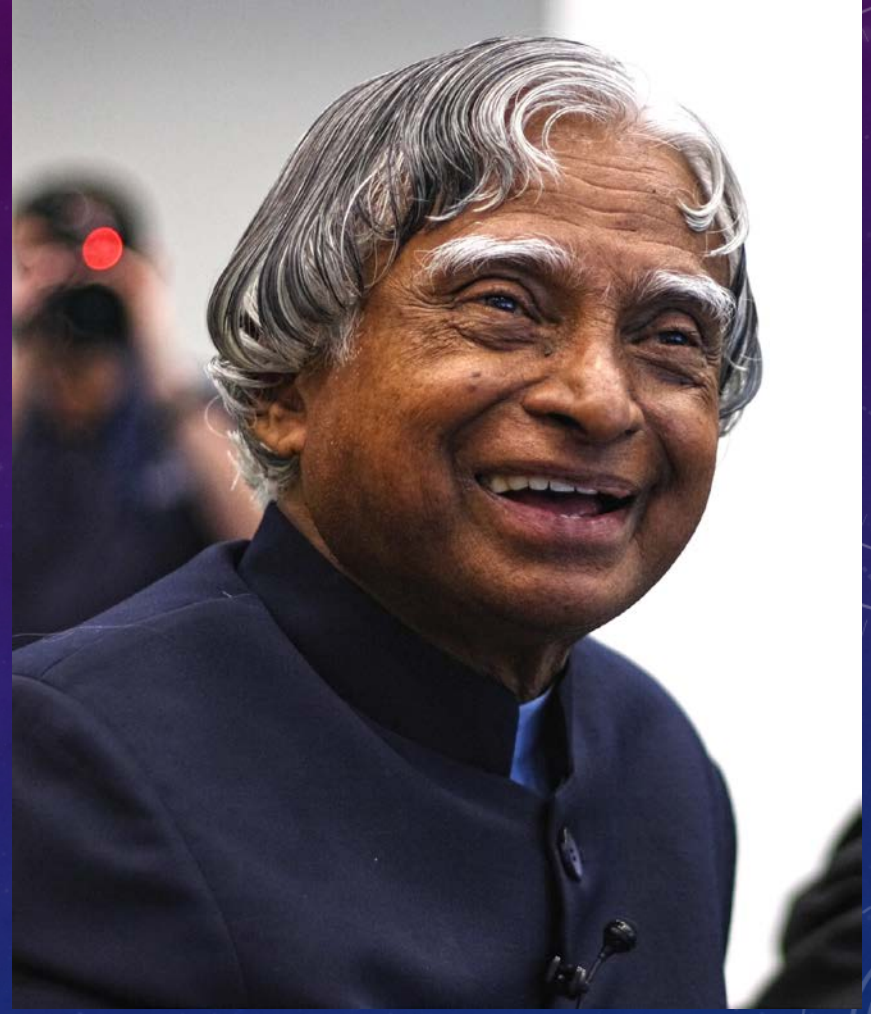
SIGNAL PROCESSING WITH FOURIER ANALYSIS:

NOVEL ALGORITHMS AND APPLICATIONS

Dr. Amartya Sen
The Argumentative Indian: Writings on Indian History, Culture and Identity, 2006.



(Late) Dr. A. P. J. Abdul Kalam
Wings of Fire, 1999.
The Missile Man of India

OVERVIEW

- Robust Registration using Spherical Polar Fourier Transform and Spherical Harmonics
- Nonuniform Polar and Spherical Discrete Fourier Transform
- Frequency Estimation from nonuniform samples
- Real time fast phasor estimation
- Applications and future work
- Conclusions
- Q & A

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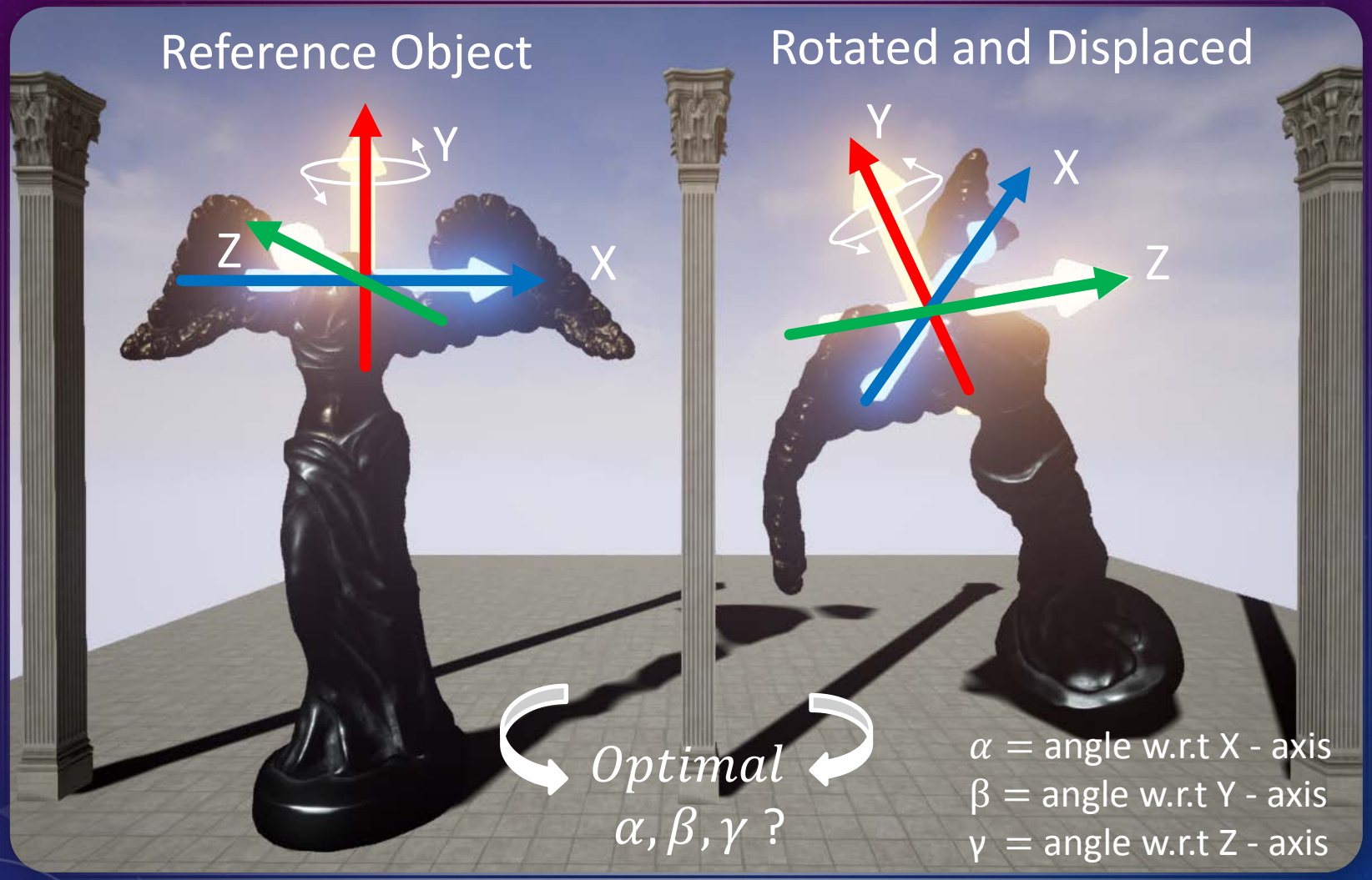


WHAT IS THE PROBLEM ?

- Given two 3D objects find the rotation and displacement to align them optimally

WHY IS THE PROBLEM HARD ?

01

On the face of it the problem looks relatively easy. Is there a reliable algorithm ?

02

Is the algorithm stable to spatial and time varying illumination disturbances ?

03

How to optimally align 3D objects in a noisy environment of additive nature?

04

How to do robust registration in the presence of partial overlap ?

05

How to perform this operation when there is missing data or holes ?

WHAT IS IN THE LITERATURE ?

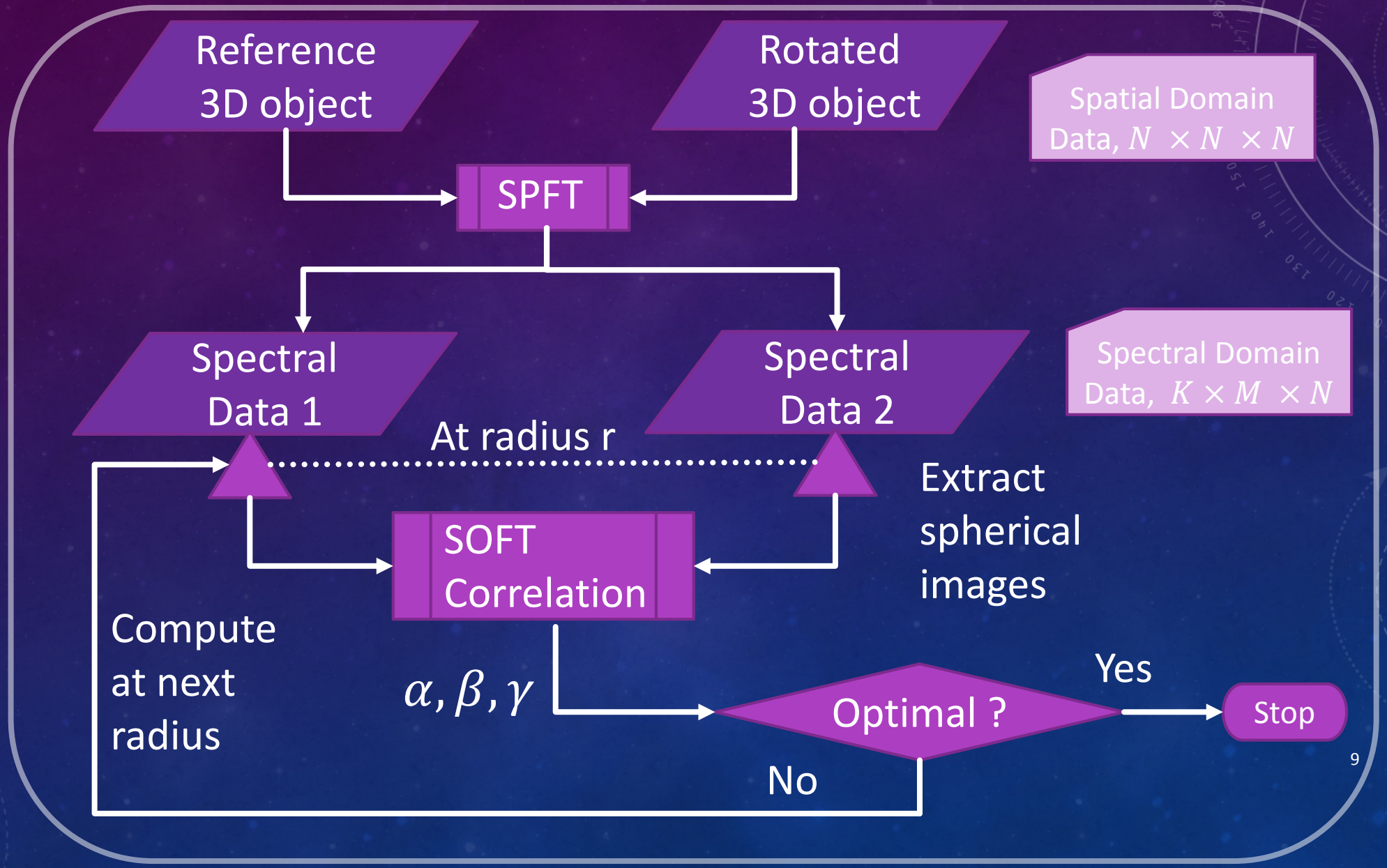
- Problem is not considered solved – still under active research.
- Due to issues like missing data, partial overlap, noisy data, and objects with intra-class variations and symmetries in 3D objects obtained from scans.
- Salah Althloothi, Mohammad H. Mahoor, and Richard M. Voyles, “A robust method for rotation estimation using spherical harmonics representation.” Image Processing, IEEE Transactions on 22, no. 6 (2013): 2306-2316.
- Bulow, Henning, and Andreas Birk, “Spectral 6DOF registration of noisy 3D range data with partial overlap.” Pattern Analysis and Machine Intelligence, IEEE Transactions on 35, no. 4 (2013): 954-969.
- Yang, Jiaolong, Hongdong Li, Dylan Campbell, and Yunde Jia, “Go-ICP: A Globally Optimal Solution to 3D ICP Point-Set Registration,” IEEE Trans. Pattern Analysis and Machine Intelligence (2016).

WHAT IS YOUR APPROACH ?

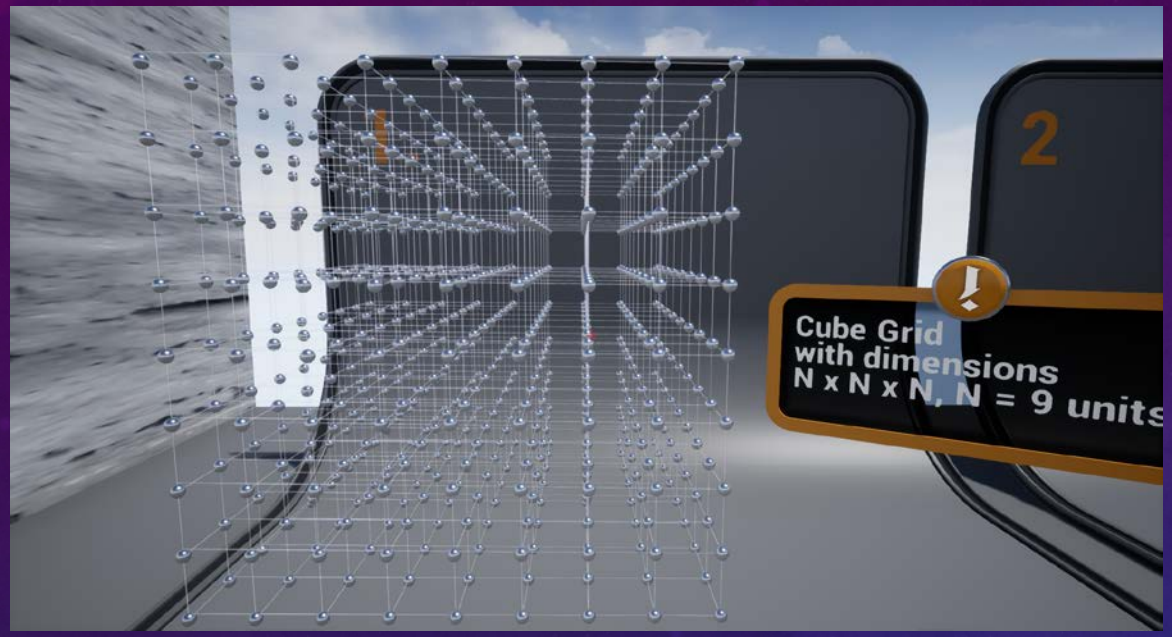
Intuitions

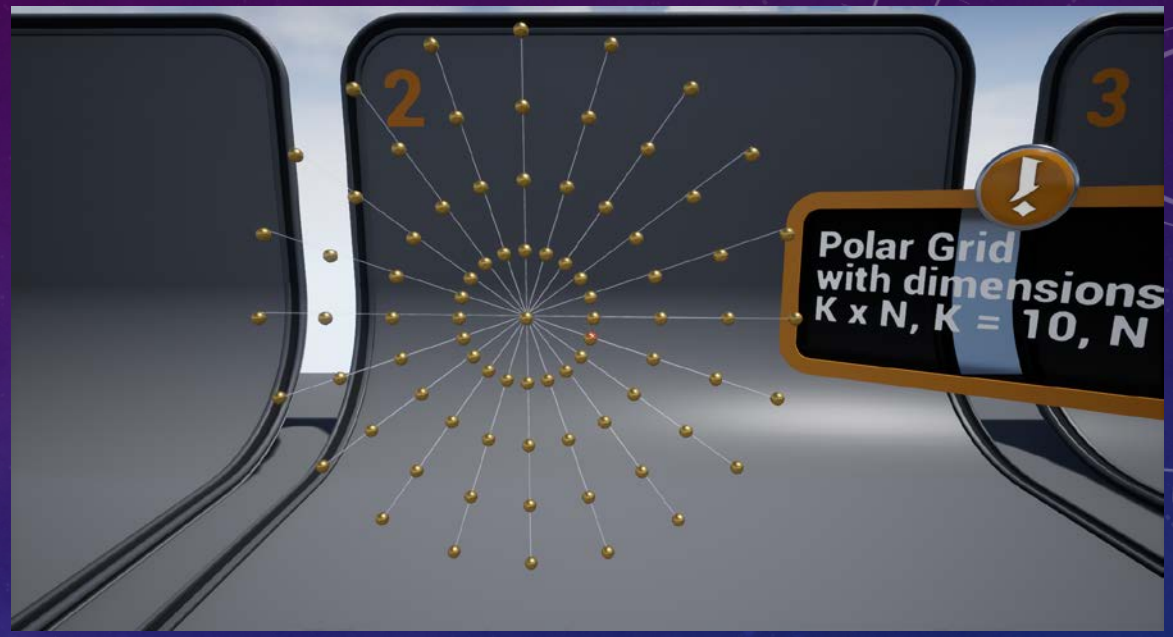
- We recently introduced an algorithm called Spherical Polar Fourier Transform (SPFT).
- SPFT has the unique advantage of computing equiangular data points in the Fourier domain.
- Spectral data at multiple radii from SPFT gives us robust rotation estimation with Spherical Harmonics based correlation
- Global Method

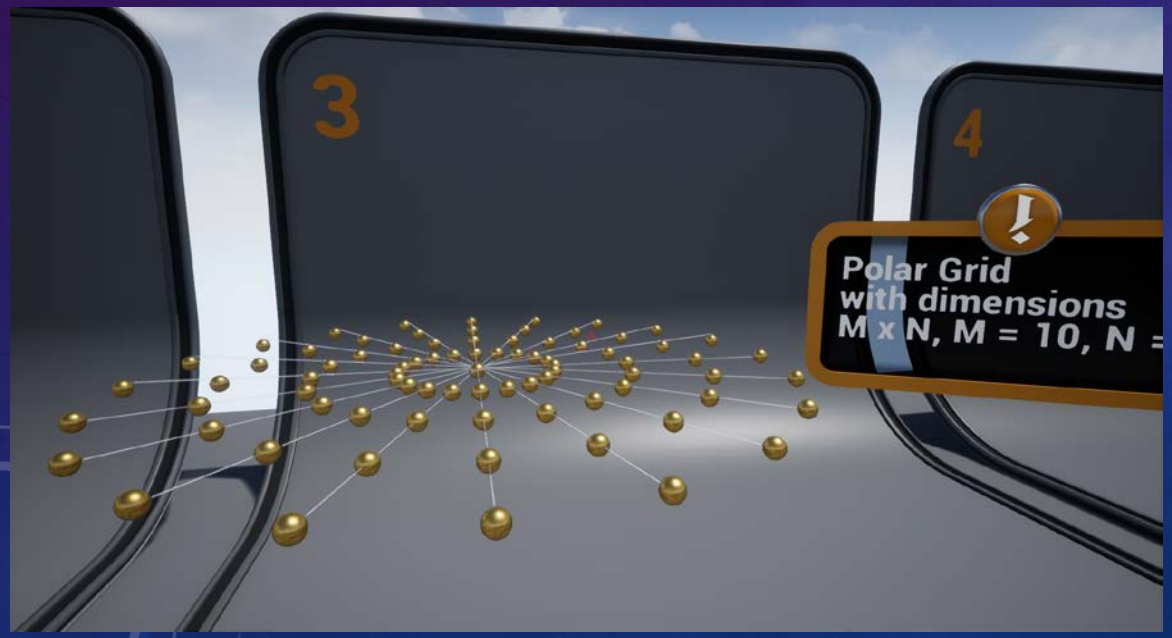
HOW DO YOU SOLVE IT ?

