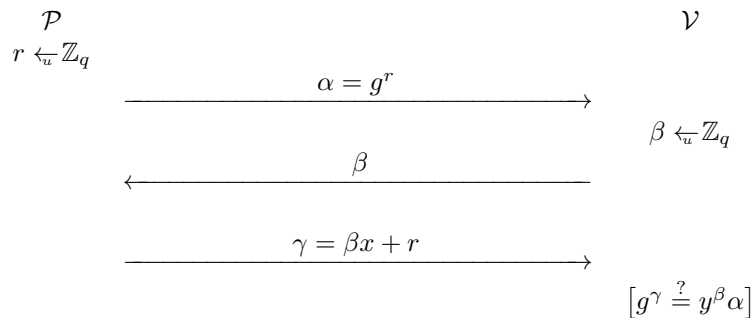


Exercise (Knowledge-extraction for Schnorr identification scheme). *The Schnorr identification scheme is directly based on the discrete logarithm problem. The identification scheme is a honest verifier zero-knowledge proof that the prover knows x such that $g^x = y$ in a group \mathbb{G} of size q . The protocol itself is following.*



Show that if an honest t -time prover \mathcal{P}^* that can convince the honest verifier with probability ε on average over all $y \in \mathbb{G}$ can also solve the discrete logarithm problem well enough.

Solution. We can define a knowledge extractor $\mathcal{K}^{\mathcal{P}^*}(y)$ for finding discrete logarithm and then a self-randomised algorithm that uses $\mathcal{K}^{\mathcal{P}^*}$ on re-randomised problem instance. Consider the success probability for fixed time-bound on the running-time $\mathcal{K}^{\mathcal{P}^*}$.

Hint. Take the standard knowledge extraction construction with success probability one and expected running-time $\Theta(1/(\varepsilon - \kappa))$. Fix a time-bound such that the extractor would succeed with probability $1/2$ when the ε is the average success probability of \mathcal{P}^* . Now note that for each y the success probability of \mathcal{P}^* ε_y can be different. Estimate what is the success probability of the knowledge extractor \mathcal{K} with the fixed time-bound. Use random self reducibility to even this into average value for each challenge y .