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RESEARCH ARTICLE

Determination of effective stress parameter of unsaturated soils: A Gaussian process regression approach

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ABSTRACT This article examines the capability of Gaussian process regression (GPR) for prediction of effective stress parameter (χ) of unsaturated soil. GPR method proceeds by parameterising a covariance function, and then infers the parameters given the data set. Input variables of GPR are net confining pressure (σ_3), saturated volumetric water content (θ_s), residual water content (θ_r), bubbling pressure (h_b), suction (s) and fitting parameter (λ). A comparative study has been carried out between the developed GPR and Artificial Neural Network (ANN) models. A sensitivity analysis has been done to determine the effect of each input parameter on χ. The developed GPR gives the variance of predicted χ. The results show that the developed GPR is reliable model for prediction of χ of unsaturated soil.

KEYWORDS unsaturated soil, effective stress parameter, Gaussian process regression (GPR), artificial neural network (ANN), variance

1 Introduction

Effective stress (σ') is an important parameter for determination of shear strength parameter of unsaturated soil. Geotechnical engineers use the following equation for determination of σ' of unsaturated soil [1].

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w), \tag{1}$$

where σ is total stress, u_a is pore air pressure, u_w is pore water pressure and χ is an effective stress parameter. The determination of χ is an imperative task for determination of σ' of unsaturated soil. Geotechnical engineers use different methods for prediction of χ [2,3]. The available methods are not reliable for all types of unsaturated soils [4,5]. Ajdari et al. [6] successfully used Artificial Neural Network (ANN) for determination of χ. However, ANN has some limitations such as black box approach, arriving at local minimum, low generalization capability, etc. [7,8].

This article employs Gaussian process regression (GPR) for prediction of χ based on matric suction, net mean stress and Soil Water Characteristic Curve (SWCC) parameters. GPR is a probabilistic, non-parametric model [9]. In GPR, different kinds of prior knowledge can be applied. Researchers have successfully used GPR for solving different problems in engineering [10–13]. The purpose of this paper is twofold. First, to present the procedure of prediction of χ using the GPR. Second, to carry out a comparative study with the ANN model developed by Ajdari et al. [6]. This article uses the database collected by Ajdari et al. [6]. The paper is organized as follows: Section 2 presents the GPR for prediction of χ. Section 3 describes the analysis of results and discussion. Finally, Section 4 draws the conclusions from this article.

2 Details of GPR

This section will give a brief introduction of GPR model. The details of GPR are given by Williams and Rasmussen [14]. Let us consider the following noise dataset.

$$D = \{(x_i, y_i)\}_{i=1}^{N}, \tag{1}$$

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where x is input and y is output and N is the number of data. In this study, net confining pressure (σ_3), saturated volumetric water content (θ_s), residual water content (θ_r), bubbling pressure (h_b), suction (s) and fitting parameter (λ) are used as input variables of the GPR. The output of GPR is χ. So, $x = [\sigma_3, \theta_s \theta_r, h_b, s, \lambda]$ and $y = [\chi]$.

It is assumed that the above data are generated from the following equation.

$$y_i = f(x_i) + \varepsilon_i, \hat{\xi} N(0, \sigma^2), \tag{2}$$

where ε is Gaussian noise term.

The joint distribution of y is given by the following equation.

$$P(y) = N\left(0,\ K(x,x) + \sigma^2 I\right), \tag{3}$$

where $K(x, x)$ is kernel function and I is identity matrix.

For a test input x^*, GPR defines a Gaussian predictivs distribution over the output y^* with mean

$$\mu = k_*^{\mathrm{T}} [K(x,x) + \sigma^2 I]^{-1} y, \tag{4}$$

and variance

$$\sum = K(x^*, x^*) - K_*^{\mathrm{T}} [K(x,x) + \sigma^2 I]^{-1} K_*, \tag{5}$$

where T is transpose.

