How a CDCL SAT Solver works

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Abstract

- Demonstrating how a modern SAT solver works on a small example.
- Target Algorithm
  - Conflict-Driven Clause Learning (CDCL)
  - 1-UIP learning + far-backtracking
  - details are omitted (decision heuristics, efficient data structure, etc.,)

An Example Problem

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Decide

Implication Graph

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Decide

x1

Implication Graph

x1@1

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Decide

x1
|
x2

Implication Graph

x1@1

x2@2

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9

Decide

x1
|
x2
|
x3

Implication Graph

x3@3

x1@1

x2@2

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9

Decide

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x2@2

x4@4

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x2@2

x4@4

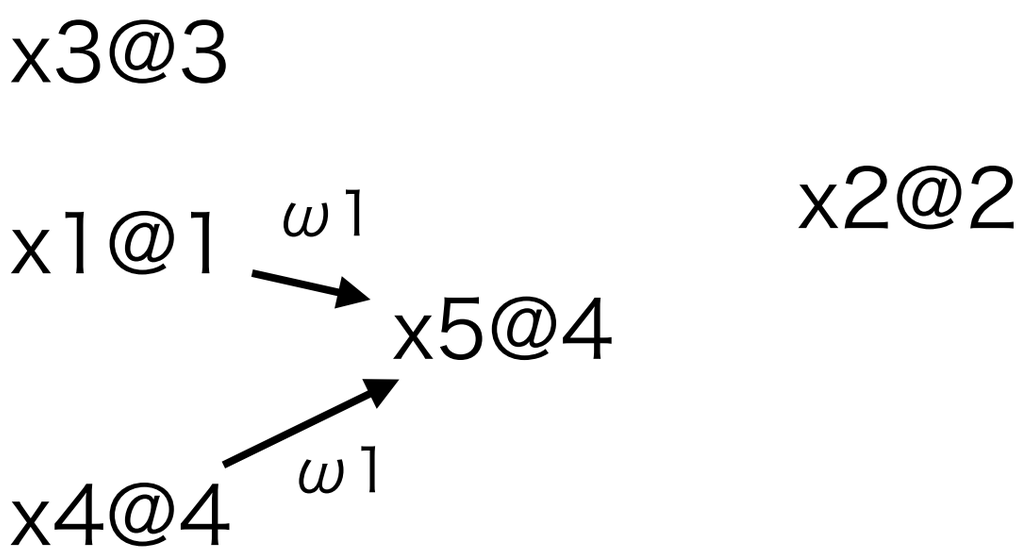
Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



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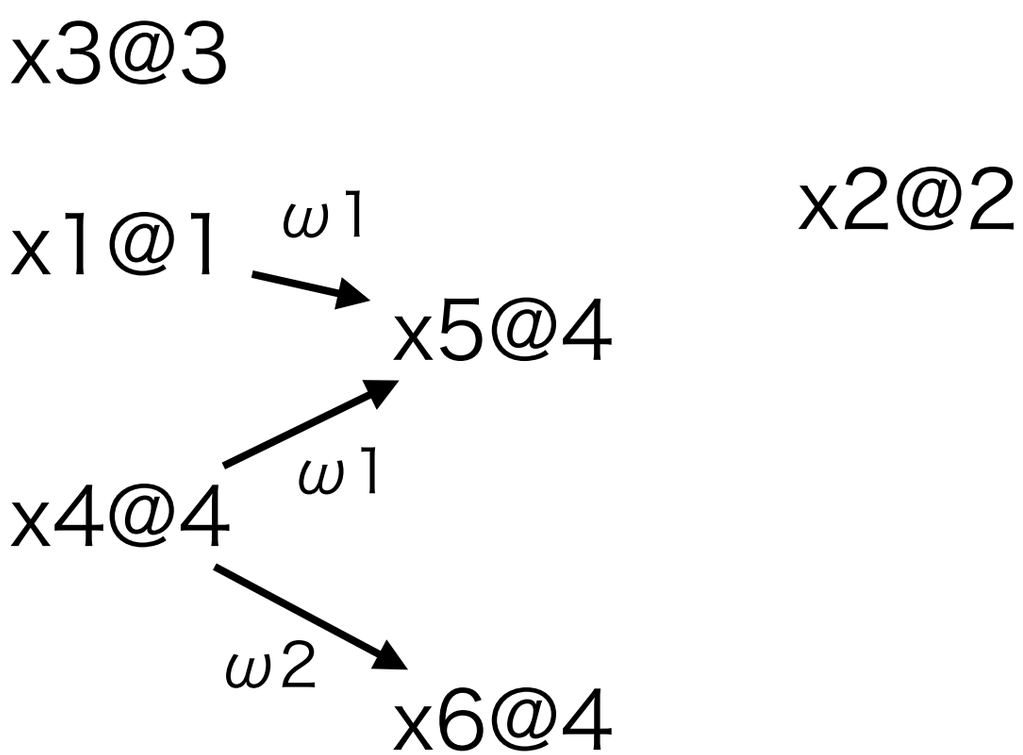
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



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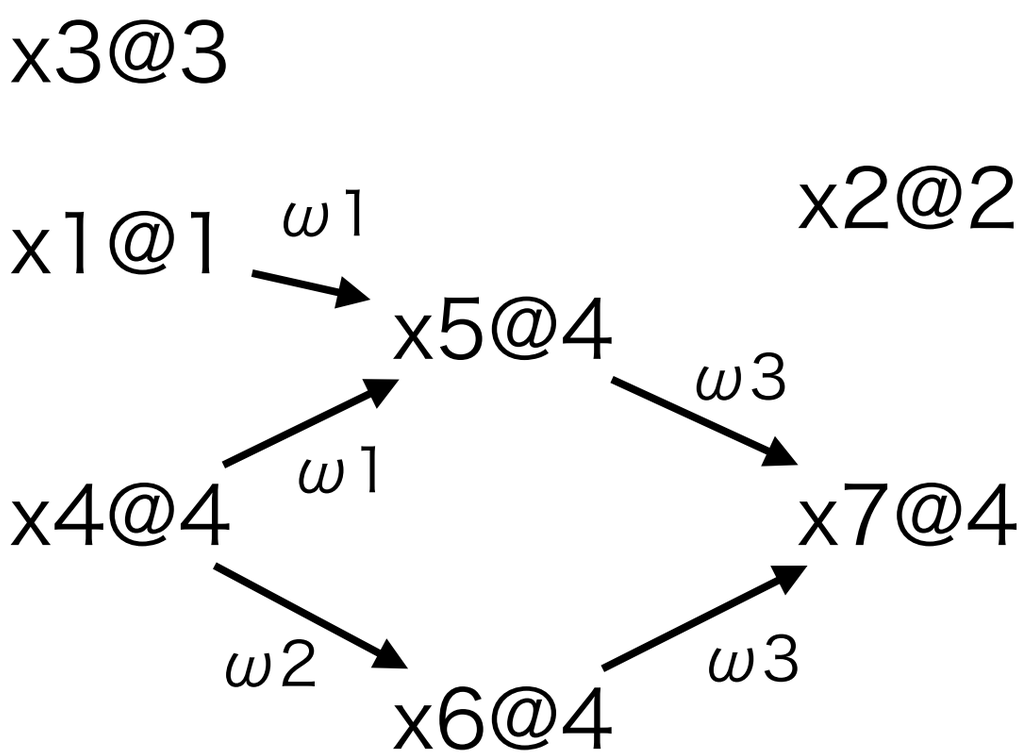
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



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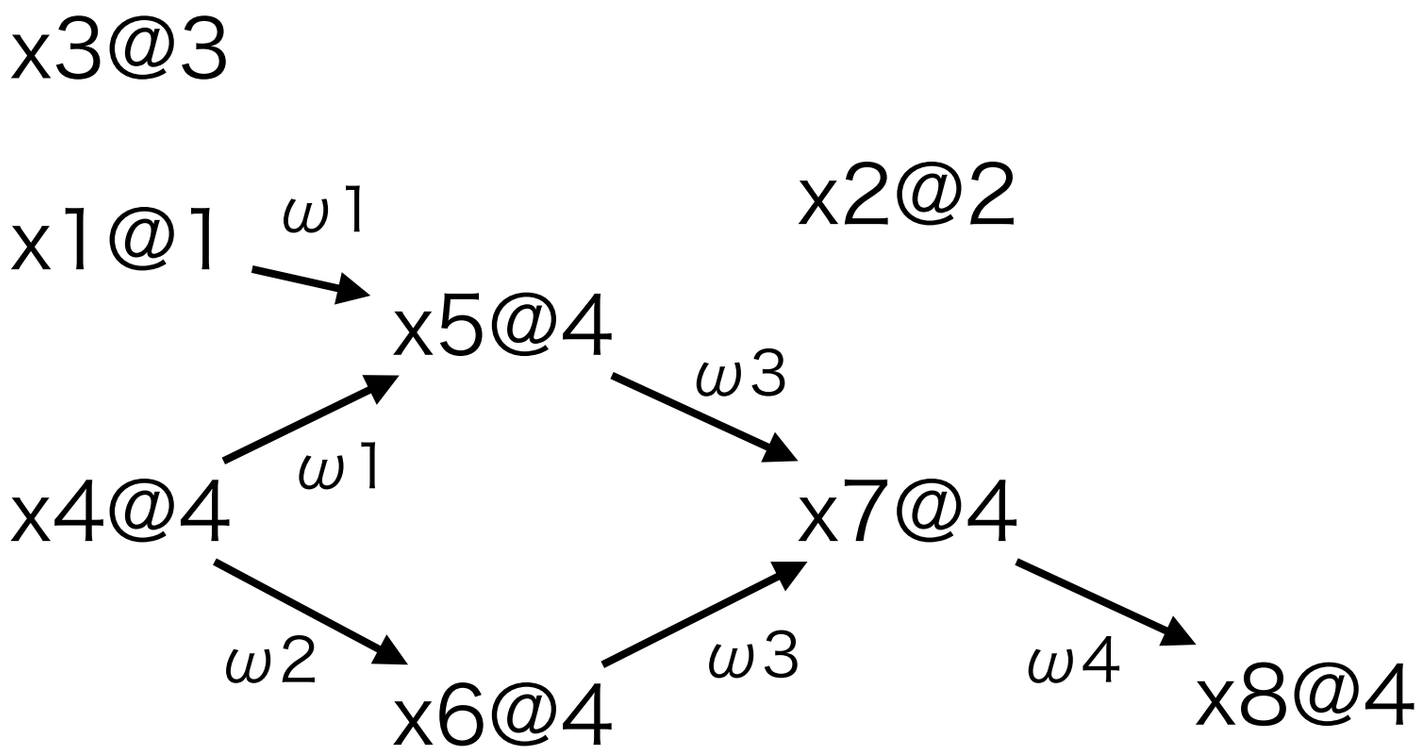
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

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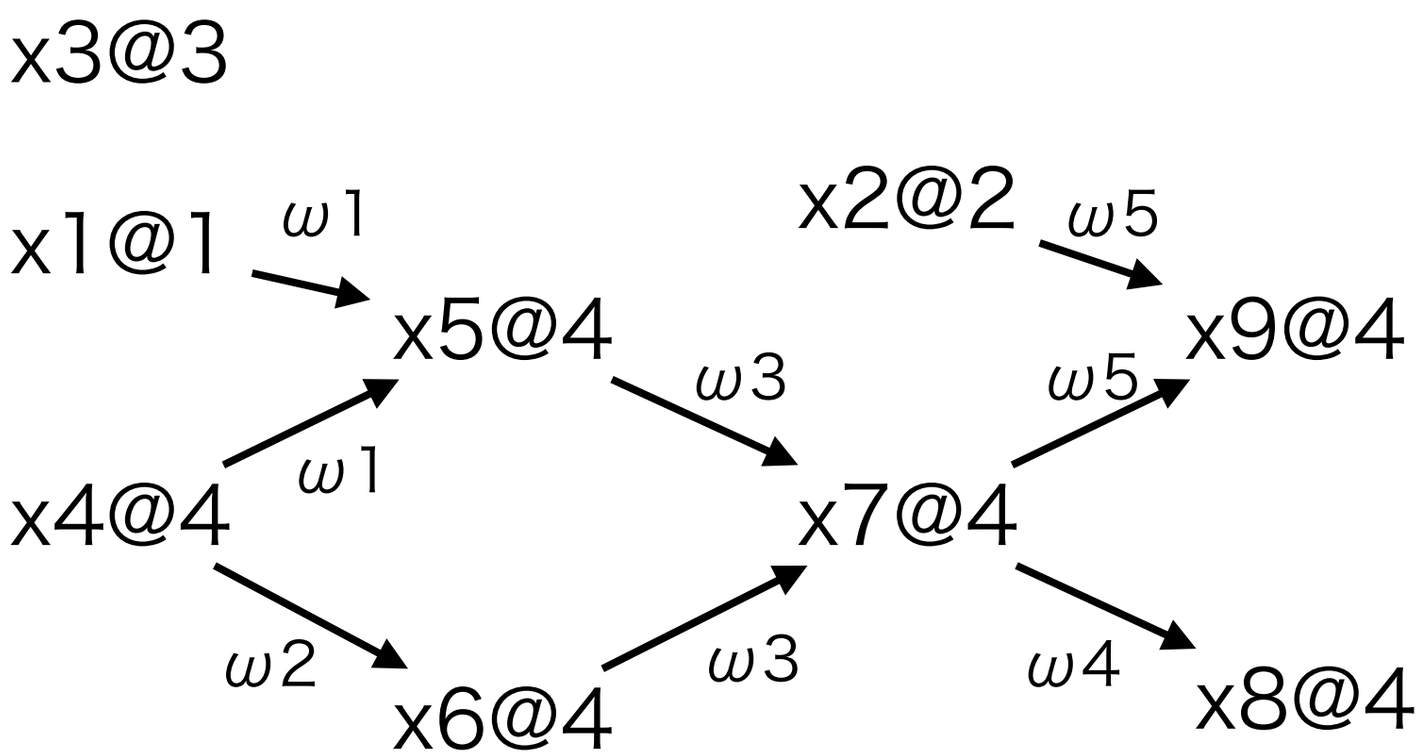
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

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- $\omega 2$: ¬x4 ∨ x6
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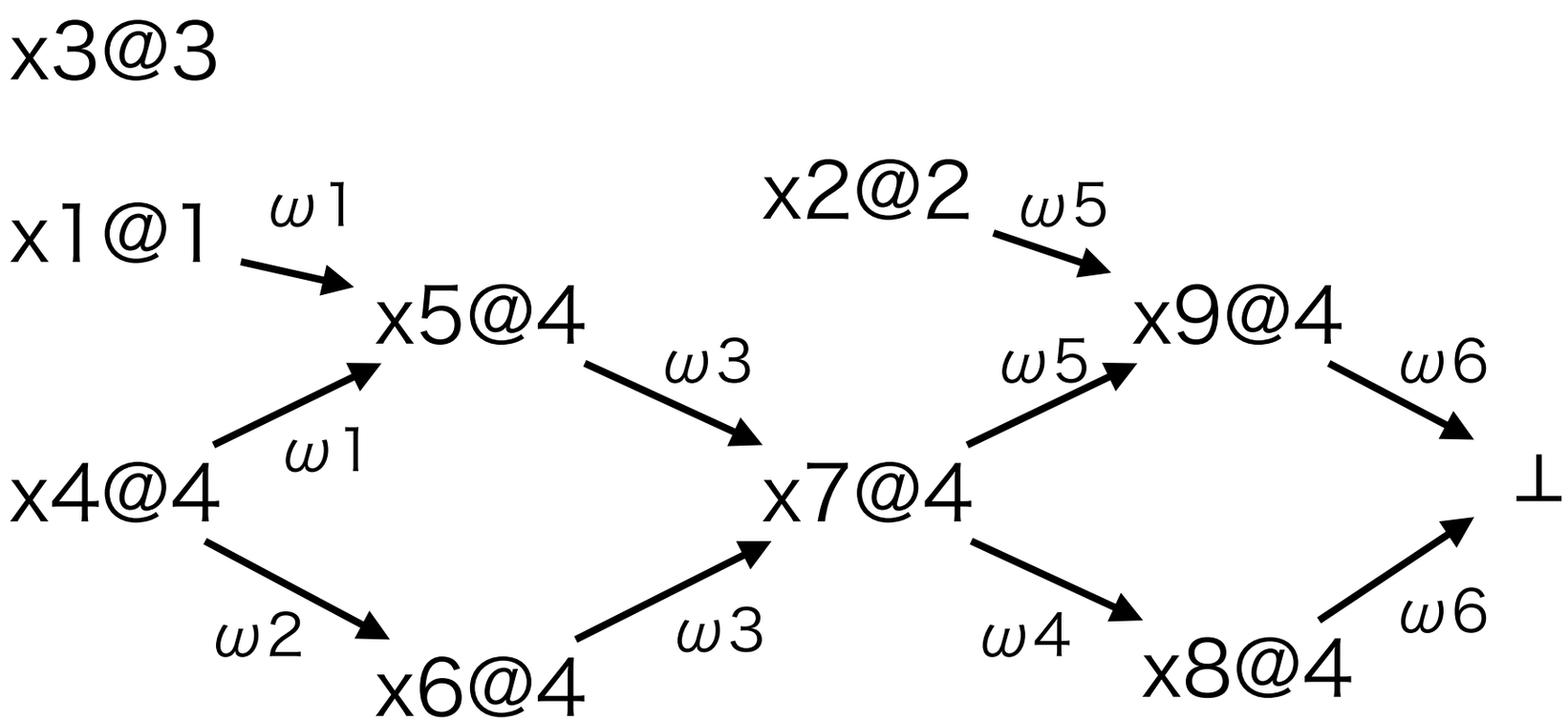
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

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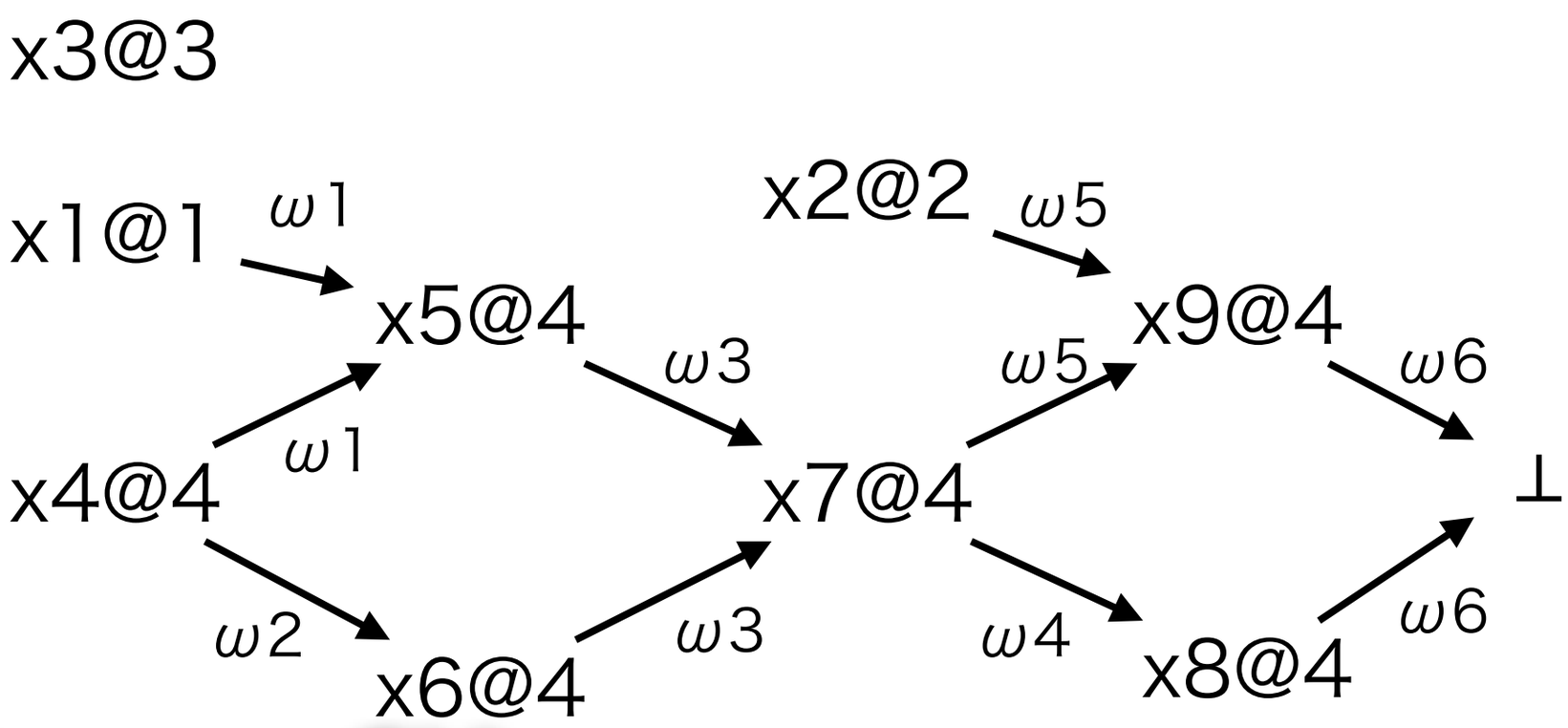
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph



