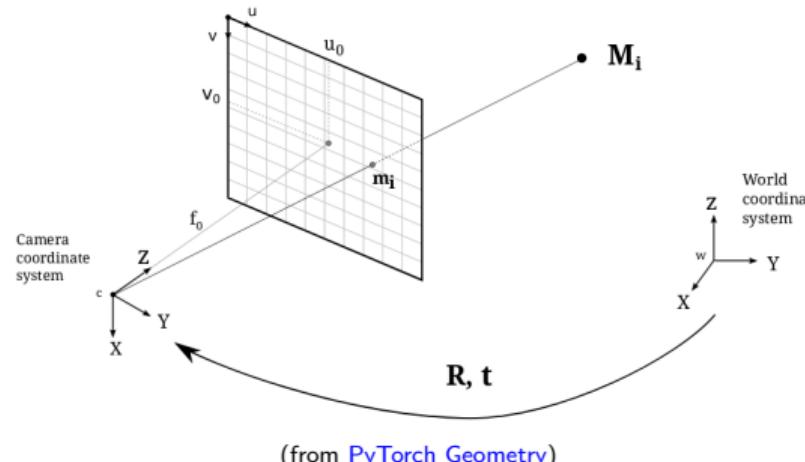
⇒ comparison between human eye and pinhole camera:



## 1.3. 3D projection on 2D plane

Image = 3D world projection on 2D

⇒ projection using the **pinhole camera** model:



Perspective transformation:

$$s \ m' = K[R|t]M' \quad (1)$$

$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (2)$$

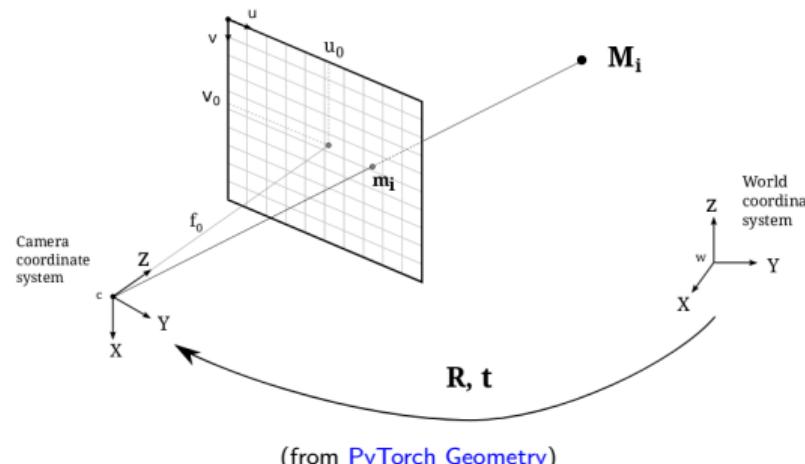
where:

- $M'$  = 3D point in space with coordinates  $[X, Y, Z]^T$  expressed in Euclidean coordinates
- $m'$  = projection of the 3D point  $M'$  onto the image plane with coordinates  $[u, v]^T$  expressed in pixel units
- $K$  = camera calibration matrix (a.k.a intrinsics parameters matrix)
  - $f_x, f_y$  = focal lengths expressed in pixel units
  - $u_0, v_0$  = coordinates of the optical center (aka principal point), origin in the image plane
- $[R|t]$  = joint rotation-translation matrix (a.k.a. extrinsics parameters matrix), describing the camera pose, and translating from world coordinates to camera coordinates

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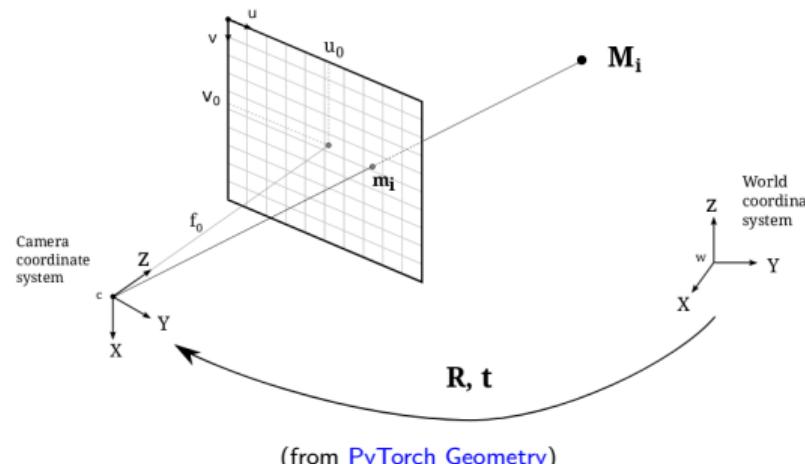
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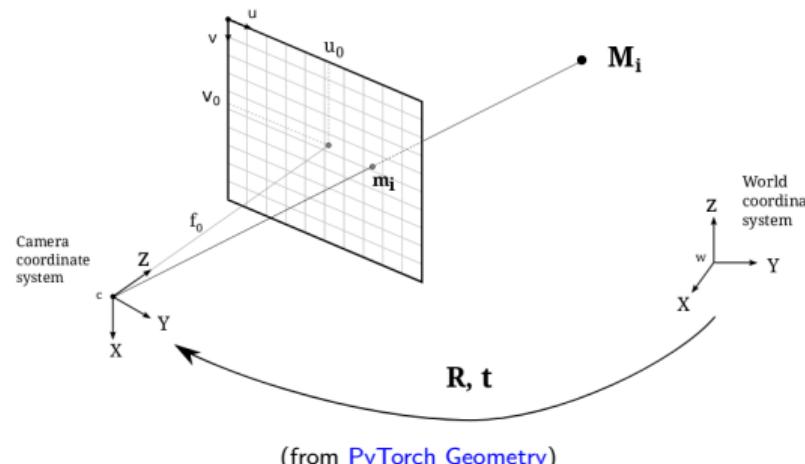
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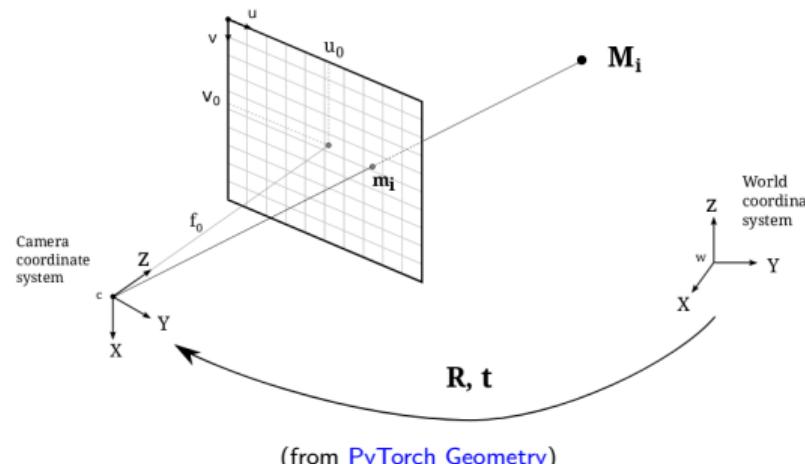
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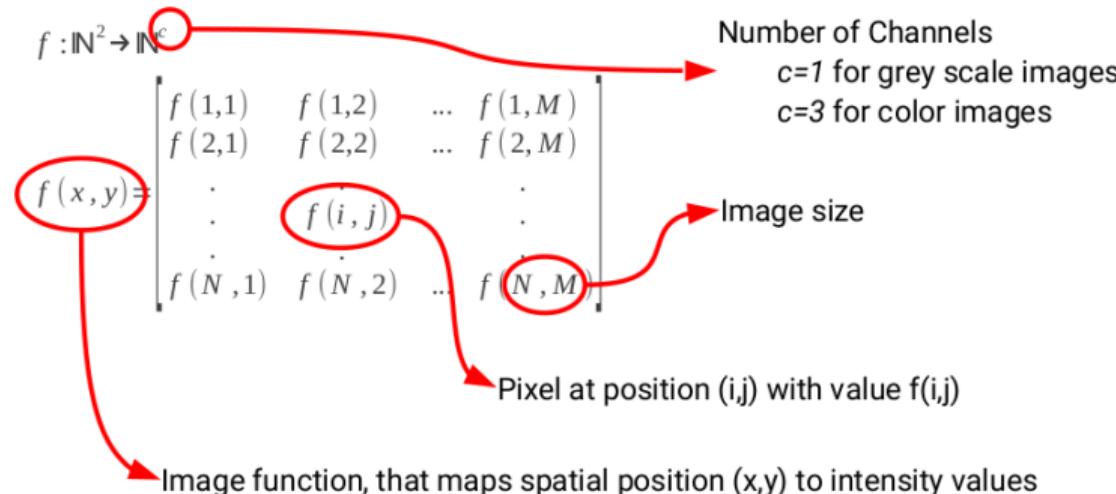
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⇒ digital image function  $f(x, y)$