To develop GPR mode, a suitable covariance function is required. This article uses radial basis function ($K(x_i, x) = \exp\left[-\frac{(x_i - x)(x_i - x)^{\mathrm{T}}}{2s^2}\right]$, where s is the width of radial basis function and T is transpose) as covariance function. The GPR model uses the same training and testing data set as used by Ajdari et al. [6]. The data are normalized between 0 and 1. The following equation is used for normalization:

$$d_{\text{normalized}} = \frac{(d - d_{\min})}{(d_{\max} - d_{\min})}, \tag{6}$$

where d is input or output data, $d_{\min}$ = minimum value of the entire dataset, $d_{\max}$ = maximum value of the entire dataset, and $d_{\text{normalized}}$ = normalized value of the data.

A sensitivity analysis has been carried out to determine the effect of each input parameter on χ. It is performed by varying each of input parameter, one at a time, at a constant rate of 30%. The analysis has been done on training data set. The percent change of output is calculated for the change of input parameter. The sensitivity (S) of each input parameter is calculated from the following formula [15]:

$$S(\%) = \frac{1}{N} \sum_{j=1}^{N} \left(\frac{\%\text{change in ouput}}{\%\text{change in input}} \right)_j \times 100, \tag{7}$$

where N is the number of data. MATLAB is used to develop the GPR.

3 Results discussion

The design values of ε and width(s) of radial basis function have been determined by trial and error approach. The developed GPR gives best performance at $\varepsilon = 0.05$ and $s = 0.20$. Therefore, the design value of ε and s is 0.05 and 0.20 respectively. The performance of training and testing data set has been determined by using the design values of ε and s.

Figures 1 and 2 illustrates the performance of training and testing data set respectively. The performance of developed GPR has been assessed in terms of coefficient of determination (R^2). For a good model, the value of R^2 is close to one. It is observed from Figs. 1 and 2 that the value of R^2 is close to one. Therefore, the developed GPR predicts χ reasonable well.

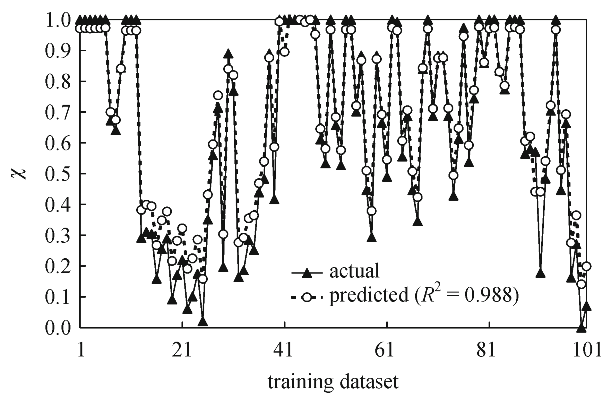

Fig. 1 Performance of training dataset

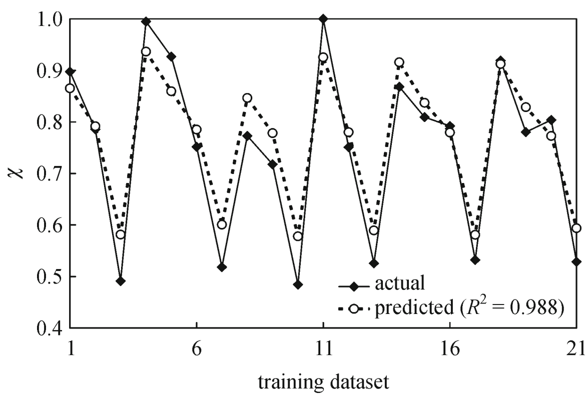

Fig. 2 Performance of testing dataset

A comparative study has been carried out between the developed GPR and ANN model developed by Ajdari et al. [6]. Comparison has been done in terms of R^2.