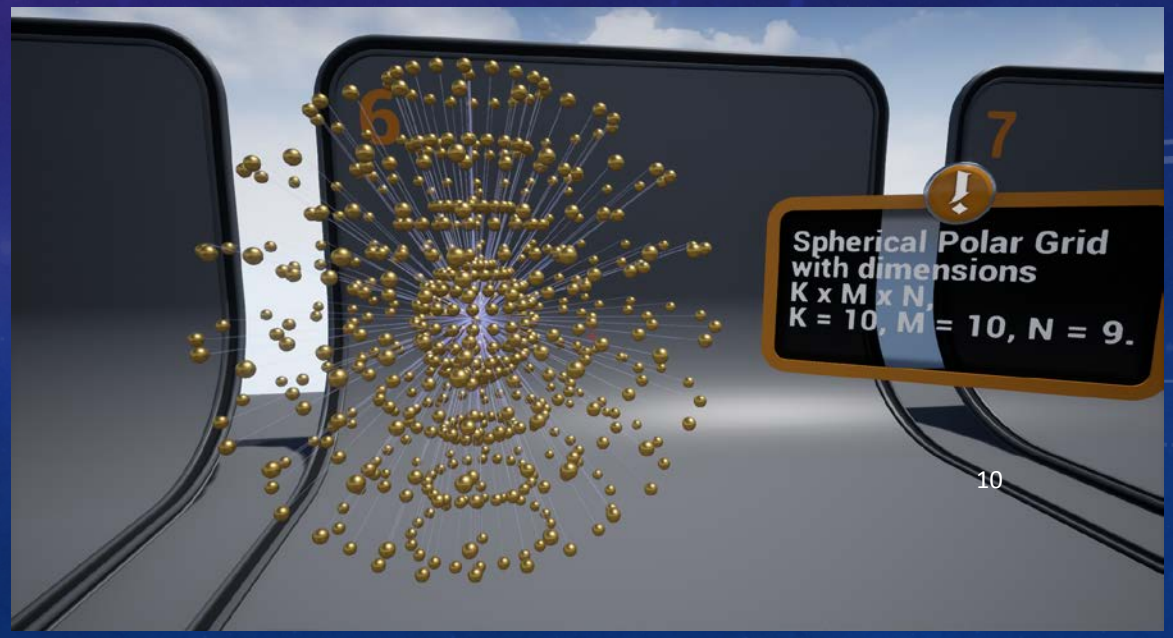


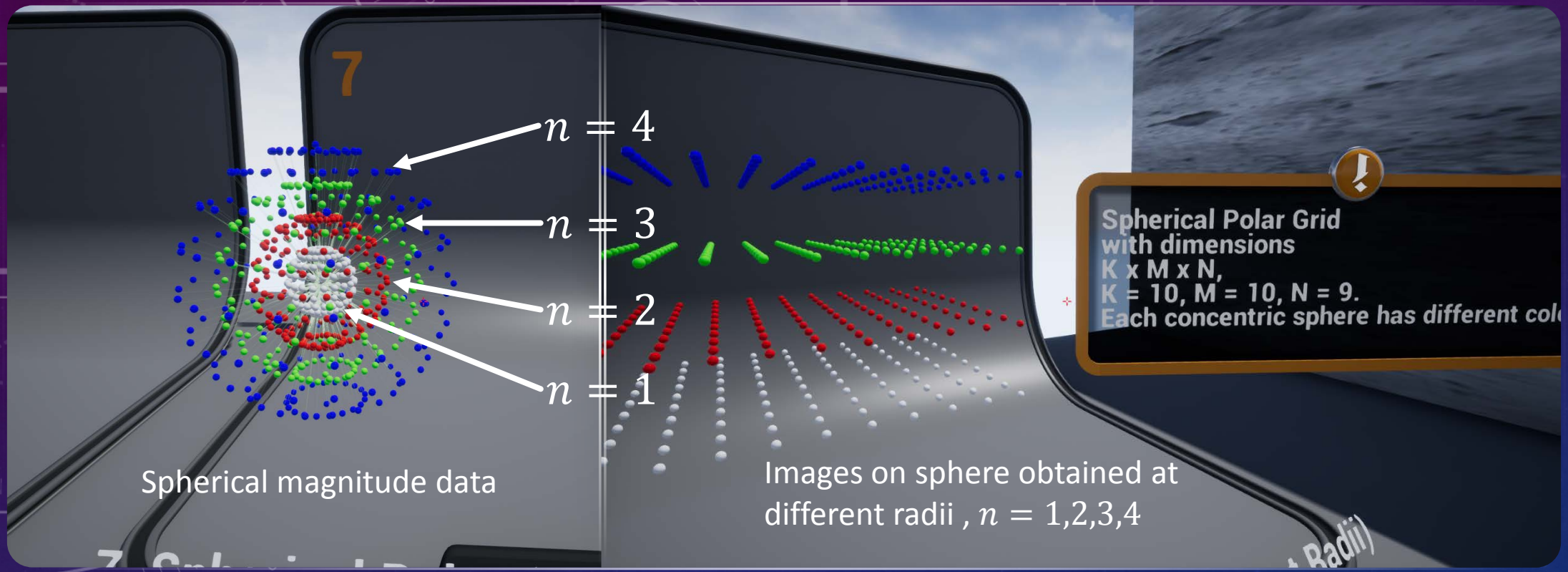
TRANSFORMATION – SPATIAL TO FOURIER DOMAIN





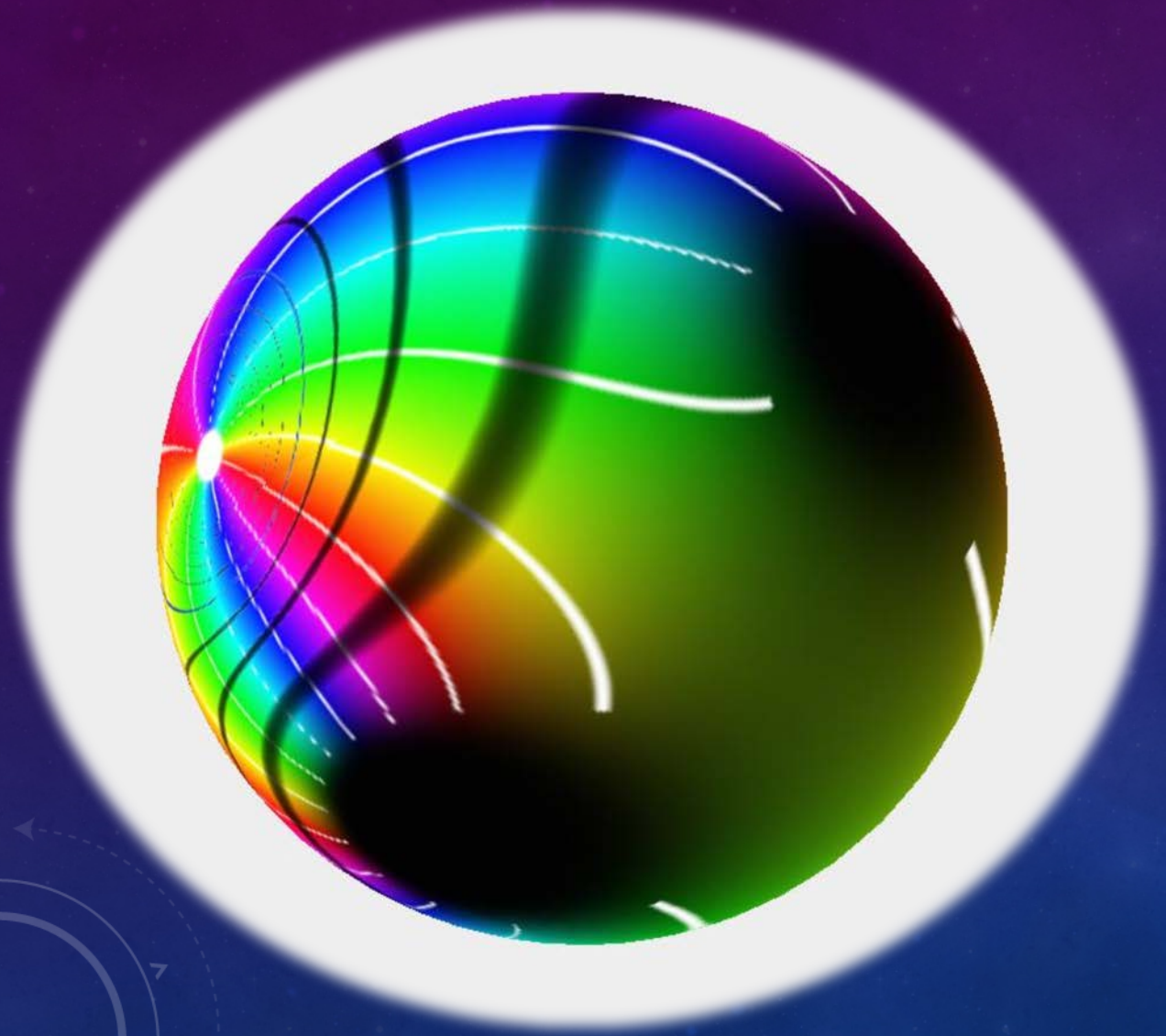




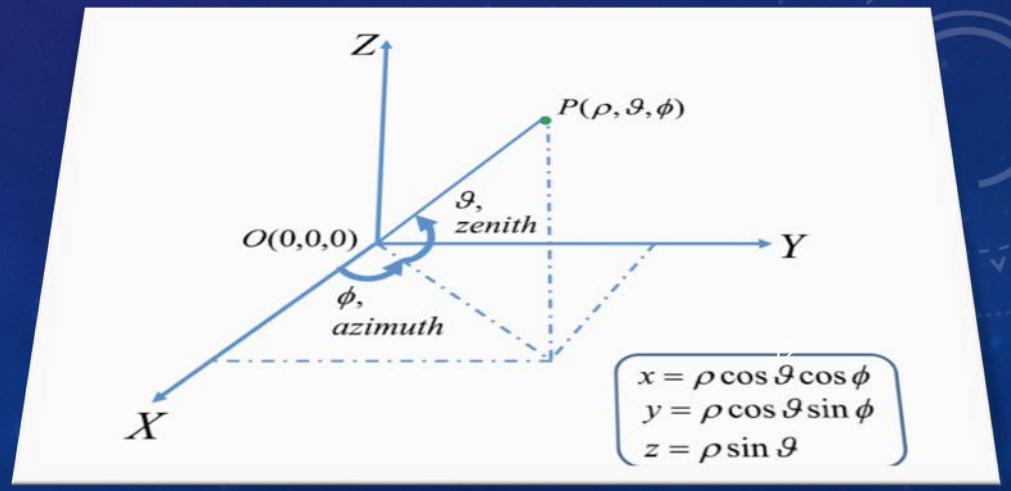


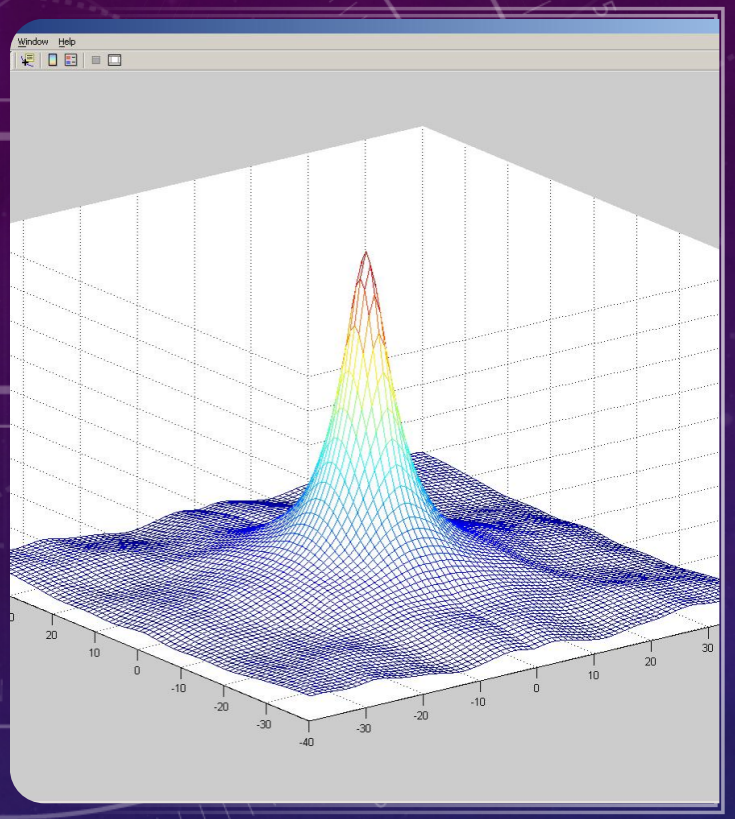
MULTI-LAYERED / MULTI-SPECTRAL APPROACH

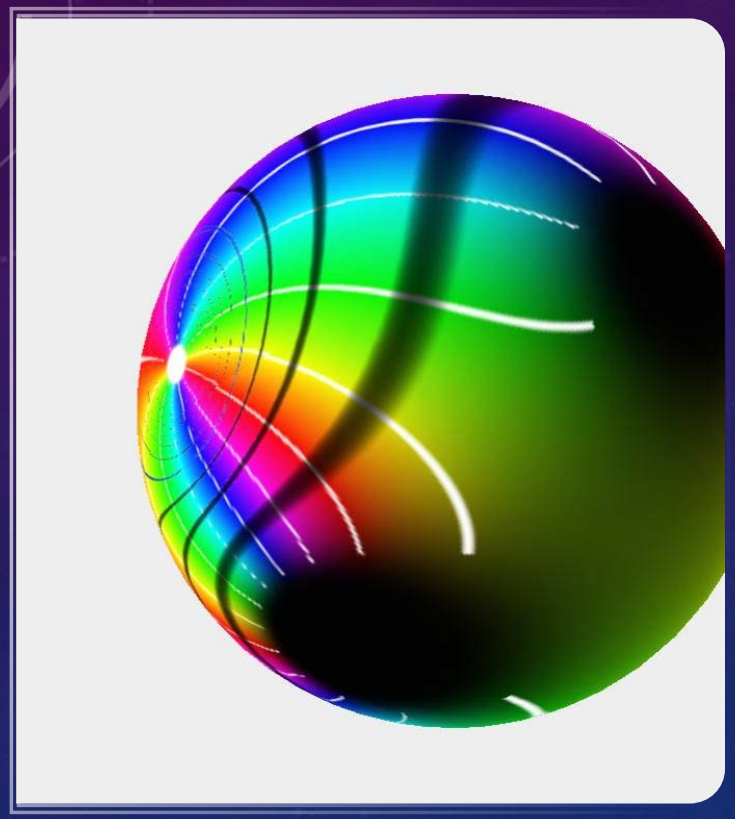
FOURIER TRANSFORM ON $SO(3)$ OR SOFT



- Fact : We have a function defined on the sphere, sampled on a regular $\mathrm{N} \times \mathrm{N}$ grid of angles of elevation and azimuth, the forward and inverse spherical harmonic transforms can be computed in $\mathrm{O}(\mathrm{N}^2 \log \mathrm{N})$.







CORRELATION OF FUNCTIONS ON SPHERES USING SOFT

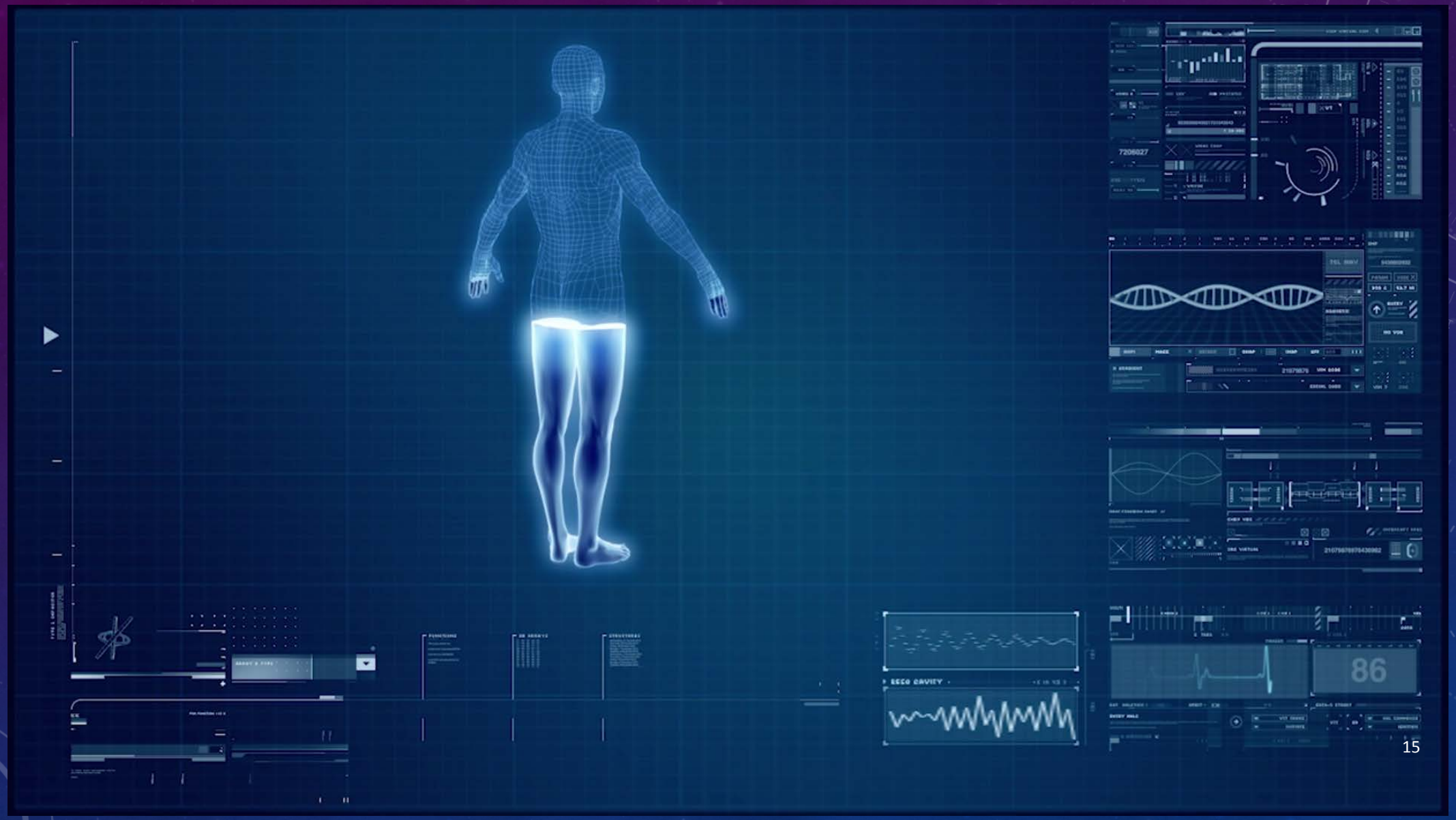
CORRELATION COMPLEXITY

The complexity of correlating two spherical functions sampled on a regular grid (equi-angular) of size $\mathrm{N} \times \mathrm{N}$ is:

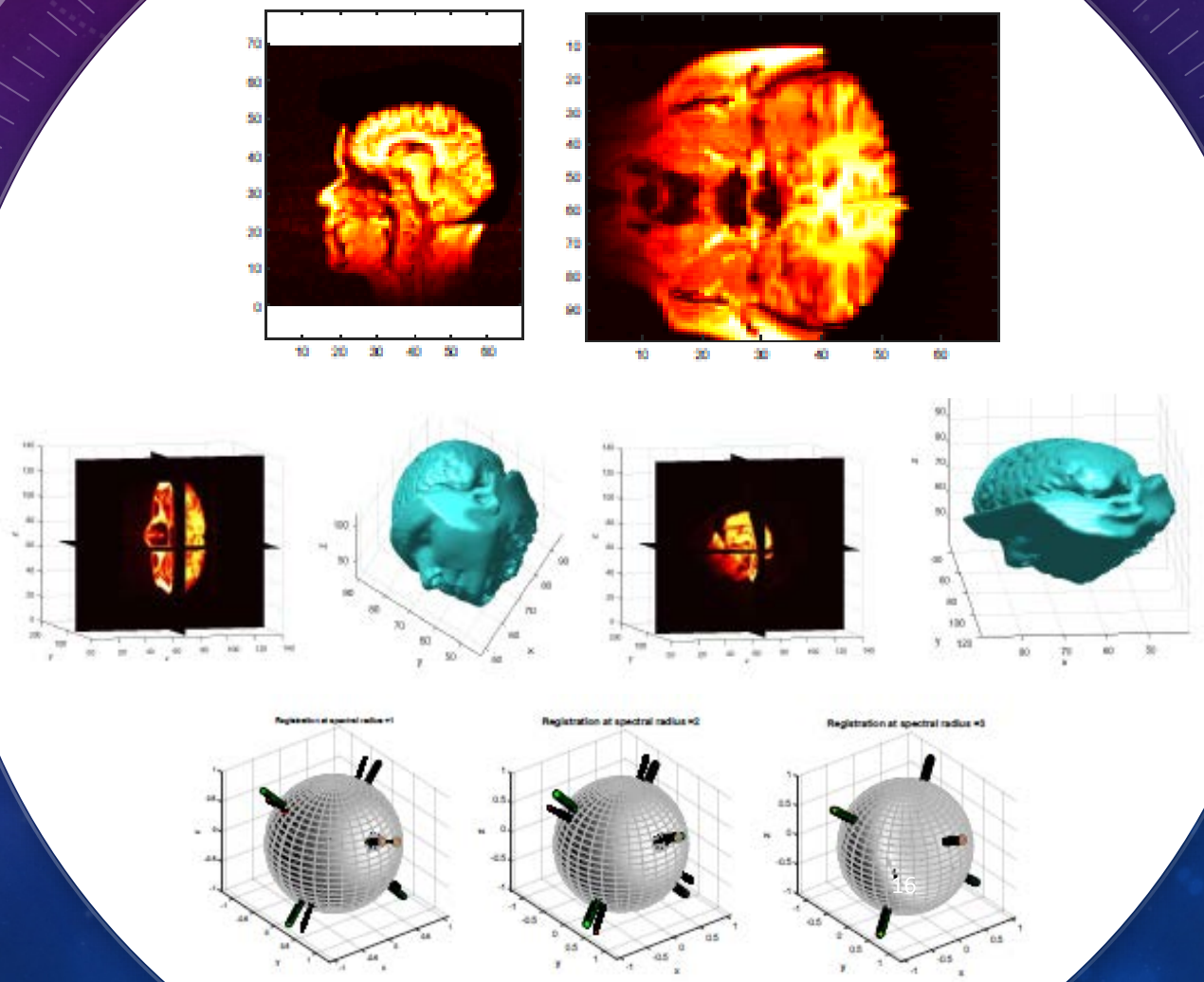
- Forward spherical harmonic transform : $\mathrm{O}(\mathrm{N}^2 \log_2 \mathrm{N})$
- Multiplying frequency terms : $\mathrm{O}(\mathrm{N}^3)$
- Inverse Wigner-D transform : $\mathrm{O}(\mathrm{N}^4)$

