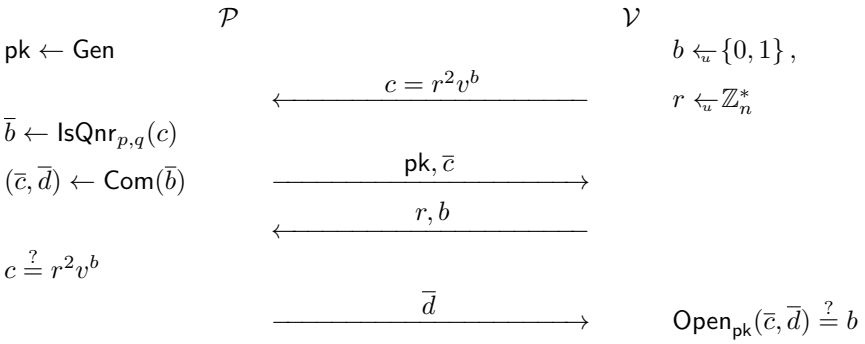
MTAT.07.003 Cryptology II
Spring 2012 / Exercise session ?? / Example Solution

Exercise (Soundness of the QNR-ZKD protocol). *Let n be a composite number with a factorisation $n = pq$ known to the prover $\mathcal{P}$. Let $v \in \mathbb{Z}_n^*$ be a number for which the prover wants to prove that it is quadratic non-residue. Let $(\mathsf{Gen}, \mathsf{Com}, \mathsf{Open})$ be a perfectly binding and computationally hiding commitment. Show that the the following zero-knowledge protocol*



[show soundness]

where the verifier $\mathcal{V}$ releases r, b is simulatable. For that construct first a semi-efficient algorithm $\mathcal{K}^{\mathcal{V}_*}$ for extracting r, b that correspond to initial message c. Next show that the following simulator construction

$\mathcal{V}_\circ(\phi)$
- $\omega \leftarrow \Omega$
- $c \leftarrow \mathcal{V}_*(\phi; \omega)$
- $(b_*, r_*) \leftarrow \mathcal{K}^{\mathcal{V}_*}(\phi; \omega)$
- $\alpha \leftarrow \mathcal{V}_*$
- $\beta \xleftarrow{u} \mathcal{B}$
- $\gamma \leftarrow \mathcal{V}_*(\beta)$
- if $\mathsf{Ver}(\alpha, \beta, \gamma) = 0$ then ***return*** $\mathcal{V}_*(\bot)$
- if $\beta_* \neq \bot$ then ***return*** $\mathcal{V}_*(b_*)$
- else ***return*** $\bot$

can create an output distribution $\psi_\circ$ that is computationally $(t_\circ, \varepsilon_\circ)$-distant from the output distribution of malicious verifier $\mathcal{V}_*$ that interacts with the honest prover. Also, estimate how the running-time of the simulator depends on the desired distance $\varepsilon_\circ$.

Solution.