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⇒ digital image function  $f(x, y)$

		columns									
		0	1	2	3	4	5	6	7	8	9
rows	0	24	67	103	87	79	176	138	94	180	
	1	98	53	66	226	14	34	241	240	24	143
	2	228	107	60	58	144	251	137	93	86	130
	3	155	108	132	159	129	141	245	211	100	11
	4	91	187	67	135	49	175	193	61	14	183
	5	199	251	80	2	121	105	222	147	226	63
	6	181	27	56	238	113	158	176	47	167	109
	7	38	172	18	192	184	162	181	202	17	72
	8	13	106	30	17	53	68	178	232	91	219
	9	211	181	78	11	185	204	106	131	70	

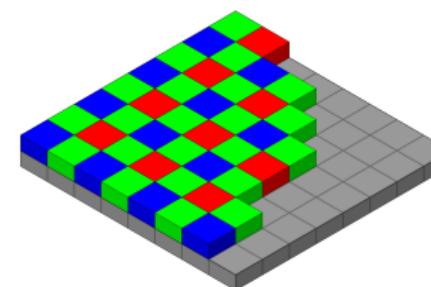
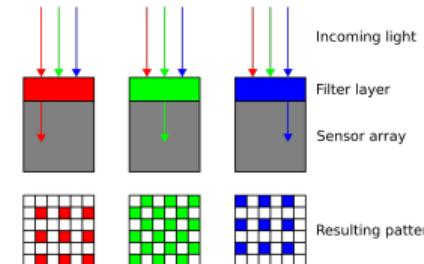
Typical ranges:

- $\text{uint8} = [0-255]$   
(8 bits = 1 byte =  $2^8 = 256$  values per pixel)
- $\text{float32} = [0-1]$   
(32 bits = 4 bytes =  $4.3 \times 10^9$  values per pixel)

## 1.4. color image

How do we record colors?

⇒ **Bayer Filter**: color filter array for arranging RGB color filters on a square grid of photosensors

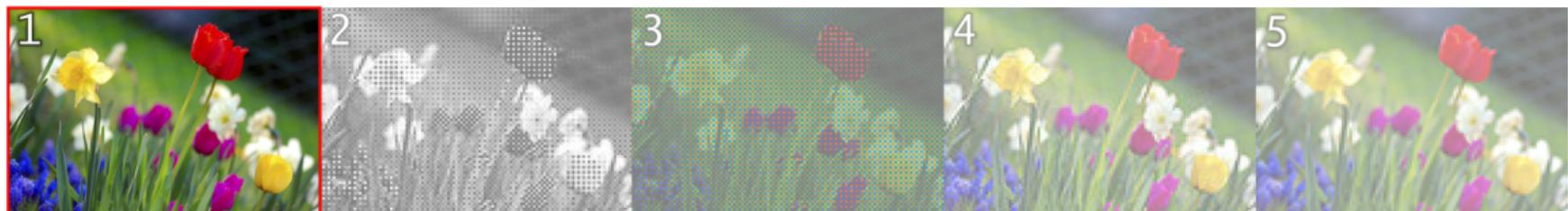


(source [wikipedia](#))

## 1.4. color image

How do we record colors?

