

**Exercise (Separation between CDH and DDH).** Let  $\mathbb{G}$  be a finite additive group of prime order  $q$  such that all elements  $y \in \mathbb{G}$  can be expressed as multiples of  $g \in \mathbb{G}$ .

- Then the Computational Diffie-Hellman (CDH) problem is following. Given  $x = a \cdot g$  and  $y = b \cdot g$ , find a group element  $z = ab \cdot g$ .
- Then the Decisional Diffie-Hellman (DDH) problem is the following. For any triple  $x, y, z \in \mathbb{G}$ , you must decide whether it is a Diffie-Hellman triple or not.
- The group  $\mathbb{G}$  has a bilinear pairing  $\langle \cdot, \cdot \rangle : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_*$  when following equalities hold

$$\begin{aligned}\langle x_1 + x_2, y \rangle &= \langle x_1, y \rangle \cdot \langle x_2, y \rangle \\ \langle x, y_1 + y_2 \rangle &= \langle x, y_1 \rangle \cdot \langle x, y_2 \rangle\end{aligned}$$

and the pairing is efficiently computable and non-degenerate, i.e.,  $\langle g, g \rangle \neq 1$ .

Prove that  $(t, \varepsilon)$ -CDH group with a bilinear pairing cannot be DDH group.

**Solution.**