Google Code Jam 2014

– Qualification Round –

1 Cookie Clicker

Translating the problem into mathematical language we have to minimize the following formula with respect to $k \in \mathbb{N}$:

$$t(k) = C\sum_{i=0}^{k-1}\frac{1}{2+i\,F} + \frac{X}{2+k\,F}$$

Note the difference of two consecutive element:

$$\Delta t(k) = t(k+1) - t(k) = C\left(\sum_{i=0}^{k}\frac{1}{2+i\,F} - \sum_{i=0}^{k-1}\frac{1}{2+i\,F}\right) + X\left(\frac{1}{2+(k+1)\,F} - \frac{1}{2+k\,F}\right)$$

$$\Delta t(k) = \frac{C}{2+k\,F} + \frac{X}{2+(k+1)\,F} - \frac{X}{2+k\,F}$$

Conjecture: $t(k)$ sequence is a strictly monotonic decreasing sequence for some finite integer $K_1 \geq 0$ and then it's a strictly monotonic increasing sequence which limit goes to infinity for $k > K_2$, so for at least one K integer $t(k)$ is minimal.

Thus $\Delta t(k < K_1) < 0$ and $\Delta t(k > K_2) > 0$, so we want to solve $\Delta t(k) = 0$

$$k = \frac{x}{c} - \frac{2}{f}$$

But this expression does not guarantee that k is integer, so let's take $k = \lfloor \frac{x}{c} - \frac{2}{f} \rfloor$ [1]. With a given k we can substitute to the first formula to get the result.

2 Deceitful War

The problem mention that Ken follows an *optimal strategy*. In this game the optimal strategy is to maximize the won point number. It is easy to show that trying to "beat" every Naomi's weight with the possible lightest weight and if he can't beat it, letting it go with his lightest weight can maximize his point – if they both play War.

2.1 War

According to the previous strategy anything what Naomi call will be tried to be beaten. Let's investigate what happens if Naomi calls in ascending order her weights:

When Ken have to use his heaviest weight he won't be able to defeat more weights, so the remaining number of weights would score for Naomi. In other order of calls Ken would use the same (if the lightest still available) or heavier (if he used already the lightest) weights when beating Naomi's given weight. Reordering Naomi calls doesn't change (is used the lightest) the final score or decrease it (when not the lightest was used), but it will no way increase her score.

The steps of a very basic algorithm determining the final score in War. Basically it's call Naomi's weights in ascending order and check if Ken can beat it. The ones, which was used in a round become marked.

1. Sort both of the weight sequences and set unmarked all of the weights
2. $i \leftarrow 1$; *score* $\leftarrow$ length of the sequence
3. Select Naomi's ith weight
4. $j \leftarrow 1$

[1] I'm not sure why not rounding, but this do the job (anyway integer solution should be somewhere around the fraction value)

5. Is Ken's jth weight is not marked and heavier than Naomi's ith weight?
   (a) Yes: mark Ken's jth weight and $score \leftarrow score - 1$ and if $i \leq n$ go back to 3.
   (b) No: $j \leftarrow j + 1$ and if $j \leq n$ go back to 5.

2.2 Deceitful War

Naomi knowing Ken's strategy, that he tries to beat every beatable weight with the possible lightest weight of his, can easily kick out Ken's heaviest weights with her lighter weight. Let's propose Ken has K_h and Naomi has $N_l < K_h$ weight. Naomi can lie that her N_l weight is $K_h - \Delta$ [2]. So Ken would use his K_h weight to beat Naomi's N_l. When Naomi wants to score with N_h she must lie her weight to be the heaviest in the game (e.g. heaviest $+\Delta$), so Ken would call his lightest one K_l. It's optimal for Naomi when she use his lightest weight to beat Ken's heaviest weight which doesn't exceed her weight.

Let's investigate what if Naomi call weights in ascending order. She must consider in every round that is there any weight which can be beaten with her currently selected weight. If there's one lie it as the heaviest weight of the current round$+\Delta$; if there's not lie it as Ken's heaviest weight$-\Delta$.

A basic algorithm determining the final score in Deceitful War:

1. Sort both of the weight sequences and set unmarked all of the weights
2. $i \leftarrow 1$; $score \leftarrow 0$
3. Select Naomi's ith weight
4. $j \leftarrow 1$
5. Is Ken's jth weight is not marked and lighter than Naomi's ith weight?
   (a) Yes: mark Ken's jth weight and $score \leftarrow score + 1$ and if $i \leq n$ go back to 3.
   (b) No: $j \leftarrow j + 1$ and if $j \leq n$ go back to 5.

2.3 Generally about War and Deceitful War (without proof)

Let's denote U Naomi's weight set and V Ken's weight set. Create G bipartite graph with these U, V sets as vertex sets and connect with edge $n \in U$ and $k \in V$ according to a game specific rules. The score will be the maximum bipartite matching of G.

2.3.1 War

The rule: connect with edge $n \in U$ and $k \in V$ if $n < k$.

2.3.2 Deceitful War

The rule: connect with edge $n \in U$ and $k \in V$ if $n > k$.

[2] Because of the discrete nature of the problem there will be always infinitely many Δ for which $K_h - \Delta$ only beatable with K_h