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1. Original scene
2. Output of a  $120 \times 80$ -pixel sensor with a Bayer filter
3. Output color-coded with Bayer filter colors
4. Reconstructed image after interpolating missing color information (a.k.a. demosaicing)
5. Full RGB version at  $120 \times 80$ -pixels for comparison

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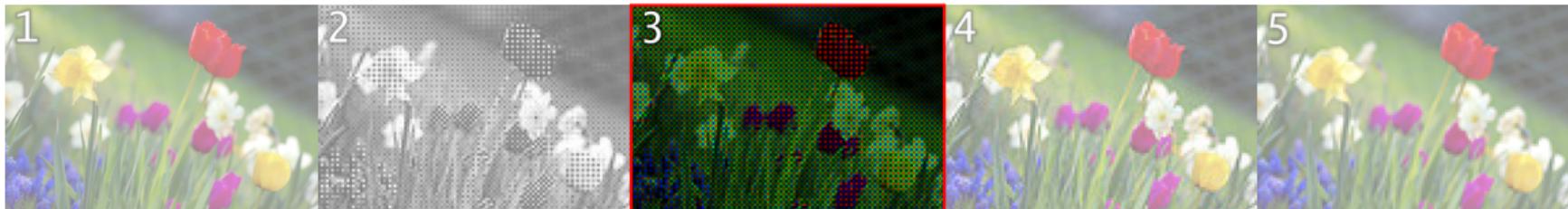


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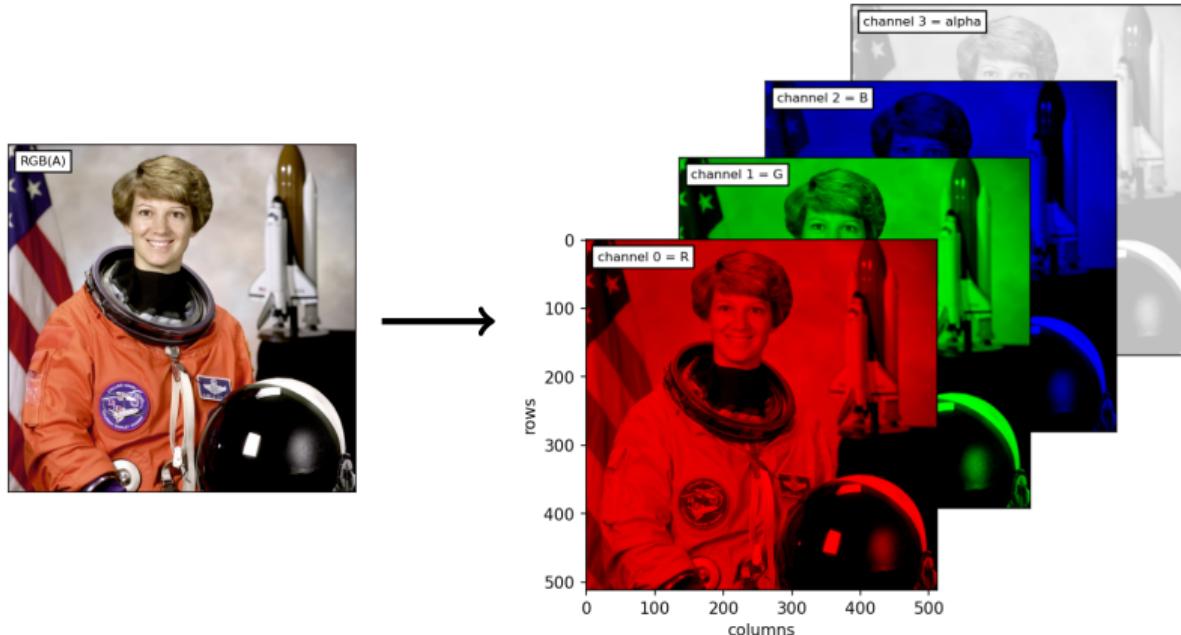


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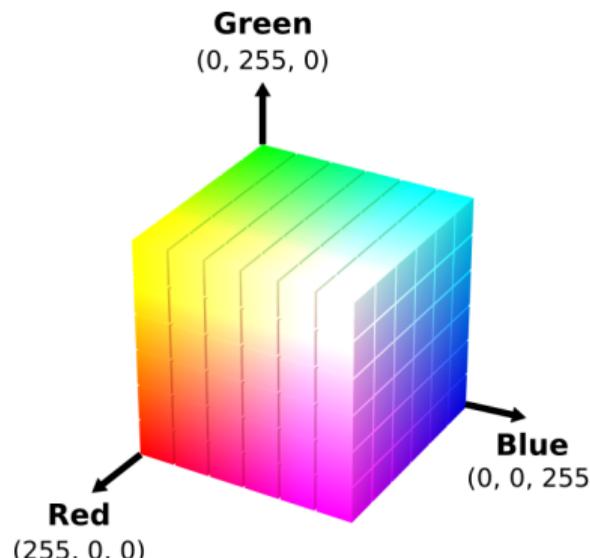
⇒ color image = 3D tensor in colorspace

- **RGB** = Red + Green + Blue bands (.JPEG)
- **RGBA** = Red + Green + Blue + Alpha bands (.PNG, .GIF, .BMP, TIFF, .JPEG 2000)

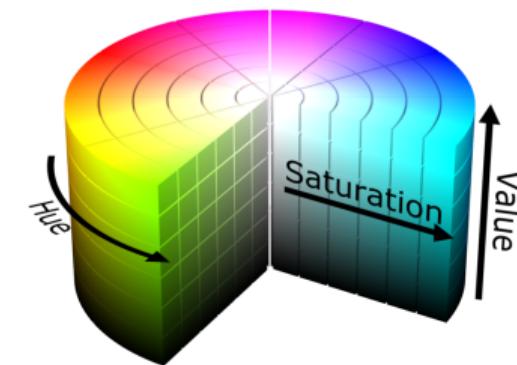


Other ways to represent the color information?