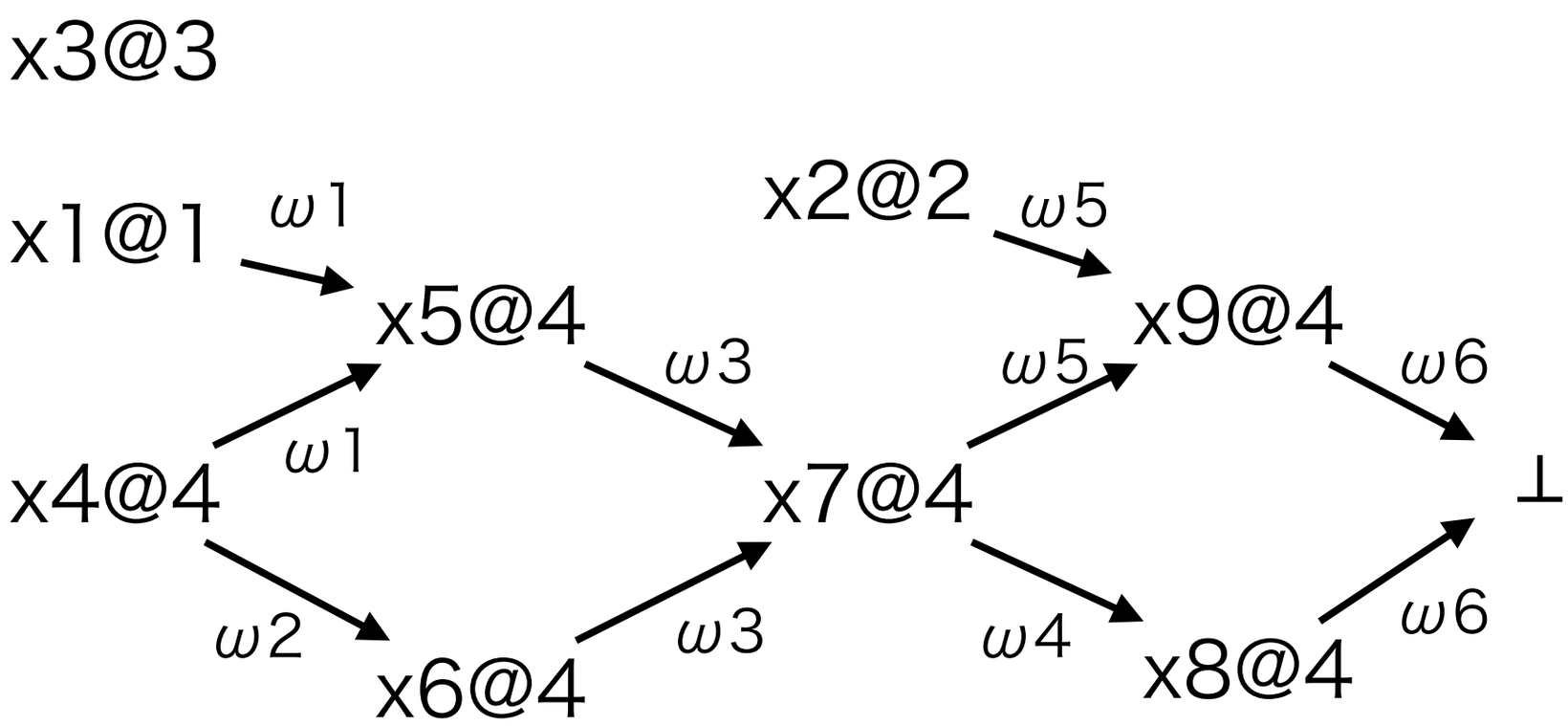
Conflict

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Diagnose

Implication Graph



Clause DB

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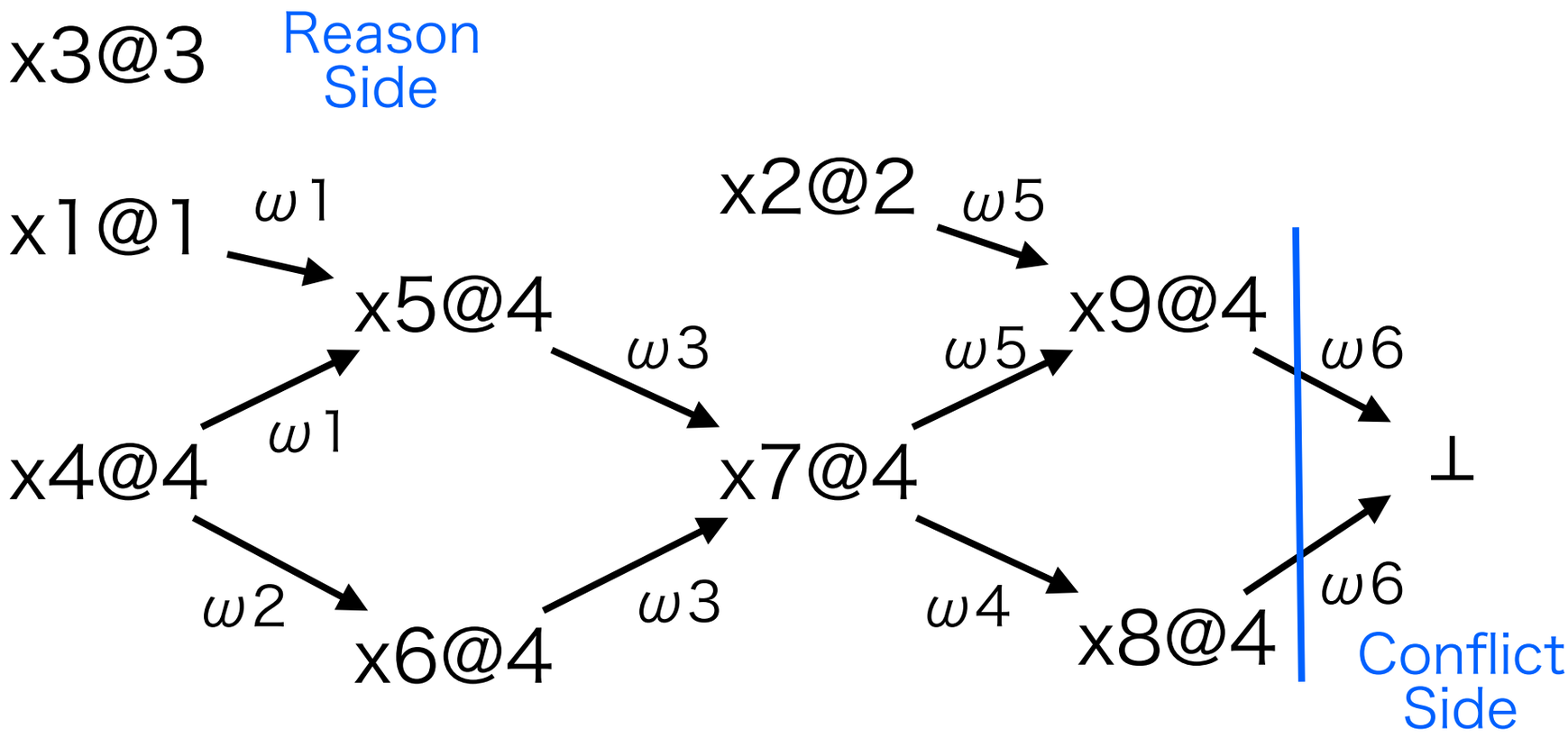
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9

Diagnose

$\omega 6$:
$\neg x8 \vee \neg x9$

Implication Graph



Clause DB

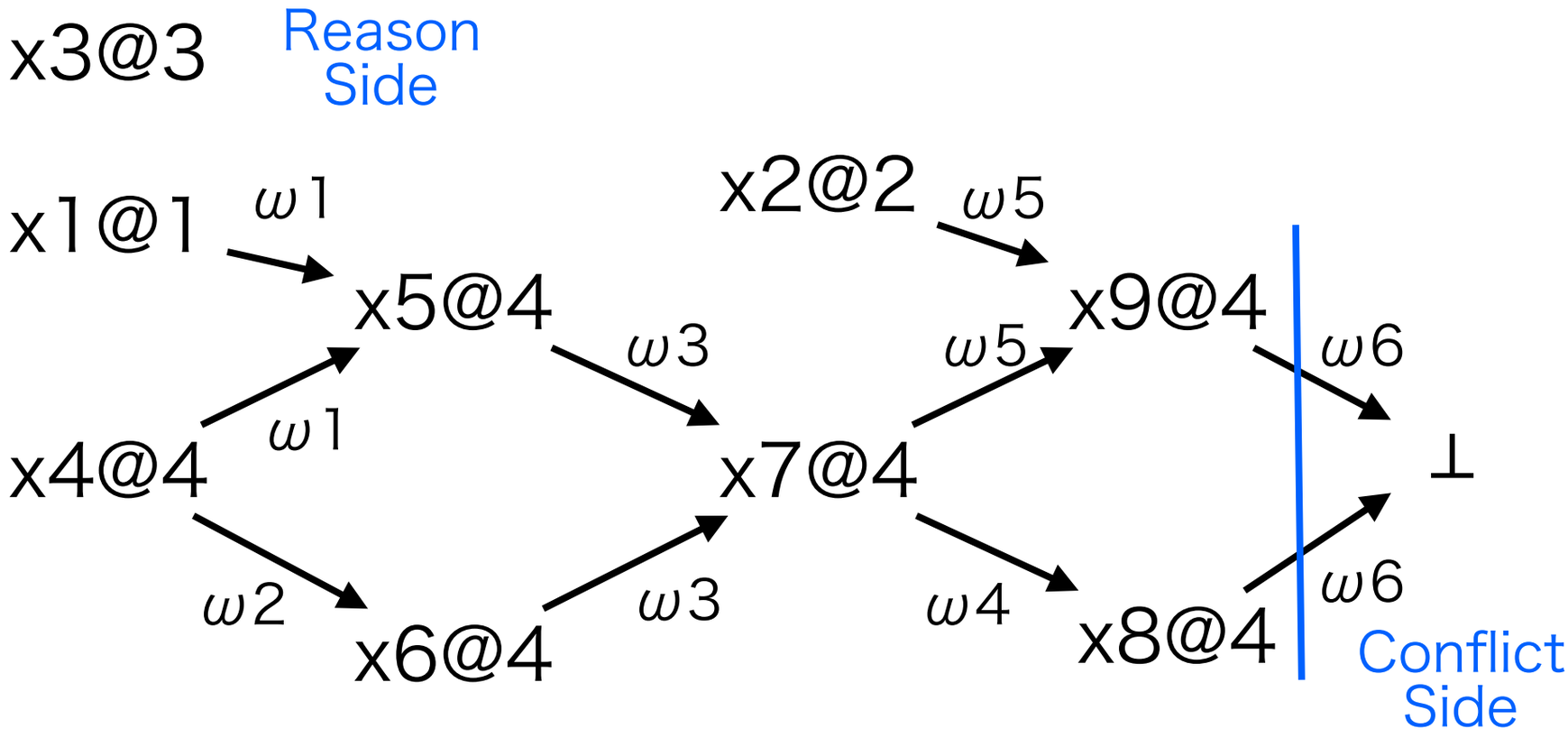
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

$\omega 6$: $\neg x8 \vee \neg x9$

$\omega 4$: $\neg x7 \vee x8$

Implication Graph



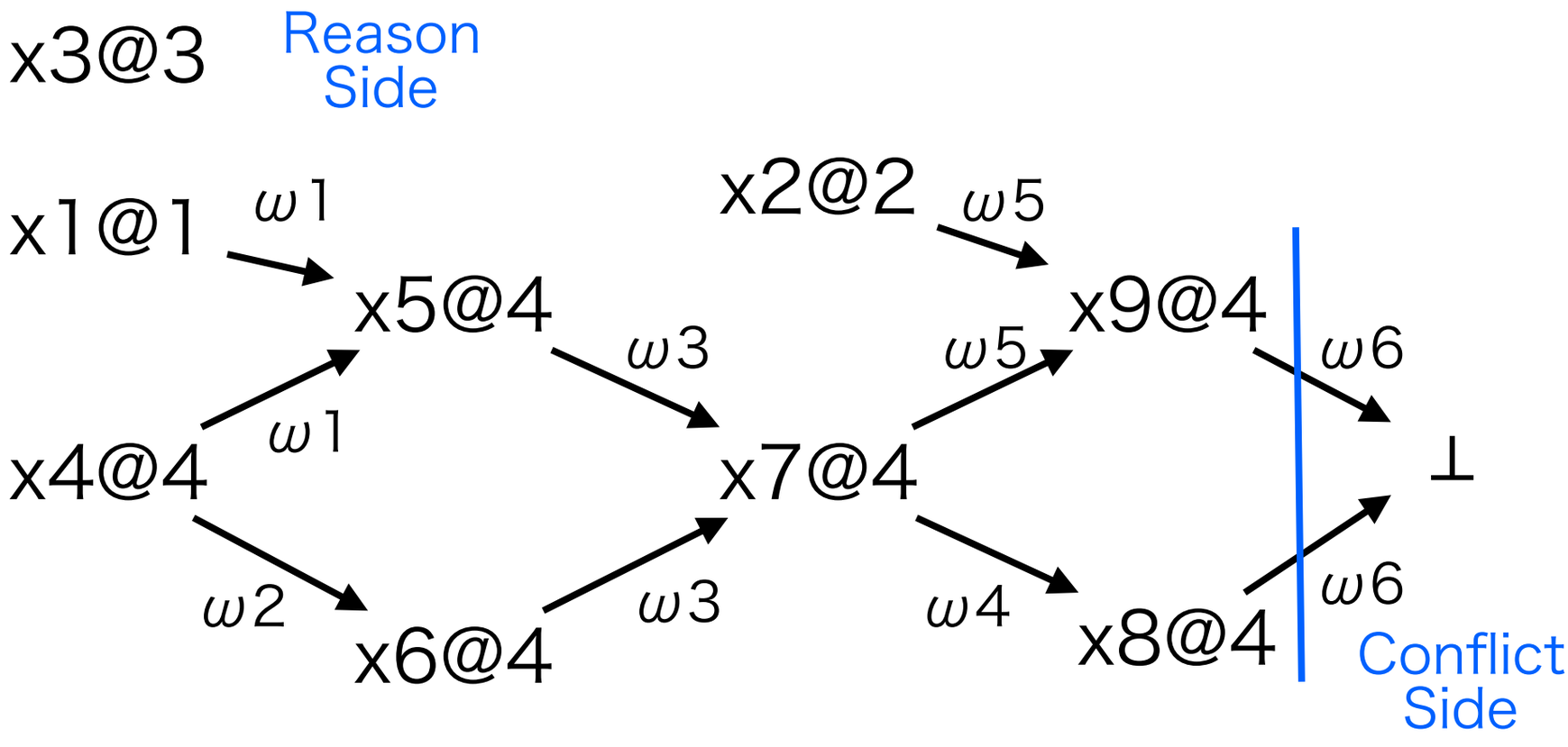
Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

$\omega 6$:	$\omega 4$:
$\neg x8 \vee \neg x9$	$\neg x7 \vee x8$

Implication Graph



Clause DB

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- $\omega 2$: $\neg x4 \vee x6$
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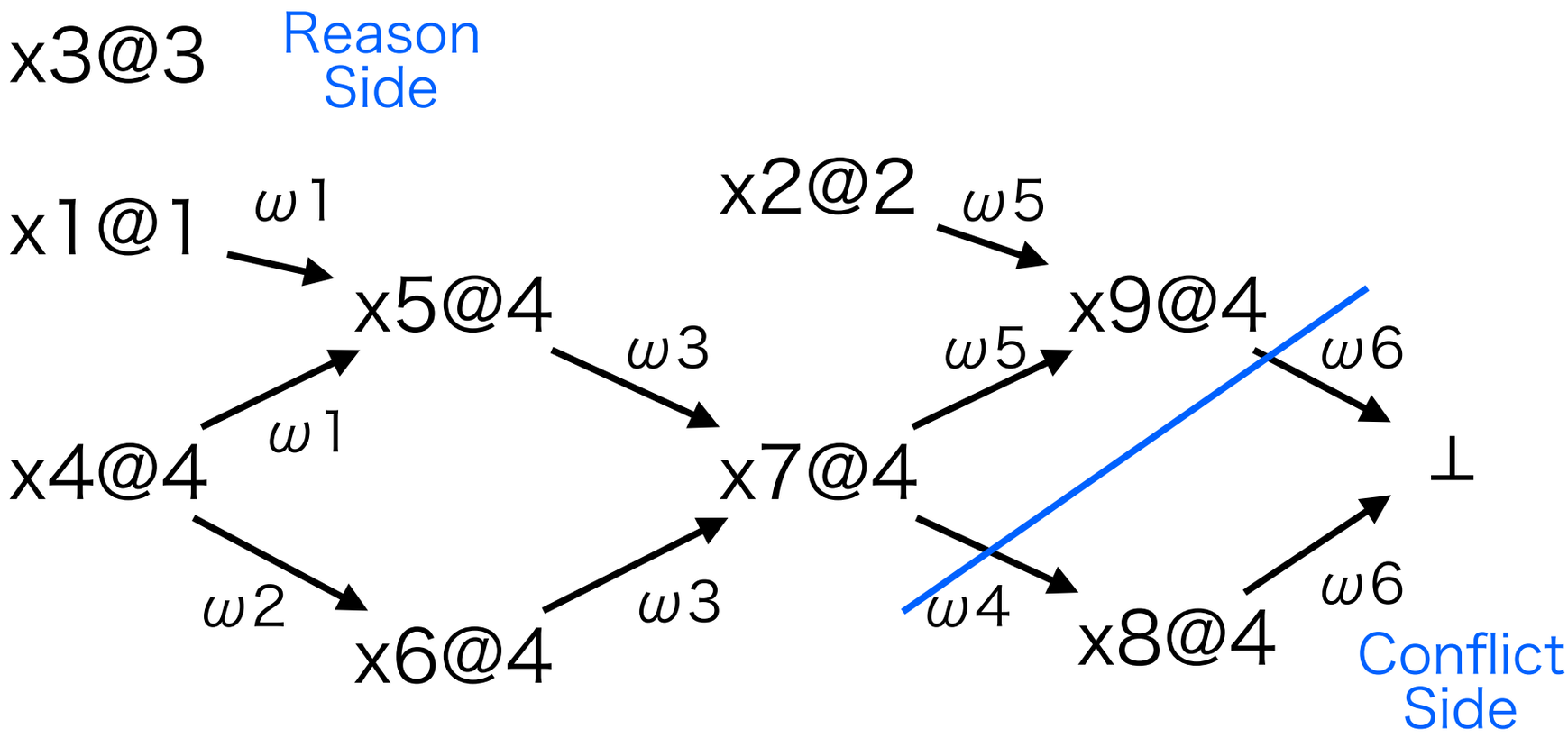
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

$\omega 6$: $\neg x8 \vee \neg x9$ $\quad$ $\omega 4$: $\neg x7 \vee x8$

$\neg x7 \vee \neg x9$

Implication Graph



Clause DB

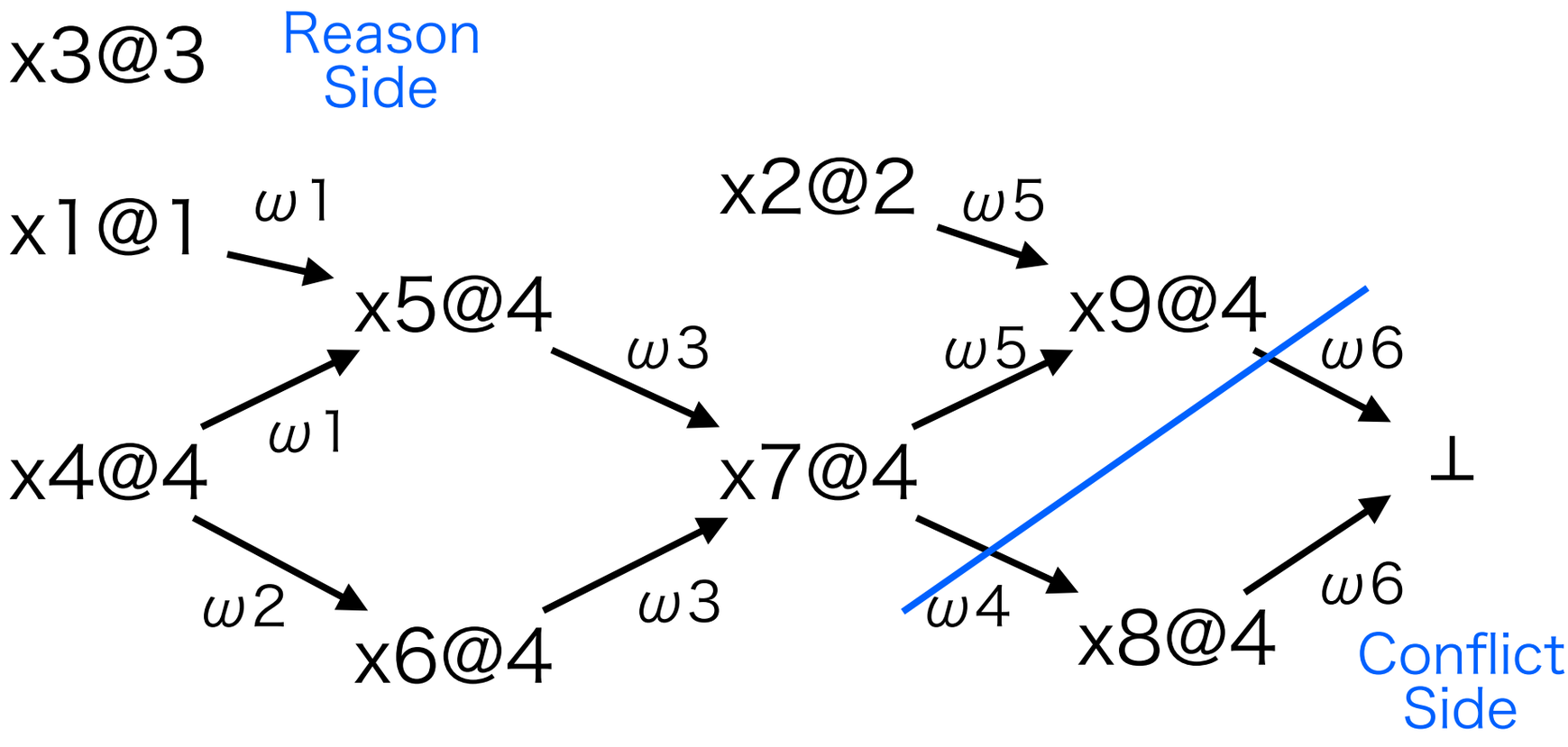
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

$\omega 6$: $\neg x8 \vee \neg x9$ $\quad$ $\omega 4$: $\neg x7 \vee x8$

$\neg x7 \vee \neg x9$ $\quad$ $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$

Implication Graph



Clause DB

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- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

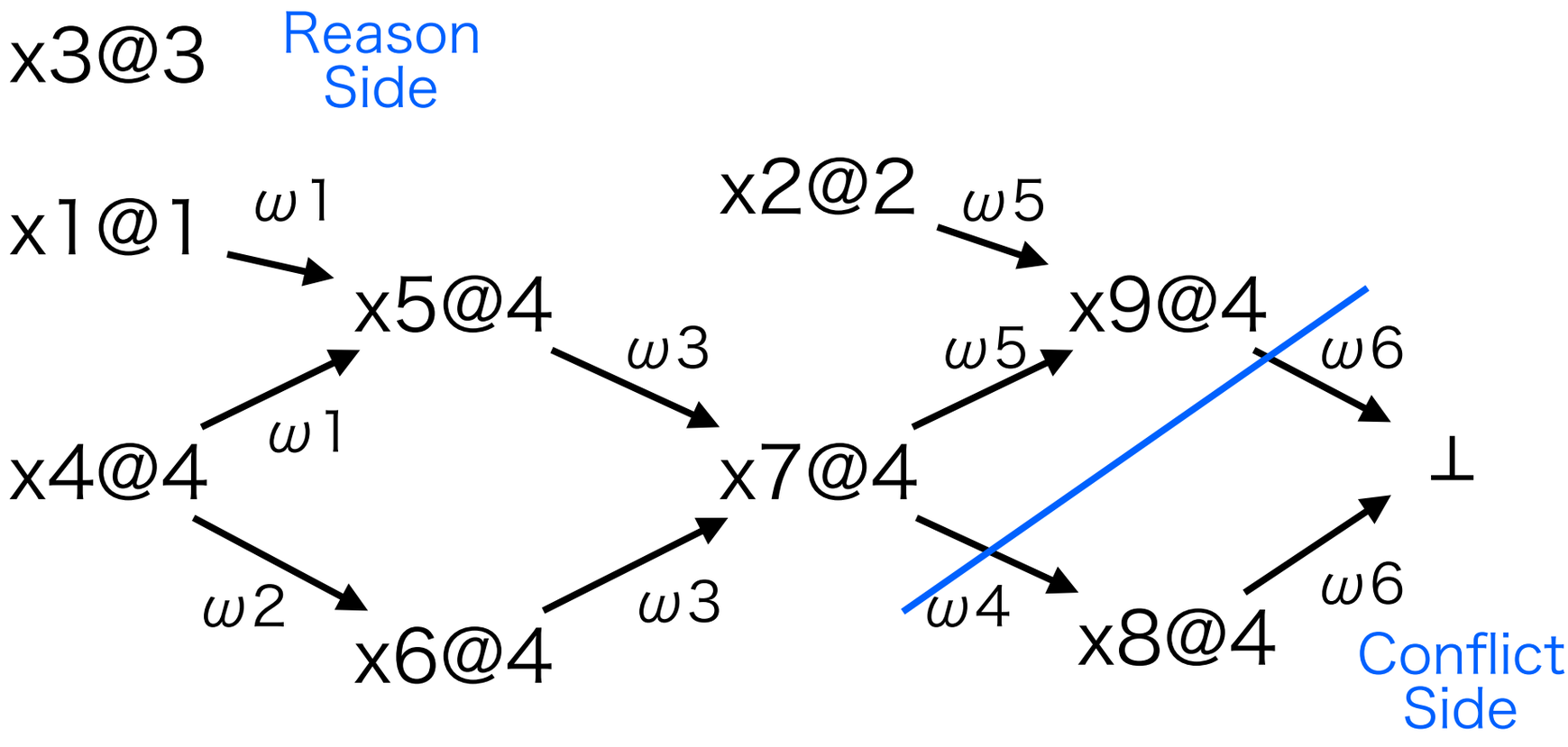
$\omega 6$: $\neg x8 \vee \neg x9$

$\omega 4$: $\neg x7 \vee x8$

$\neg x7 \vee \neg x9$

$\omega 5$: $\neg x2 \vee \neg x7 \vee x9$

Implication Graph



Clause DB

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- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

