

Exercise (Alternative definitions for IND-CPA security). *Estimate computational distance between following games under the assumption that $(\text{Gen}, \text{Enc}, \text{Dec})$ is (t, ε) -IND-CPA secure cryptosystem.*

1. *Left-or-right games (LOR security games)*

$$\begin{array}{cc} \mathcal{G}_0^A & \mathcal{G}_1^A \\ \left[\begin{array}{l} \text{sk} \leftarrow \text{Gen} \\ \text{For } i = 1, \dots, q \text{ do} \\ \quad \left[\begin{array}{l} (m_0^i, m_1^i) \leftarrow \mathcal{A} \\ \text{Give } \text{Enc}_{\text{sk}}(m_0^i) \text{ to } \mathcal{A} \end{array} \right] \\ \text{return the output of } \mathcal{A} \end{array} \right. & \left[\begin{array}{l} \text{sk} \leftarrow \text{Gen} \\ \text{For } i = 1, \dots, q \text{ do} \\ \quad \left[\begin{array}{l} (m_0^i, m_1^i) \leftarrow \mathcal{A} \\ \text{Give } \text{Enc}_{\text{sk}}(m_1^i) \text{ to } \mathcal{A} \end{array} \right] \\ \text{return the output of } \mathcal{A} \end{array} \right. \end{array}$$

2. *Real-or-random games (ROR security games)*

$$\begin{array}{cc} \mathcal{G}_0^A & \mathcal{G}_1^A \\ \left[\begin{array}{l} \text{sk} \leftarrow \text{Gen} \\ \text{For } i = 1, \dots, q \text{ do} \\ \quad \left[\begin{array}{l} m^i \leftarrow \mathcal{A} \\ \text{Give } \text{Enc}_{\text{sk}}(m^i) \text{ to } \mathcal{A} \end{array} \right] \\ \text{return the output of } \mathcal{A} \end{array} \right. & \left[\begin{array}{l} \text{sk} \leftarrow \text{Gen} \\ \text{For } i = 1, \dots, q \text{ do} \\ \quad \left[\begin{array}{l} m_0^i \leftarrow \mathcal{A}, m_1^i \xleftarrow{u} \mathcal{M} \\ \text{Give } \text{Enc}_{\text{sk}}(m_1^i) \text{ to } \mathcal{A} \end{array} \right] \\ \text{return the output of } \mathcal{A} \end{array} \right. \end{array}$$

Moreover, we can use these security games to define (t, ε) -LOR security and (t, ε) -ROR security. Prove that security against these security notions also implies IND-CPA security.

Solution. We split the proof into separate blocks each dedicated to a single subgoal.

SUBPROOF IND-CPA \Rightarrow LOR-SECURITY

SUBPROOF LOR-SECURITY \Rightarrow IND-CPA

SUBPROOF IND-CPA \Rightarrow LOR-SECURITY

SUBPROOF ROR-SECURITY \Rightarrow IND-CPA