### RGB colorspace



### HSV colorspace

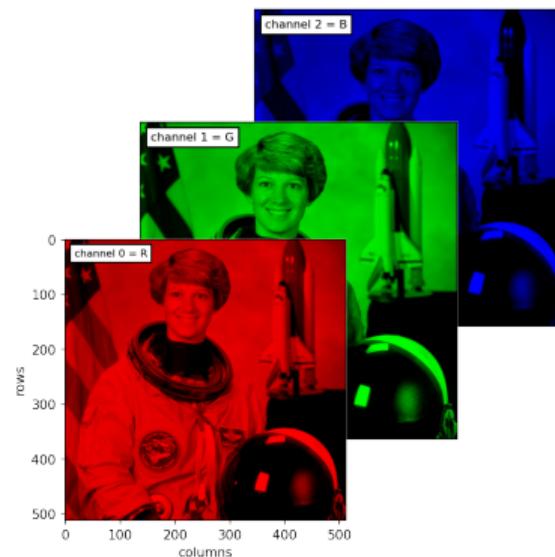


- Hue (H) = [0-360] ⇒ shift color
- Saturation (S) = [0-1] ⇒ shift intensity
- Value (V) = [0-1] ⇒ shift brightness

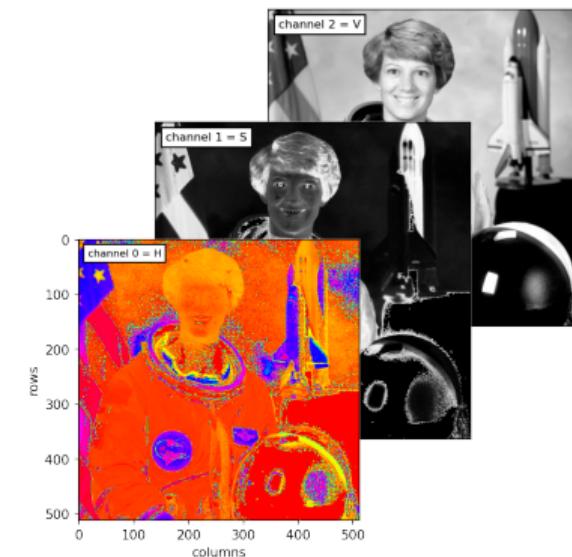
## 1.5. color spaces

3D tensor with different information

### RGB colorspace



### HSV colorspace



## 1.5. color spaces

- more saturation S

⇒ more intense colors



- more value V

⇒ brighter colors

- shift hue H

⇒ shift color

## 1.5. color spaces

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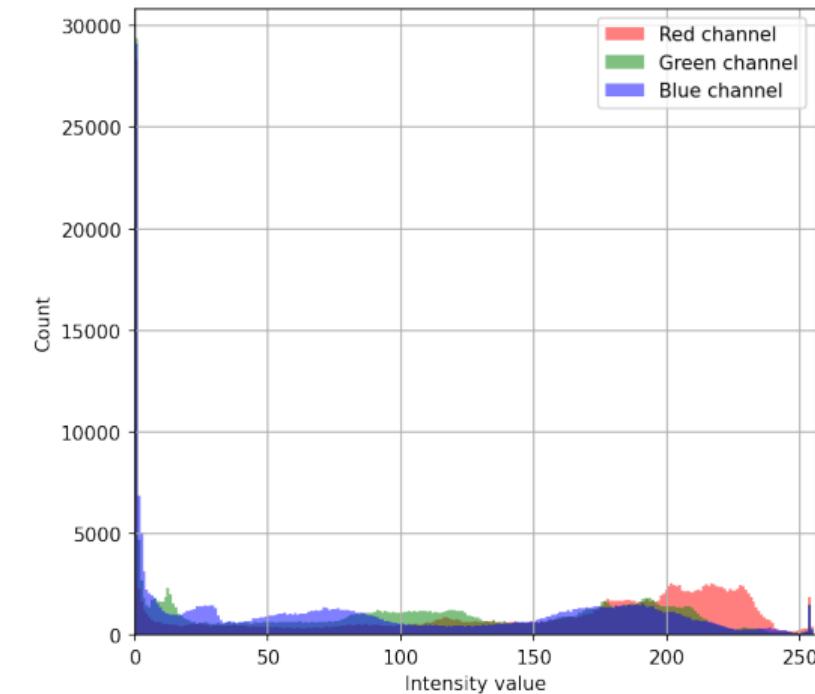
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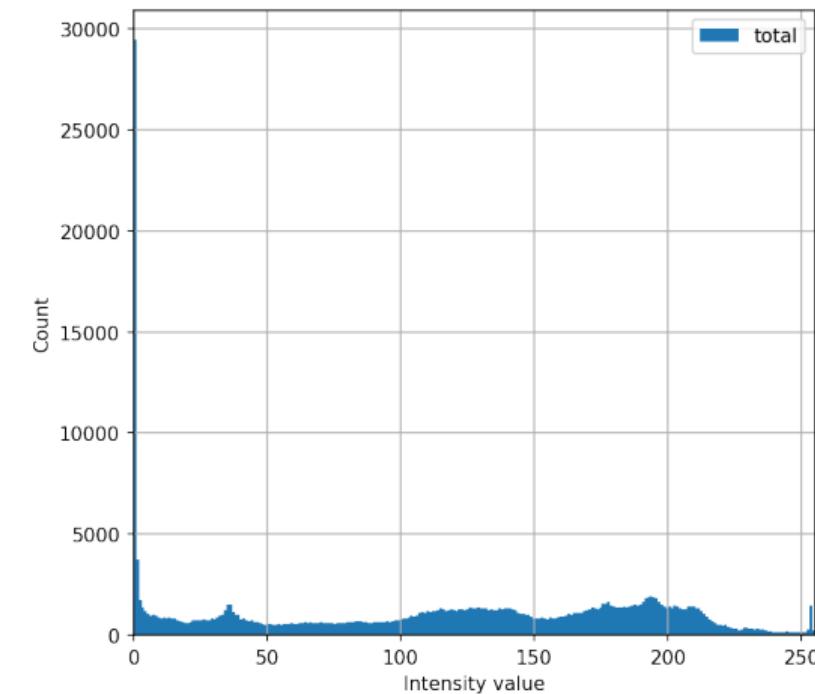
## 1.6. image histogram

Histogram of pixel values in each band:



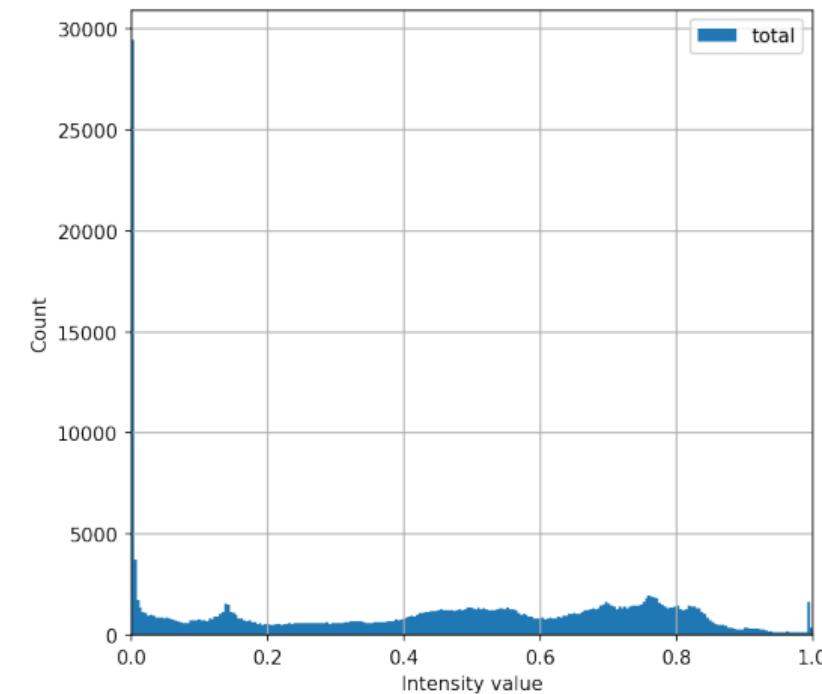
## 1.6. image histogram

Histogram of pixel values after conversion from RGB (3-bands) to gray-scale (1-band):



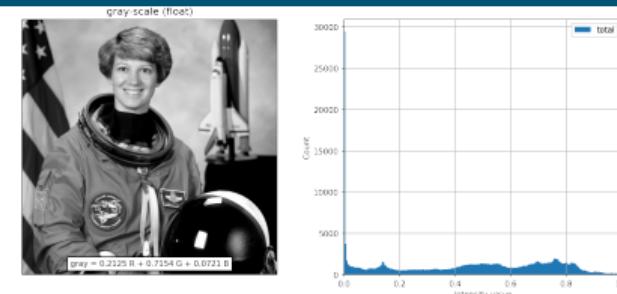
## 1.6. image histogram

Histogram of pixel values after conversion to float values (range [0-1])



## 1.6. image histogram

- original gray-scale

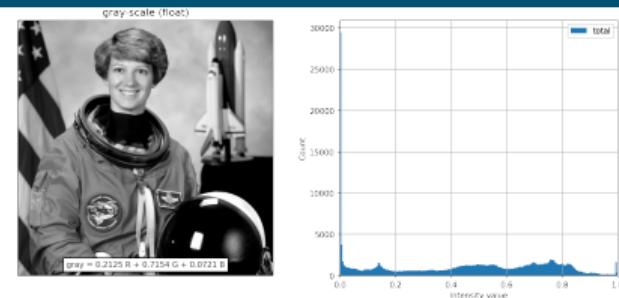


- histogram rescale to 10-90 percentiles  
⇒ contrast stretching

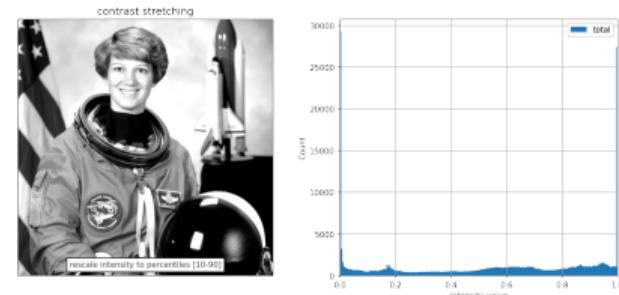
- histogram equalize  
⇒ spread out the most frequent intensity values

## 1.6. image histogram

- original gray-scale



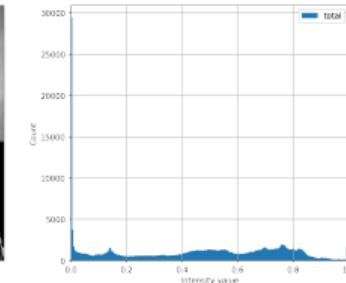
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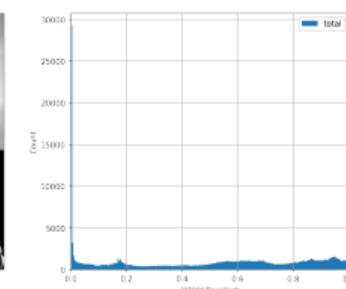
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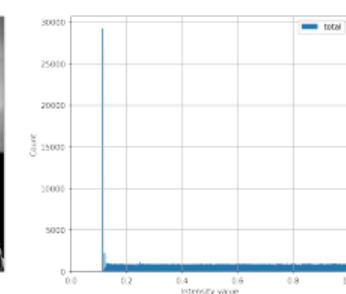
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1. What is a digital image?

**2. Point operations**

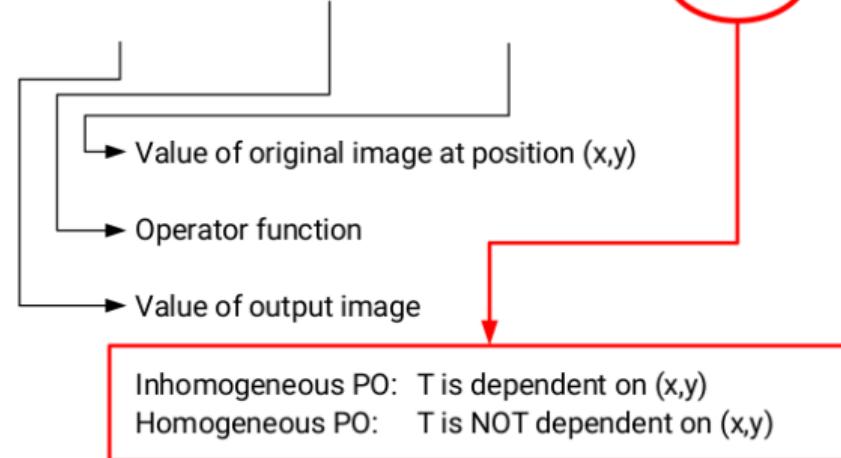
1. homogeneous point operations
2. inhomogeneous Point Operations

