

**Exercise (Separation between one-wayness and collision-resistance).** *Let  $\mathcal{H}$  be  $(t, \varepsilon)$ -oneway function family with an input domain  $\mathcal{X}$  and output range  $\mathcal{Y}$ . Define a new hash function family  $\mathcal{H}^*$  with the input domain  $\mathcal{X} \cup \{x_0, x_1\}$  and output range  $\mathcal{Y} \cup \{y_0\}$  such that the function family is still one-way but is not collision resistant.*

**Solution.** Let  $x_0, x_1$  be outside of the range  $\mathcal{X}$  and  $y_0$  outside the range. Then we can implant a known collision to each hash function  $h^* \in \mathcal{H}^*$  by defining ...