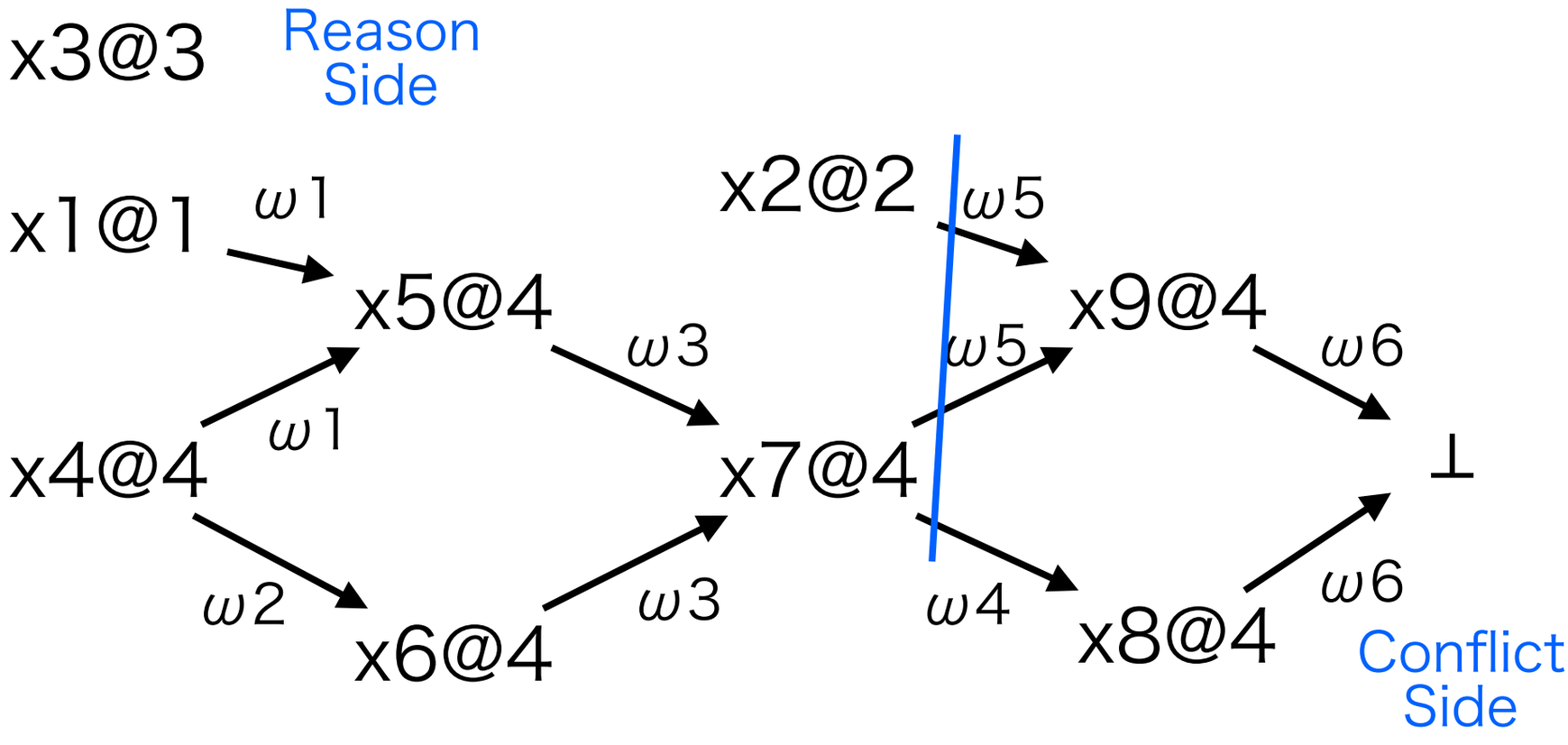
Diagnose

$\omega 6$: $\neg x8 \vee \neg x9$ $\quad$ $\omega 4$: $\neg x7 \vee x8$

$\neg x7 \vee \neg x9$ $\quad$ $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$

$\neg x2 \vee \neg x7$

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

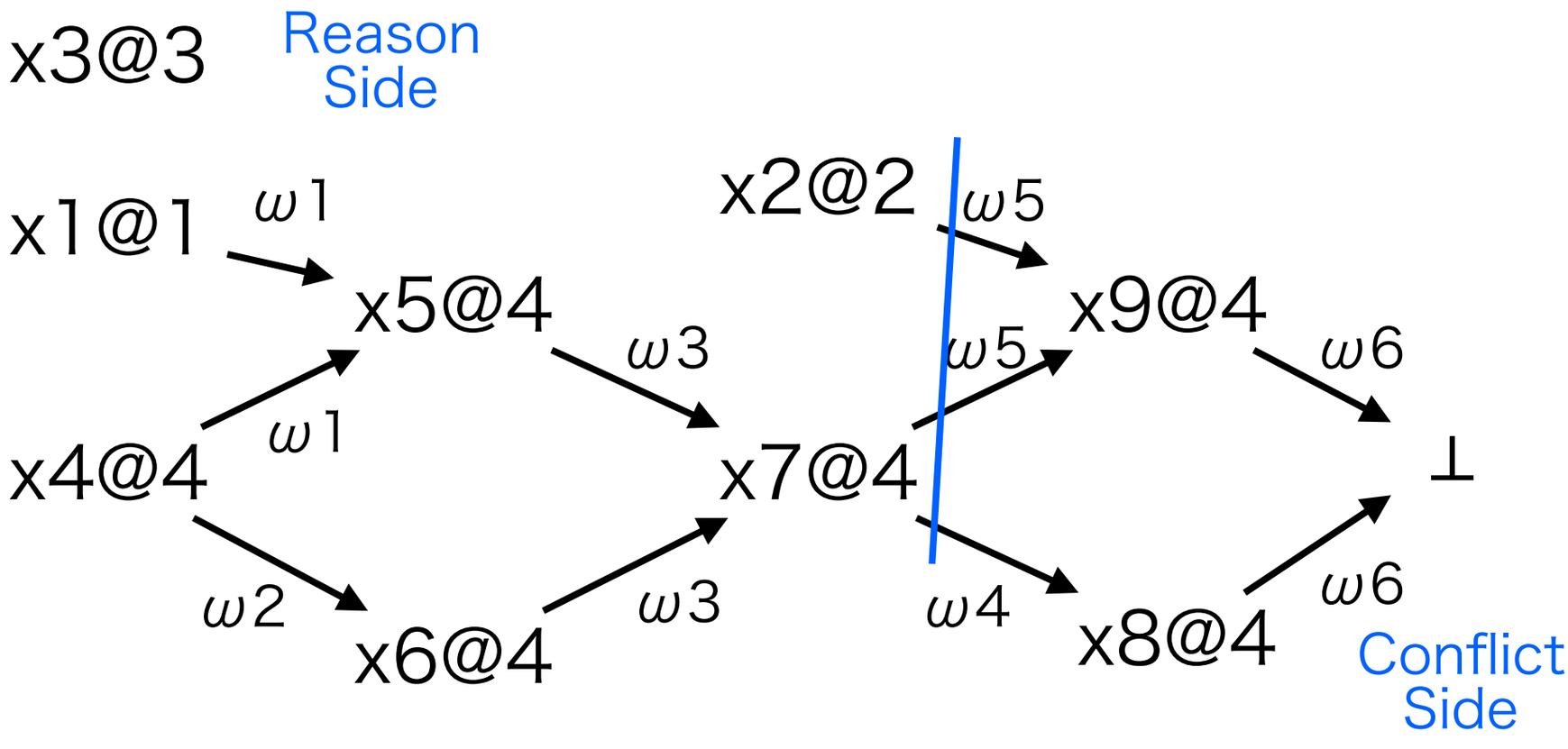
$\omega 6$: $\neg x8 \vee \neg x9$ $\quad$ $\omega 4$: $\neg x7 \vee x8$

$\neg x7 \vee \neg x9$ $\quad$ $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$

$\neg x2 \vee \neg x7$

It contains one literal in the current decision level (i.e. 4).

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$

Diagnose

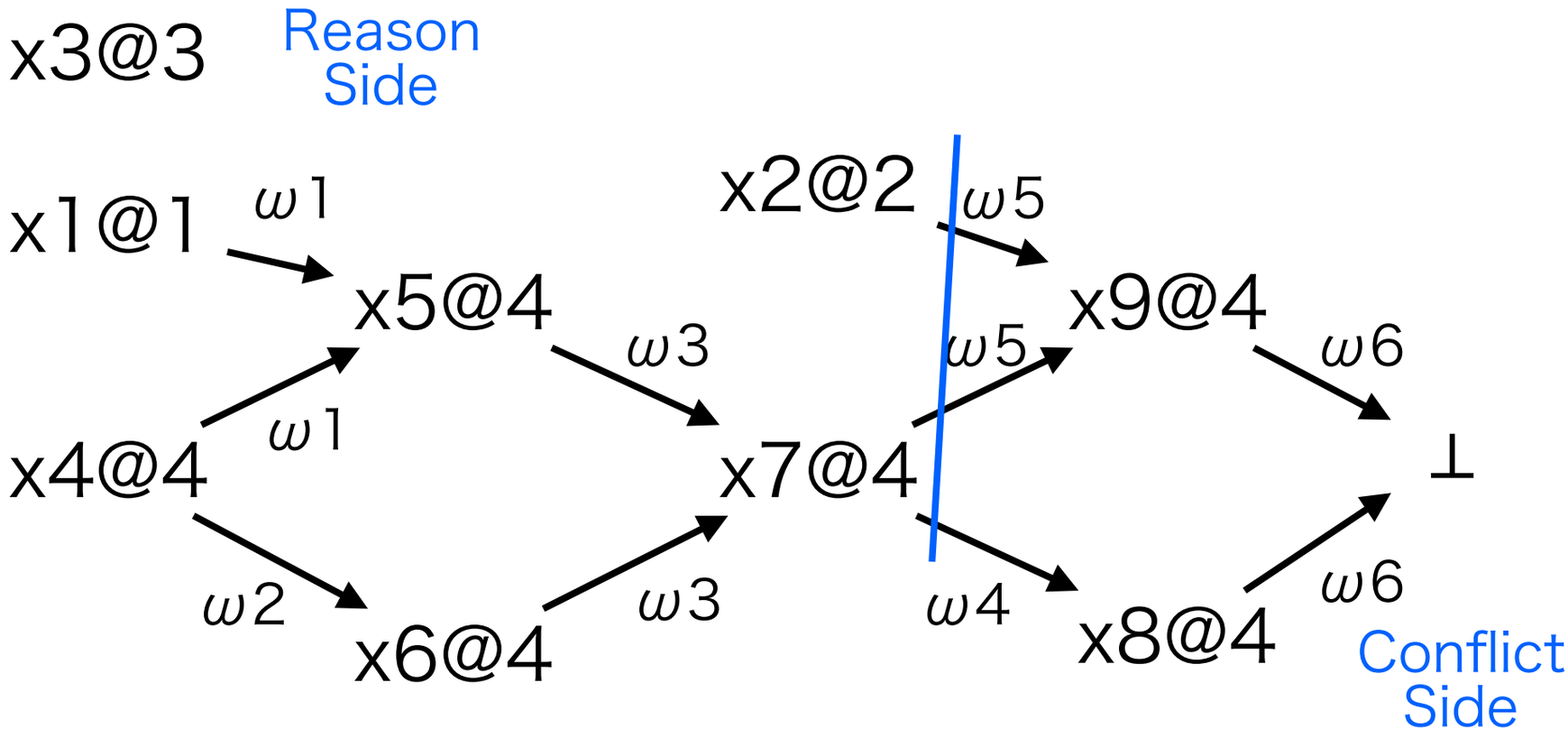
$\omega 6$: $\neg x8 \vee \neg x9$ $\quad$ $\omega 4$: $\neg x7 \vee x8$

$\neg x7 \vee \neg x9$ $\quad$ $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$

$\neg x2 \vee \neg x7$

It contains one literal in the current decision level (i.e. 4).

Implication Graph



Clause DB

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- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

New clause is learnt !

Backtrack

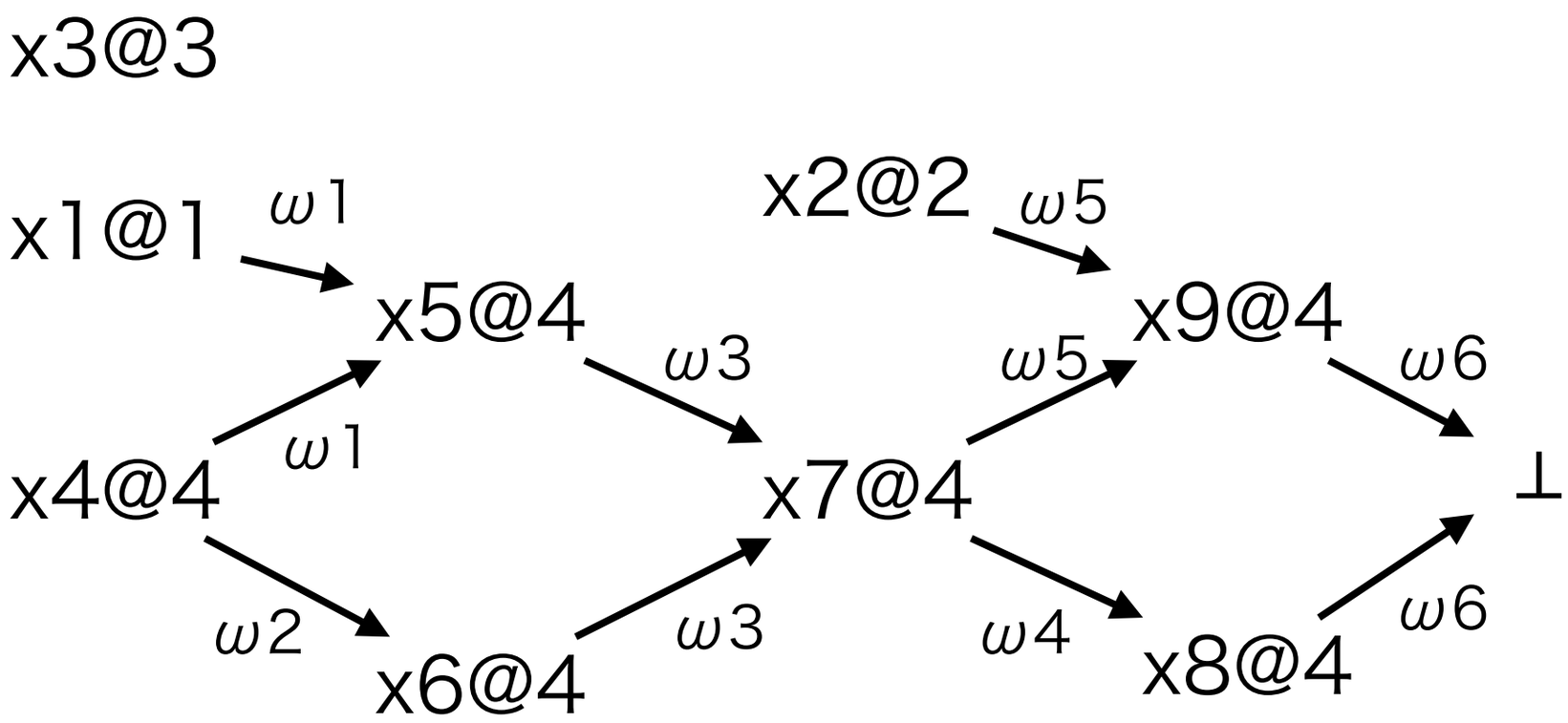
Learnt clause:
¬x2 ∨ ¬x7

x2 was set at level 2
x7 was set at level 4

Assertion level of this clause (which is the second highest level) is 2.

⇒ Backtrack to level 2!

Implication Graph



Clause DB

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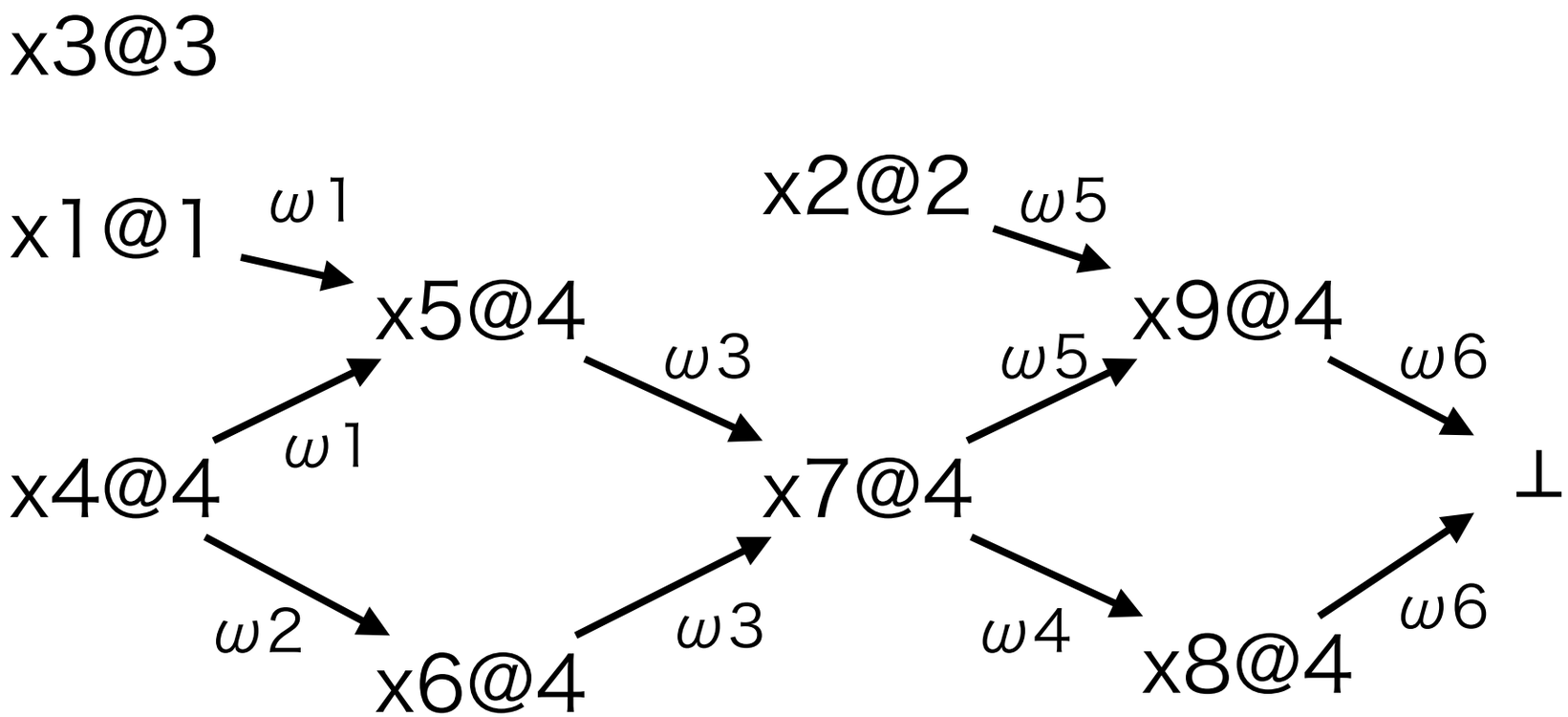
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

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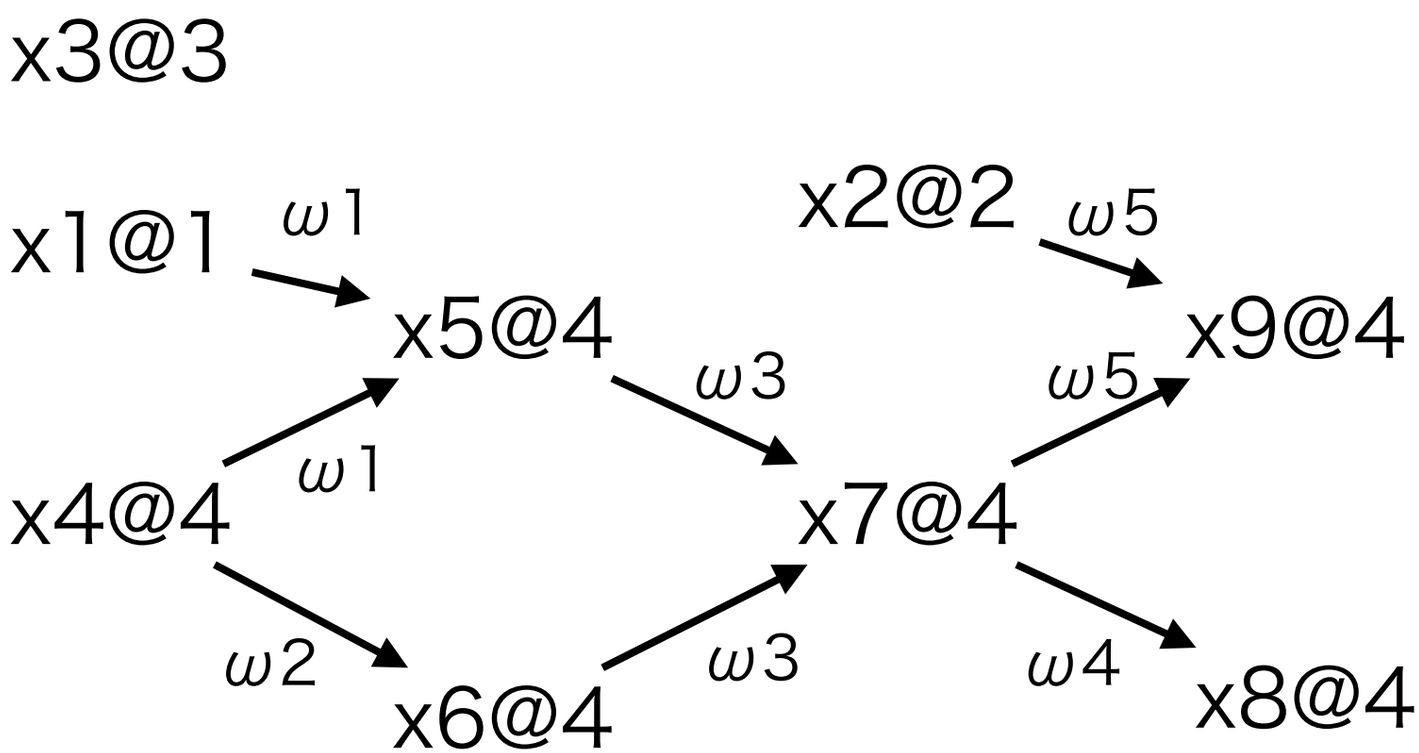
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

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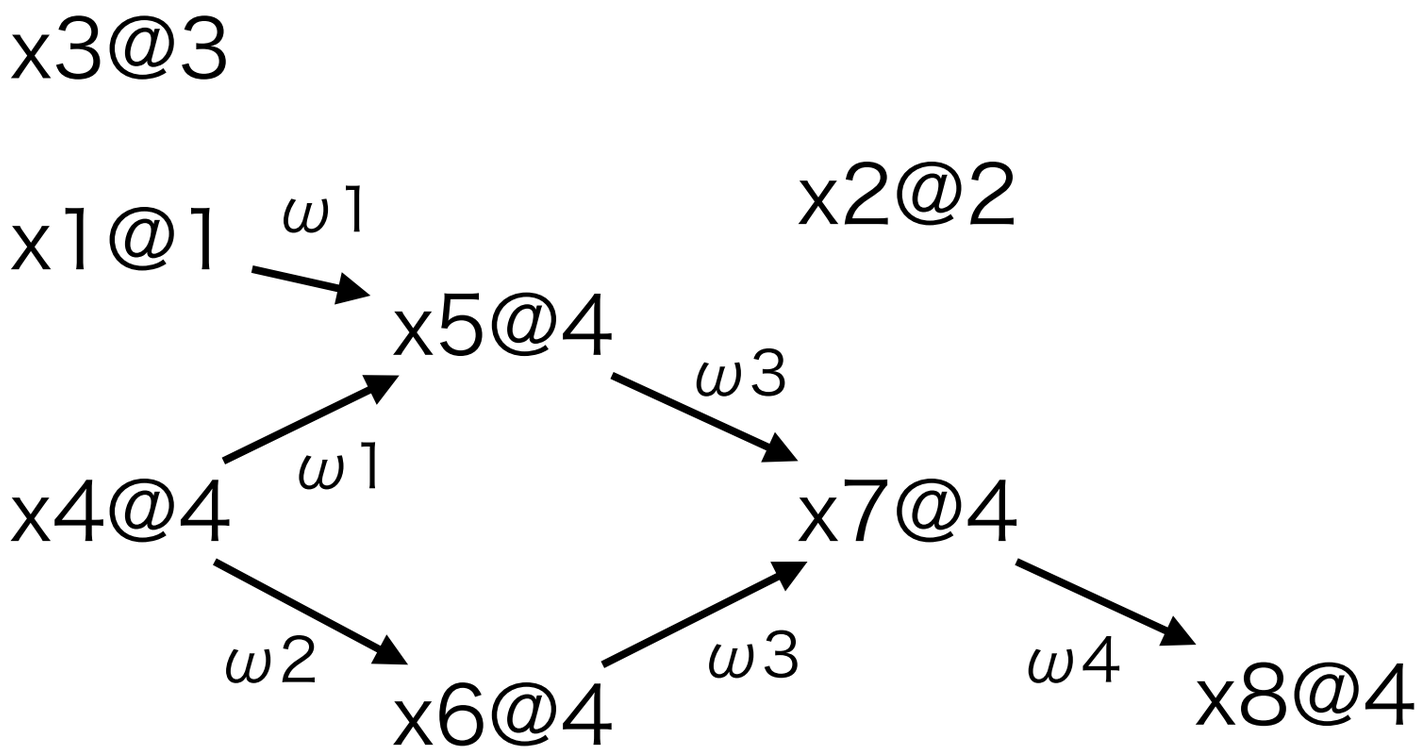
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