Total computation complexity of correlation of spherical images is : $\mathrm{O}(\mathrm{N}^4)$

BIOMEDICAL IMAGING APPLICATIONS – 3D

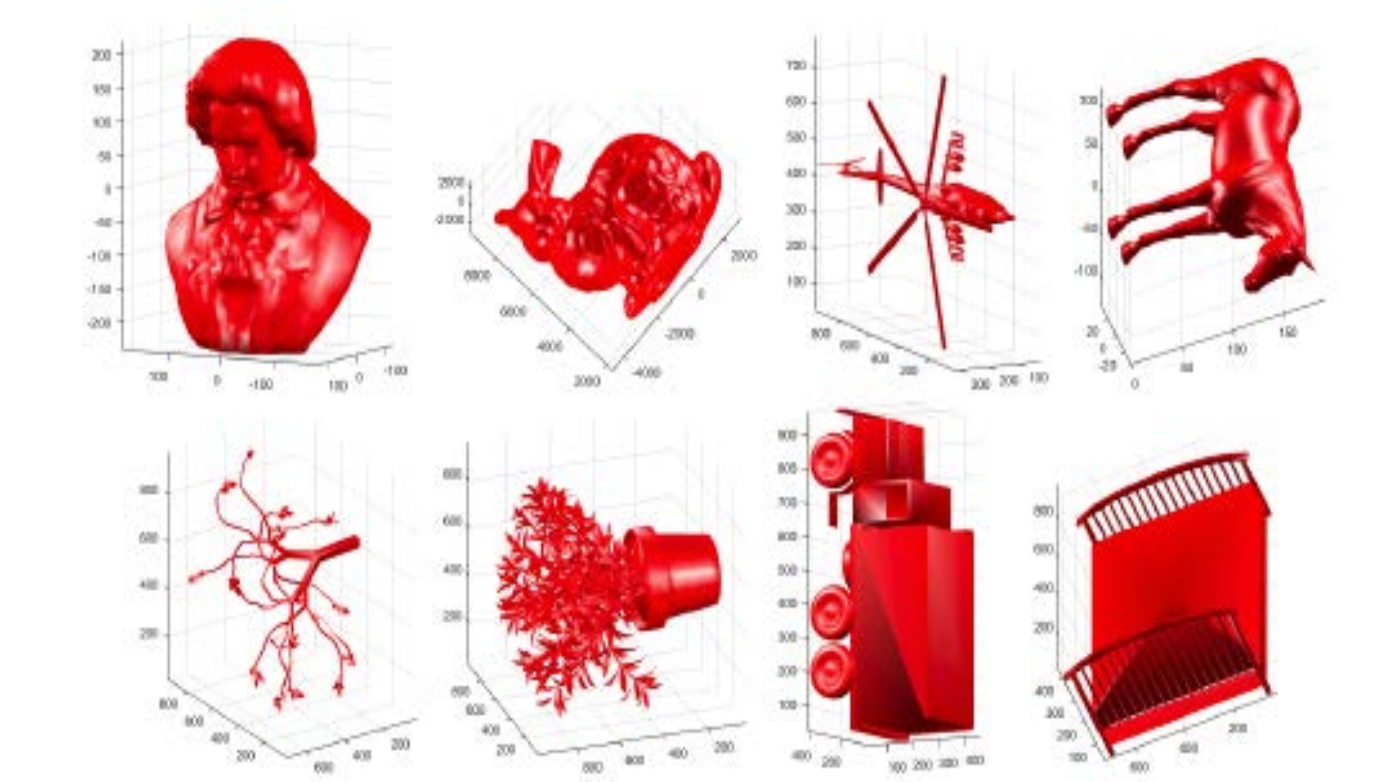


3D ROTATION ESTIMATION – MRI DATASET

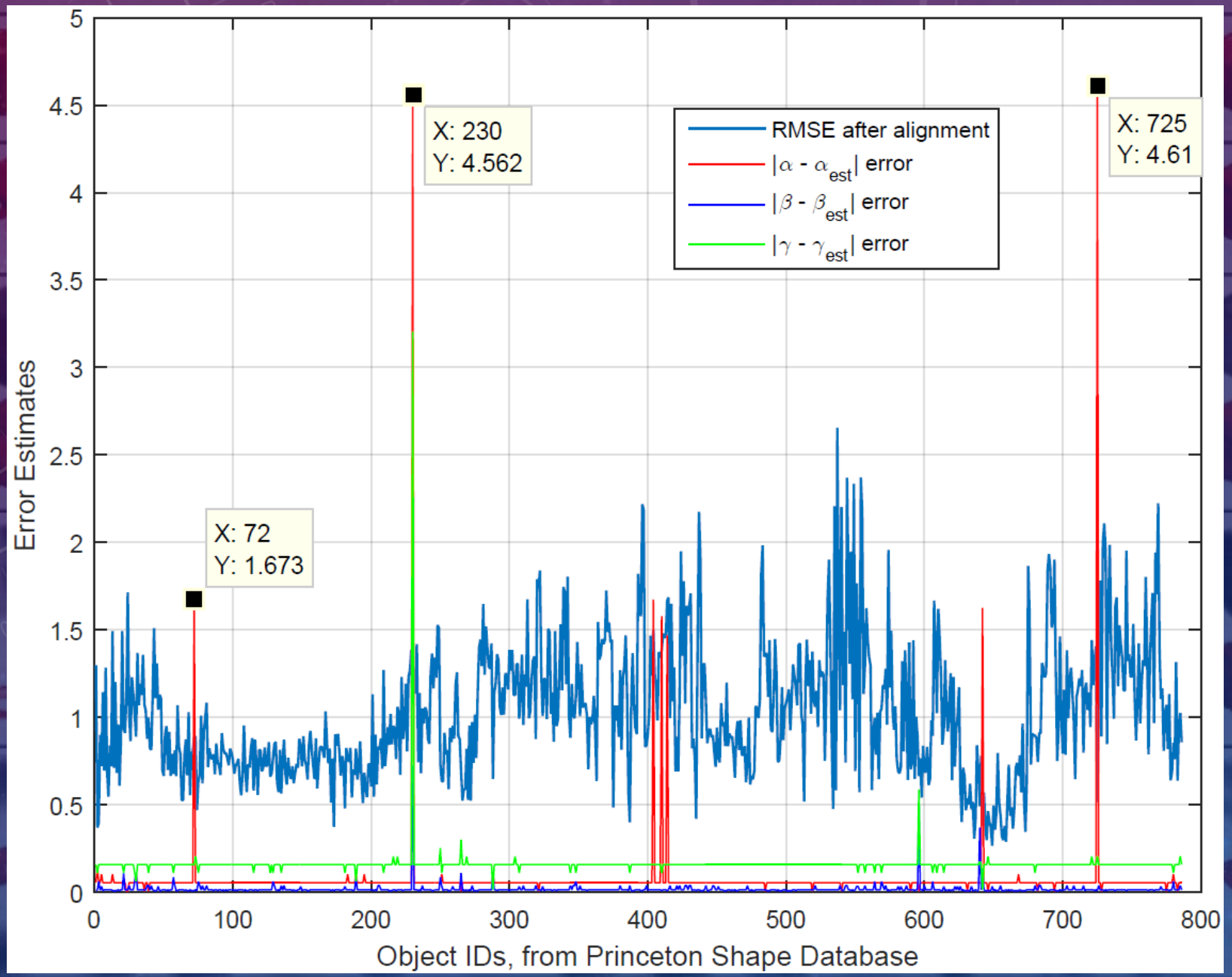


PRINCETON SHAPE BENCHMARK DATASET

- Large database for 3D models
- Hundreds of 3D models collected from different web domains
- Objects are natively encoded as vertices and faces
- Voxelizations required for the use of our method

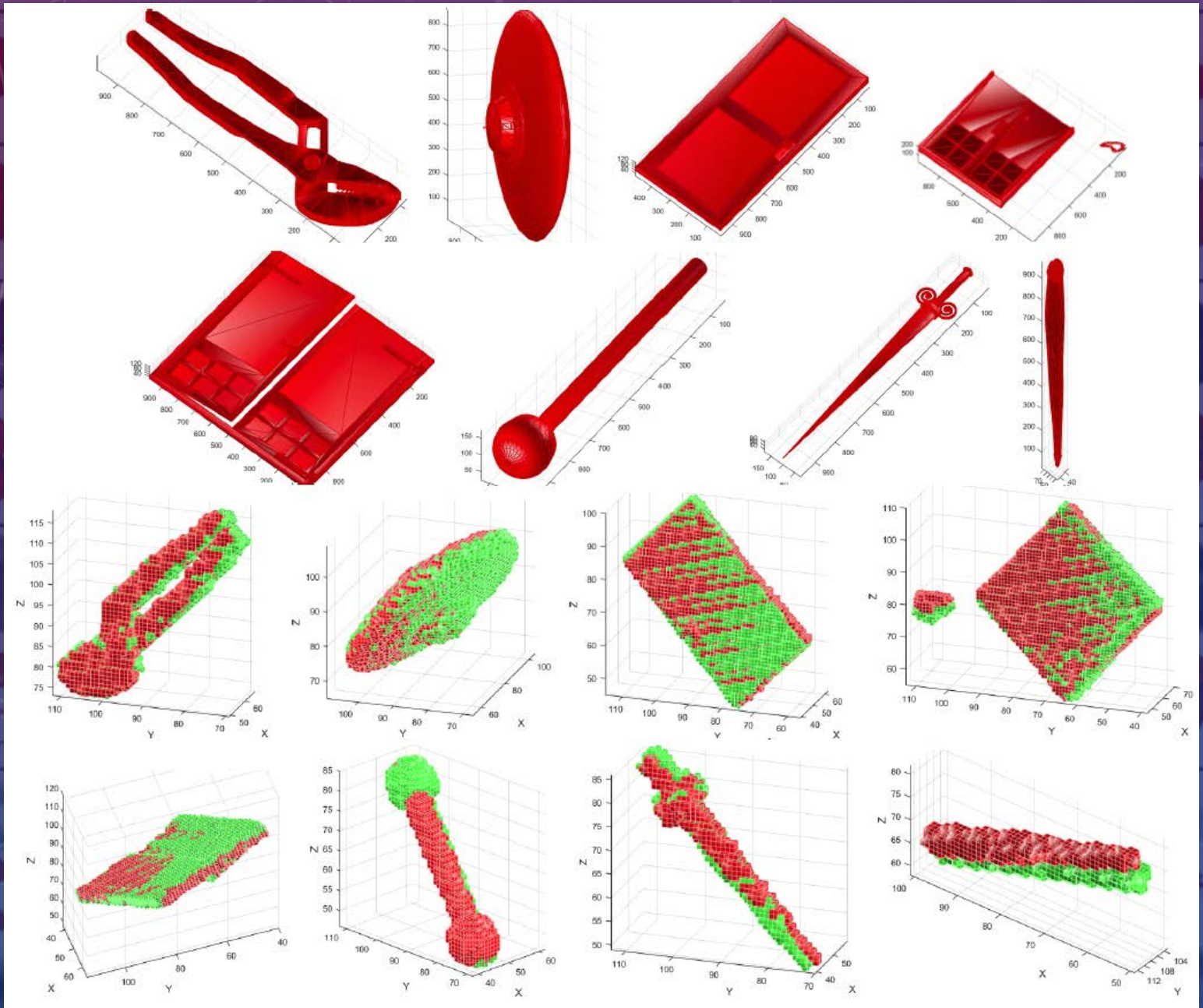


PRINCETON SHAPE BENCHMARK DATASET



PRINCETON SHAPE BENCHMARK DATASET

- Anomalies due to $\pi/2\pi$ ambiguity
- Mapping spectrum of elongated and flat objects leads to ambiguity
- Intensity shape & information helps



CRASHED CAR DATASET – CAPTURED BY A DISASTER ROBOT

EXPERIMENTAL RESULTS AND CONCLUSIONS

Table I: Effects of perturbation on the surface vertices.

SNR(dB)	$\lvert\alpha\rvert_{\text{error}}$	$\lvert\beta\rvert_{\text{error}}$	$\lvert\gamma\rvert_{\text{error}}$	RMSE
10	0.1861	0.0893	0.2547	0.1893
15	0.0858	0.0558	0.1210	0.0915
20	0.0144	0.0057	0.0542	0.0325
30	0.0479	0.0057	0.0207	0.0303
35	0.0190	0.0057	0.0207	0.0165
40	0.0144	0.0057	0.0542	0.0325

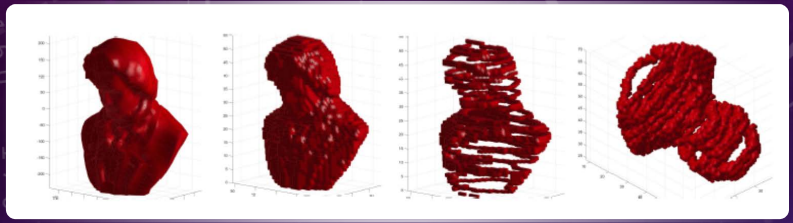
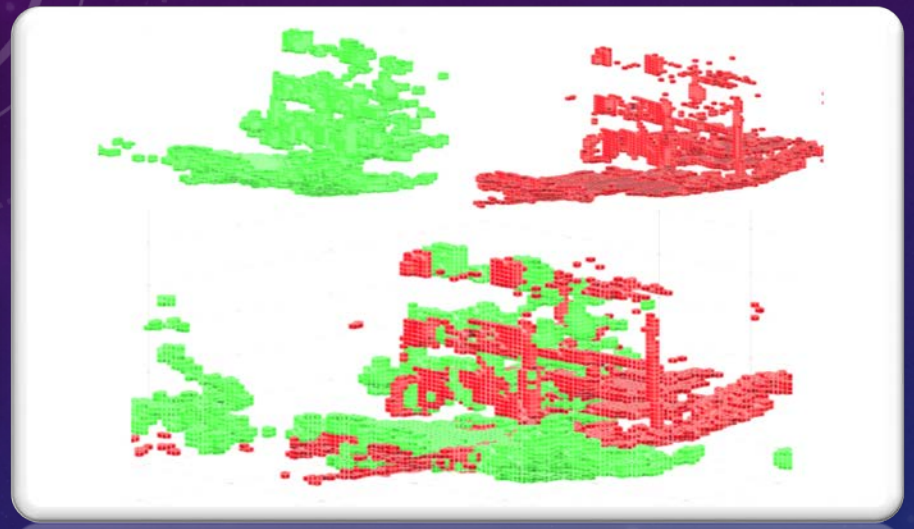


Table II: Performance in case of holes/missing or partial data.

Method	$> 80\%$	$> 60\%$	$> 40\%$	$> 30\%$	$> 20\%$
SPFT+	0.0980	0.0980	0.1091	0.1030	0.1872
RESH	0.0045	0.6832	1.4060	1.0378	1.4637



CONTRIBUTIONS

- The proposed solution is extremely robust to noise, partial overlap or missing data.
- Automatic technique, no hole-filling, de-noising, or interpolations needed.
- Superior performance in terms of registration compared to three state of the art methods.

KEY REFERENCES

- Syed Alam Abbas, Qiyu Sun, and Hassan Foroosh, "An Exact and Fast Computation of Discrete Fourier Transform for Polar and Spherical Grid", IEEE Transactions on Signal Processing, Volume: 65, Issue: 8, April 15, 2017.
- Syed Alam Abbas, and Hassan Foroosh, "Robust 3D Registration using Spherical Polar Fourier Transform and Spherical Harmonics", IEEE Transactions on Pattern Analysis and Machine Intelligence, Submitted on October 16, 2016.

SUBMITTED TO THE JOURNAL OF IEEE TRANS. ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE.

Robust 3D Registration using Spherical Polar Fourier Transform and Spherical Harmonics

Syed Alam Abbas, *Student Member, IEEE* and Hassan Foroosh, *Senior Member, IEEE*

***Abstract*—The analysis of affine transformations in the frequency domain exploits the well-known property whereby the translational displacement in this domain can be factored out and separately estimated from its affine matrix. This paper presents a new and effective method for estimating up to 7 degrees of freedom (DOF), i.e. similarity transformation, for 3D volume registration using Fourier transformed data. The proposed method provides a closed-form solution for the 3D rotation estimation problem, which is the most difficult part in a 3D similarity transformation, without any recurrence relations or searching for point correspondences between the two volumes/objects. We present the spectral registration technique using a fast and algebraically exact discrete 3D spherical polar Fourier transform, which gives us the spectral data defined on the spheres at multiple radii. Complete stacks of such layers are processed to find the best 3D rotation parameters using correlations on spheres with spherical harmonics, making it very accurate and robust to large rotations, noise, holes, missing data, and partial overlap. We compare the proposed technique with the state-of-the-art methods for rotation estimation and demonstrate the effectiveness of our solution with extensive experimentation, including a set of scanned MRI images, a crashed car parking dataset, and the Princeton shape benchmark dataset with hundreds of 3D objects.**

***Index Terms*—3D Registration, 3D Rotation Estimation, Non-uniform Fast Fourier Transform, Spherical Harmonics, Affine Transformations, 3D Spherical Polar Fourier Transform, 2D Polar Fourier Transform, Spherical Correlations, Phase Correlation, Sub-pixel registration.**

I. Introduction

Image or data registration is the process of spatially aligning two or more datasets of an object or a scene. The goal is to estimate the most optimal geometric transformation between 3D point clouds or volumes acquired by different sensors or from different viewpoints, with the goal of enabling the fusion of information and/or the estimation of the motion of sensors.

Registration is a widely studied problem that still remains an important topic of research, with a diverse range of algorithms and methods proposed in the literature for volume/point-set registration, see [1]-[14], and references therein.

It is, for instance, crucial for object pose alignment in computer vision and used for registration of volumetric data in medical imaging. It is also used for a wide range of tasks such as object grasping, manipulation, object identification, self-localization, the interpolation of trajectory of 3D orientations, robot kinematics, flight simulation, structure from motion, 3D pose recovery of objects, and motion capture in computer vision, and robotics.

The algorithms proposed in the literature could be classified into *spatial domain* and *frequency domain* methods depending on whether the registration algorithm is operated using spatial or frequency information. While the frequency domain methods are mostly global feature-less registration algorithms, the spatial domain methods can be further classified into *local* and *global* feature based methods. In this paper, we focus only on the global frequency domain methods.

A vast amount of literature is present for registration techniques in the Fourier transformed domain. The Fourier spectrum information acts as a global, feature-less and robust descriptor [15] of a signal (e.g. 3D volume), it has the following advantages over the spatial registration algorithms:

- **Robustness**: Fourier based methods are robust and are known to be more stable to additive noises, distortions, spatial and time varying illumination disturbances, since they use all the information available rather than the content associated with only a sets of features.
- **Reliability**: Compared to feature based methods, they are less sensitive to any or specific control parameters. In a sense, the parameters in Fourier based registration techniques determine only the runtime and precision.
- **Seperability**: As it is well known, due to the nature of the FT, we can achieve full separability of affine matrix and the translation parameters of a complete affine transformation, which is not possible with spatial domain techniques, see the 2D affine theorem in [16], [17] and the *Fourier shift* theorem.
- **Speed**: They have a low computational complexity, require no initializations, and take a fixed time period for registration.
- **Versatility**: The phase correlation technique known as the phase-only matched filter (POMF) [18]-[20], is adopted in frequency based registration algorithms to deal with partially overlapped datasets [6].

For a Euclidean 3D transformation between the two volumes/objects, the problem of estimating the rotation between two 3D objects is still under active research since issues such as missing data, partial overlap, noisy data, and objects with intraclass variations and symmetries have not been completely solved [21]. Some techniques are better suited to deal with only surface data [21], while others are designed for a full volumetric data [22]. In this paper, we focus on the volume data since surface data can be considered a subset of it. In 3D robotic mapping, the localization of the poses of the 3D

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Manuscript received October 10, 2016.