Figure 3 shows the bar chart of R^2 of the ANN and GPR models. It is clear from Fig. 3 that the developed GPR gives better performance than the ANN. There are two tuning parameters (Gaussian noise and s) in the GPR

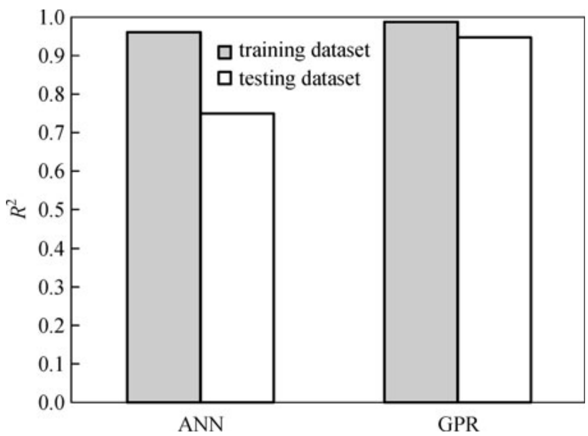

Fig. 3 Comparisons between ANN and GPR

model. However, ANN uses many tuning parameters such as number of hidden layers, number of neurons in the hidden layer, transfer function, epochs, etc.

Figure 4 shows the result of sensitivity analysis. The developed GPR is simple, practical and powerful bayesian tool for data analysis. It is observed from Fig. 4 that s has maximum effect on the predicted χ. H_b has minimum effect on the predicted χ. The developed GPR gives the variance of the predicted χ. Figures 5 and 6 depict the variance of predicted χ for training and testing data set respectively. The obtained variance can be used for determination of uncertainty.

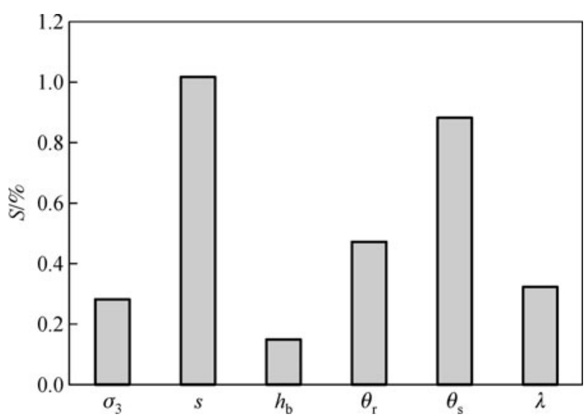

Fig. 4 Sensitivity analysis of the input parameters

4 Conclusion

This article has described GPR for prediction of χ of unsaturated soil. The developed GPR gives encouraging performance. It proves his ability for predicting χ by using the proper value of Gaussian noise and s. It gives better performance than the ANN model. The predicted variance gives the corresponding risk. Sensitivity analysis shows that s is most important parameter for prediction of χ. It can be concluded that GPR can be used to solve different problems in geotechnical engineering.

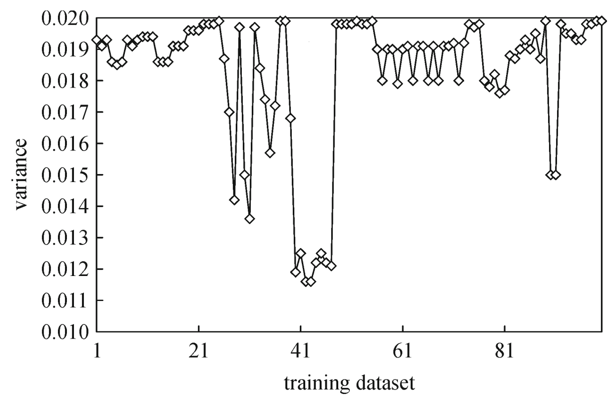

Fig. 5 Variance of training dataset

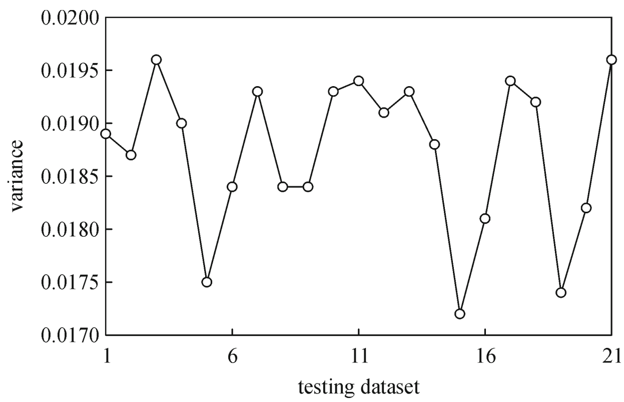

Fig. 6 Variance of testing dataset

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