3. Image processing levels

4. Image manipulation with Python

## Point operations

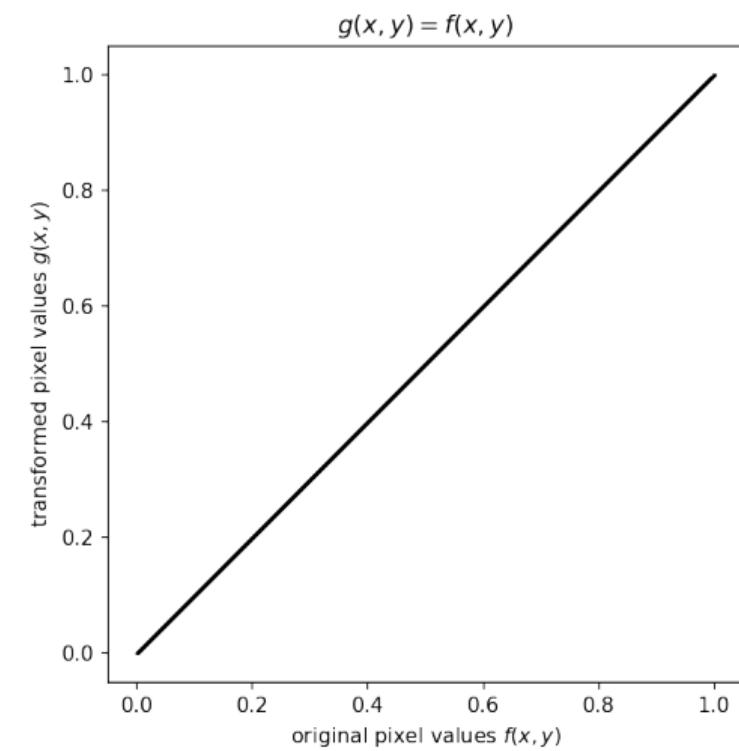
$$g(x, y) = T(f(x, y), x, y)$$



## 2.1. homogeneous point operations

## Homogeneous Point Operations (does not depend on pixel position)

identity



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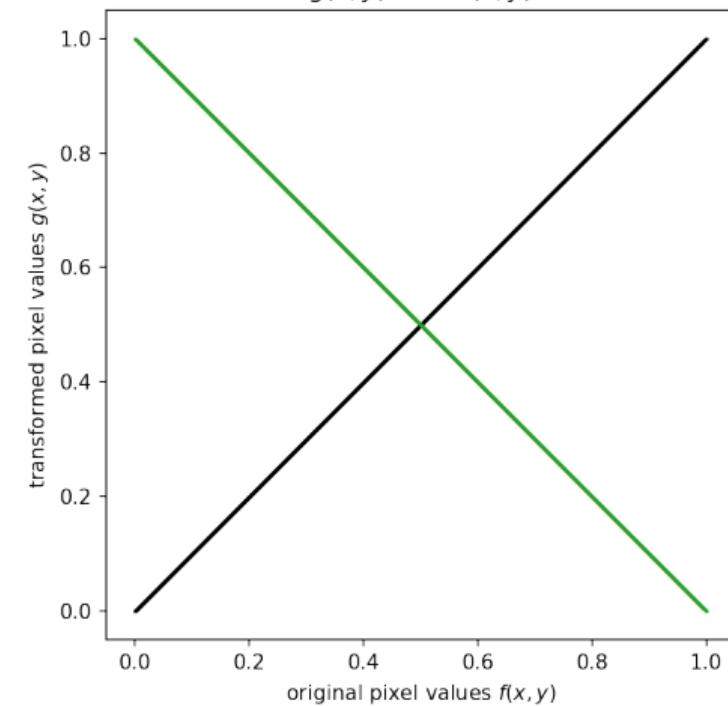
identity



inverse



$$g(x, y) = 1 - f(x, y)$$



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identity



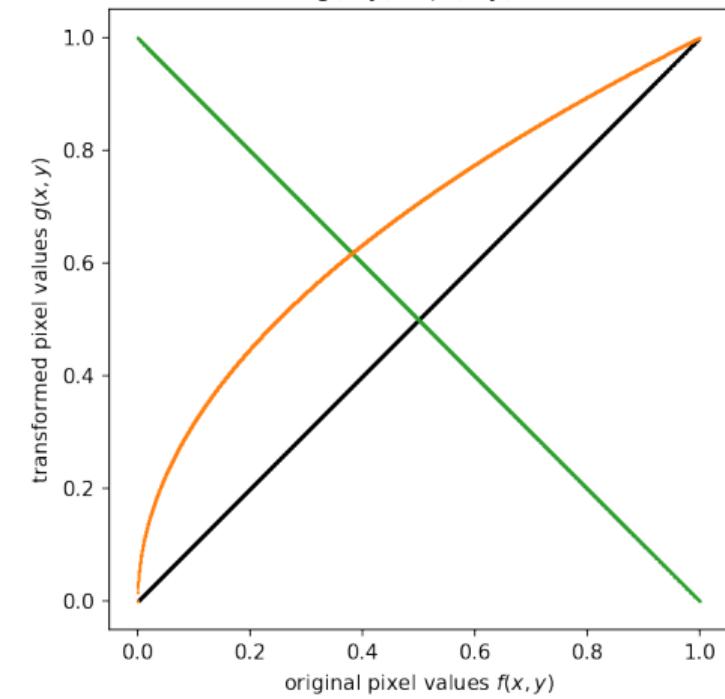
inverse



square root



$$g(x, y) = \sqrt{f(x, y)}$$



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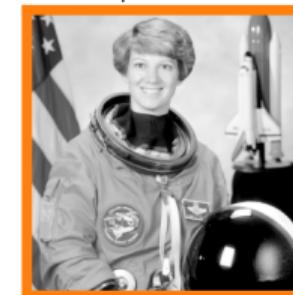
identity