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POLAR FOURIER TRANSFORM – 2D

$f(r,c)$ is the 2D matrix with r, c as rows and cols.

$$F(m,n) = \sum_{c=-\frac{N}{2}}^{\frac{N}{2}} \sum_{r=-\frac{N}{2}}^{\frac{N}{2}} f(r,c) e^{-i\frac{2\pi n}{N+1}(r\cos\theta_m + c\sin\theta_m)},$$

where,

$-\frac{N}{2} \leq n \leq \frac{N}{2} \rightarrow$ radial spacing,

$\theta_m = m\Delta\theta, 0 \leq m \leq M-1, \Delta\theta = \frac{180^0}{M} \rightarrow$ angular spacing

SPHERICAL POLAR FOURIER TRANSFORM – 3D

$f(r,c,d)$ is the 3D matrix with r, c, d as rows, cols and depths.

$$F(k,m,n) = \sum_{d=-\frac{N}{2}}^{\frac{N}{2}} \sum_{c=-\frac{N}{2}}^{\frac{N}{2}} \sum_{r=-\frac{N}{2}}^{\frac{N}{2}}$$

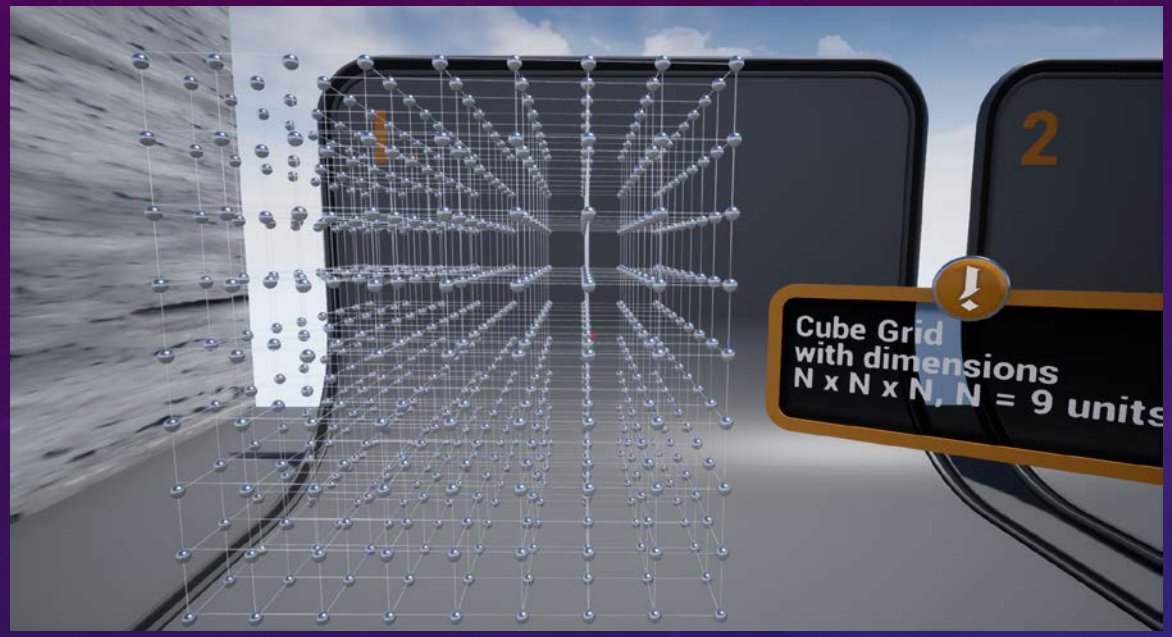
$$f(r,c,d)e^{-i\frac{2\pi n}{N+1}(r\cos\theta_k\cos\varphi_m + c\cos\theta_k\cos\varphi_m + d\sin\theta_k)},$$

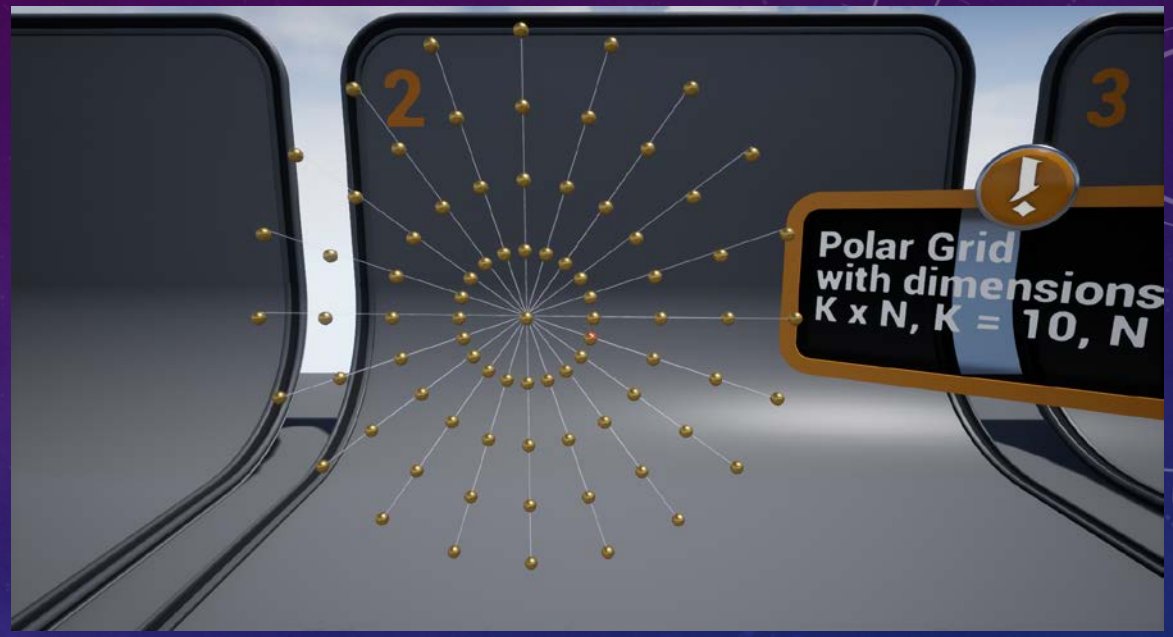
where, $-\frac{N}{2} \leq n \leq \frac{N}{2} \rightarrow$ radial spacing,

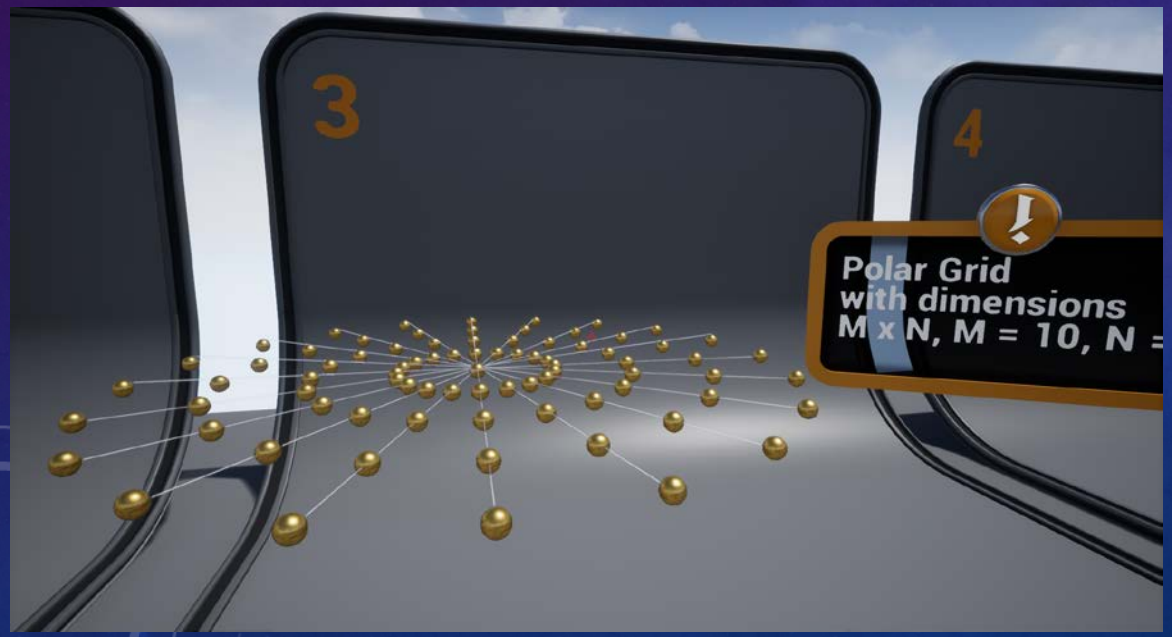
$$\theta_k = k\Delta\theta, 0 \leq k \leq K-1, \Delta\theta = \frac{180^0}{K} \rightarrow \text{zenith angles}$$

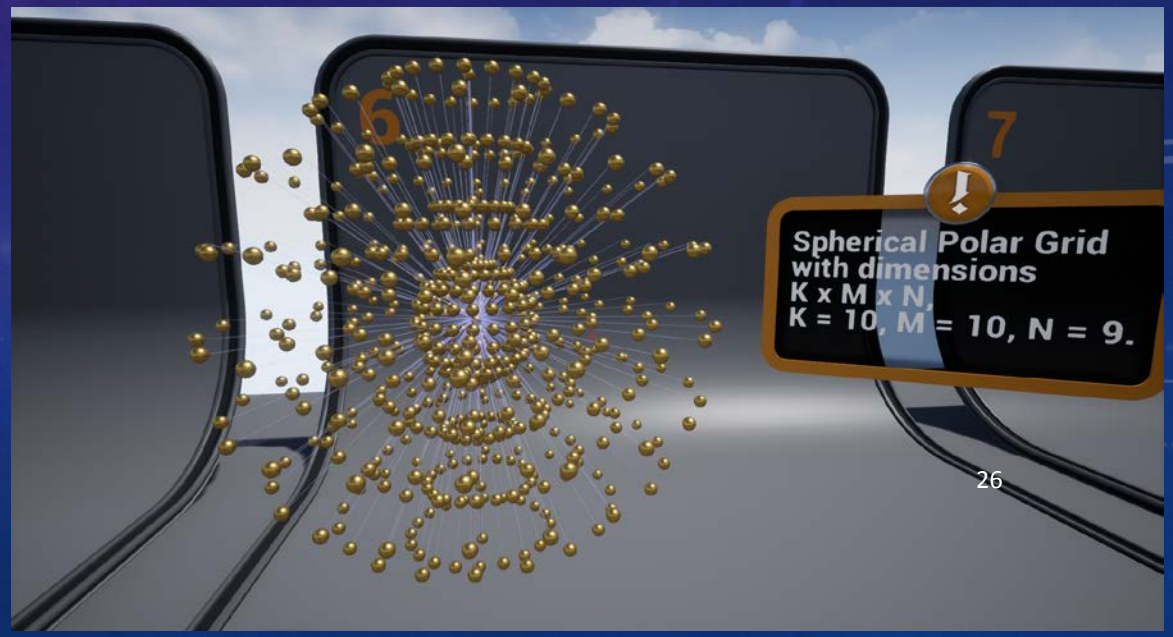
$$\varphi_m = m\Delta\varphi, 0 \leq m \leq M-1, \Delta\varphi = \frac{180^0}{M} \rightarrow \text{azimuth angles}$$

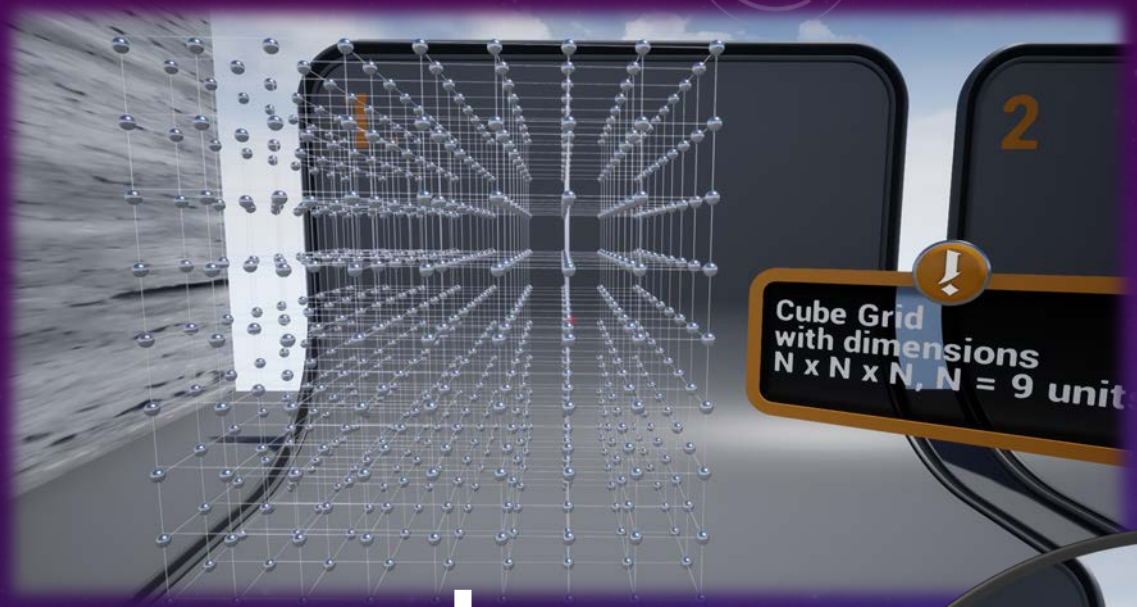
TRANSFORMATION – SPATIAL TO FOURIER DOMAIN



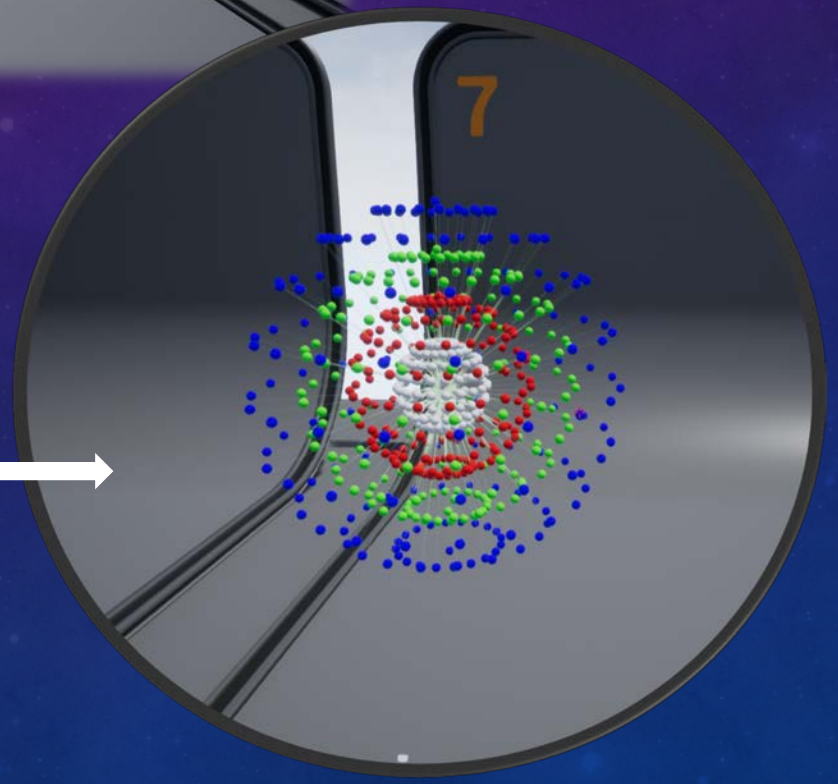








Special Fourier Transform



WHAT IS THE PROBLEM ?

- Given 2D/3D matrix of spatial information find the corresponding Fourier domain info – 2D PFT / 3D SPFT.

WHY IS THE PROBLEM HARD ?

01

On the face of it the problem looks relatively easy. Is there a fast and exact algorithm ?

02

Is there a true discrete analog to its continuous form transformation?

03

What about the inverse, is there a good algorithm?

04

What about the speed and accuracy of the forward and inverse transforms?

05

Is there some conditioning required or special structure of the matrix to be exploited ?

WHAT IS IN THE LITERATURE ?

- Prevailing wisdom was that there exists no fast and exact solution to the problem of such nonuniform Fourier transforms. So build your own …
- Numerous attempts can be seen in the literature trying to improve its accuracy through more and more sophisticated approximation procedures.
- A. Averbuch, R. R. Coifman, D. L. Donoho, M. Israeli, and J. Walden, “Fast slant stack: A notion of Radon transform for data in a Cartesian grid which is rapidly computible, algebraically exact, geometrically faithful and invertible,” Dept. Statist., Stanford Univ., Stanford, CA, USA, 2001 – later in SIAM.
- A. Averbuch, R. R. Coifman, D. L. Donoho, M. Elad, and M. Israeli, “Fast and accurate polar Fourier transform,” Appl. Comput. Harmon. Anal., vol. 21, no. 2, pp. 145–167, 2006.
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WHAT IS IN THE LITERATURE ?

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- A. Averbuch, R. R. Coifman, D. L. Donoho, M. Israeli, and Y. Shkolnisky, “A framework for discrete integral transformations I—The pseudopolar Fourier transform,” SIAM J. Sci. Comput., vol. 30, no. 2, pp. 764–784, 2008.
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- J. Miao, F. Frster, and O. Levi, “Equally sloped tomography with oversampling reconstruction,” Phys. Rev. B, vol. 72, no. 5, 2005, Art. no. 052103.

WHAT IS IN THE LITERATURE ?

Dave Donoho

Statistics Department Stanford

Amir Averbuch

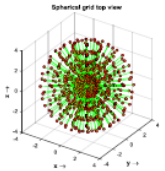
CS Department Tel-Aviv University

Ronald Coifman

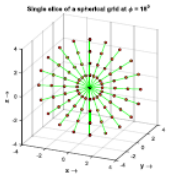
Math. Department Yale

HOW DO YOU SOLVE IT ? – USE 1D FRFT + D&C

Spherical Grid

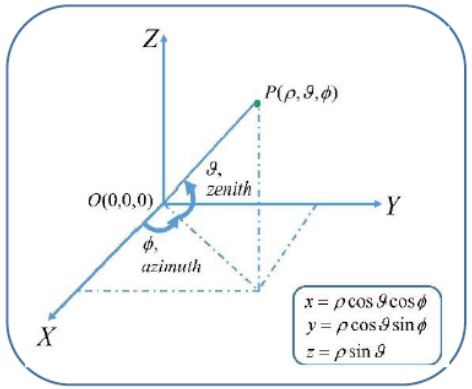


Level q Polar slice

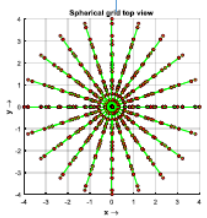


$q\Delta\phi, azimuth$

Spherical coordinates



$(q\Delta\phi, 180-q\Delta\phi)$



$(90-q\Delta\phi, 90+q\Delta\phi)$

X-axis oriented slices

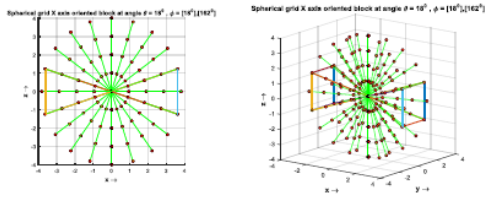
Y-axis oriented slices

Level p lines

$p\Delta\theta, zenith$

XX block

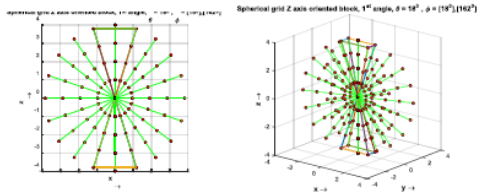
$(p\Delta\theta, 180-p\Delta\theta)$



$$\alpha_{xx}^{pq} = \cos(p\Delta\theta)\cos(q\Delta\phi)$$
$$\beta_{xx}^{pq} = \cos(p\Delta\theta)\sin(q\Delta\phi)$$
$$\gamma_{xx}^{p} = \sin(p\Delta\theta)$$

XZ block

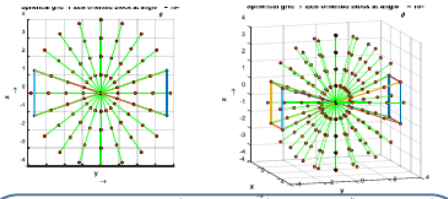
$(90-p\Delta\theta, 90+p\Delta\theta)$



$$\gamma_{xz}^{p} = \cos(p\Delta\theta)$$
$$\alpha_{xz}^{pq} = \sin(p\Delta\theta)\cos(q\Delta\phi)$$
$$\beta_{xz}^{pq} = \sin(p\Delta\theta)\sin(q\Delta\phi)$$

YY block

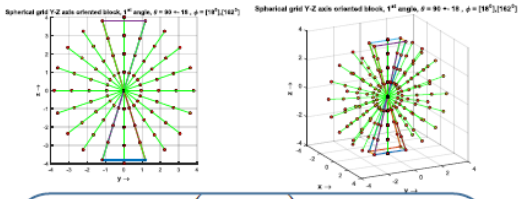
$(p\Delta\theta, 180-p\Delta\theta)$



$$\beta_{yy}^{pq} = \cos(p\Delta\theta)\cos(q\Delta\phi)$$
$$\alpha_{yy}^{pq} = \cos(p\Delta\theta)\sin(q\Delta\phi)$$
$$\gamma_{yy}^{p} = \sin(p\Delta\theta)$$

YZ block

$(90-p\Delta\theta, 90+p\Delta\theta)$



$$\gamma_{yz}^{p} = \cos(p\Delta\theta)$$
$$\alpha_{yz}^{pq} = \sin(p\Delta\theta)\sin(q\Delta\phi)$$
$$\beta_{yz}^{pq} = \sin(p\Delta\theta)\cos(q\Delta\phi)$$

COMPARISON RESULT

- Our complexity :

$$O\left(\frac{M}{4} \times N^2 \log_2 N\right)$$

- State of the art NFFT complexity :

$$O\left((\sigma N)^2 \log_2(\sigma N) + \rho^2 MN\right)$$

where,

$\rho =$ Kaiser-Bessel func cutoff

$\sigma \geq 2$, oversampling

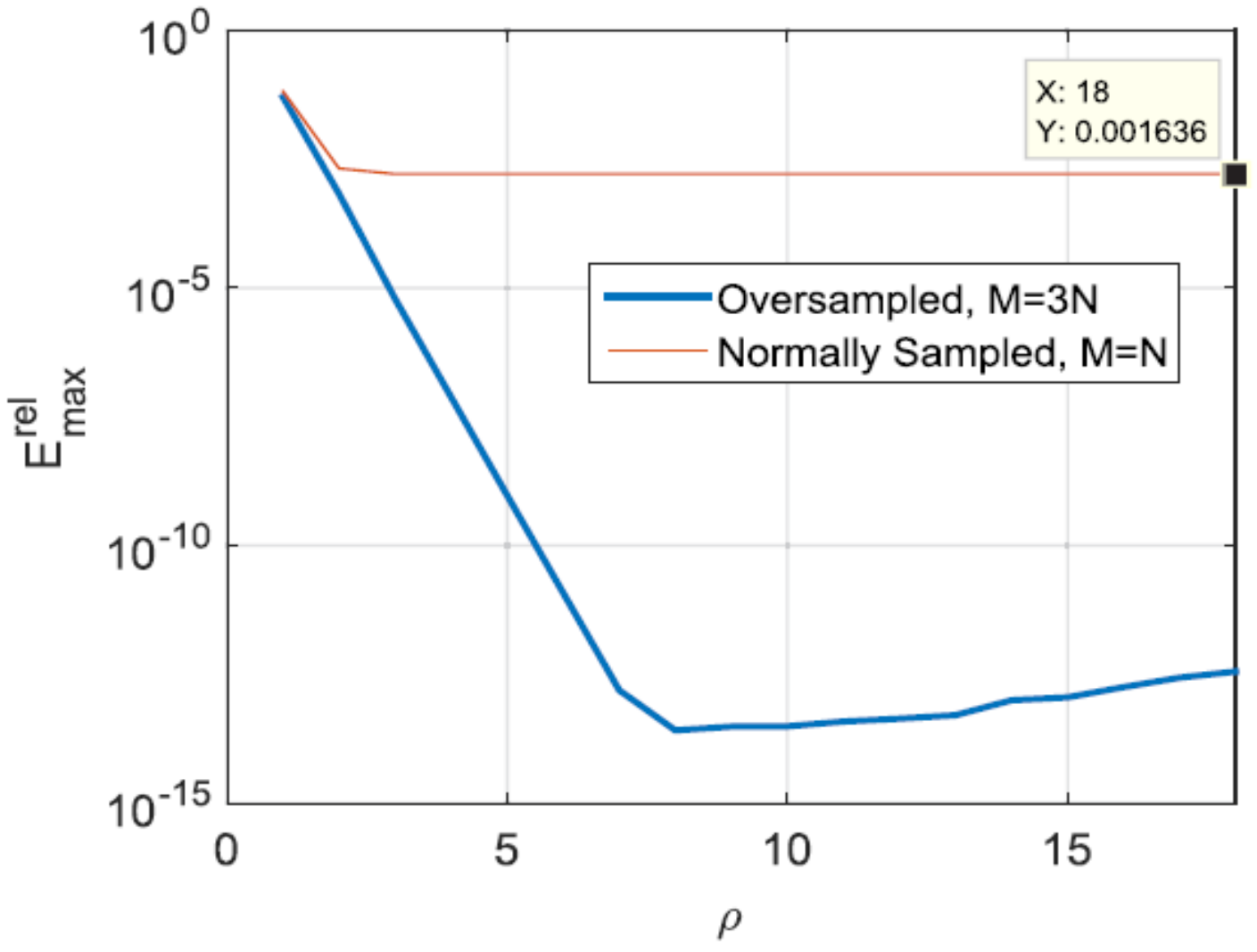


Fig. 5. In this simulation, the accuracy and computational complexity of NFFT are compared with the proposed algorithm for the polar grid.

