Exercise (Security of simultaneous message exchange protocol). Analyse security of the following simplistic protocol for simultaneous message exchange

where bits x_1 and x_2 are private protocol inputs and a triple of algorithms (Gen, Com, Open) is a commitment scheme Com with appropriate properties. The dashed line denotes sub-protocol for fixing the commitment parameters. In the following, we assume that the protocol has a trusted setup where parameter generation is done by a trusted third party. Consider security only against static malicious corruption.

Solution.

RIGHT IDEAL IMPLEMENTATION. As the first party \mathcal{P}_1 can refuse to open its input based on the opponents input x_2 , we must consider the idealised functionality where the first party \mathcal{P}_1 is in the dominant position:

- (E) OUTPUT EQUIVOCATION BASED ON TRUSTED SETUP. We can use equivocal commitments to bypass problems in the output equivocation phase. But this leads to a setting with a trusted setup.
 - \bullet Construct the corresponding simulator for malicious \mathcal{P}_2 by modifying the input extraction and output equivocation blocks
 - Prove that the corresponding simulator achieves the desired goal. That is, the joint output distributions are identical in the real and ideal world.
- (F) INPUT EXTRACTION BASED ON TRUSTED SETUP. The simulation efficiency for a malicious \mathcal{P}_1^* depends on the size of \mathcal{X}_2 as the input extractor needs to iterate over all potential inputs x_2 to unlock the commitment. This problem can be bypassed if we use trusted setup with extractable commitment schemes.

- Construct the corresponding simulator for malicious \mathcal{P}_1 by modifying the input extraction block so that its efficiency does not depend on the size of \mathcal{X}_2 .
- Prove that the corresponding simulator achieves the desired goal. That is, the joint output distributions are identical in the real and ideal world.