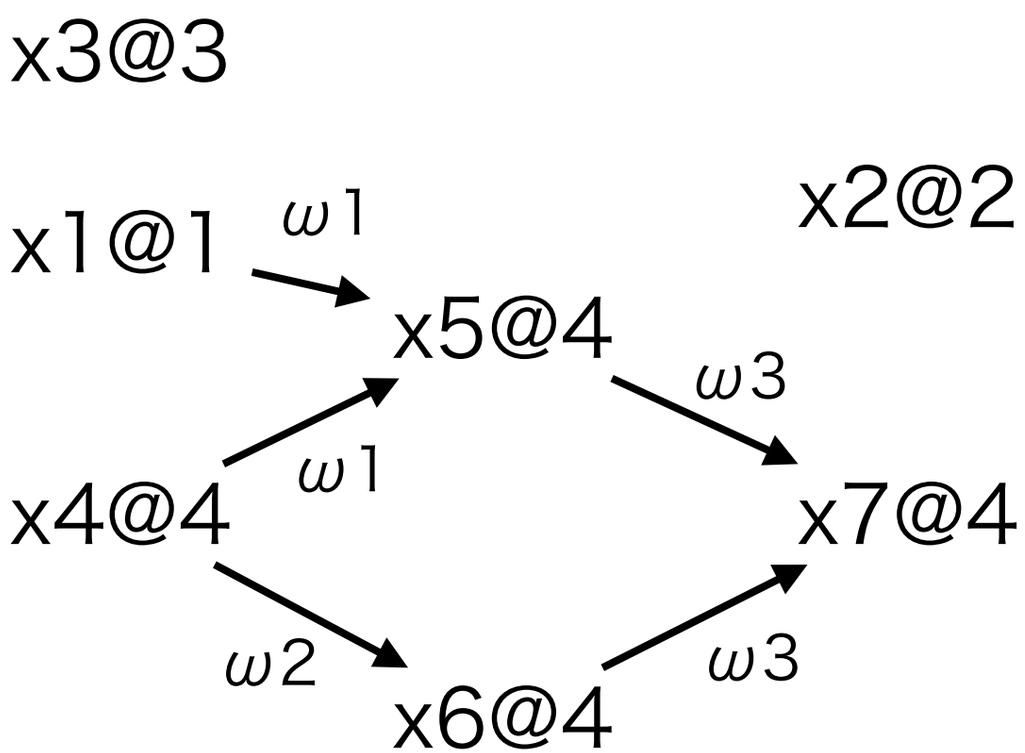
Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



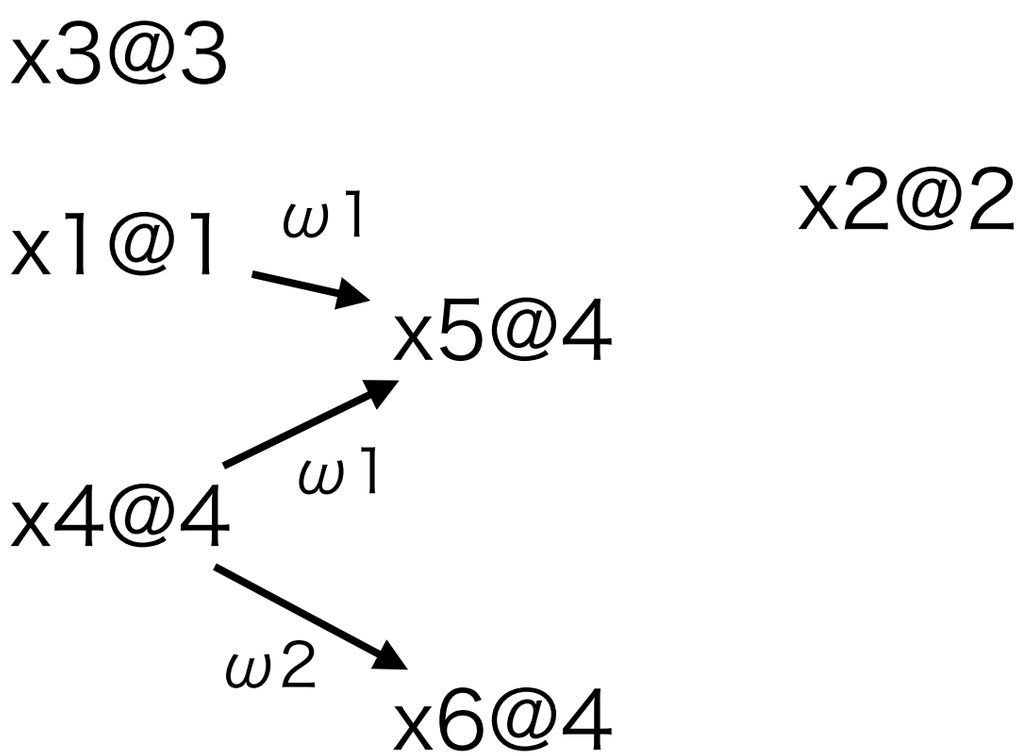
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



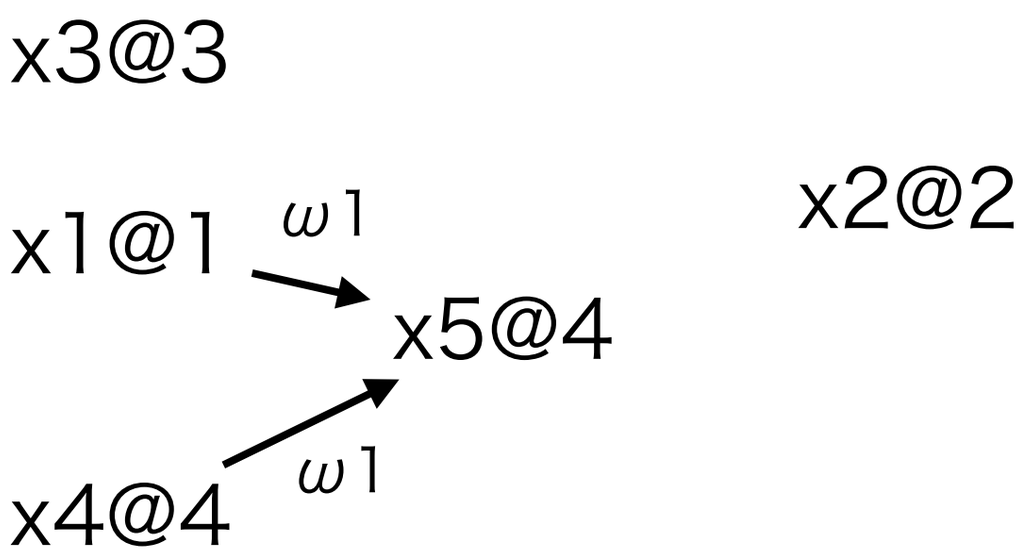
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph



Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x2@2

x4@4

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph

x3@3

x1@1

x2@2

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Backtrack

x1
|
x2
|
x3
|
x4

Implication Graph

x1@1

x2@2

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph

x1@1

x2@2

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7

Deduce

x1
|
x2
|
x3
|
x4

Implication Graph

x1@1

$\omega 8$
x2@2 → ¬x7@2

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Decide

x1
|
x2
|
x3
|
x4

Implication Graph

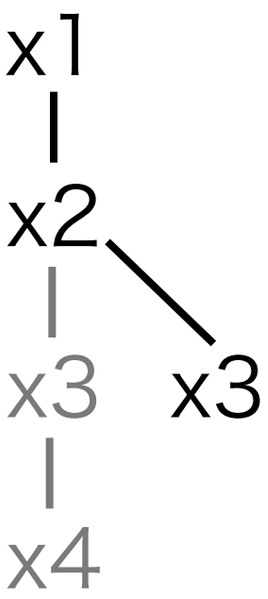
x1@1

$\overset{\omega 8}{\text{x2@2} \rightarrow \neg\text{x7@2}}$

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Decide



Implication Graph

x3@3

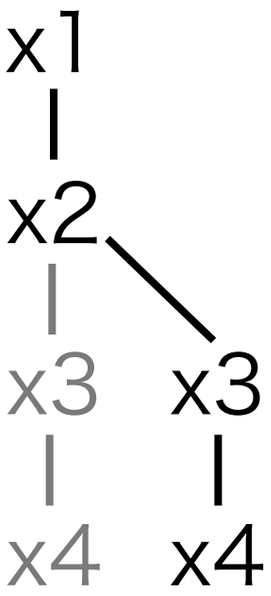
x1@1

$$x2@2 \xrightarrow{\omega 8} \neg x7@2$$

Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Decide



Implication Graph

x3@3

x1@1

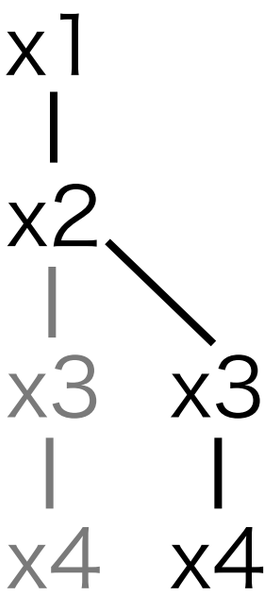
x4@4

$$x2@2 \xrightarrow{\omega 8} \neg x7@2$$

Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Deduce



Implication Graph

x3@3

x1@1

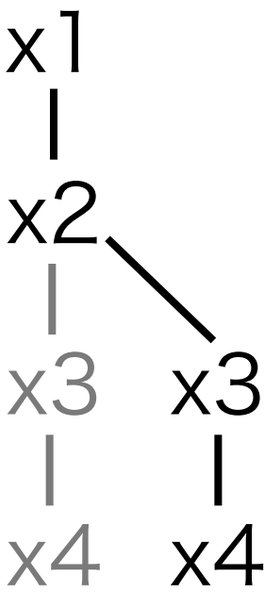
x4@4

$\overset{\omega 8}{\text{x2@2} \rightarrow \neg\text{x7@2}}$

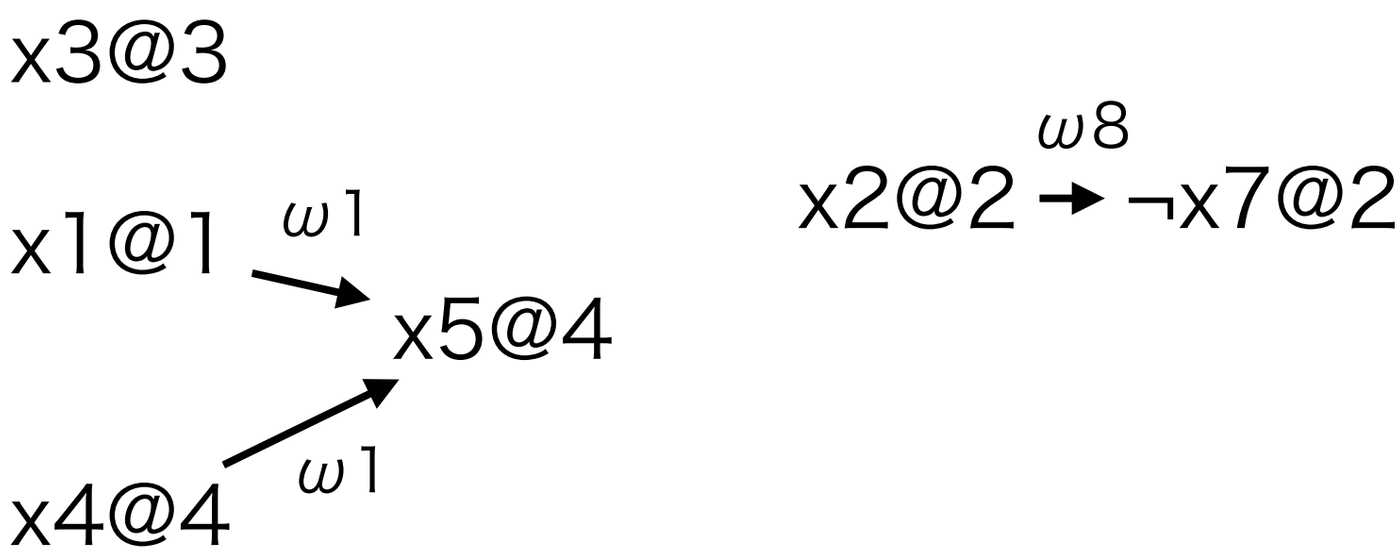
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Deduce



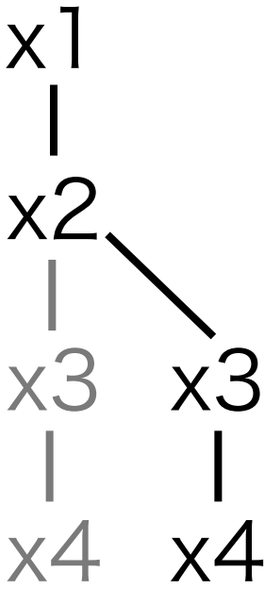
Implication Graph



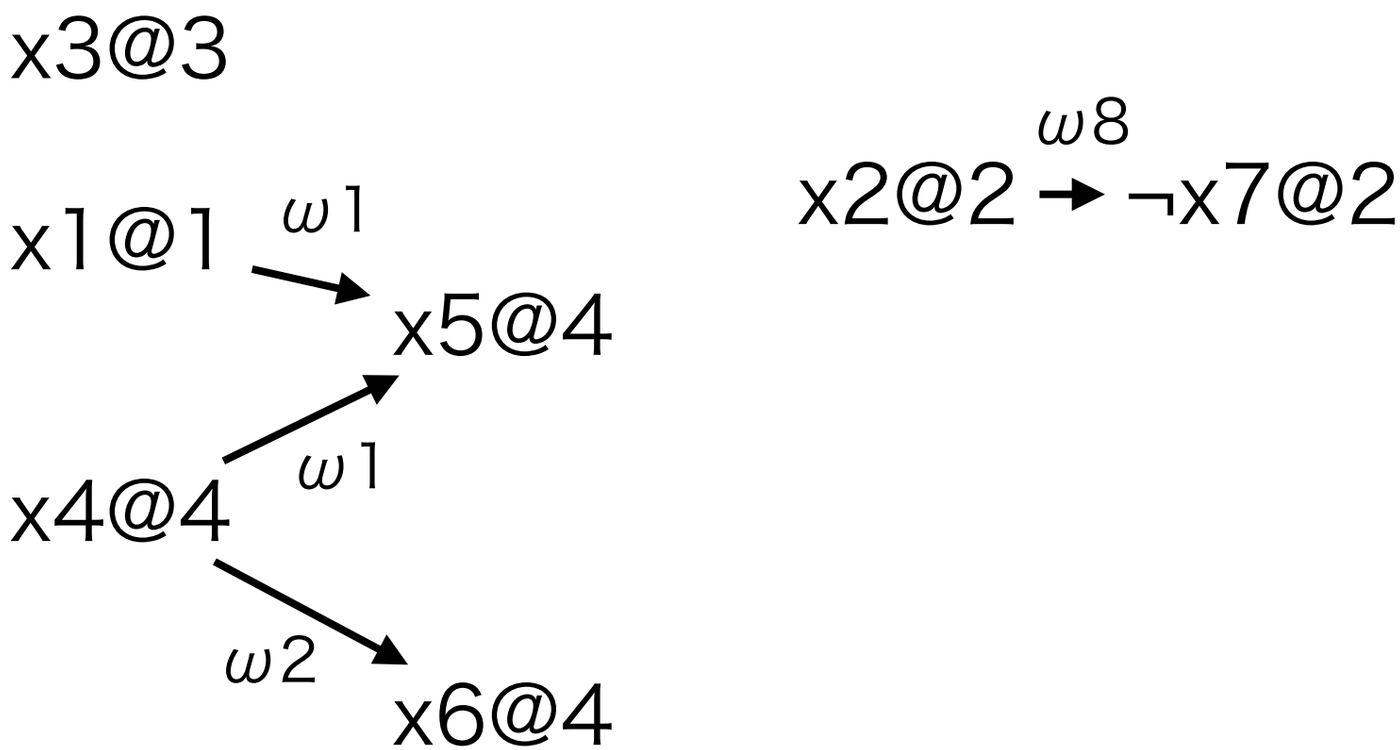
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Deduce



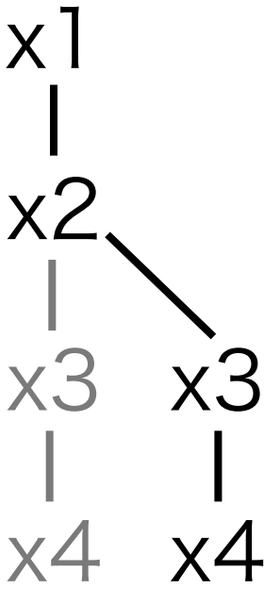
Implication Graph



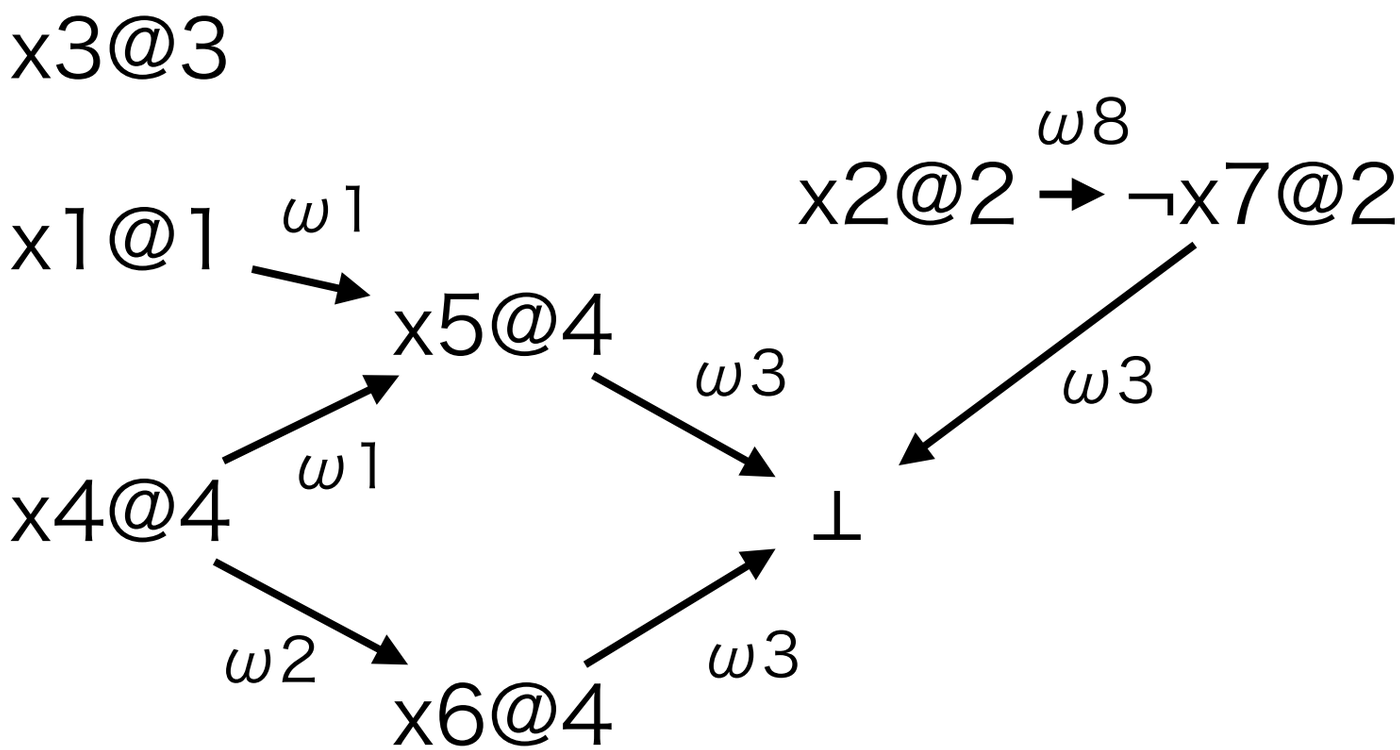
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Deduce



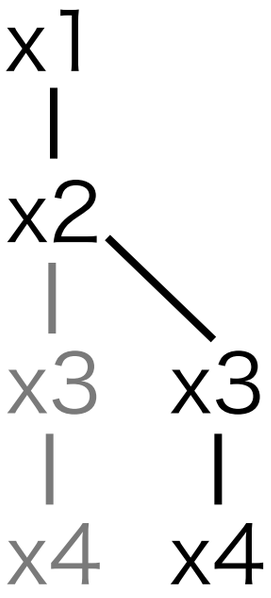
Implication Graph



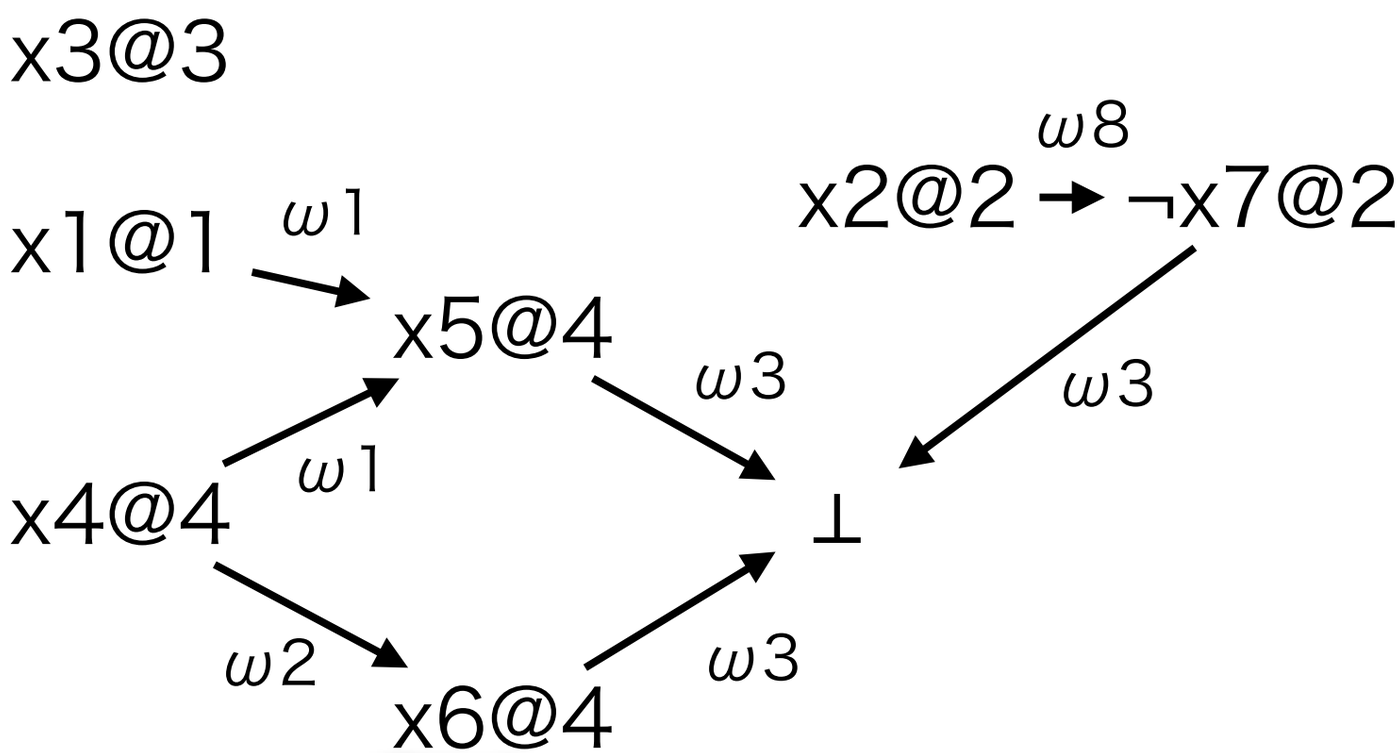
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Deduce