NORMALLY SAMPLED COMPARISON RESULT

- Consider for $M, N = 128$, our relative accuracy is 10^{-14}, compared to 10^{-3}.

- Our complexity : $O\left(\frac{M}{4} \times N^2 \log_2 N\right)$,

$$O(32 \times 128^2 \log_2 128) = O(224 \times 128^2)$$

- State of the art NFFT complexity : $O\left((\sigma N)^2 \log_2(\sigma N) + \rho^2 MN\right)$,

$$O(4(128)^2 \log_2(256) + 18^2 \times 128^2)$$
$$= O(324 \times 128^2 + Const)$$

where,

$\rho =$ Kaiser-Bessel function cutoff parameter

$\sigma \geq 2$, oversampling

INVERSE SPFT/PFT - ANALYSIS

The proposed PFT / SPFT can be written as a simple linear operator

$$A_p x = y$$

Let A_p^H represent the corresponding adjoint linear operator.

$$A_p^H \check{y} = \check{x}$$

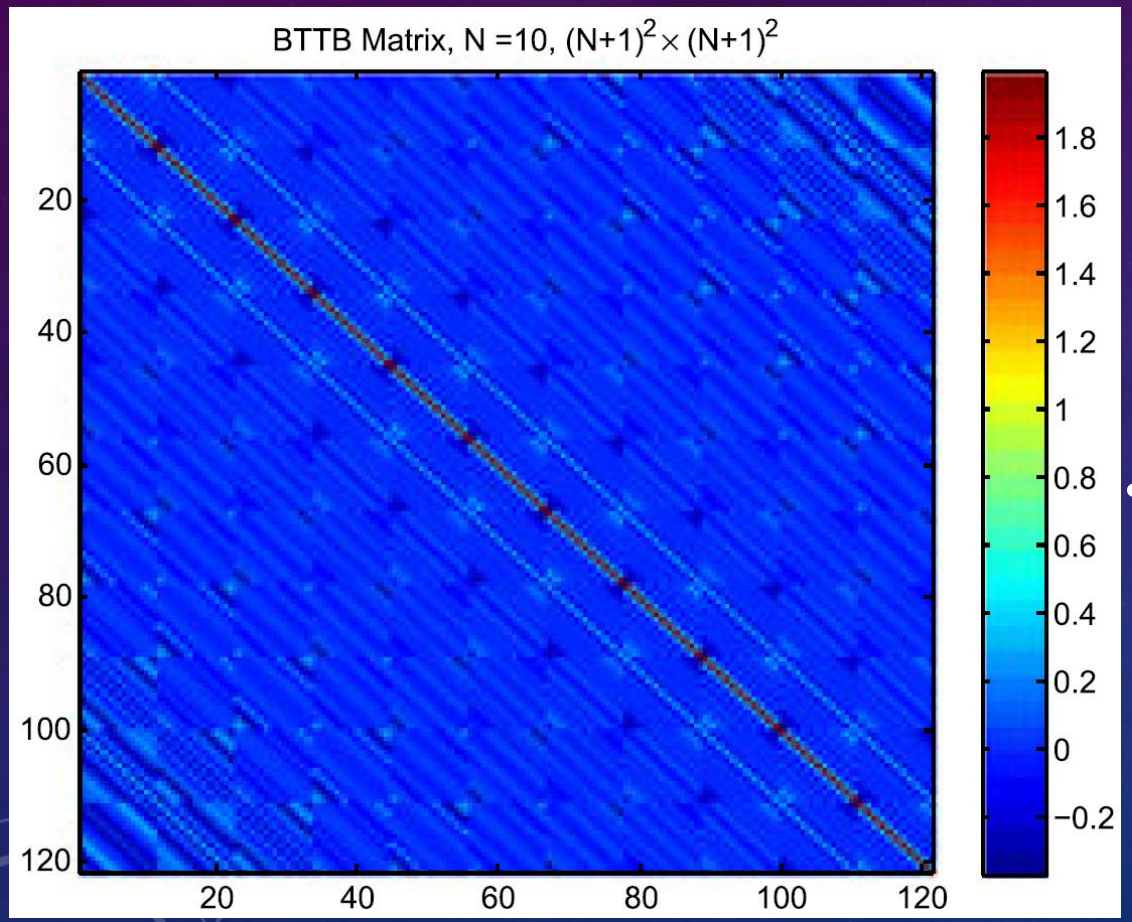
Then the inverse is computed by simply applying the adjoint.

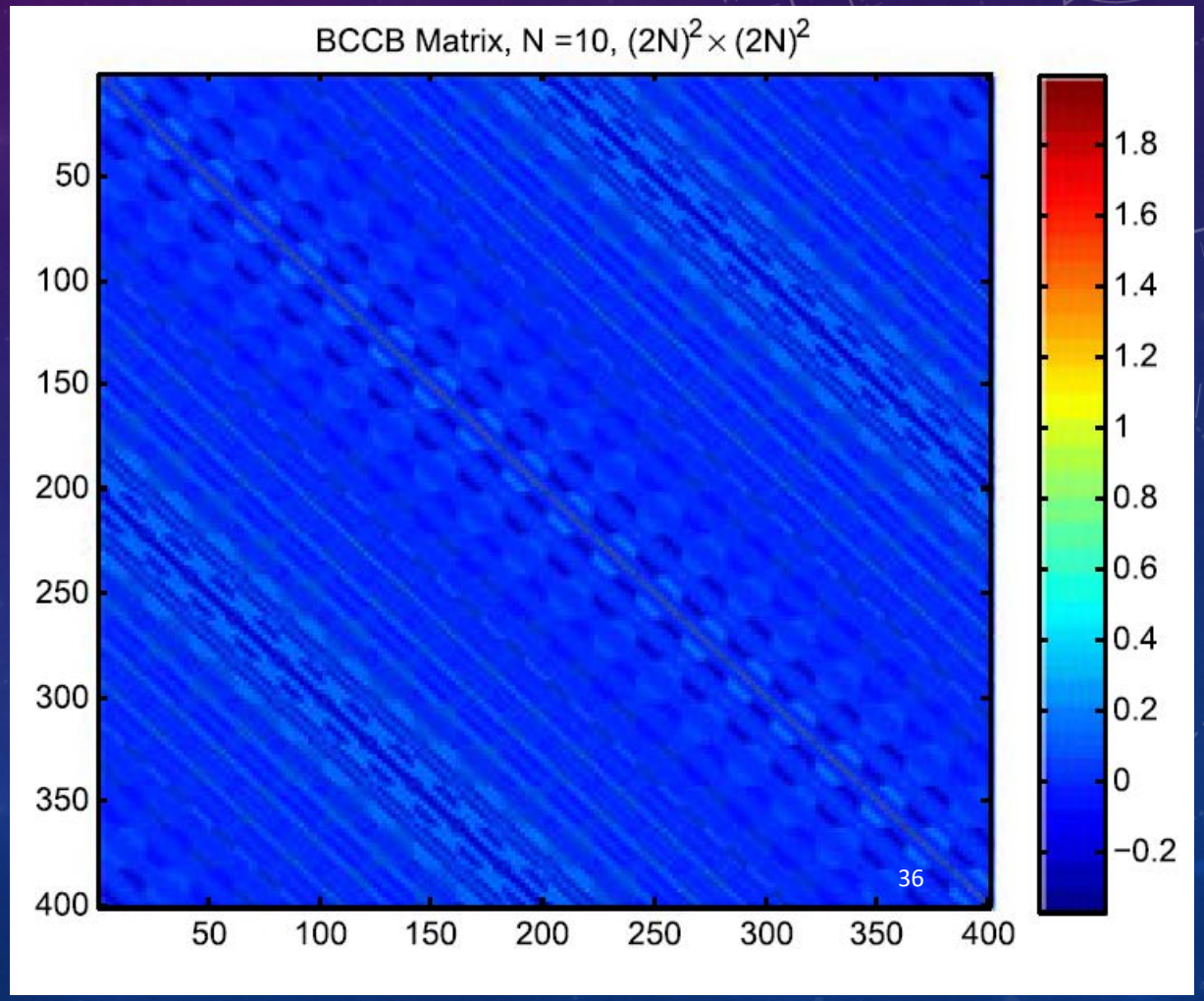
$$A_p^H A_p x = A_p^H y \rightarrow Ax = b$$

The matrix A, due to the nature of polar and spherical grids has a special structure called Block-Toeplitz-Toeplitz-block (BTTB).

INVERSE SPFT/PFT - STRUCTURE

The matrix $A \rightarrow \tilde{A}$ needs to be converted from BTTB to BCCB.





INVERSE SPFT/PFT – SPECTRAL DECOMPOSITION

The larger operator $\tilde{A}$ is a BCCB matrix with a very special spectral decomposition.

$$\tilde{A} = F^H D F$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & 0 & \cdots \\ 0 & \lambda_2 & 0 & \cdots \\ 0 & 0 & \lambda_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where,

$F = 2D/3D$ unitary DFT matrix (equi-spaced) &

$D =$ Eigenvalues of $\tilde{A}$ - Diagonal matrix

INVERSE SPFT/PFT – SOLUTION SUMMARY

- Phase 1: Offline preparation and factorization

$$\tilde{A} = F^H D F$$

- Phase 2: Online fast iterative solution.

$$Ax = b$$

Computation complexity for iterative inversion – CG per iteration

$O(N^2 \log_2 N)$ for an $N \times N$ image.

$O(N^3 \log_2 N)$ for an $N \times N \times N$ volume.

Storage cost only - D

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An Exact and Fast Computation of Discrete Fourier Transform for Polar and Spherical Grid

Syed Alam Abbas, *Student Member, IEEE*, Qiyu Sun, *Member, IEEE*, and Hassan Foroosh, *Senior Member, IEEE*

***Abstract*—Numerous applied problems of two-dimensional (2-D) and 3-D imaging are formulated in continuous domain. They place great emphasis on obtaining and manipulating the Fourier transform in polar and spherical coordinates. However, the translation of continuum ideas with the discrete sampled data on a Cartesian grid is problematic. There exists no exact and fast solution to the problem of obtaining discrete Fourier transform for polar and spherical grids in the literature. In this paper, we develop exact algorithms to the above problem for 2-D and 3-D, which involve only 1-D equispaced fast Fourier transform with no interpolation or approximation at any stage. The result of the proposed approach leads to a fast solution with very high accuracy. We describe the computational procedure to obtain the solution in both 2-D and 3-D, which includes fast forward and inverse transforms. We find the nested multilevel matrix structure of the inverse process, and we propose a hybrid grid and use a preconditioned conjugate gradient method that exhibits a drastic improvement in the condition number.**

***Index Terms*—Nonuniform fourier transform, fractional Fourier transform, discrete Fourier transform, 2D imaging, 3D imaging, polar coordinates, spherical coordinates, hybrid grid, conjugate gradient method, block Toeplitz matrix.**

I. Introduction

GIVEN a function $f(x,y)$ of a continuous argument $(x,y) \in \mathbb{R}^2$, its two-dimensional (2D) Fourier transform (FT) [1]–[3] is given by

$$\hat{F}(u,v) = \int_{\mathbb{R}^2} f(x,y) e^{-2\pi i(xu+yv)} dx dy, \quad u,v \in \mathbb{R}. \quad \text{(I.1)}$$

Transforming the frequency coordinates from Cartesian to polar coordinates, i.e. $(u,v) = (\rho\cos\theta, \rho\sin\theta)$, we get the continuous polar Fourier transform

$$\hat{F}(\rho,\theta) = \hat{F}(\rho\cos\theta, \rho\sin\theta) \quad \text{(I.2)}$$

with a slightly abused notation.

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Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

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Numerous applications of polar FT can be found especially in imaging. The significance of this trivial change of variables is that several fundamental procedures for manipulation of such continuous functions f (for instance rotation) can be described most elegantly in terms of polar variables. The polar FT is a powerful tool for organizing the understanding of operators and functions on the 2D continuous domain [4], [5]. Much the same can be said of higher dimensions.

The continuous domain concepts are simple and obvious, but most data in practice exists only in sampled digital form [6]. This is the case in estimating image rotation, image registration, tomography or Radon transform, synthetic aperture radar imaging and more. The prevailing belief is that there exists no such algorithm for fast and exact computation of discrete polar FT ([7], [4], [5], [15] and [16]). Several difficulties as discussed in [4] exists for constructing fast polar FT for digital data: (1) appropriate polar grid should be defined in the frequency domain; (2) a fast way is needed to evaluate or approximate the FT on these polar grid points; and (3) a fast and stable transform inversion design is required.

For modern applications in scientific computing, two existing bodies of literature contain the solution to the above formulation of polar FT using discrete data. (1) One corresponds to methods that compute the inverse Radon transform with applications in wide range of disciplines such as radar imaging, geophysical imaging, nondestructive testing, and medical imaging [8]–[19]. This approach is called the direct Fourier method. It exploits the projection-slice theorem to convert projection data to 2D Fourier data and then reconstruct by Fourier inversion. (2) The second approach treats the general problem of evaluating the FT on non-equally spaced frequency points. This problem is known as USFFT (Unequally Spaced FFT) or the NUFFT/NFFT (non-Uniform FFT) [20]–[28]. Since the frequencies are by definition unequally spaced, the problem of evaluating the polar FT can be considered as a special case of the USFFT problem in this approach.

A detailed discussion of different approaches to polar Fourier transform adopted in the literature is presented in [13], [4], [15], [16] and references therein. The method in those papers is based on a special grid, called the pseudo polar (PP) grid, where equi-spaced angular lines are replaced by equi-sloped ones and the computation is fast. This grid has been explored extensively since the 1970s [4]. For long, it has been argued that this is the best way to obtain polar-like sampling specially adapted for the digital data, and hence it has proliferated to numerous applications as a substitute for the true polar form

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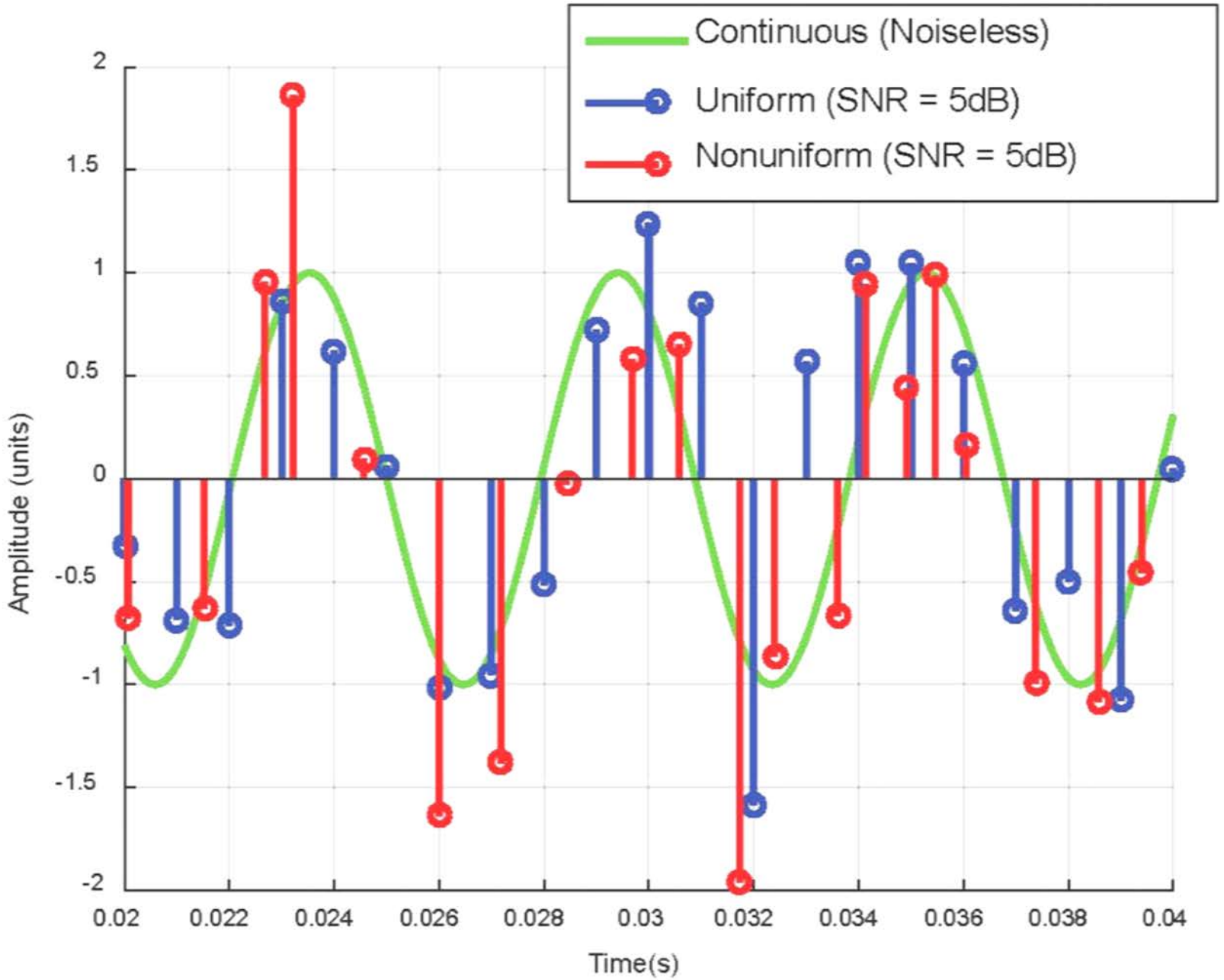
Intermission

OVERVIEW

- Robust Registration using Spherical Polar Fourier Transform and Spherical Harmonics
- Nonuniform Polar and Spherical Discrete Fourier Transform
- Frequency Estimation from nonuniform samples
- Real time fast phasor estimation
- Applications and future work
- Conclusions
- Q & A

WHAT IS THE PROBLEM ?

$$y(t) = \sum_{k=1}^{K} A_k \sin(\theta_k t + \varphi_k)$$



WHAT IS THE PROBLEM ?- NONUNIFORM

$$A = 1$$
$$\varphi = \frac{\pi}{2}$$
$$f = 170\ Hz$$
$$\Delta T = 1ms$$
$$\tau_n \in [.5,1.5]\Delta T$$

WHAT IS YOUR APPROACH TO SOLVE IT ?

Consider the following 4th order continuous autonomous dynamic system:

$$\mathrm{D}x_1 = x_2$$

$$\mathrm{D}x_2 = -2\xi\theta x_2 - \theta^2 x_1 + \theta^2 y$$

$$\mathrm{D}\theta = -\gamma(\theta^2 y - 2\xi\theta x_2)x_1$$

$$\mathrm{D}y = -2\xi\theta x_1$$

where $X = [x_1, x_2, \theta, y]$ is the state vector.