inverse



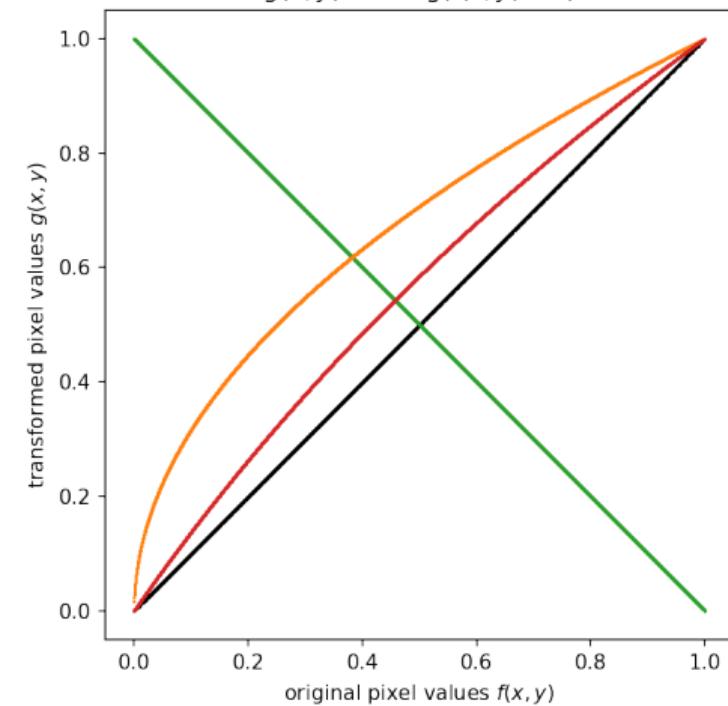
square root



logarithm



$$g(x, y) = a \cdot \log(f(x, y) + 1)$$



## 2.1. homogeneous point operations

## Homogeneous Point Operations (does not depend on pixel position)

identity



inverse



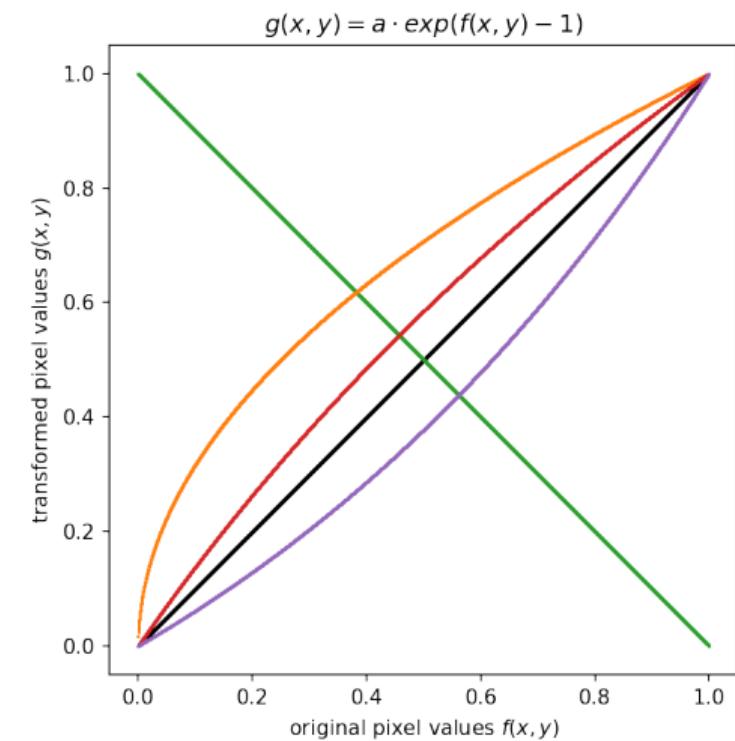
square root



logarithm



exponential



## 2.1. homogeneous point operations

## Homogeneous Point Operations (does not depend on pixel position)

identity



inverse



square root



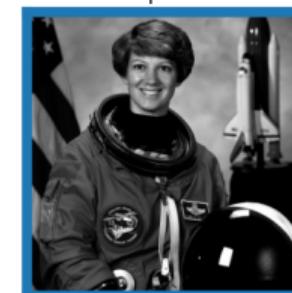
logarithm



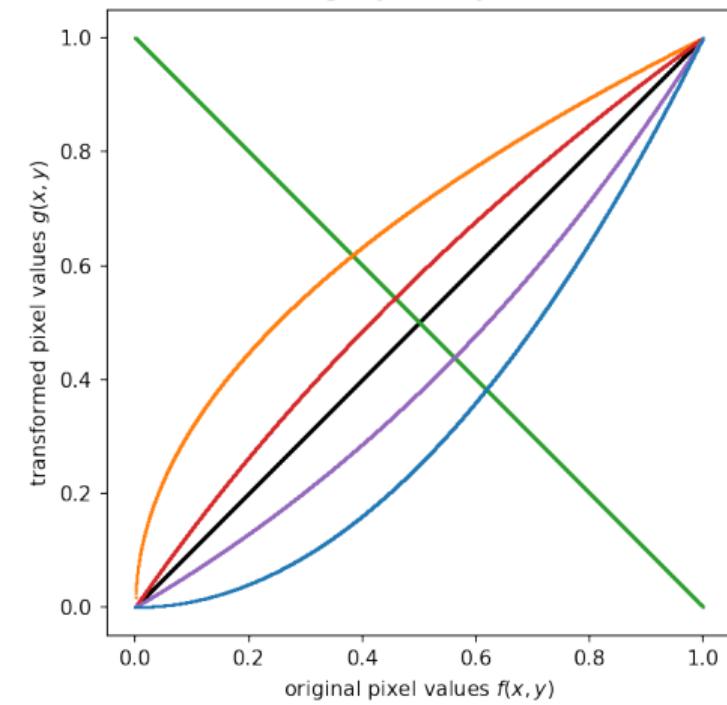
exponential



square



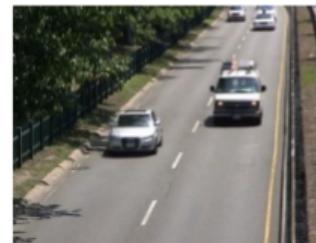
$$g(x, y) = f(x, y)^2$$



## 2.2. inhomogeneous Point Operations

Inhomogeneous Point Operations (depends on pixel position)