Implication Graph



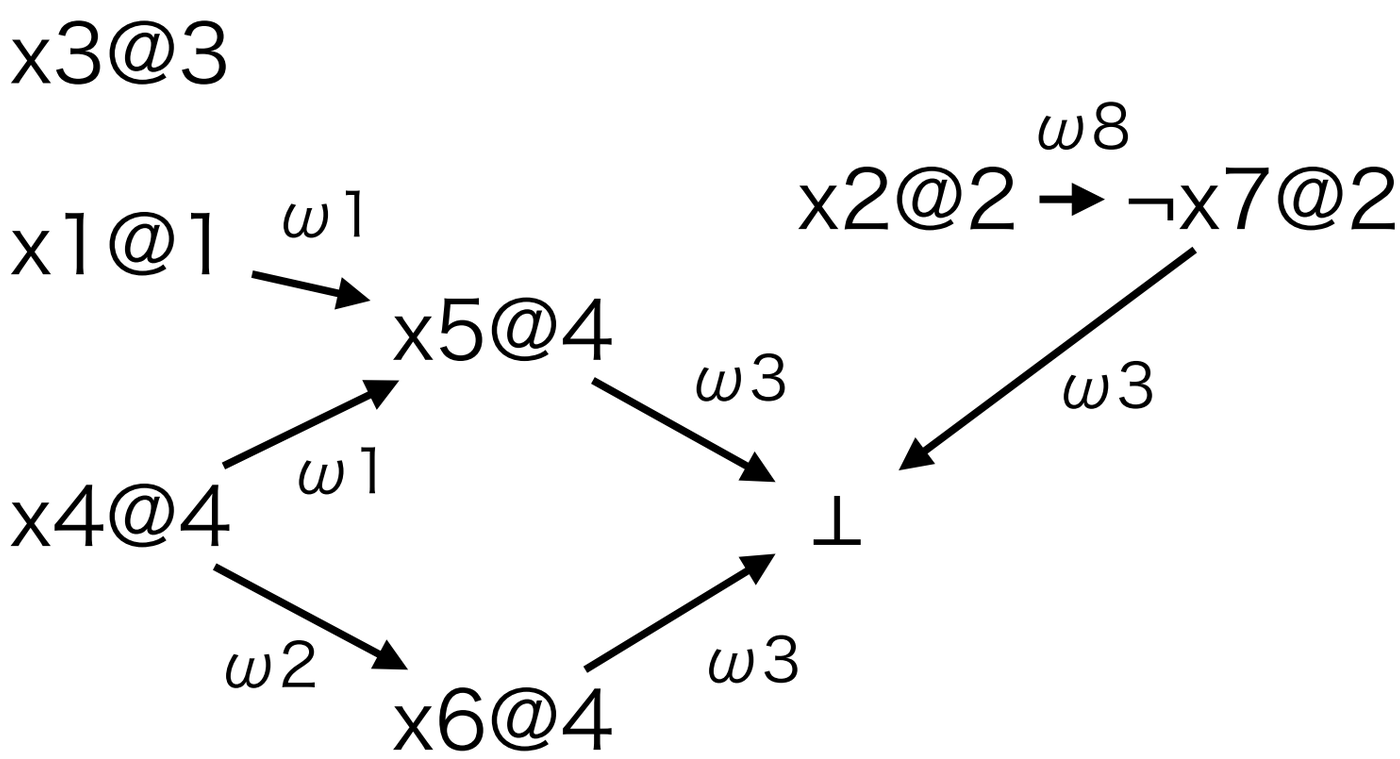
Conflict

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Diagnose

Implication Graph



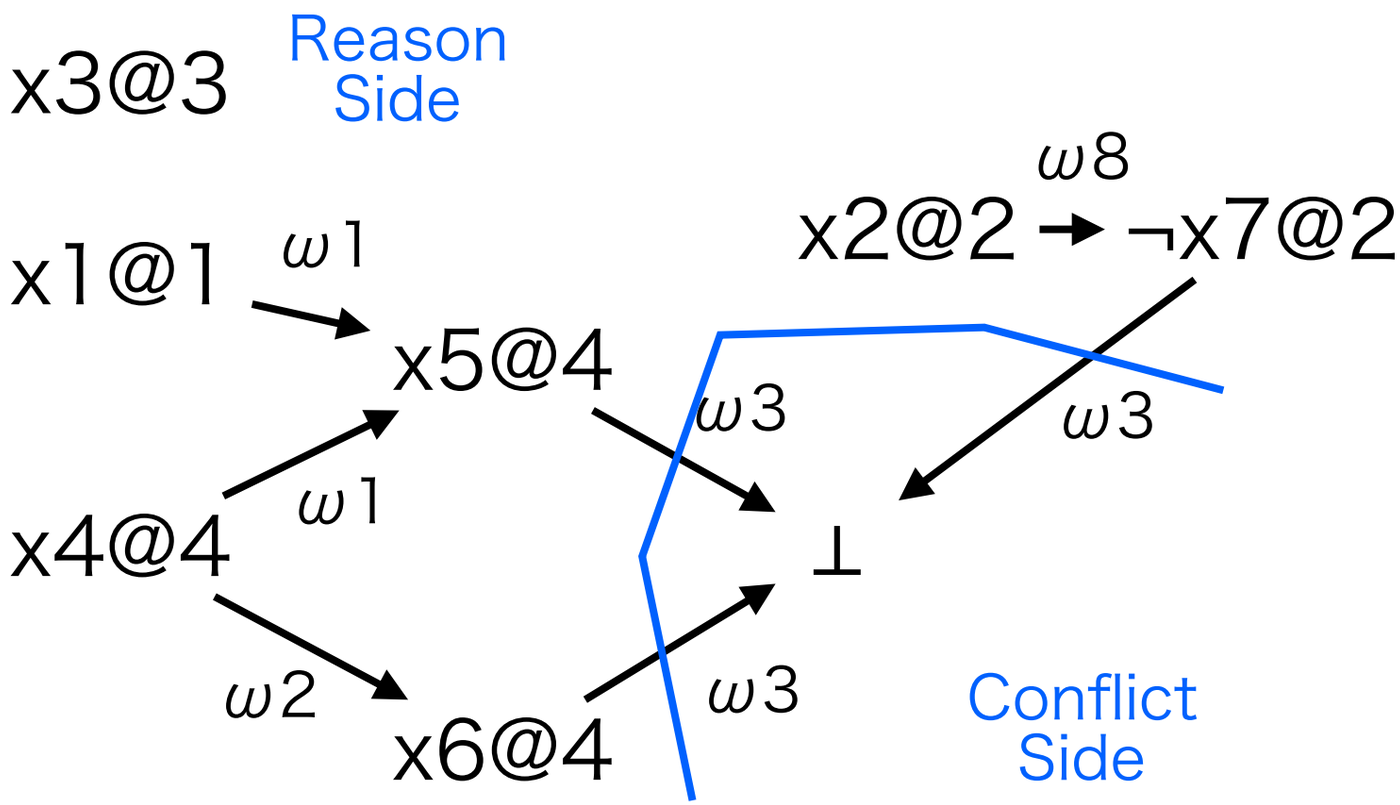
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Diagnose

$\omega 3$:
$\neg x5 \vee \neg x6 \vee x7$

Implication Graph



Clause DB

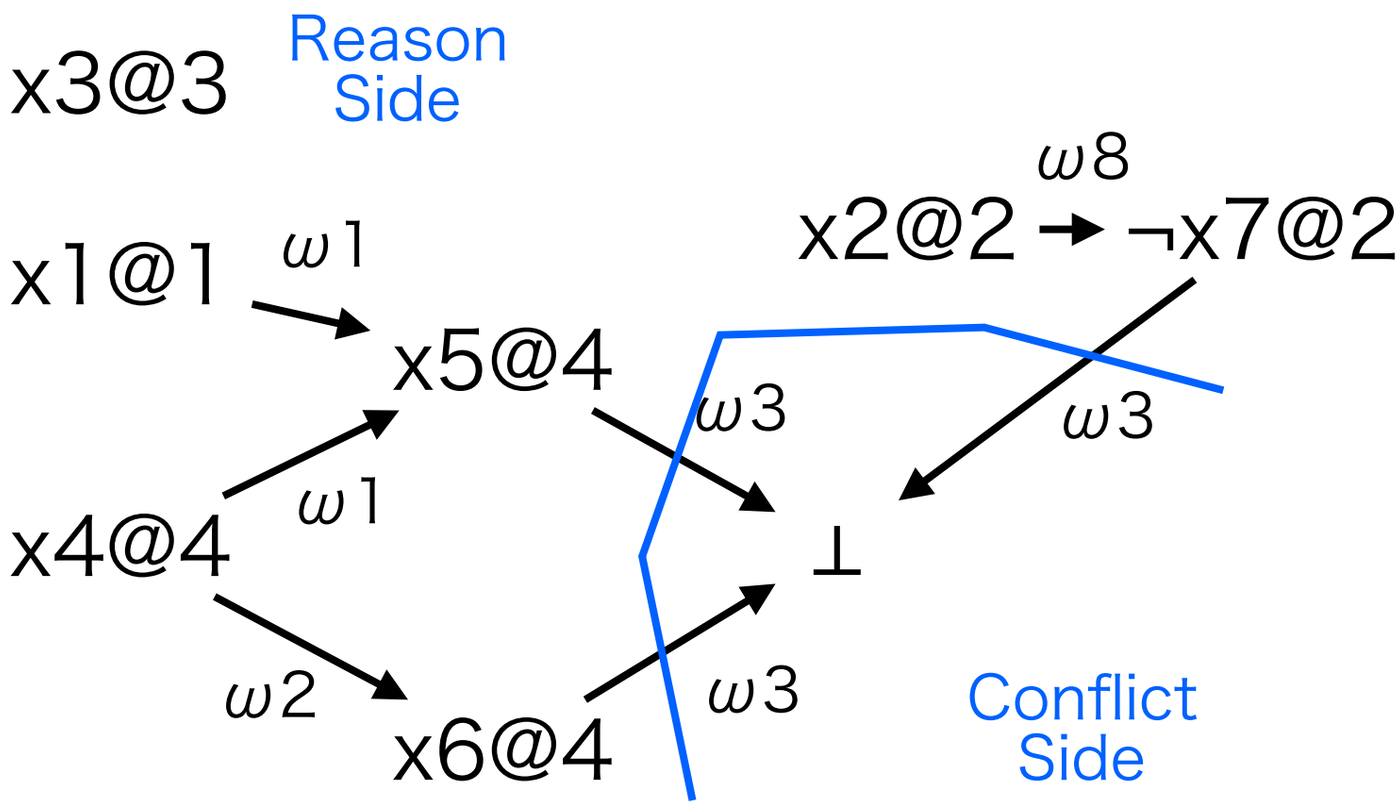
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Diagnose

$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$

$\omega 2$: $\neg x4 \vee x6$

Implication Graph



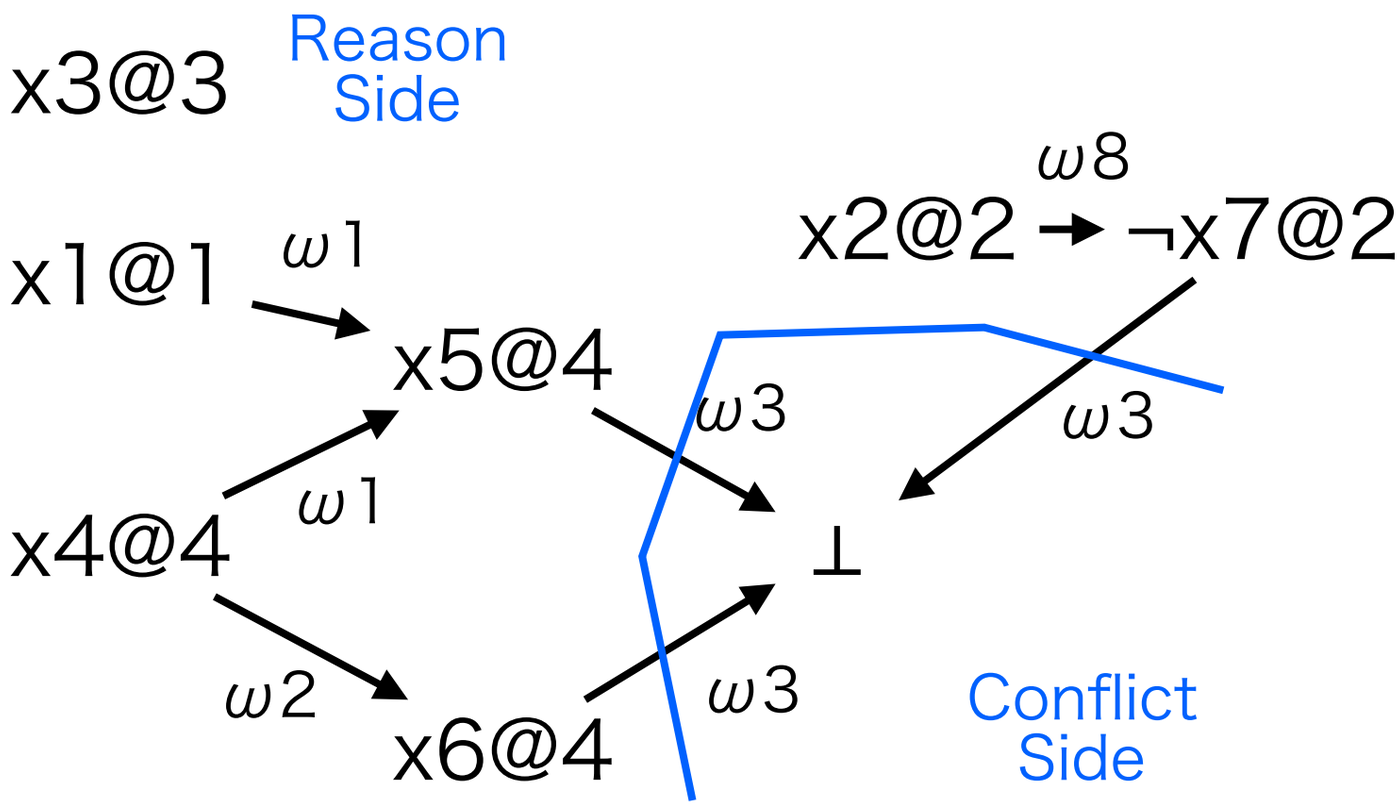
Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Diagnose

$\omega 3$: ¬x5∨¬x6∨x7	$\omega 2$: ¬x4∨x6

Implication Graph



Clause DB

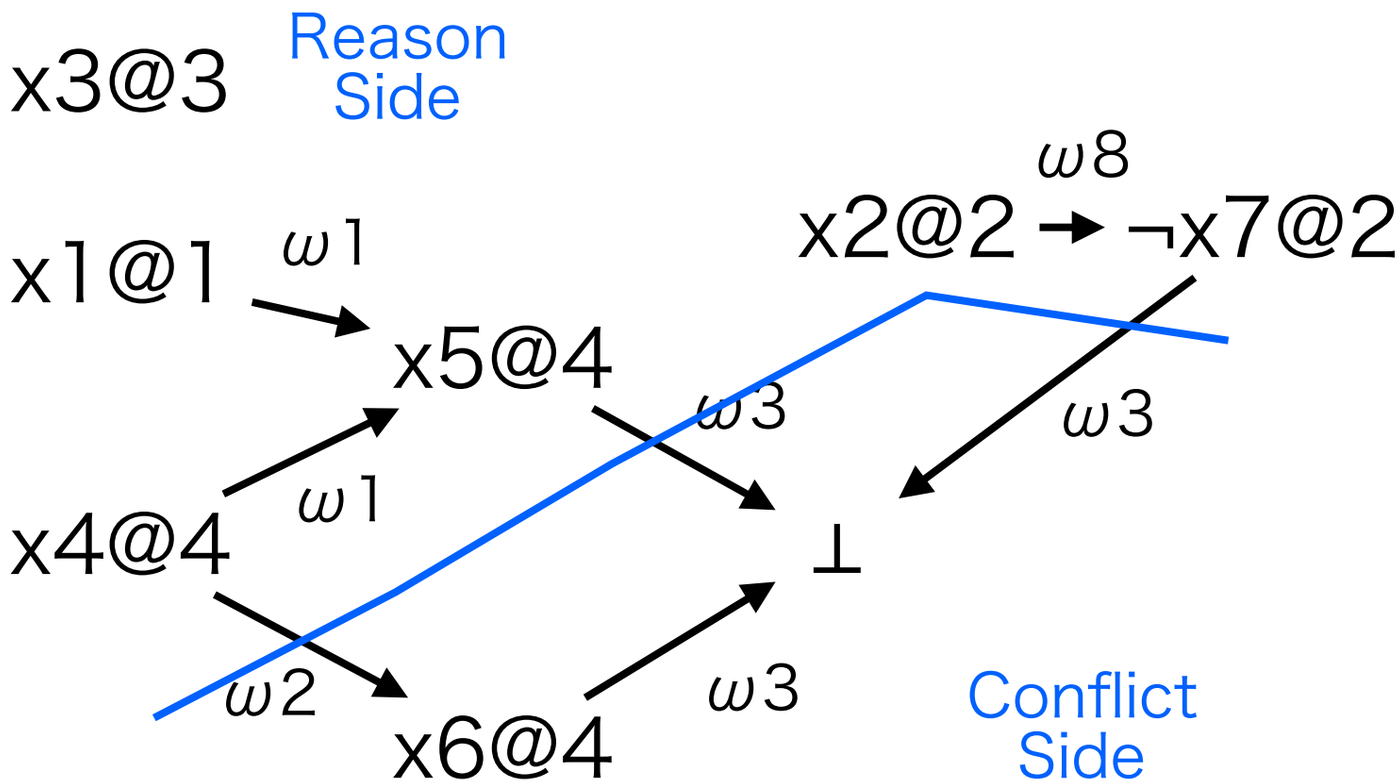
- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7

Diagnose

$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$ $\quad$ $\omega 2$: $\neg x4 \vee x6$

$\neg x4 \vee \neg x5 \vee x7$

Implication Graph



Clause DB

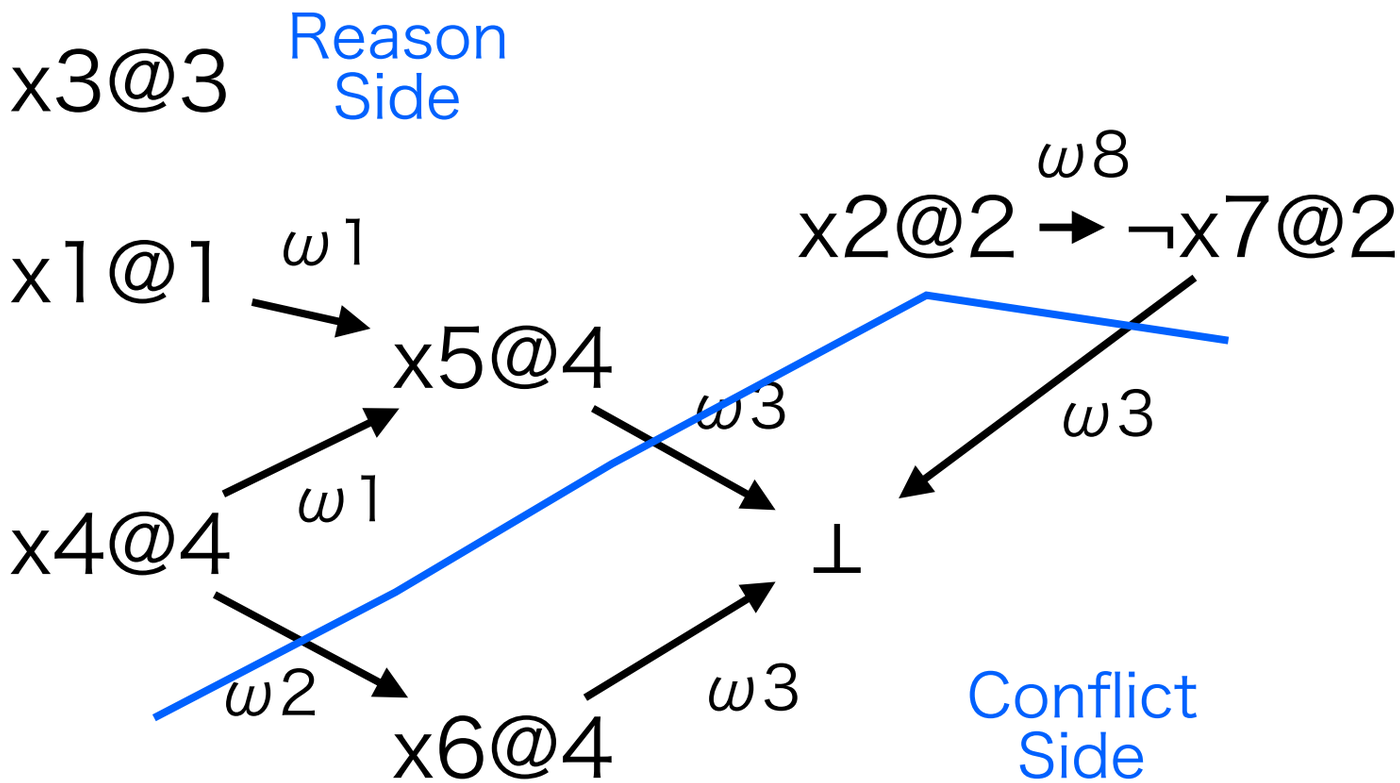
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Diagnose

$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$ $\quad$ $\omega 2$: $\neg x4 \vee x6$

$\neg x4 \vee \neg x5 \vee x7$ $\quad$ $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Diagnose

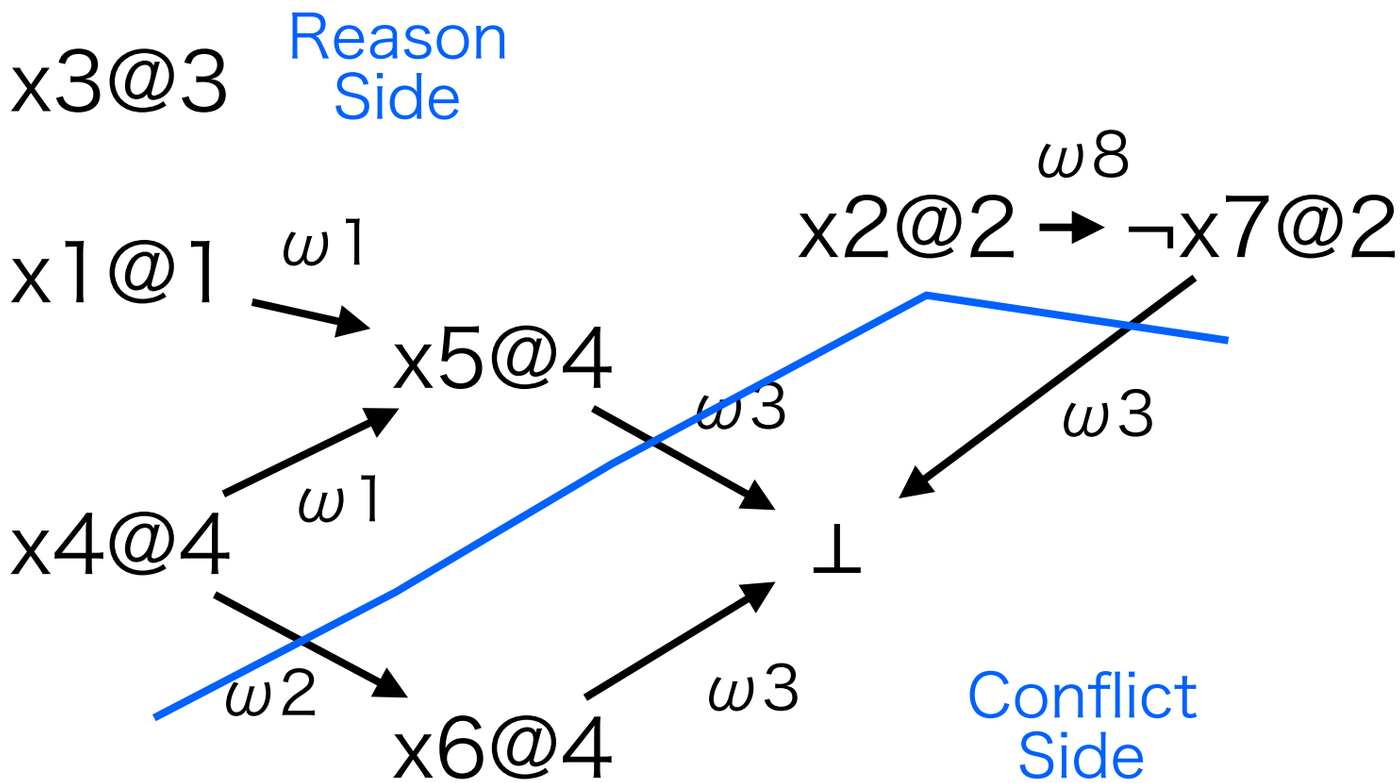
$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$