WHAT IS YOUR APPROACH TO SOLVE IT ?

Consider the linearized system near the equilibrium point, X_0 :

$$\mathrm{D}X = JX\Big|_{X_0} \qquad A = 1, t = 1$$

$$X_0 = \left[\frac{-A\cos(\theta_0 t)}{2\xi}, \frac{-A\theta_0 \sin(\theta_0 t)}{2\xi}, \theta_0, A\sin(\theta_0 t)\right]$$

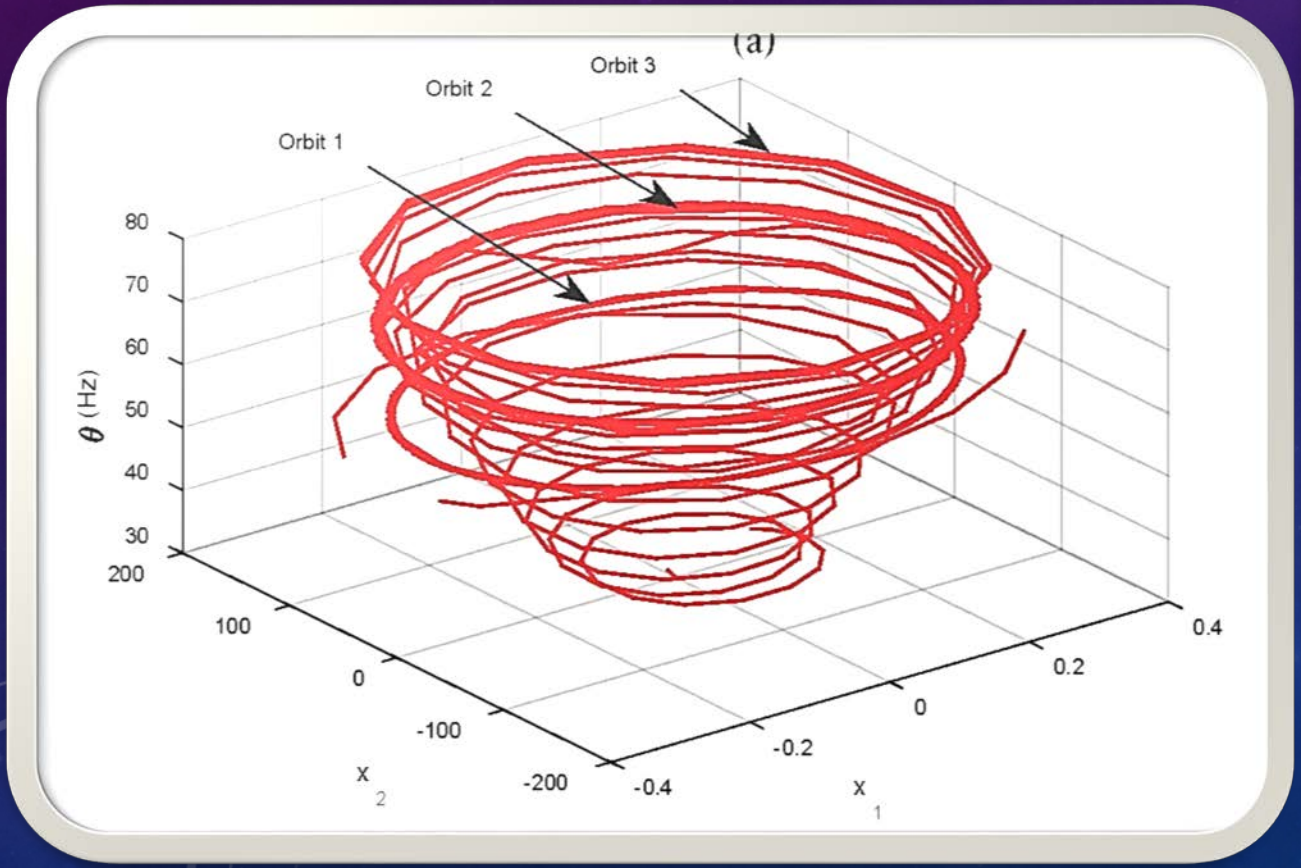
Eigenvalues :

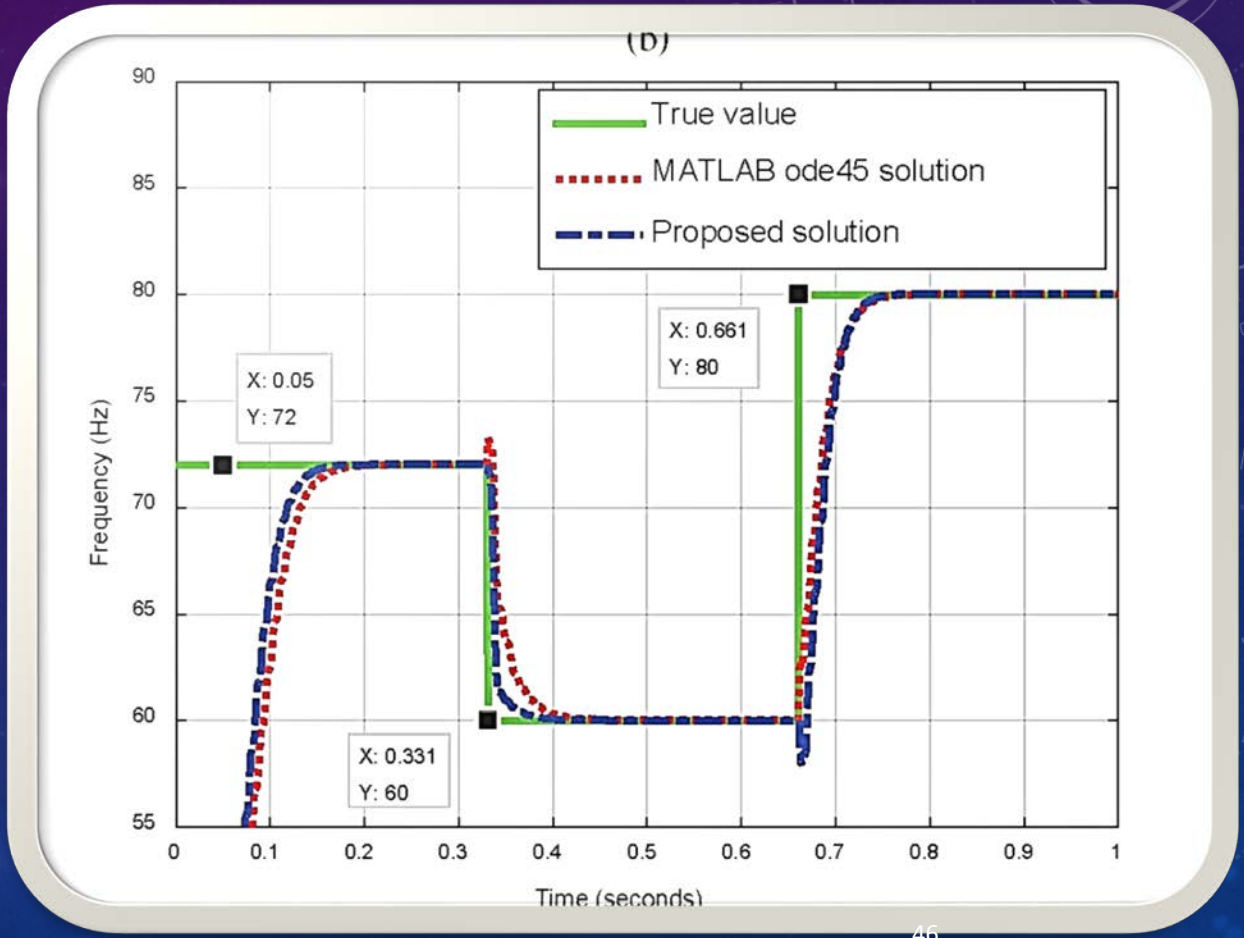
$$\lambda_1, \lambda_2 = \pm\theta_0 i,$$

$$\lambda_3, \lambda_4 = \frac{\theta_0\left(\pm\sqrt{\Delta} + 2\gamma\sin(2\theta_0) - 16\xi^2\right)}{16\xi}$$

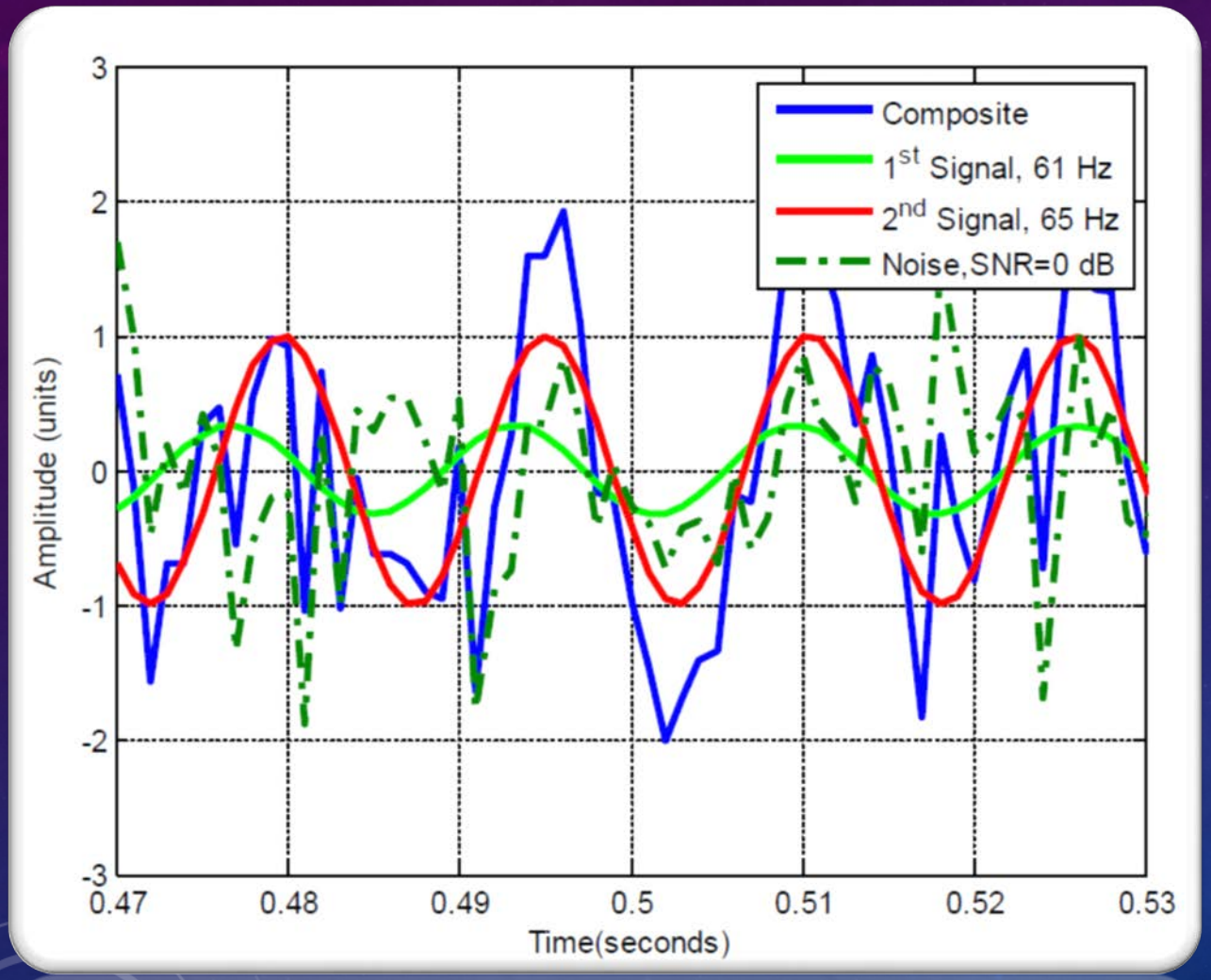
WHAT ARE YOUR RESULTS ?

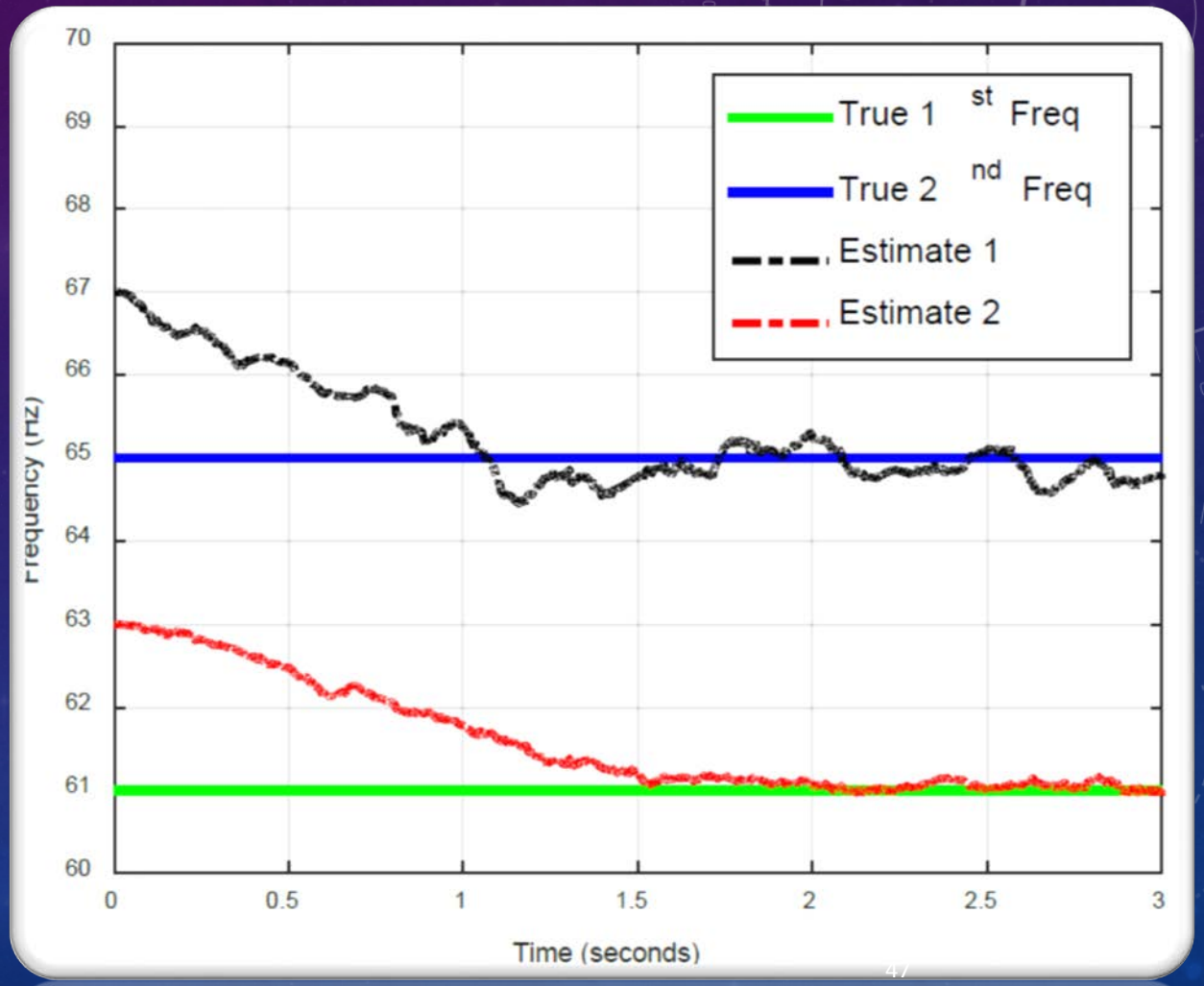
$$\mathrm{D}x_1 = x_2$$
$$\mathrm{D}x_2 = -\theta^2 x_1$$





WHAT ARE YOUR RESULTS ?





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Frequency estimation of sinusoids from nonuniform samples

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Local convergence
Tracking frequency

ABSTRACT

Sinusoid signals with multiple frequencies appear in various systems and their frequencies may carry some important features. Frequency estimation from their discrete samples is one of the fundamental problems and many frequency estimators have been proposed for uniform sampling setting. In this paper, frequency estimators based on adaptive notch filtering are proposed for nonuniform sampling setting. We observe that some dynamic systems associated with adaptive notch filters can be solved in nonuniformly sampled time steps with high accuracy. This leads us to propose a digital adaptive notch filtering method to estimate frequency of a sinusoidal signal with single frequency from its nonuniform samples. The proposed method exhibits convergent and robust frequency estimation in the presence of random sampling noises, and its variance is comparable to the Cramer–Rao lower bound in the presence of additive white noise. The above method designed for single frequency estimation could track abrupt single frequency change of an input signal, but it is not applicable directly for multiple frequency estimation. Our simulations show that the proposed estimators have robust performance for sinusoidal signals with multiple distinct frequencies, and they can be used to separate two very close frequencies of an input signal in a highly noisy sampling environment.

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1. Introduction

Consider a mixture of sinusoidal signals, whose kth component has amplitude A_k, frequency θ_k and phase ϕ_k, $1 \le k \le K$,

$$y(t) = \sum_{k=1}^{K} A_k \sin(\theta_k t + \phi_k). \tag{1.1}$$

Such sinusoidal signals are encountered in active noise and vibration control, wireless communications, audio, radar and sonar signal processing [1–4]. In telecommunication systems, the frequencies θ_k, $1 \le k \le K$, contain carrier's phase information necessary for synchronization of demodulators or other components of a receiver system.

The estimation problem of frequencies θ_k, $1 \le k \le K$, of the signal y is a fundamental problem in systems theory with many applications. It has been intensively studied in signal processing, instrumentation and measurements, and control theory. Many frequency estimators have been proposed, including adaptive notch filtering, time frequency representation, phase locked loop, eigensubspace tracking, extended Kalman filtering, internal model method, etc., see [5–8] and references therein.

Most of existing estimators are derived for uniformly sampled data $y(n\Delta T)$, $n \ge 0$, with uniform sampling frequency $1/\Delta T$, and often only for a single frequency, i.e., $K=1$. In this paper, we consider multiple frequency estimation problem of the signal y from its nonuniform samples,

$$z_n = y(T_n) + w_n, \quad n \ge 0, \tag{1.2}$$

corrupted by additive noises w_n, where T_n, $n \ge 0$, are sampling times.

Nonuniform sampling arises in many applications, such as computer graphics, frequency scanning interferometry, magnetic resonance imaging, computer tomography scans and Radon imaging [9–14]. Uniform sampling is well studied and it has been widely used in engineering applications. However in some applications, nonuniform sampling is necessary and it has better performance. For instance, in antialiasing in computer graphics [15], better results can be obtained with random sampling instead of uniform sampling. The sampling operation could be costly, and a low number of samples (but not necessarily uniform) is more desirable. For instance, in frequency scanning interferometry, the sampling effort is measured by the acquisition time at a given point, and the optimal sampling scheme is usually nonuniform [16]. Nonuniformly sampled data are harder to analyze and the related methods, even for a "simple" task of obtaining the discrete nonuniform Fourier transform, are much more difficult and often

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OVERVIEW

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WHAT IS THE PROBLEM ?

$$y(t) = \sum_{k=1}^{K} A_k \sin(\theta_k t + \varphi_k)$$

WHAT IS YOUR APPROACH TO SOLVE IT?

