

Exercise (Separation between CDH and DDH). Let \mathbb{G} be a finite additive group of prime order q such that all elements $y \in \mathbb{G}$ can be expressed as multiples of $g \in \mathbb{G}$.

- Then the Computational Diffie-Hellman (CDH) problem is following. Given $x = a \cdot g$ and $y = b \cdot g$, find a group element $z = ab \cdot g$.
- Then the Decisional Diffie-Hellman (DDH) problem is the following. For any triple $x, y, z \in \mathbb{G}$, you must decide whether it is a Diffie-Hellman triple or not.
- The group \mathbb{G} has a bilinear pairing $\langle \cdot, \cdot \rangle : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_*$ when following equalities hold

$$\begin{aligned}\langle x_1 + x_2, y \rangle &= \langle x_1, y \rangle \cdot \langle x_2, y \rangle \\ \langle x, y_1 + y_2 \rangle &= \langle x, y_1 \rangle \cdot \langle x, y_2 \rangle\end{aligned}$$

and the pairing is efficiently computable and non-degenerate, i.e., $\langle g, g \rangle \neq 1$.

Prove that (t, ε) -CDH group with a bilinear pairing cannot be DDH group.

Solution.