EX: background detection / change detection

 $f_1$  $f_i$  $f_N$ 

$$a(x, y) = \frac{1}{N} \sum_{i=0}^N f_i(x, y)$$

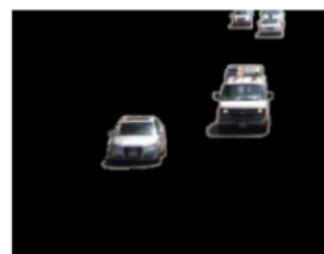
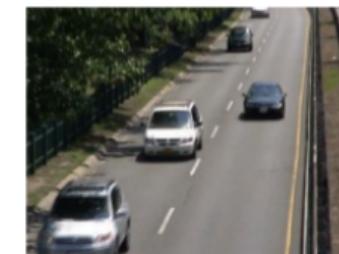
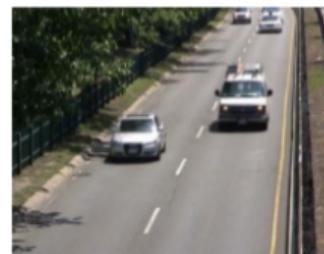
$$\begin{aligned} g_i(x, y) &= T(f(x, y), x, y) \\ &= f_i(x, y) - a(x, y) \end{aligned}$$



## 2.2. inhomogeneous Point Operations

Inhomogeneous Point Operations (depends on pixel position)

EX: background detection / change detection



1. What is a digital image?
2. Point operations
3. Image processing levels
4. Image manipulation with Python

## Image processing levels: inhomogeneous Point Operations



## Examples of processing levels:

- Low-level processing
  - image manipulation ⇒ *resizing, color adjustments, filtering, etc.*
  - feature extraction ⇒ *edges, gradients, etc.*
- Mid-level processing
  - panorama stitching
  - Structure from Motion (SfM) ⇒ 2D to 3D
  - Optical Flow ⇒ velocities
- High-level processing
  - classification ⇒ *what is in the image?*
  - detection ⇒ *where are they?*
  - segmentation (semantic or instance) ⇒ *segment image and give names*

## Examples of processing levels:

- Low-level processing

- image manipulation  $\Rightarrow$  *resizing, color adjustments, filtering, etc.*
- feature extraction  $\Rightarrow$  *edges, gradients, etc.*



hue x5



- Mid-level processing

- panorama stitching
- Structure from Motion (SfM)  $\Rightarrow$  2D to 3D
- Optical Flow  $\Rightarrow$  velocities



filter (high pass)



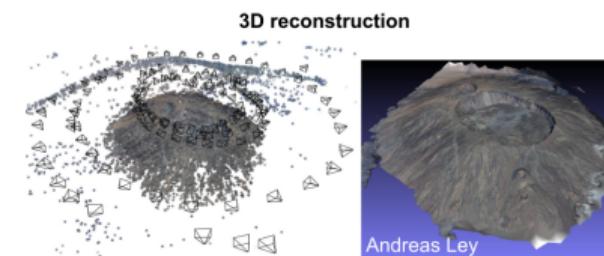
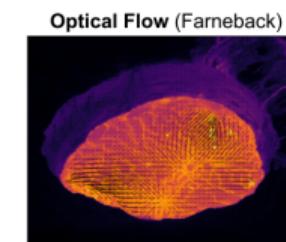
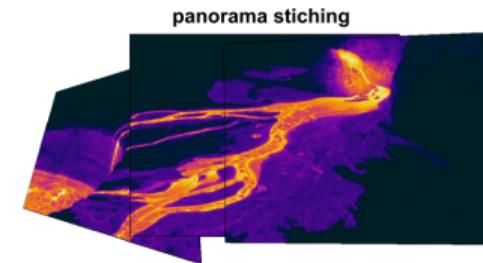
- High-level processing

- classification  $\Rightarrow$  *what is in the image?*
- detection  $\Rightarrow$  *where are they?*
- segmentation (semantic or instance)  $\Rightarrow$  *segment image and give names*

## Image processing levels

**Examples of processing levels:**

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## Image processing levels

## Examples of processing levels:

- Low-level processing

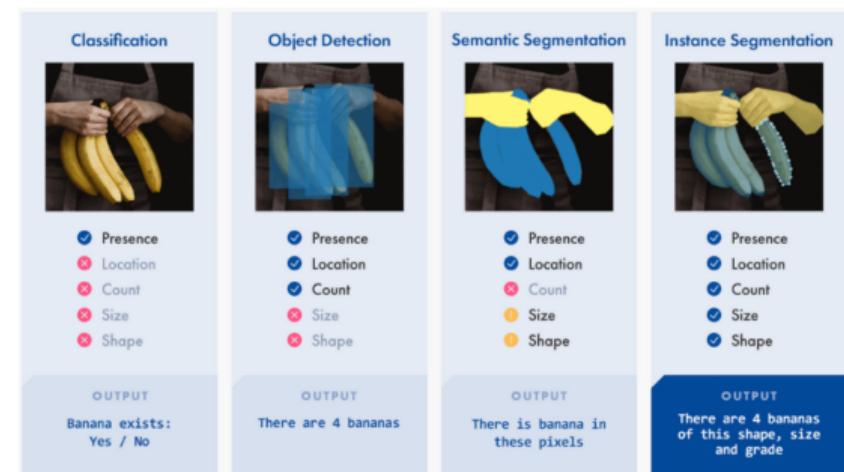
- image manipulation ⇒ *resizing, color adjustments, filtering, etc.*
- feature extraction ⇒ *edges, gradients, etc.*

- Mid-level processing

- panorama stitching
- Structure from Motion (SfM) ⇒ 2D to 3D
- Optical Flow ⇒ velocities

- High-level processing

- classification ⇒ *what is in the image?*
- detection ⇒ *where are they?*
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Credit: cloudfactory

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1. What is a digital image?
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  1. numpy tutorial
  2. exercises

## 4.1. numpy tutorial

Numpy tutorial:

⇒ Open DIP4RS\_02\_imagebasics/[DIP4RS\\_02\\_numpy-tutorial.ipynb](#)

## 4.2. exercises

Exercices:

⇒ Open DIP4RS\_02\_imagebasics/[DIP4RS\\_02\\_exercices.ipynb](#)