Problem,

$$y(n) = V_{p0}\sin(2\pi f_0 n\Delta T + \theta_0) + V_{ph}\sin(2\pi f_h n\Delta T + \theta_h)$$

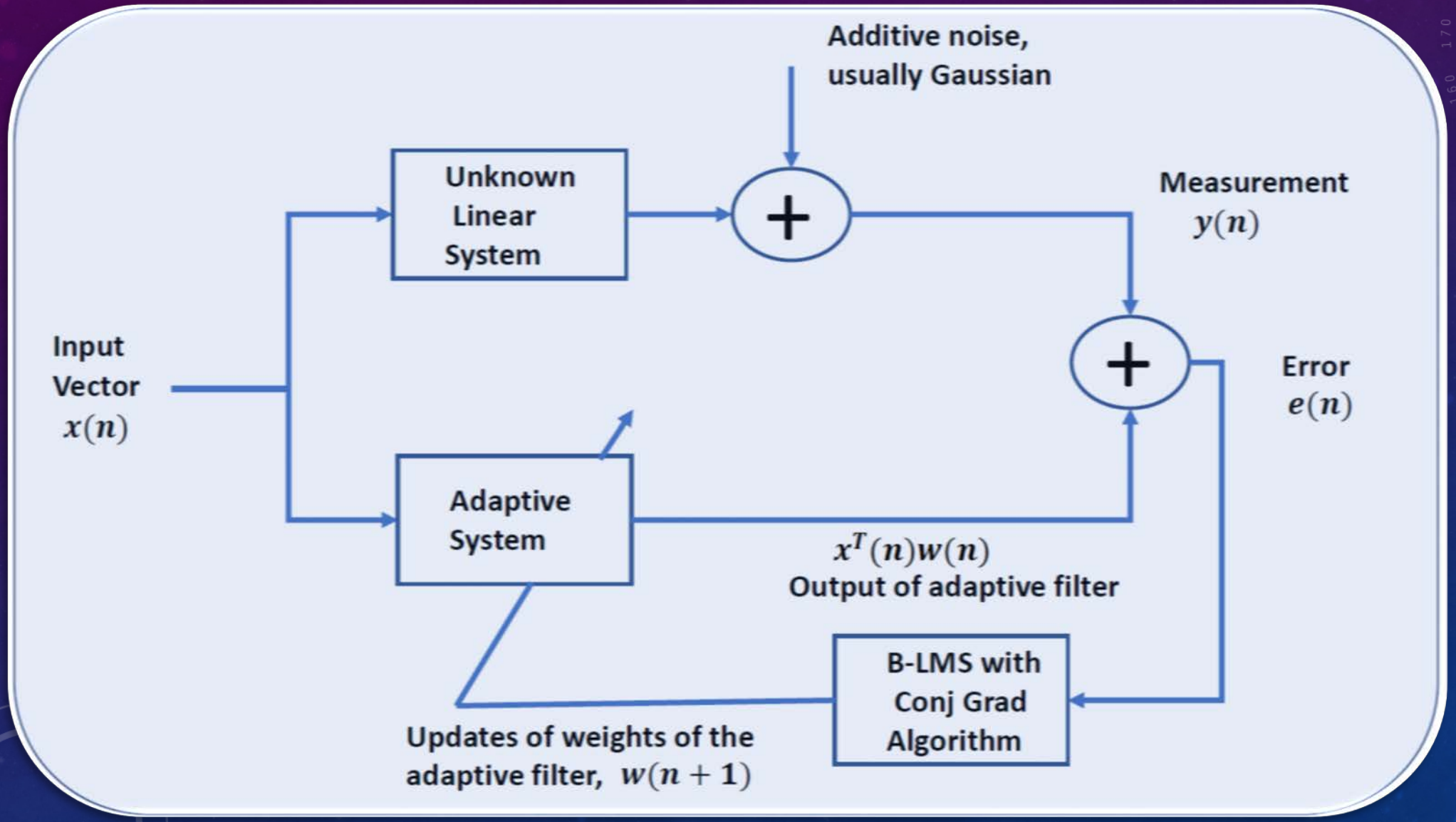
Expansion,

$$w_1(n) = V_{p0}\cos\theta_0\,, w_2(n) = V_{p0}\sin\theta_0\,,$$
$$w_3(n) = V_{ph}\cos\theta_0\,, w_4(n) = V_{ph}\sin\theta_0\,,$$
$$x_1(n) = \sin(2\pi f_0 n\Delta T), x_2(n) = \cos(2\pi f_0 n\Delta T),$$
$$x_3(n) = \sin(2\pi f_h n\Delta T), x_4(n) = \cos(2\pi f_h n\Delta T).$$

Linear solution,

$$y(n) = x_1(n)w_1(n) + x_2(n)w_2(n) + x_3(n)w_3(n) + x_4(n)w_4(n)$$

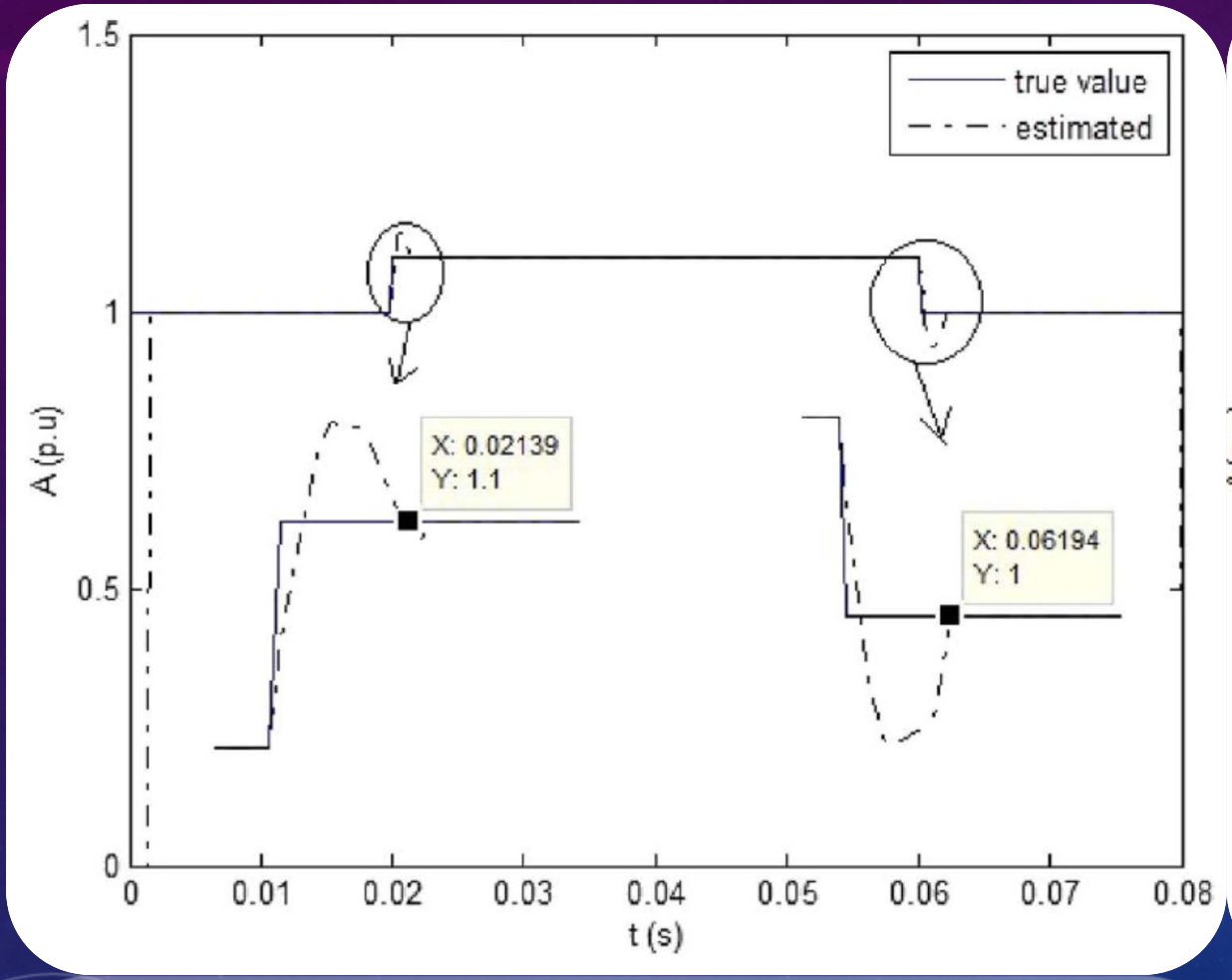
WHAT IS YOUR APPROACH TO SOLVE IT?

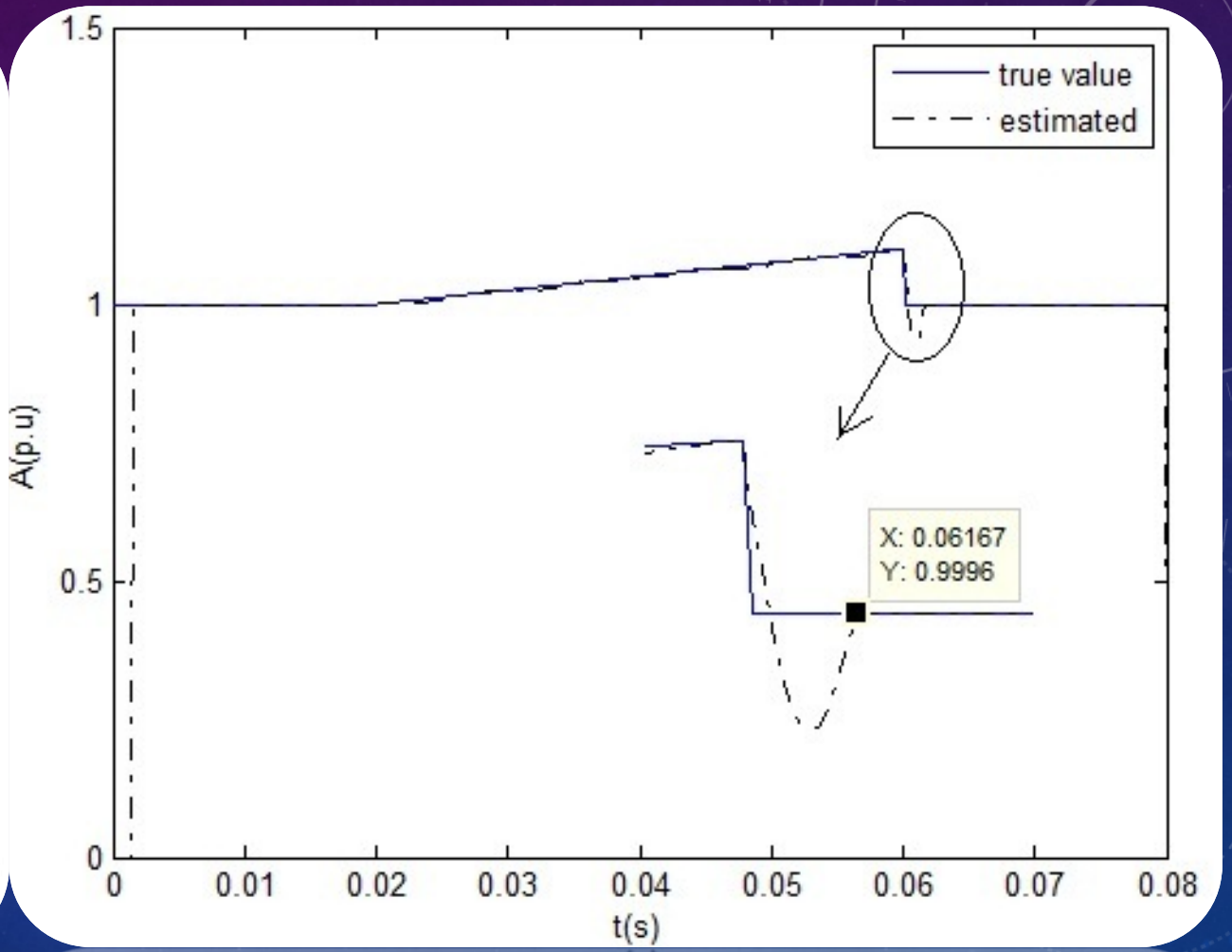


Linear solution,

$$y(n) = x_1(n)w_1(n) + x_2(n)w_2(n) + x_3(n)w_3(n) + x_4(n)w_4(n)$$

WHAT ARE YOUR RESULTS? – DETECTING FAULTS





WHAT ARE THE BENEFITS OF USING OUR APPROACH

- Simplicity – Formulated as a linear filtering problem
- Error function is quadratic in unknown weights which is the phasor information.
- Very accurate – if a sinusoid is absent weights should converge close to zero.
- Very fast, less than quarter cycle performance.
- Online solution – works by adaptive signal processing on sample by sample basis.
- Very flexible, search harmonics or inter-harmonics at no extra modeling cost.

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A New Fast Algorithm to Estimate Real-Time Phasors Using Adaptive Signal Processing

Syed Alam Abbas, *Student Member, IEEE*

***Abstract*—Phasor magnitude and angle of various harmonic and interharmonic components of the power signal are widely used as critical variables and performance indices for power system applications, such as protection, relaying, and state monitoring. This paper proposes a novel adaptive algorithm for estimating the phasor parameters in real time. It uses a faster quasi-second-order optimization technique to estimate amplitude and phase and does not require any matrix inversions. It features fast response, achieves high accuracy, and involves less computational complexity than many other models and window-based methods.**

***Index Terms*—Adaptive signal processing, block least mean square, conjugate gradient-based search, decaying dc offset elimination, harmonic and interharmonic component estimation, phasor amplitude value, phasor angle.**

I. Introduction

POWER systems in many applications require real-time measurements of phasor magnitude and angle of the fundamental component and the harmonics present in the voltage and current signals of the power line. These are parameters of critical importance for the purpose of monitoring, control, and protection. Speedy and accurate estimations are required for a proactive response under abnormal conditions and to effectively monitor and preempt any escalation of system issues.

A variety of techniques for real-time estimation of phasors has been developed and evaluated in past two decades. They are either model based, least error squared (LES), recursive least square (RLS) [1], Kalman filtering [2], or other window-based methods. They all use the stationary signal sinusoidal model. LES, RLS, and Kalman filters are more suitable for online processing since they generate time trajectories of the evolved parameter but the complexity involved is significant and the matrix has to be fixed and accurate for the model to work. Any drift from the assumed nominal frequency will render the model highly inaccurate. Some artificial-intelligence techniques, such as genetic algorithms (GAs) [3] and neural networks [4], have been used to achieve precise frequency estimation and phasor estimations over a wide range with fast response. Although better performance can be achieved by these optimization techniques, the implementation algorithm is more complex and intense in computations.

Window-based methods, such as discrete Fourier transform (DFT), short time Fourier transform (STFT), and wavelets, are also applied extensively for real-time estimation of power system amplitude and phase parameters. DFT is desirable due to its low computational requirement and fast response. However, the implicit data window in the DFT approach requires a full cycle. To improve the performance of DFT-based approaches, some enhancements have been proposed. Adaptive methods based on the feedback loop by tuning the sample interval [5], adjusting the data window length [6], changing the nominal frequency used in DFT iteratively [7], correcting the gains of the orthogonal filters [8], and tuning the weighted factor [9] recursively, and compensation method to improve phasor estimation [10] are proposed. But due to the inherent limitations in such methods, at least one cycle of the analyzed signal is still required, which hardly meets the demand of high-speed response especially for protection schemes. A method using three consecutive samples of the instantaneous input signal is discussed in [11]. The noise and zero crossing issue may bring large errors to this method. The STFT-based approach has limitations in its accuracy and still requires half a cycle to respond [12]. Recursive wavelet transform (RWT), which is faster, can output phasor parameters in a quarter cycle and has been proposed recently [13]. In this method, inappropriately selecting the window length and sampling rate may cause the weighting matrix to go singular. Also, it has the inherent limitation of having more computational requirements and a higher sampling rate to achieve reasonable accuracy in short time. A combination of algorithms has also been used to overcome individual limitations [14] which can only improve the accuracy at the expense of increased complexity and additional delays.

Many techniques have been proposed to eliminate the imact of decaying dc components in phasor estimation. The conventional DFT algorithm achieves excellent performance when the signals contain only the fundamental frequency and interger harmonic frequency components since, in most cases, the currents containing decaying dc components may introduce fairly large errors in the phasor estimations [15], [16]. A digital filter that features high-pass frequency response can help filter the high-frequency noise. It performs well when its time constant matches the time constant of the exponentially decaying component. Theoretically, the decaying component can be completely removed from the original waveform once its parameters can be obtained. Based on this idea, [17] and [18] utilize additional samples to calculate the parameters of the decaying component. Reference [19] uses the simultaneous equations

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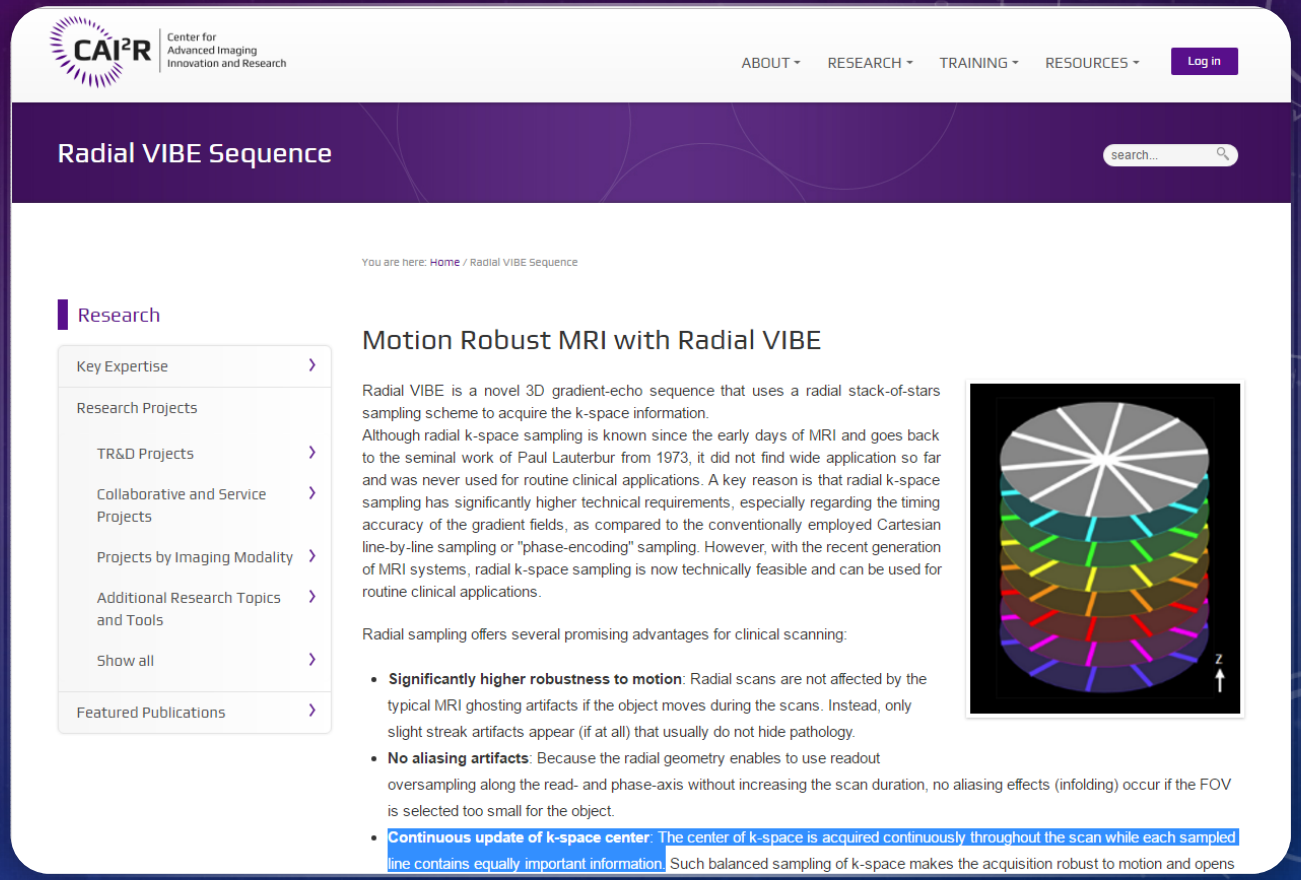
MOTION ROBUST MRI WITH RADIAL VIBE

- Radial sampling offers several promising advantages for clinical scanning.
- Significantly higher robustness to motion: Radial scans are not affected by the typical MRI ghosting artifacts.
- The center of k-space is acquired continuously throughout the scan while each sampled line contains equally important information.



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Radial VIBE Sequence

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Research

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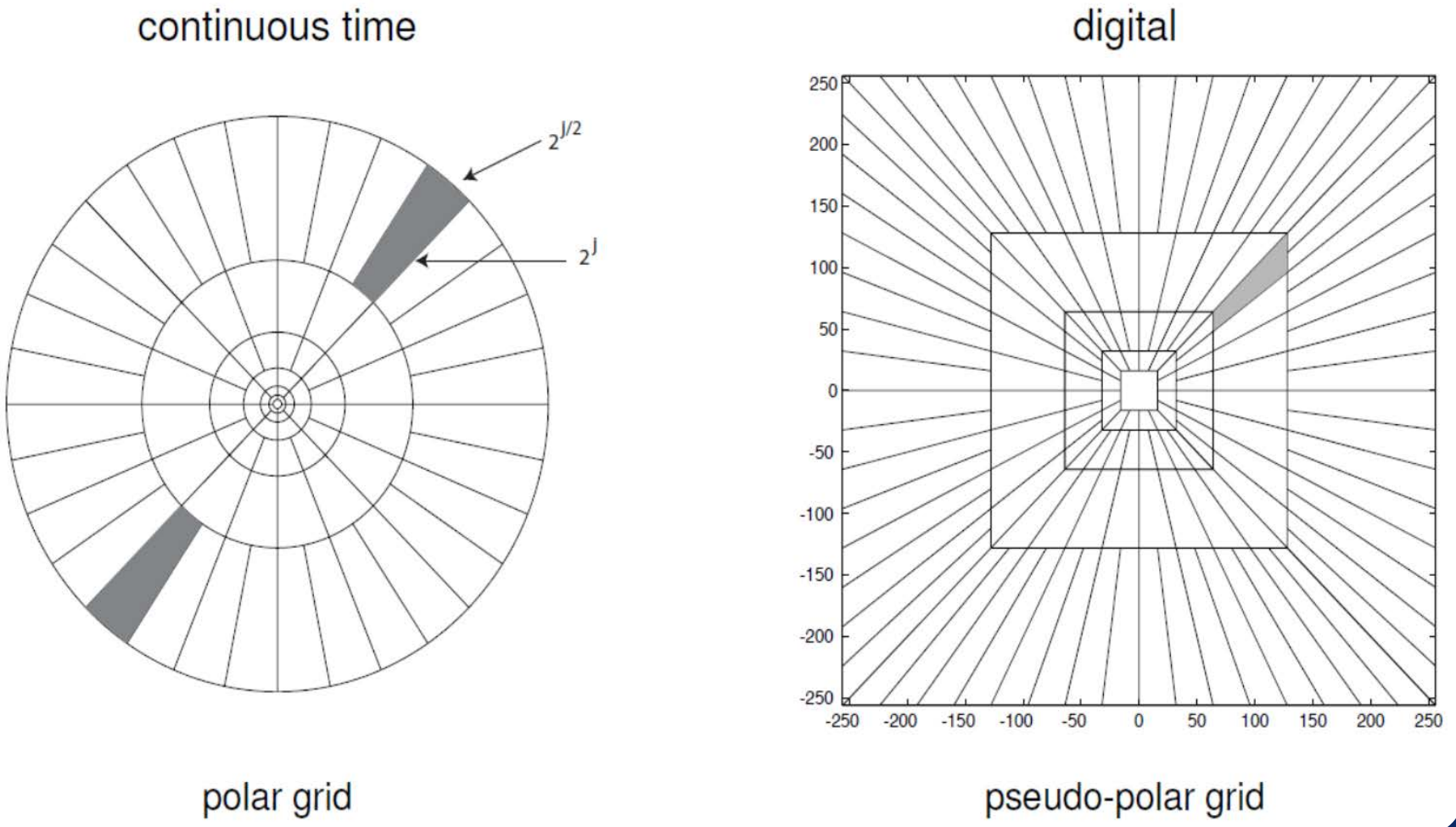
Motion Robust MRI with Radial VIBE

Radial VIBE is a novel 3D gradient-echo sequence that uses a radial stack-of-stars sampling scheme to acquire the k-space information.
Although radial k-space sampling is known since the early days of MRI and goes back to the seminal work of Paul Lauterbur from 1973, it did not find wide application so far and was never used for routine clinical applications. A key reason is that radial k-space sampling has significantly higher technical requirements, especially regarding the timing accuracy of the gradient fields, as compared to the conventionally employed Cartesian line-by-line sampling or "phase-encoding" sampling. However, with the recent generation of MRI systems, radial k-space sampling is now technically feasible and can be used for routine clinical applications.