$\omega 2$: $\neg x4 \vee x6$

$\neg x4 \vee \neg x5 \vee x7$

$\omega 1$: $\neg x1 \vee \neg x4 \vee x5$

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

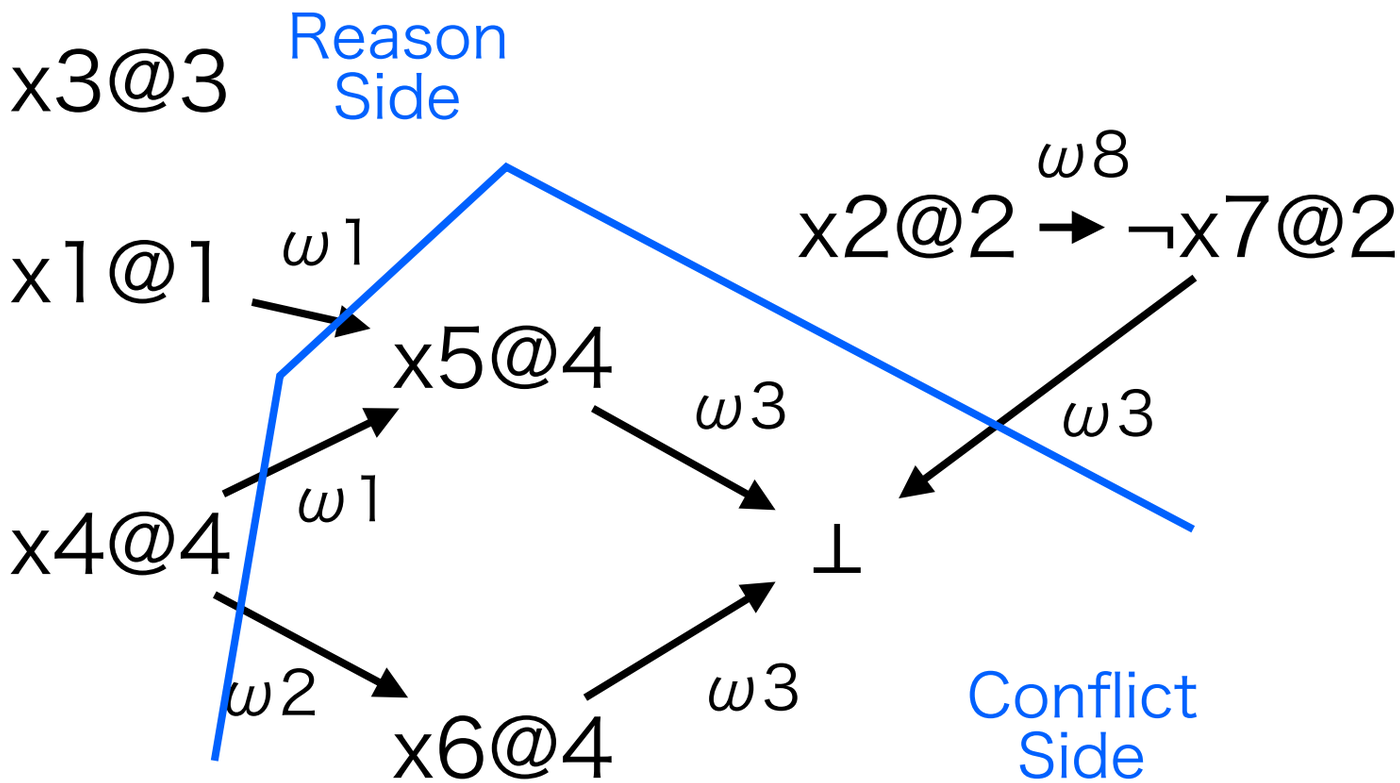
Diagnose

$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$ $\omega 2$: $\neg x4 \vee x6$

$\neg x4 \vee \neg x5 \vee x7$ $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$

$\neg x1 \vee \neg x4 \vee x7$

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Diagnose

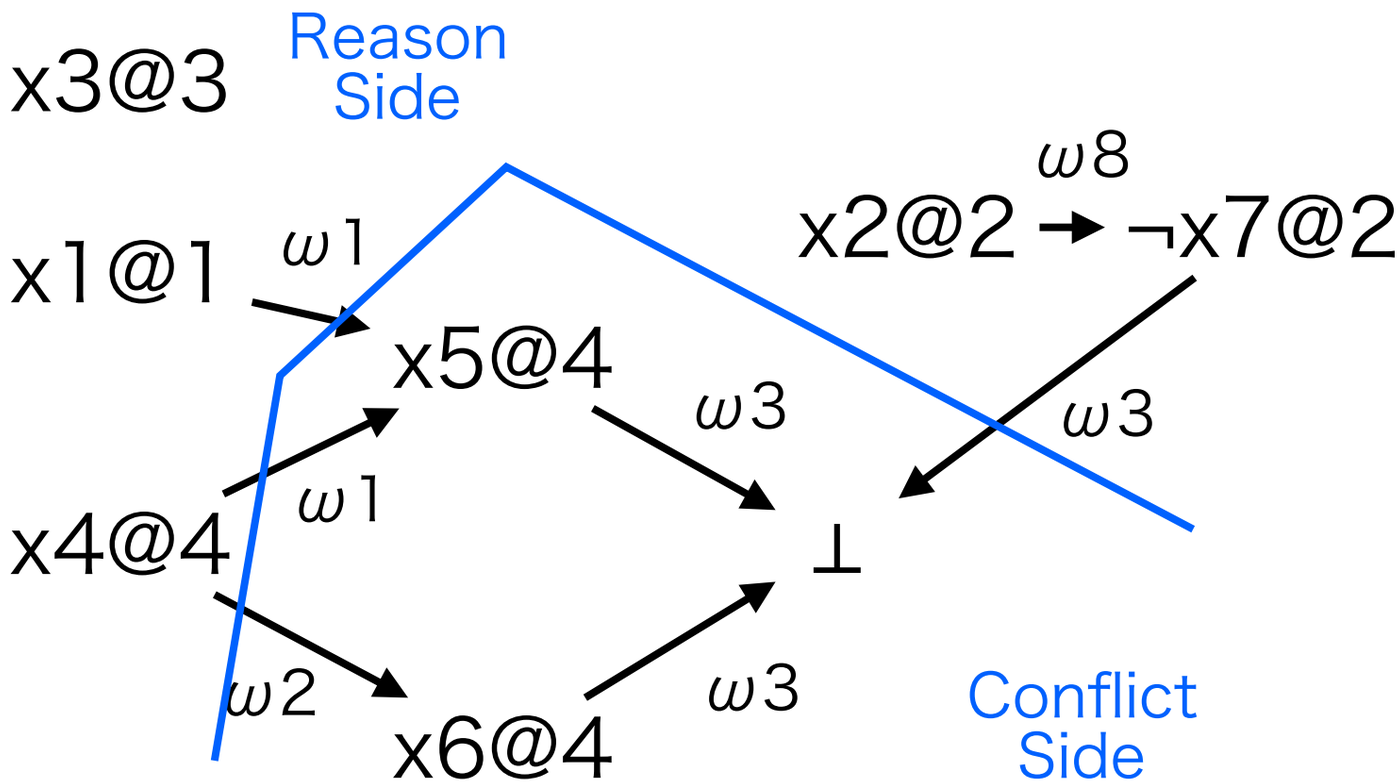
$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$ $\quad$ $\omega 2$: $\neg x4 \vee x6$

$\neg x4 \vee \neg x5 \vee x7$ $\quad$ $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$

$\neg x1 \vee \neg x4 \vee x7$

It contains one literal in the current decision level (i.e. 4).

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$

Diagnose

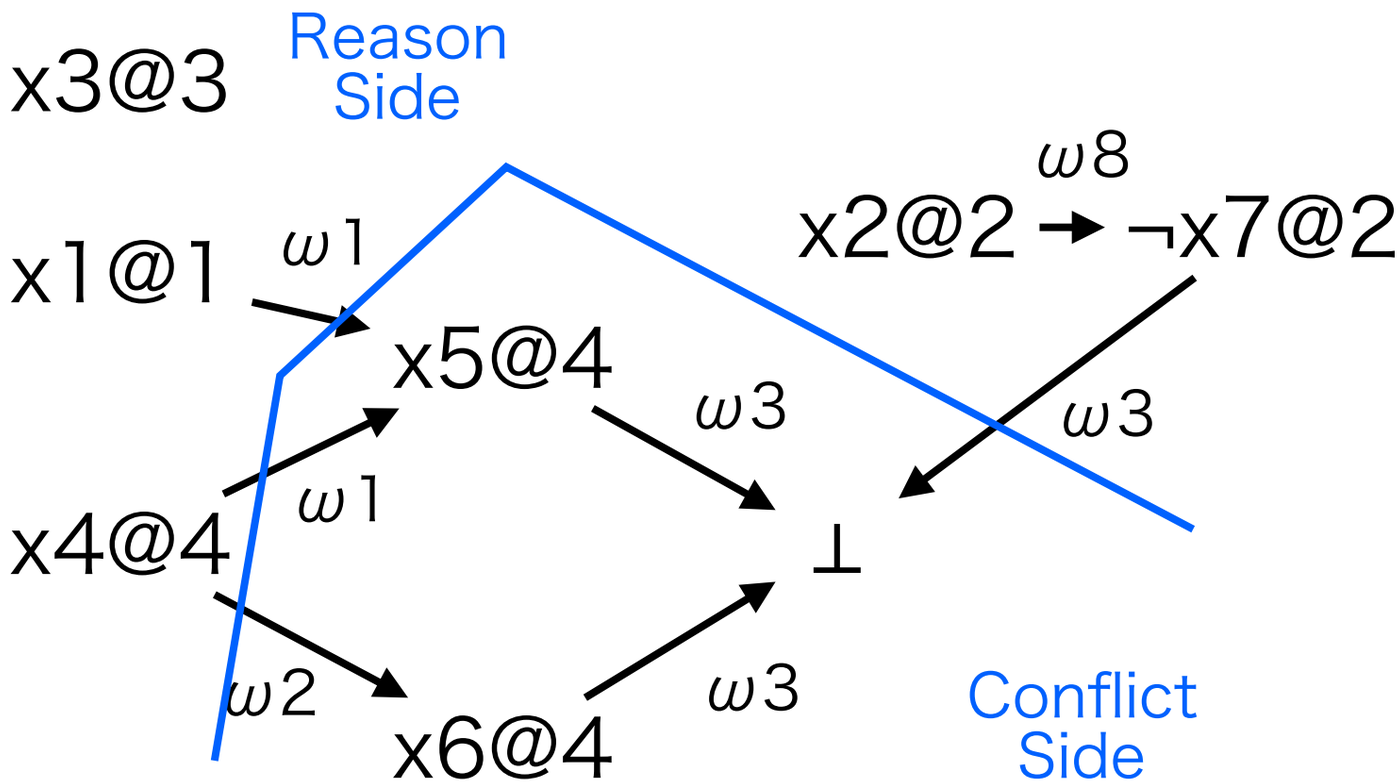
$\omega 3$: $\neg x5 \vee \neg x6 \vee x7$ $\quad$ $\omega 2$: $\neg x4 \vee x6$

$\neg x4 \vee \neg x5 \vee x7$ $\quad$ $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$

$\neg x1 \vee \neg x4 \vee x7$

It contains one literal in the current decision level (i.e. 4).

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$

New clause is learnt !

Backtrack

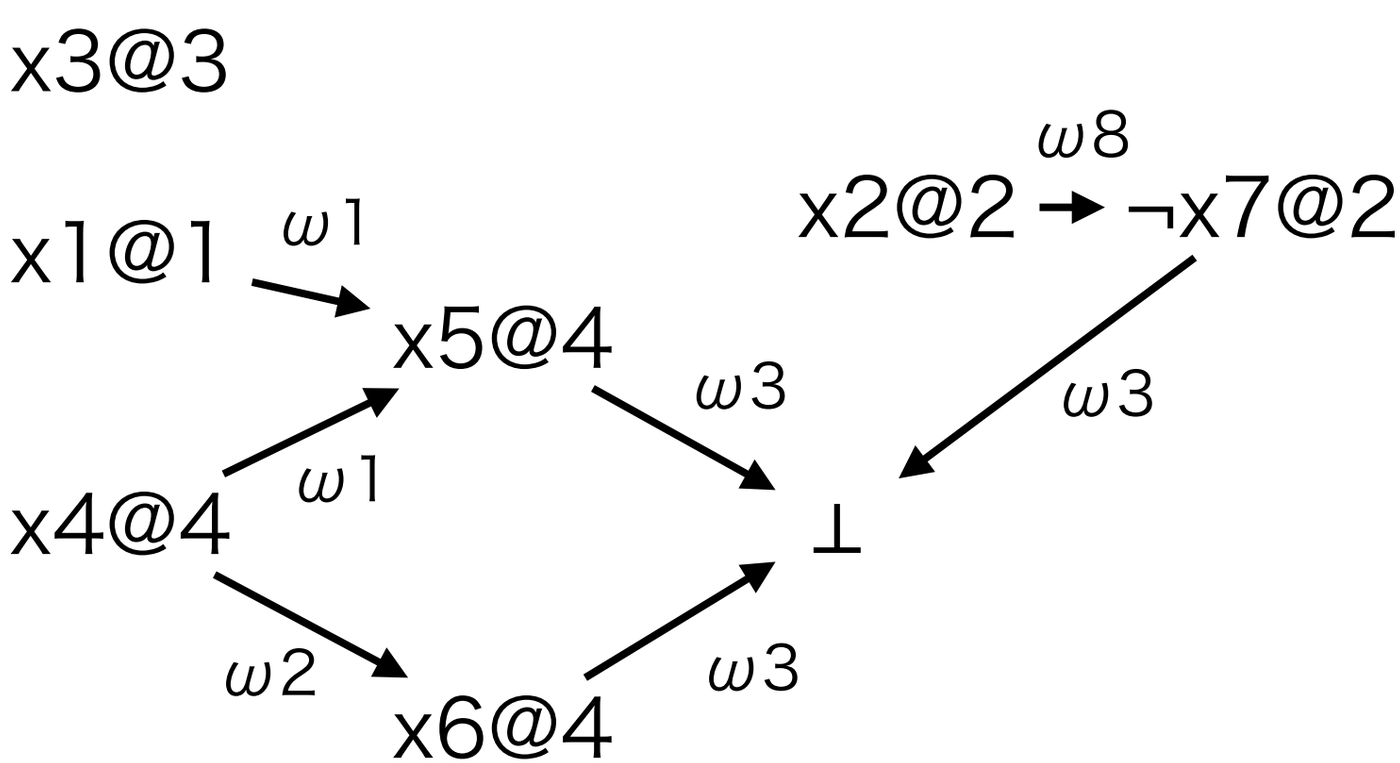
Learnt clause:
¬x1 ∨ ¬x4 ∨ x7

x1 was set at level 1
x4 was set at level 4
x7 was set at level 2

Assertion level of this clause is 2.

⇒ Backtrack to level 2!

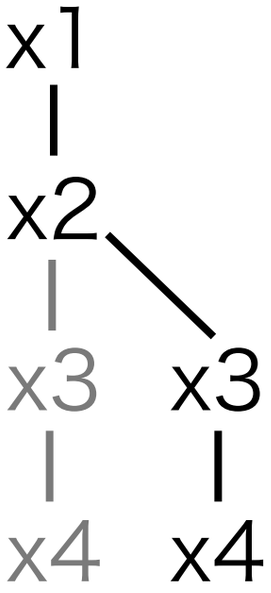
Implication Graph



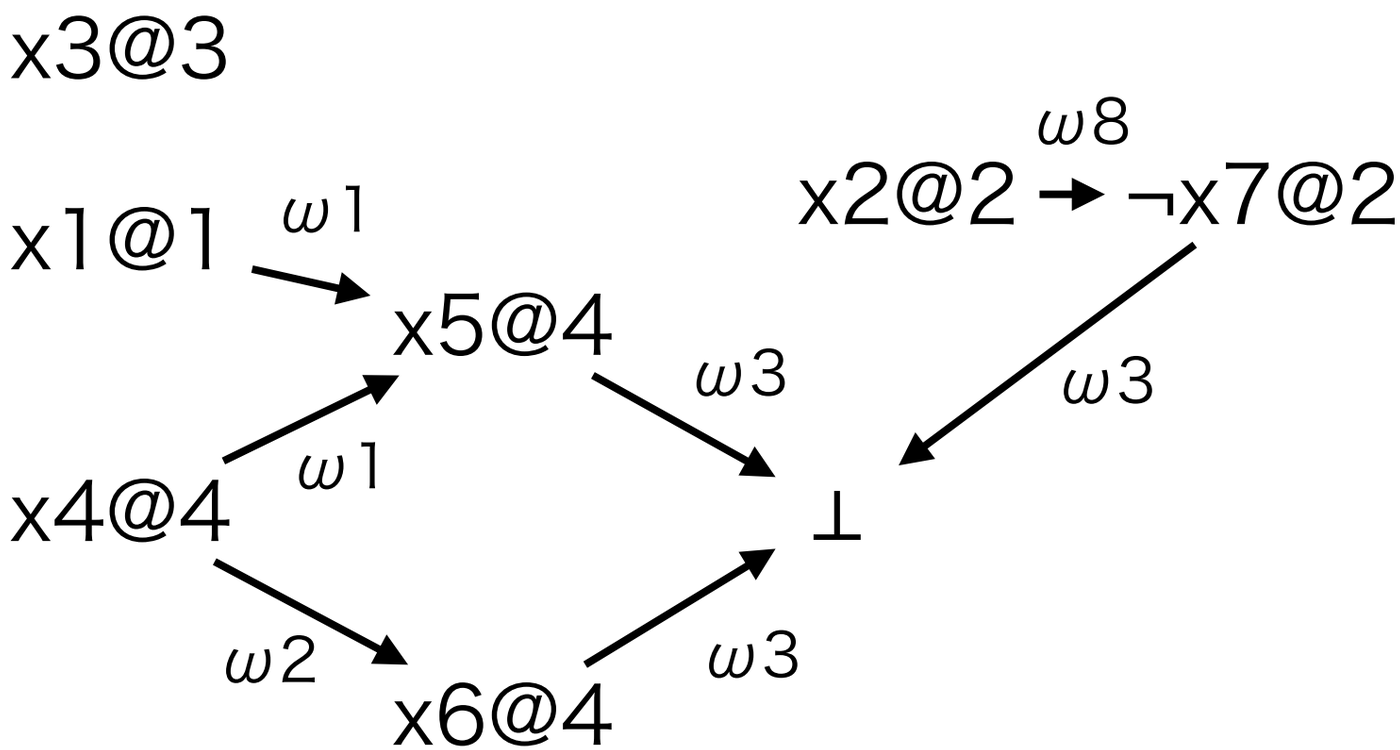
Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7

Backtrack



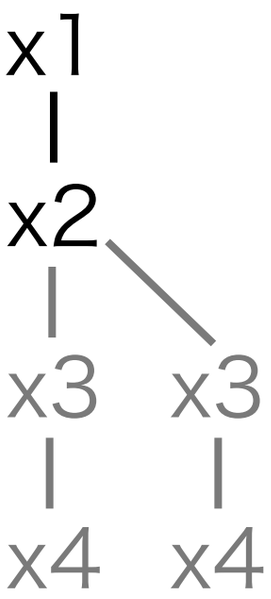
Implication Graph



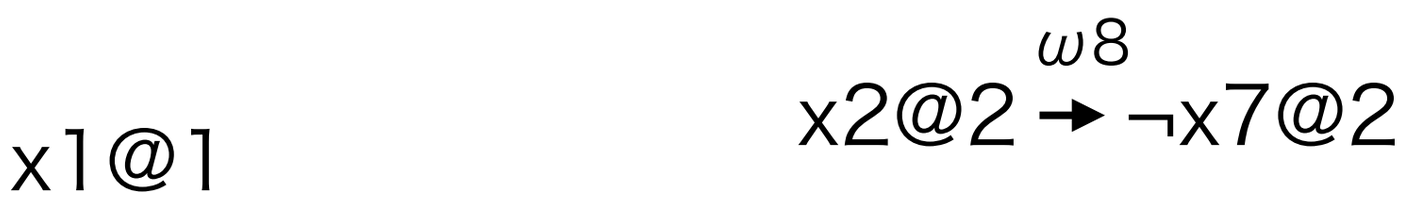
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Backtrack



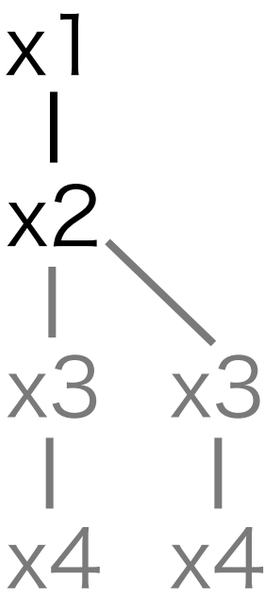
Implication Graph



Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



Implication Graph

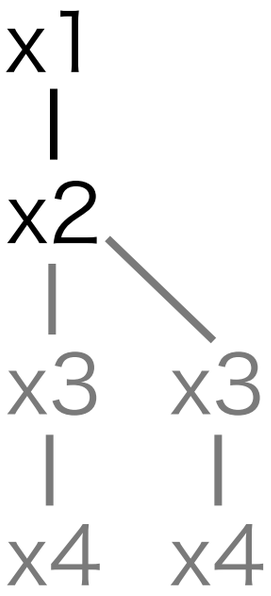
x1@1

$\omega 8$
x2@2 → ¬x7@2

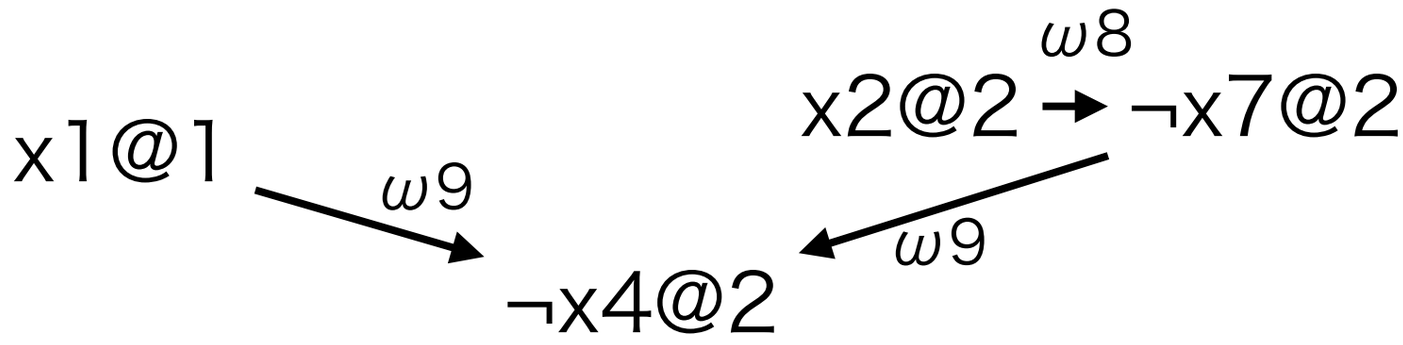
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