Radial sampling offers several promising advantages for clinical scanning:

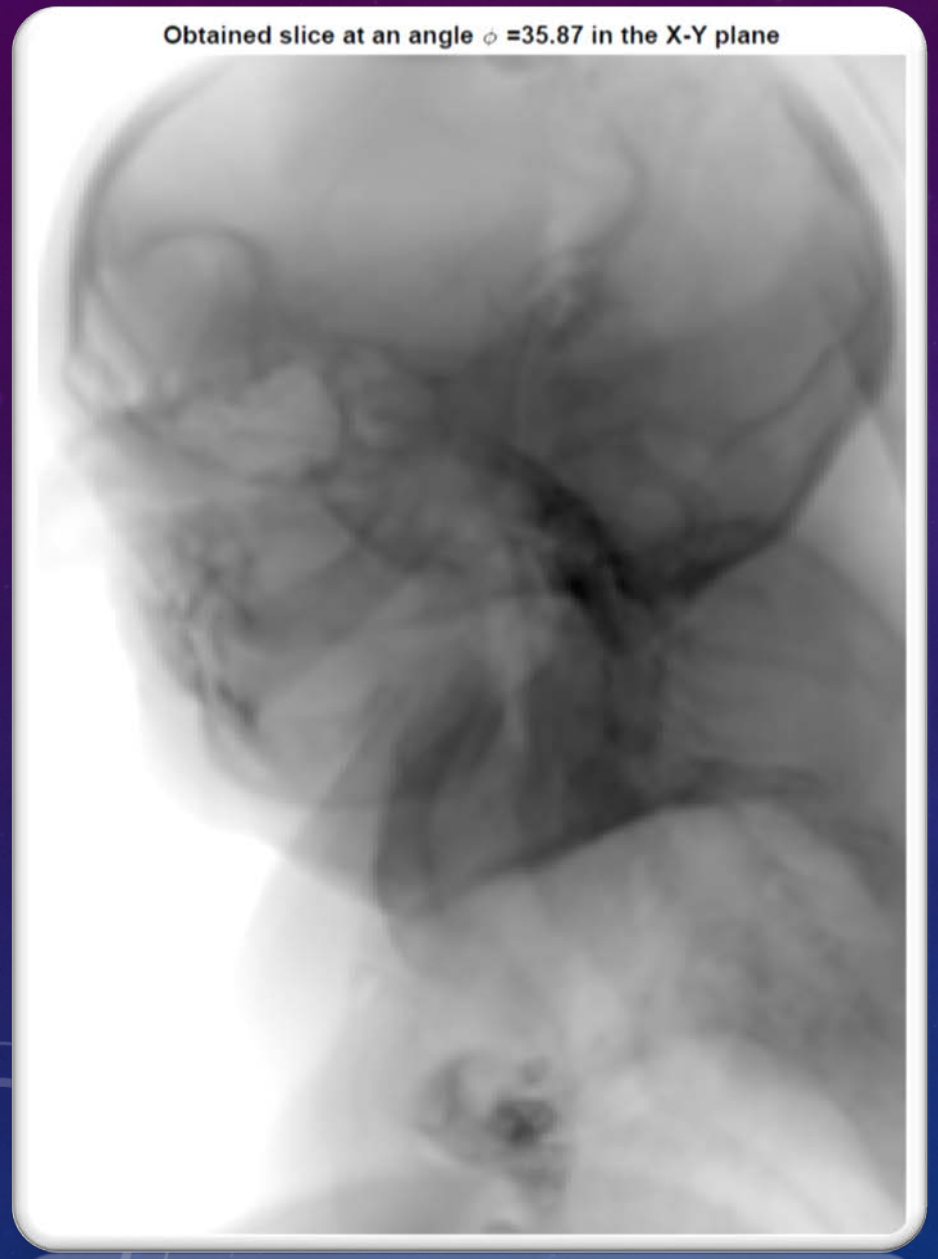
- **Significantly higher robustness to motion**: Radial scans are not affected by the typical MRI ghosting artifacts if the object moves during the scans. Instead, only slight streak artifacts appear (if at all) that usually do not hide pathology.
- **No aliasing artifacts**: Because the radial geometry enables to use readout oversampling along the read- and phase-axis without increasing the scan duration, no aliasing effects (infolding) occur if the FOV is selected too small for the object.
- **Continuous update of k-space center**: The center of k-space is acquired continuously throughout the scan while each sampled line contains equally important information. Such balanced sampling of k-space makes the acquisition robust to motion and opens

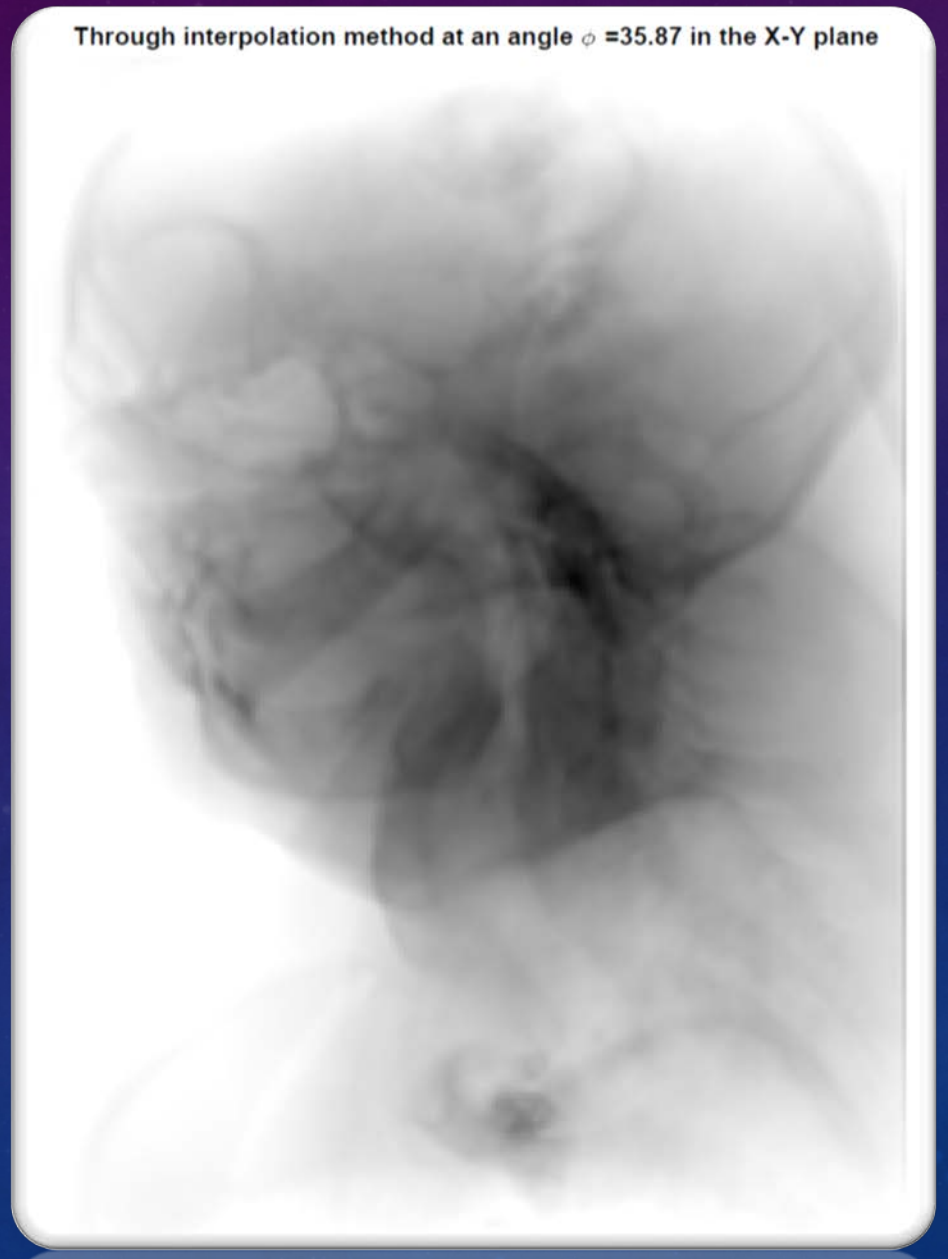
CURVELETS

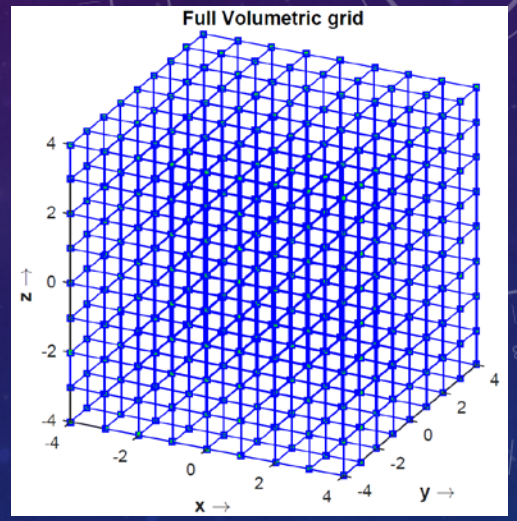


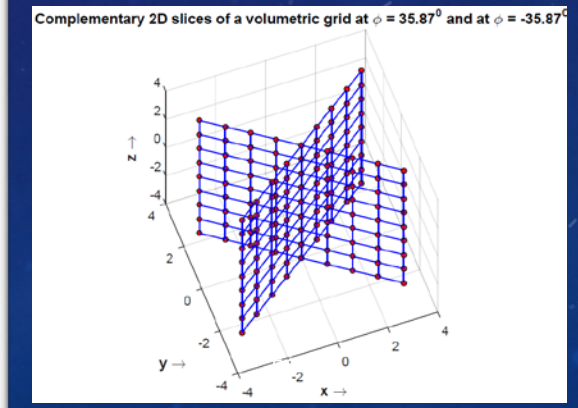
- Designed using continuous time formulation for a perfect polar grid.
- Tweaked to fit digital implementation because no fast and exact algorithm for true polar grid exists.

FOURIER VOLUME RENDERING









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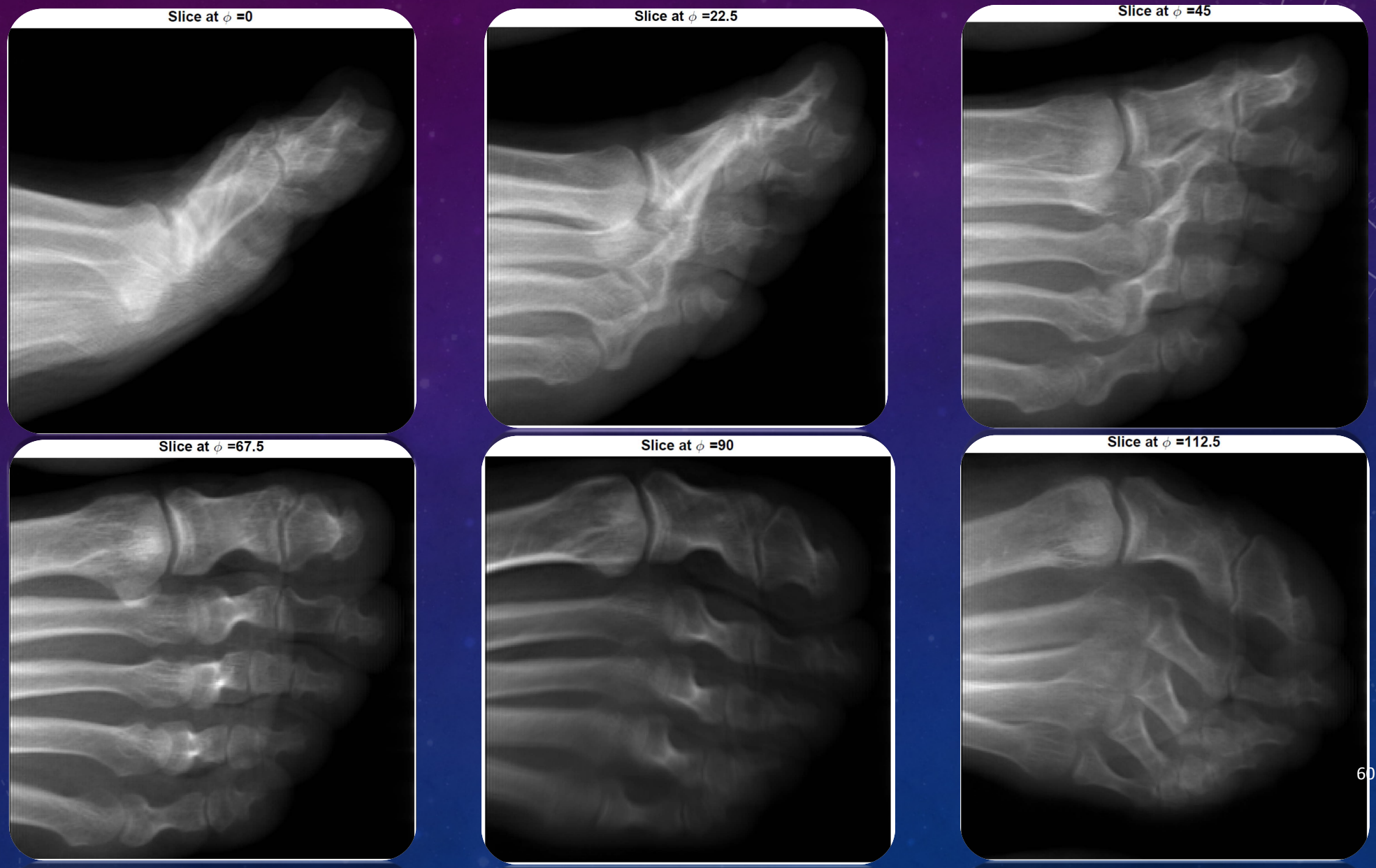

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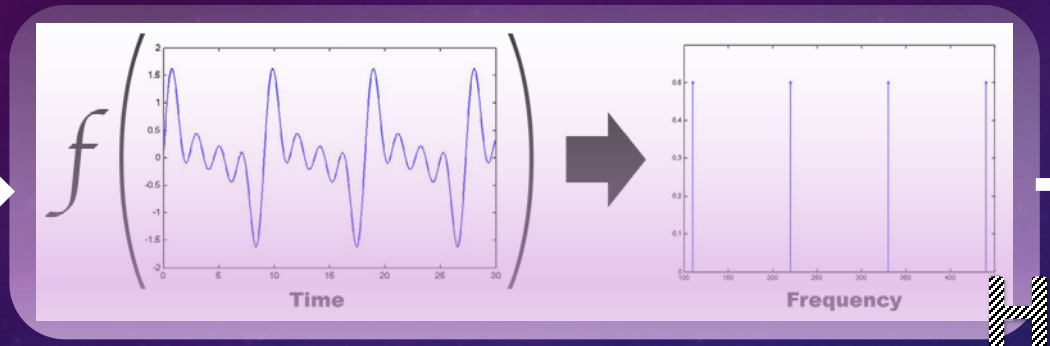
A WORD ON A.I & MACHINE LEARNING

Input

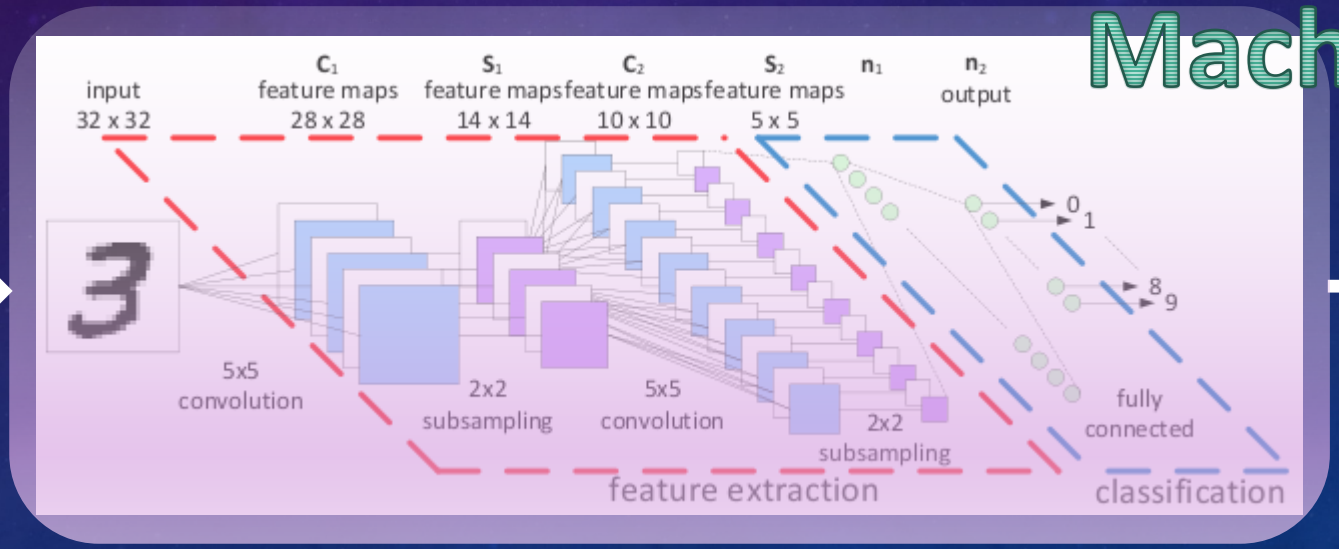


Output

Hand Engineering

Machine Learning

Input



Output

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Hand Engineering

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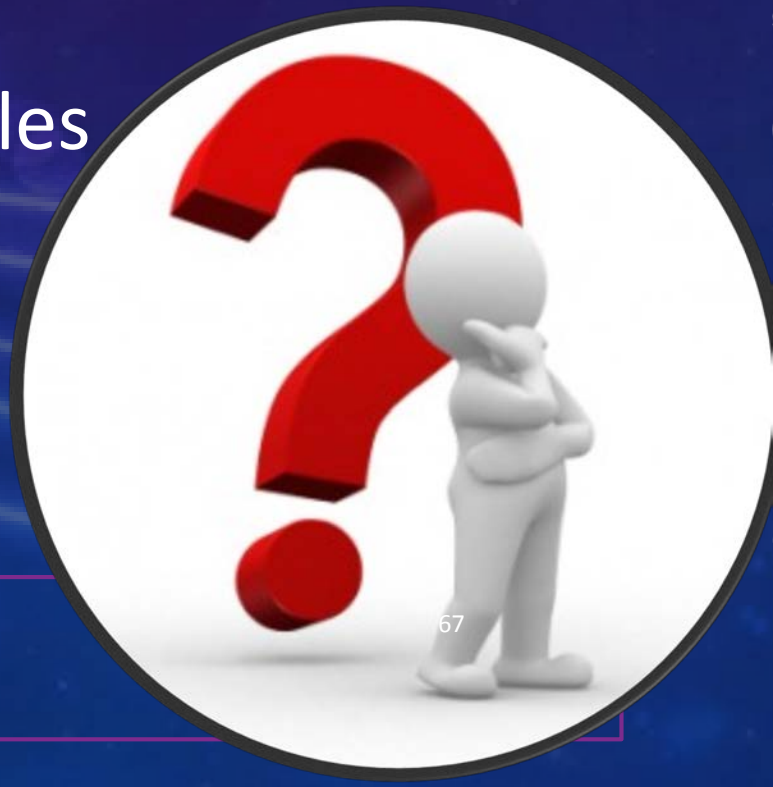
Hand Engineering

HOW SHOULD YOU DO RESEARCH ?

- Do you have an independent and critical mind ? Ph.D. is probably for you only if you are committed now and will remain committed.
- Possess/Acquire strong analytical skills.
- Study a topic you like/ or are very good at. You don't always have to like a topic to be good at, but when you do, the topic will like you.
- Identify a problem that is likely to make an "impact."
- Come up with an "elegant", "smart" , "innovative" solution.
- Find the best journal to publish your results. Always start with the best …

Fluke or by chance an action results in something new.

6

OVERVIEW

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