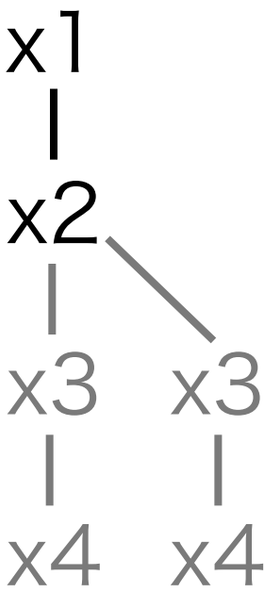
Implication Graph



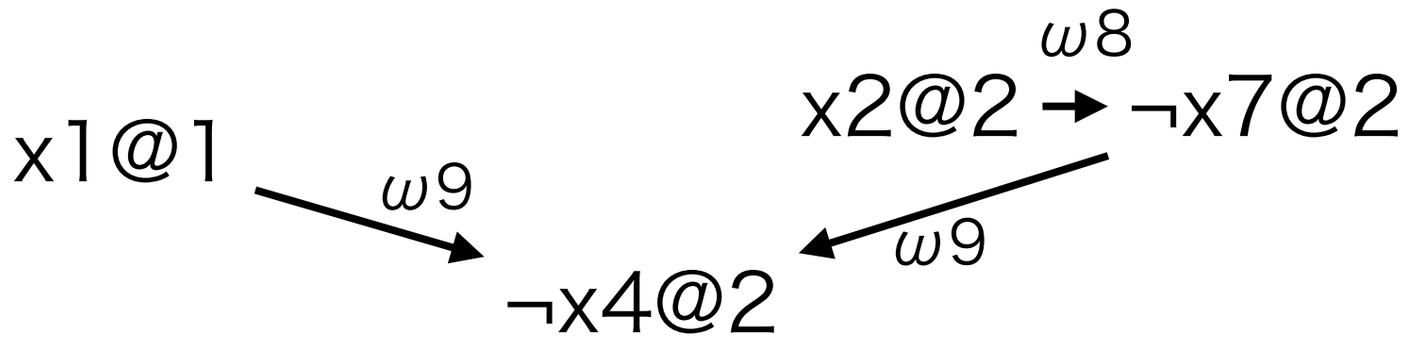
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Decide



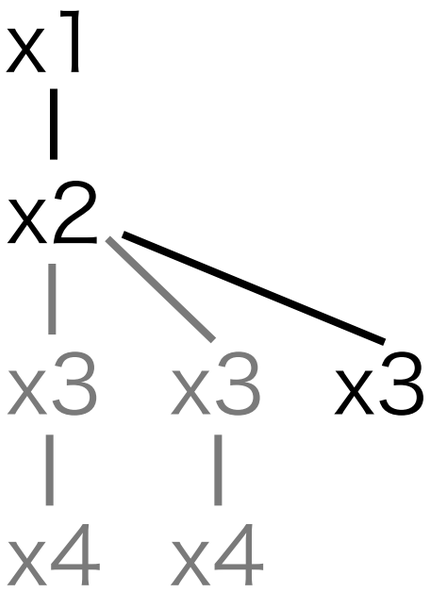
Implication Graph



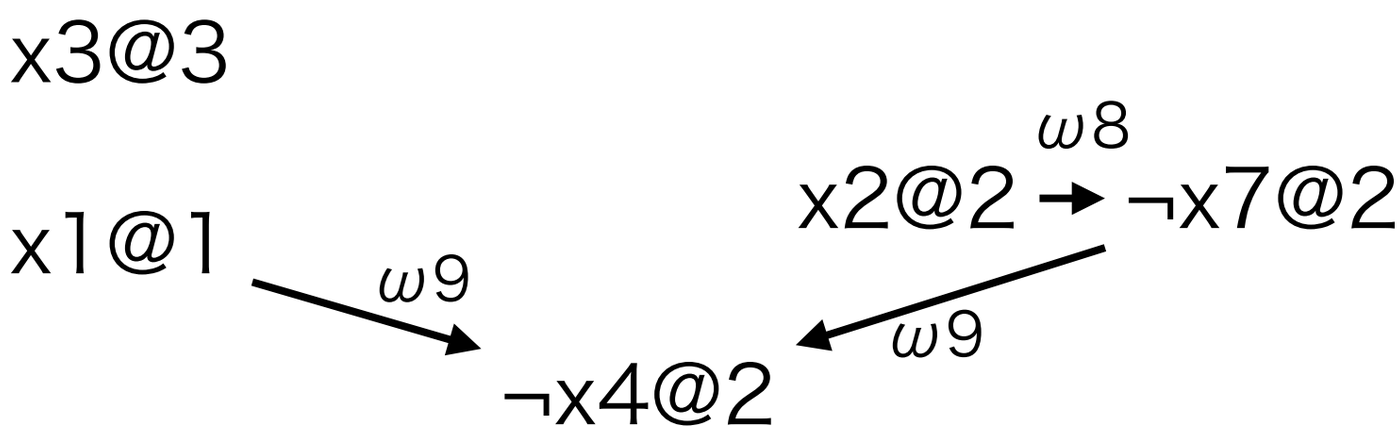
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Decide



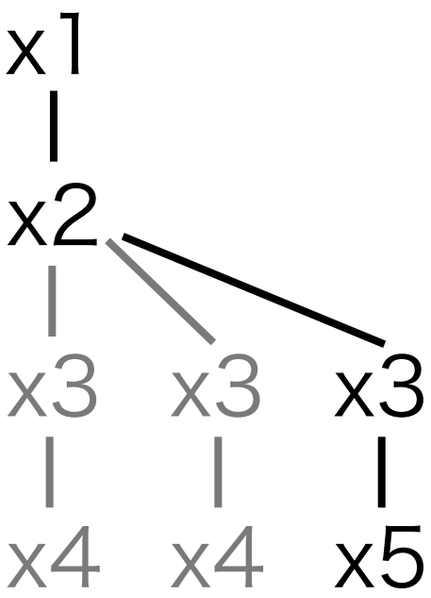
Implication Graph



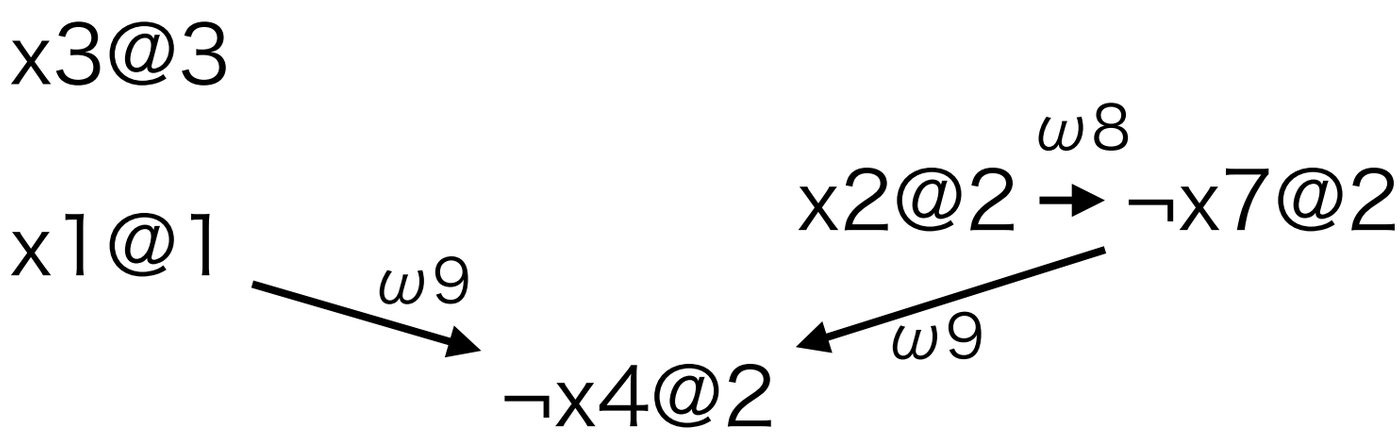
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Decide



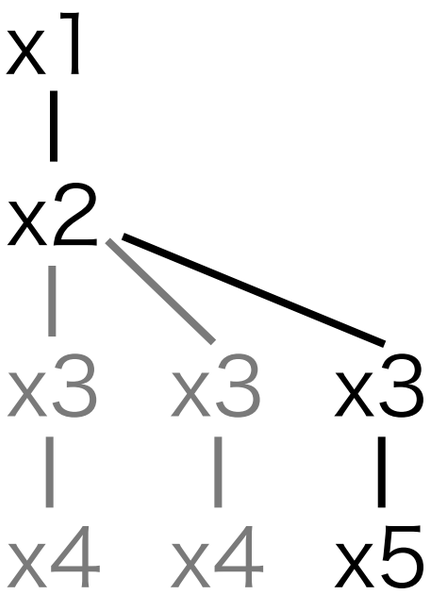
Implication Graph



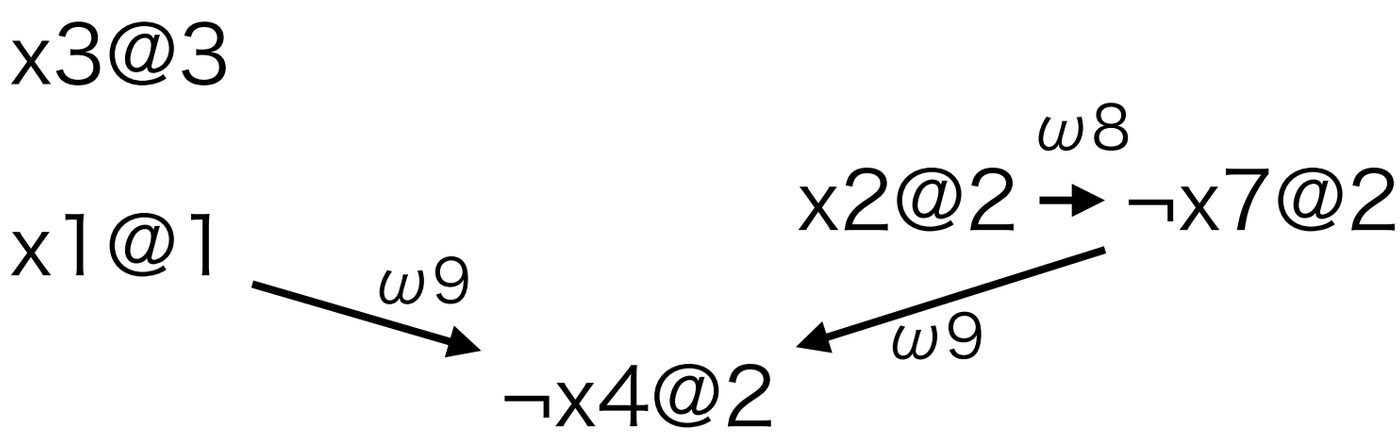
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



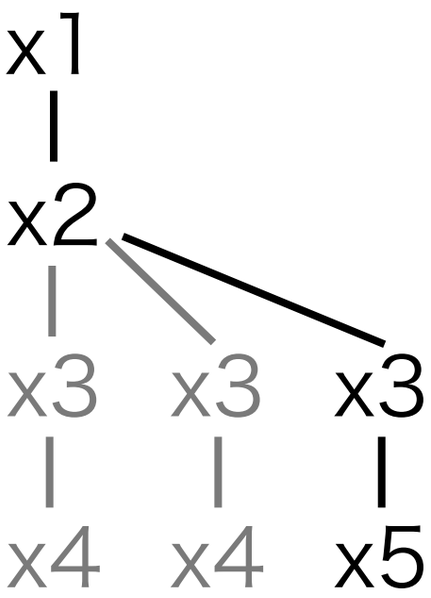
Implication Graph



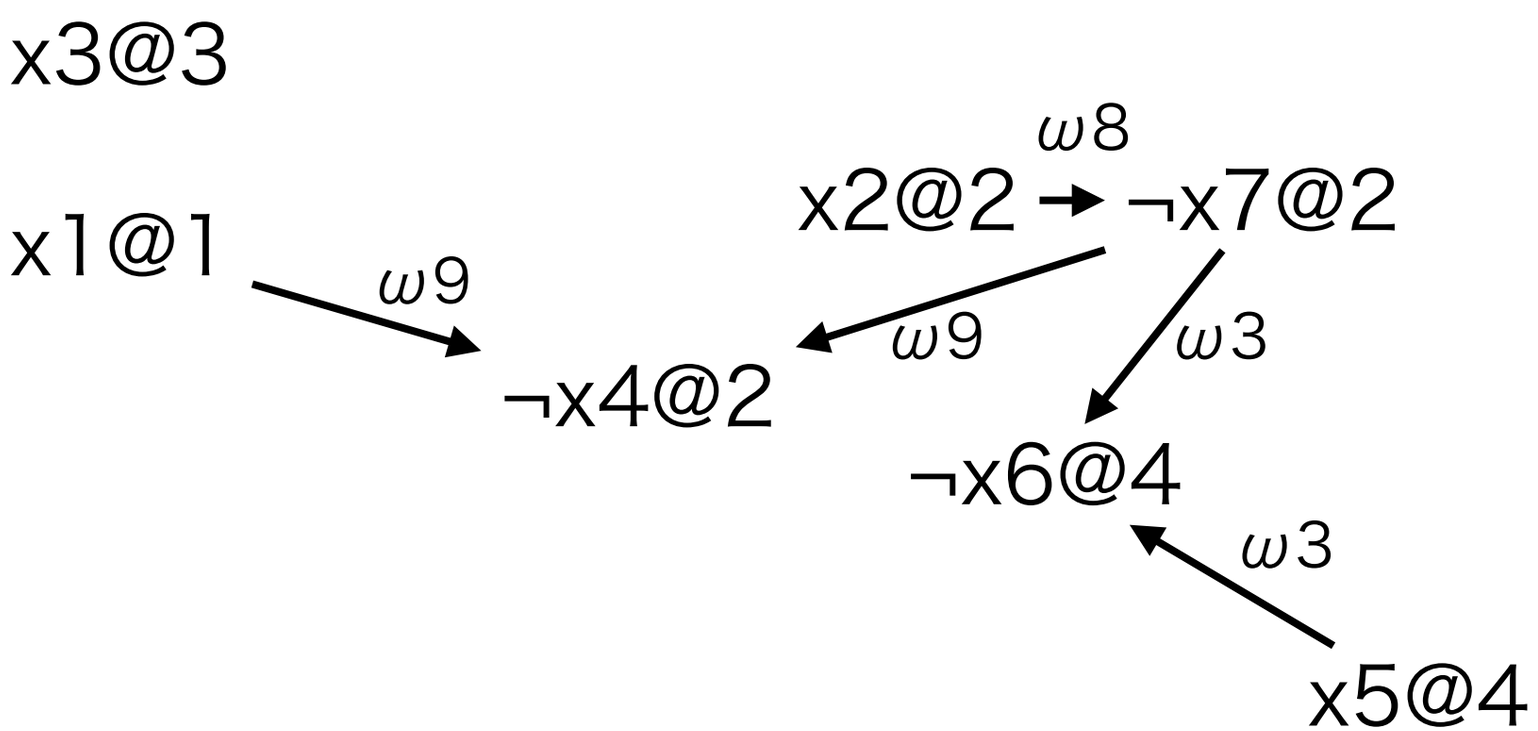
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



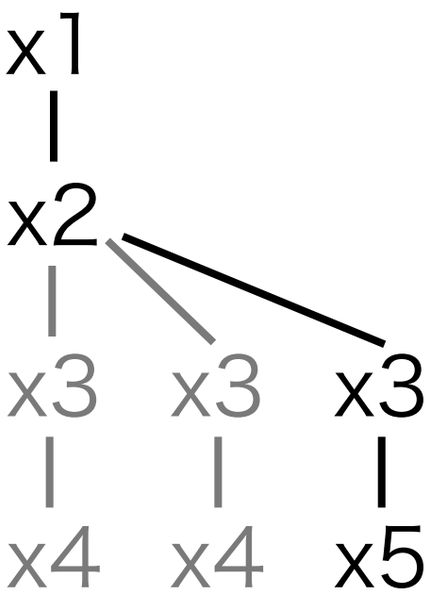
Implication Graph



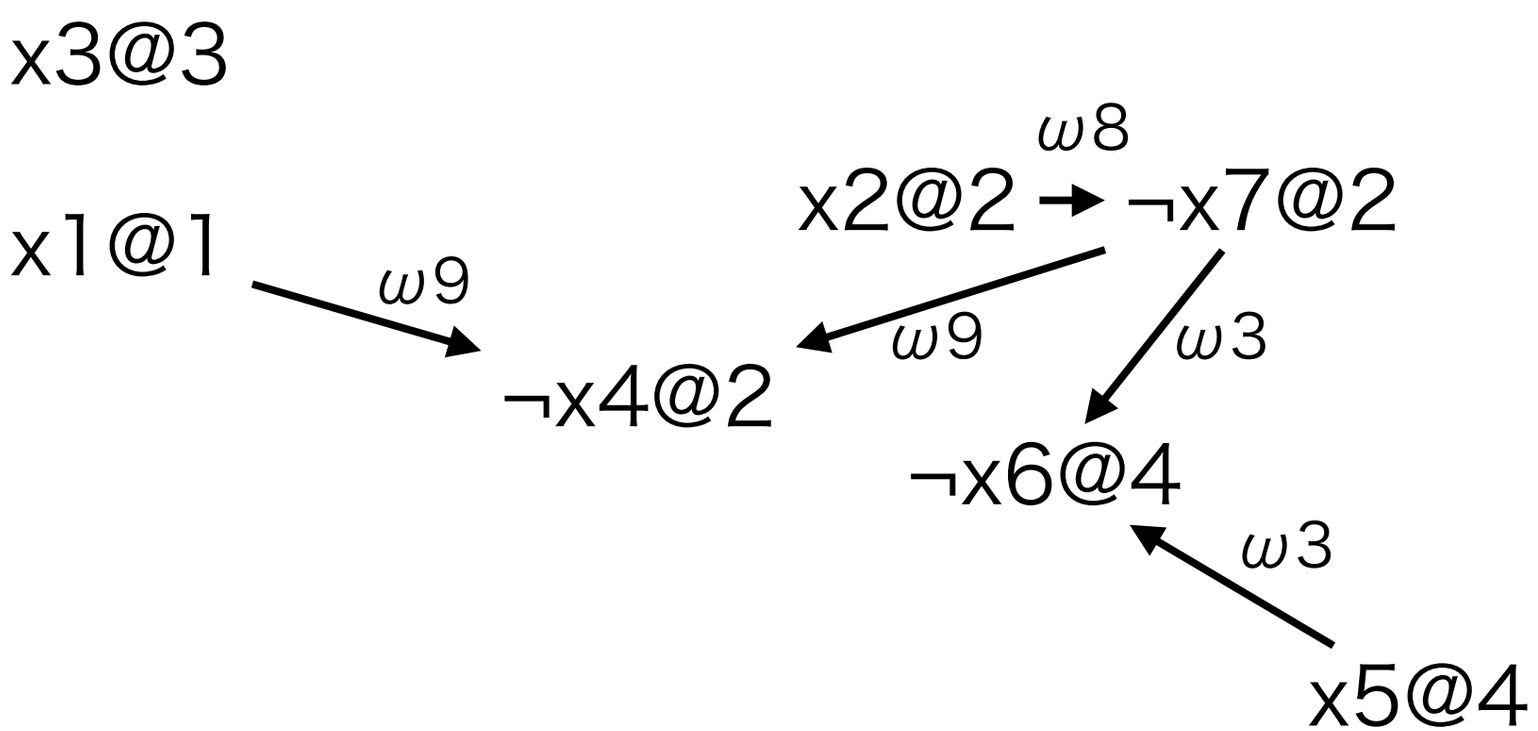
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Decide



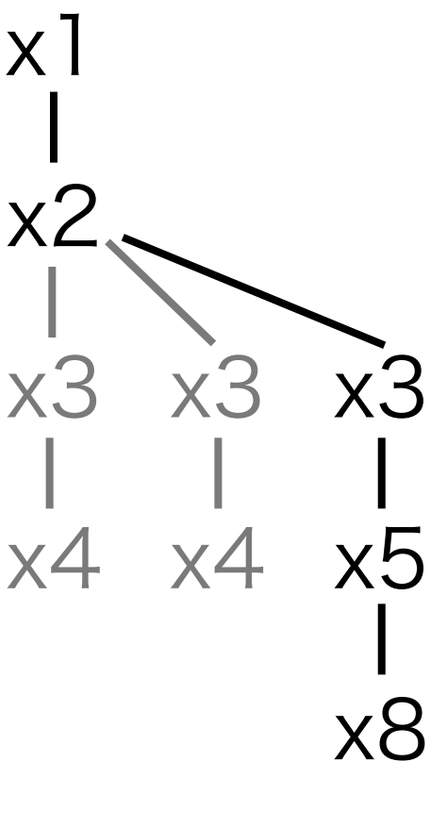
Implication Graph



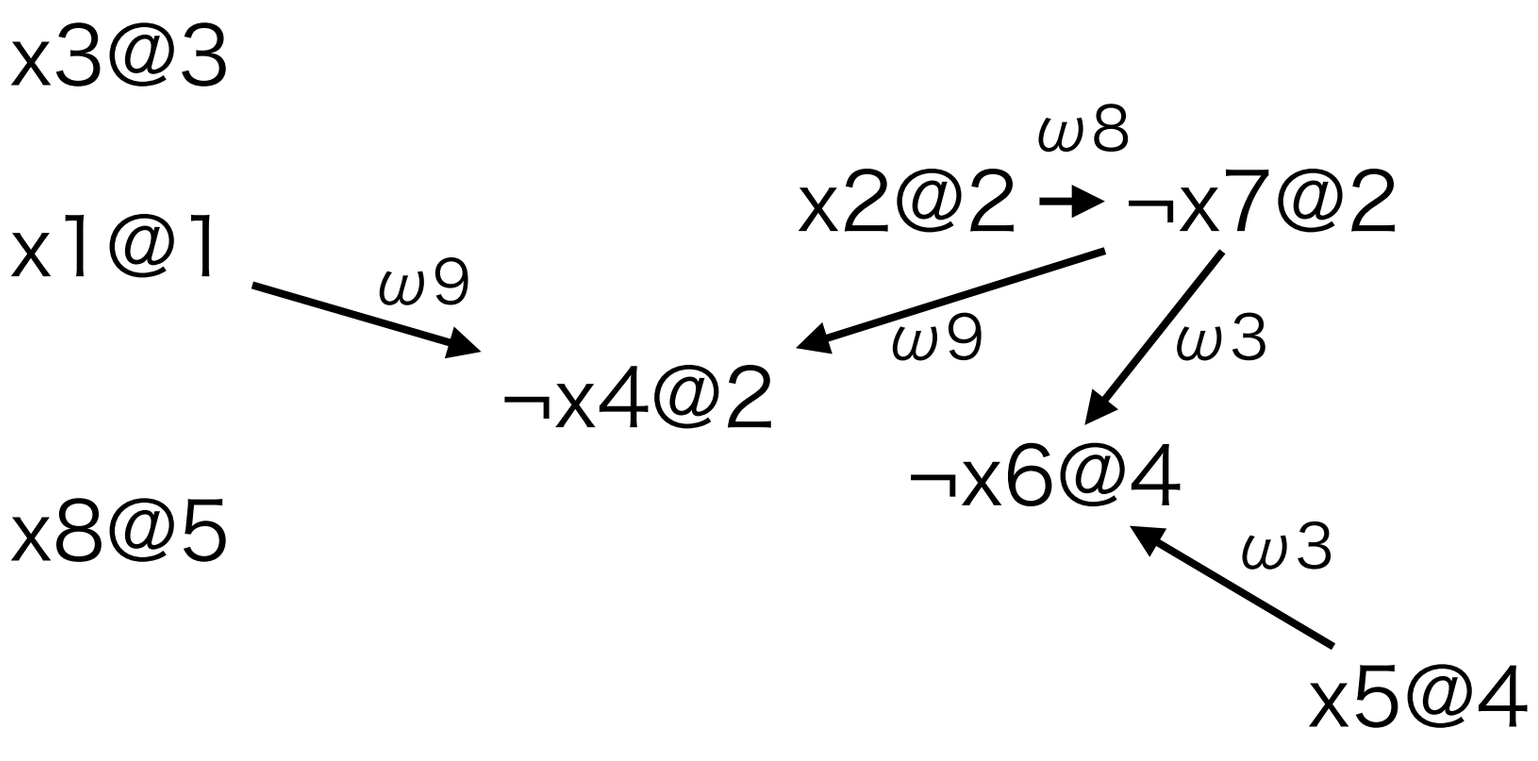
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Decide



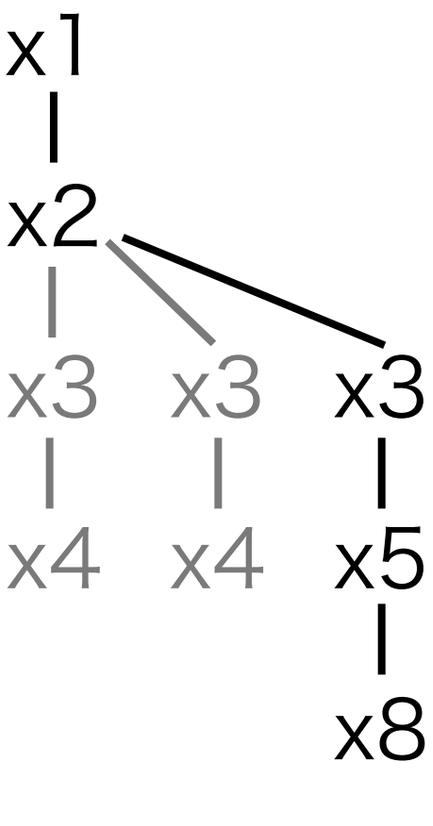
Implication Graph



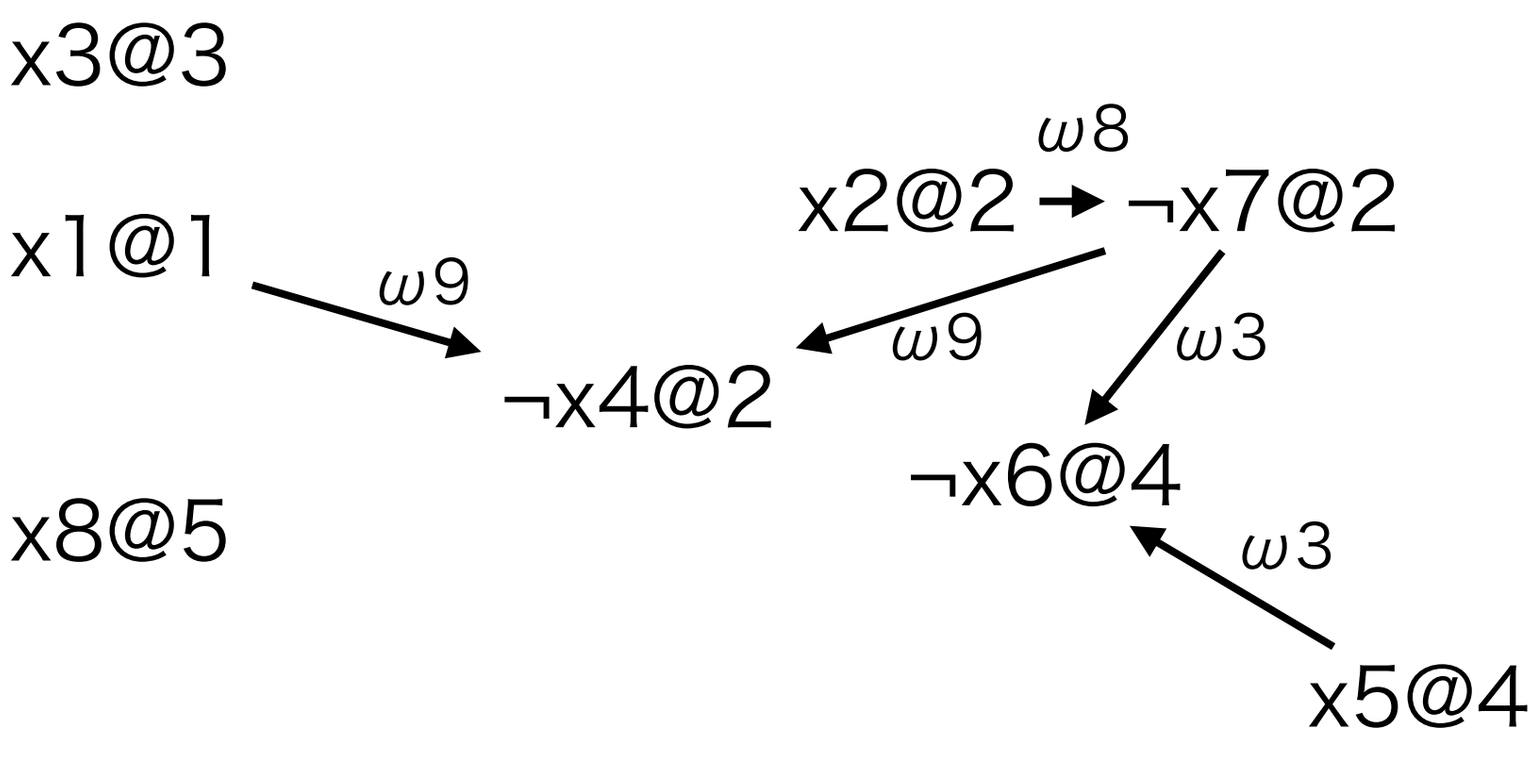
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



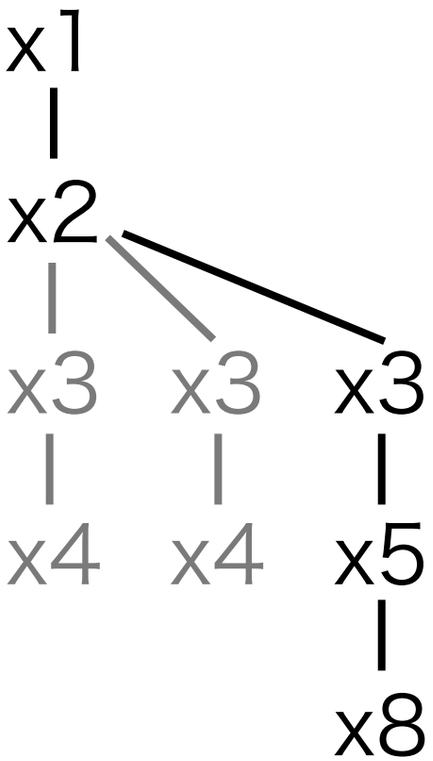
Implication Graph



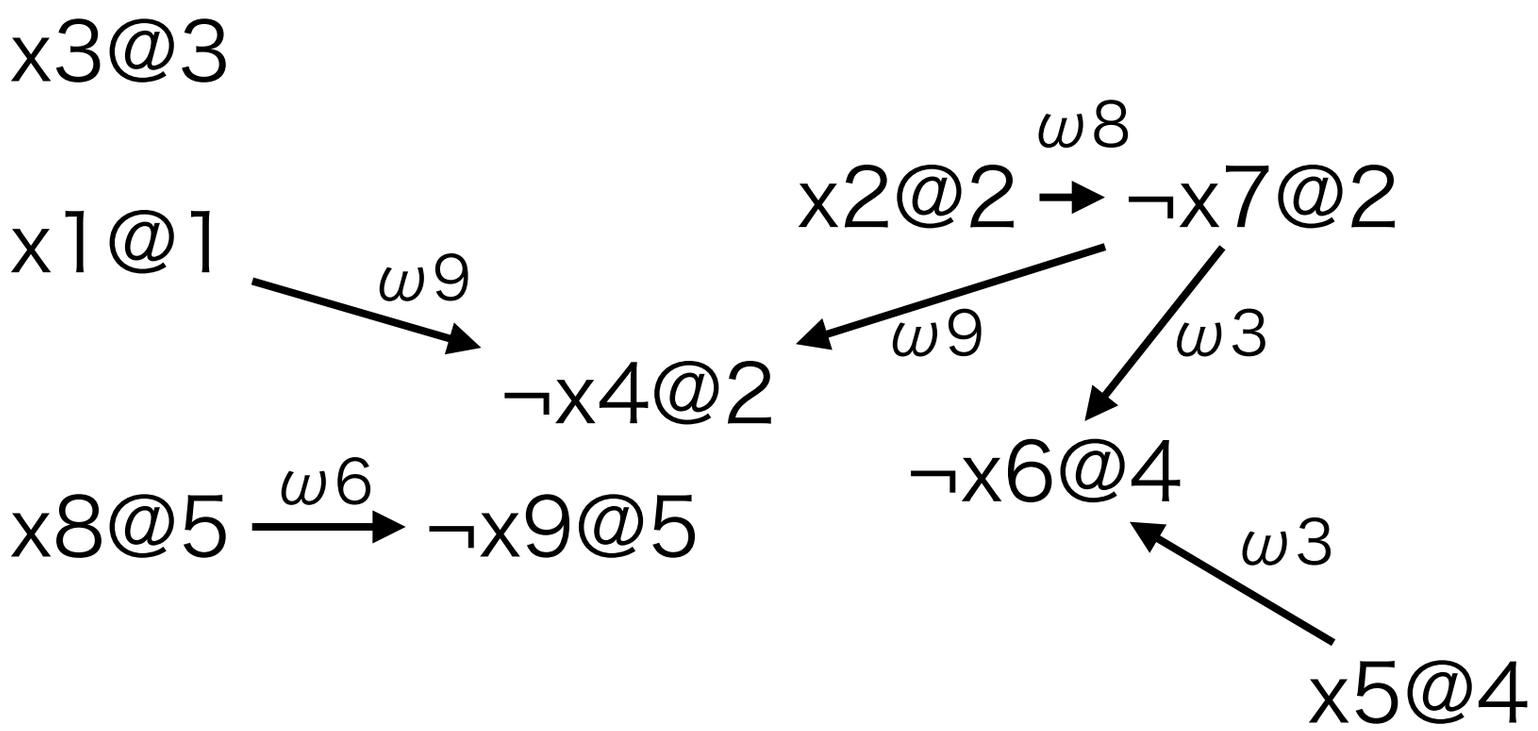
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



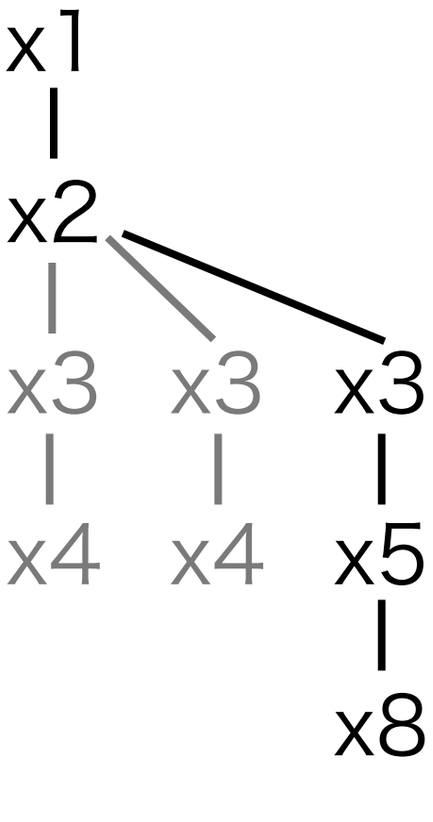
Implication Graph



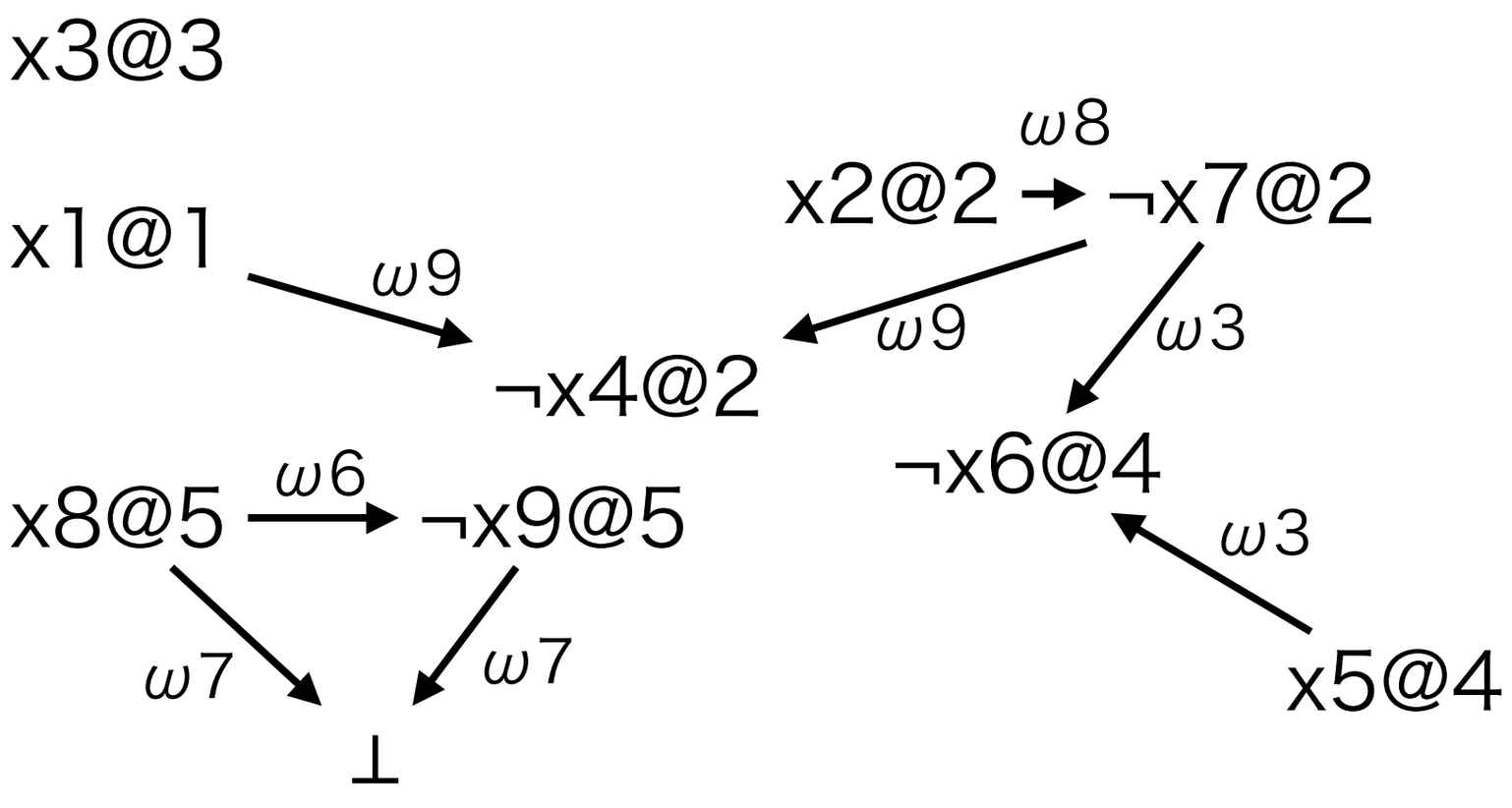
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



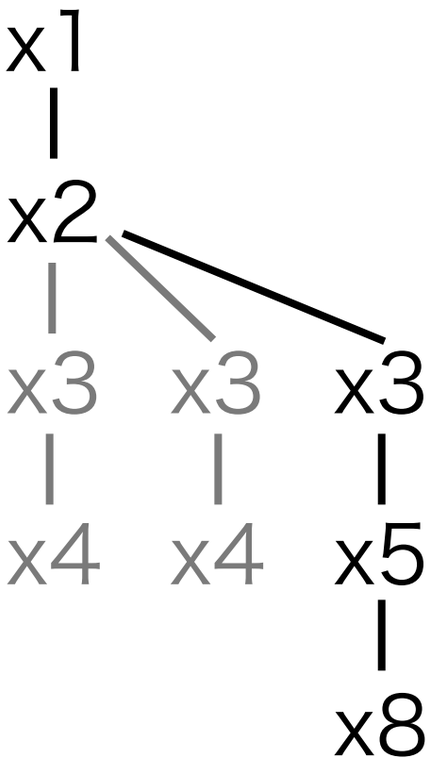
Implication Graph



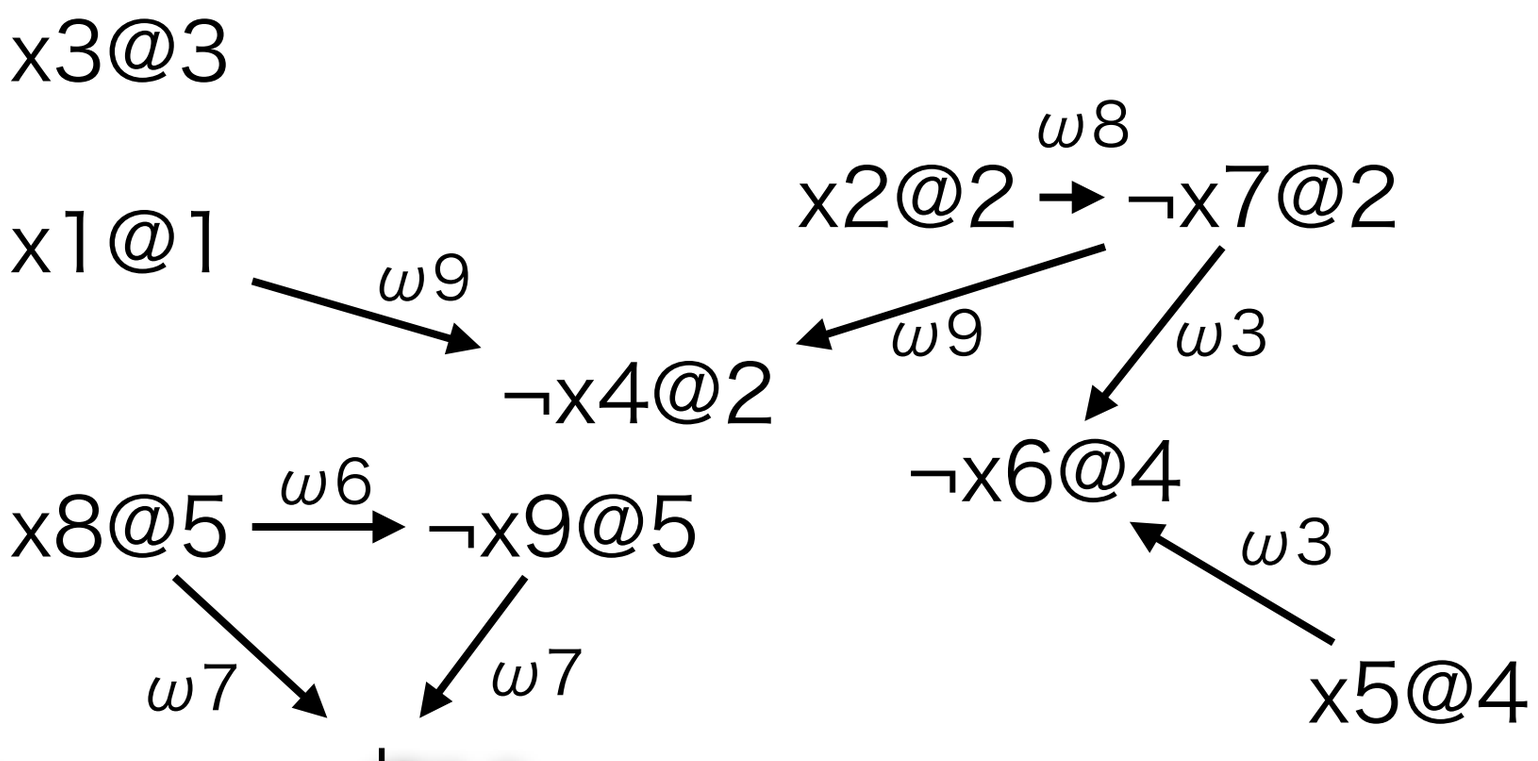
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Deduce



Implication Graph

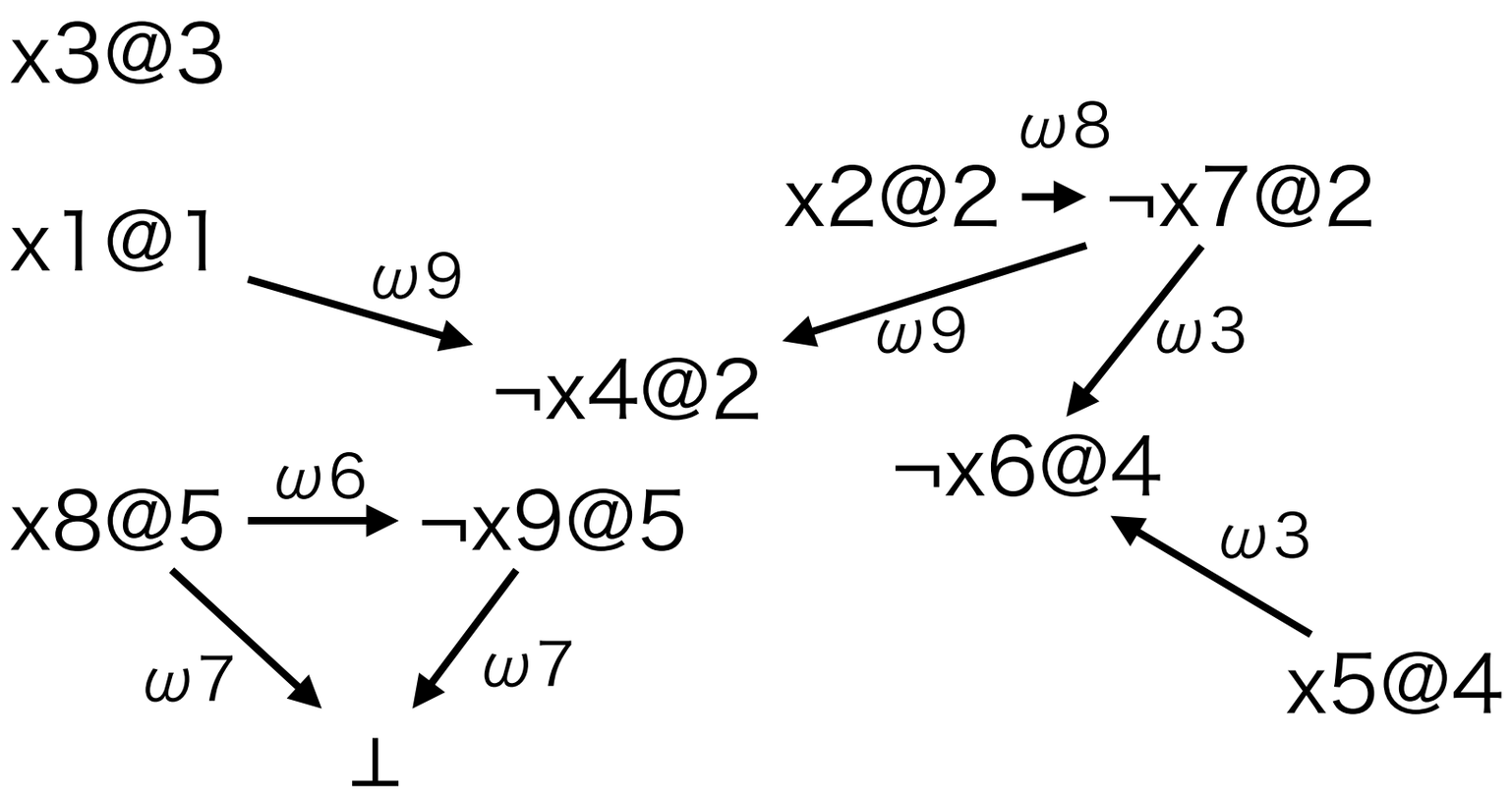


Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Diagnose

Implication Graph



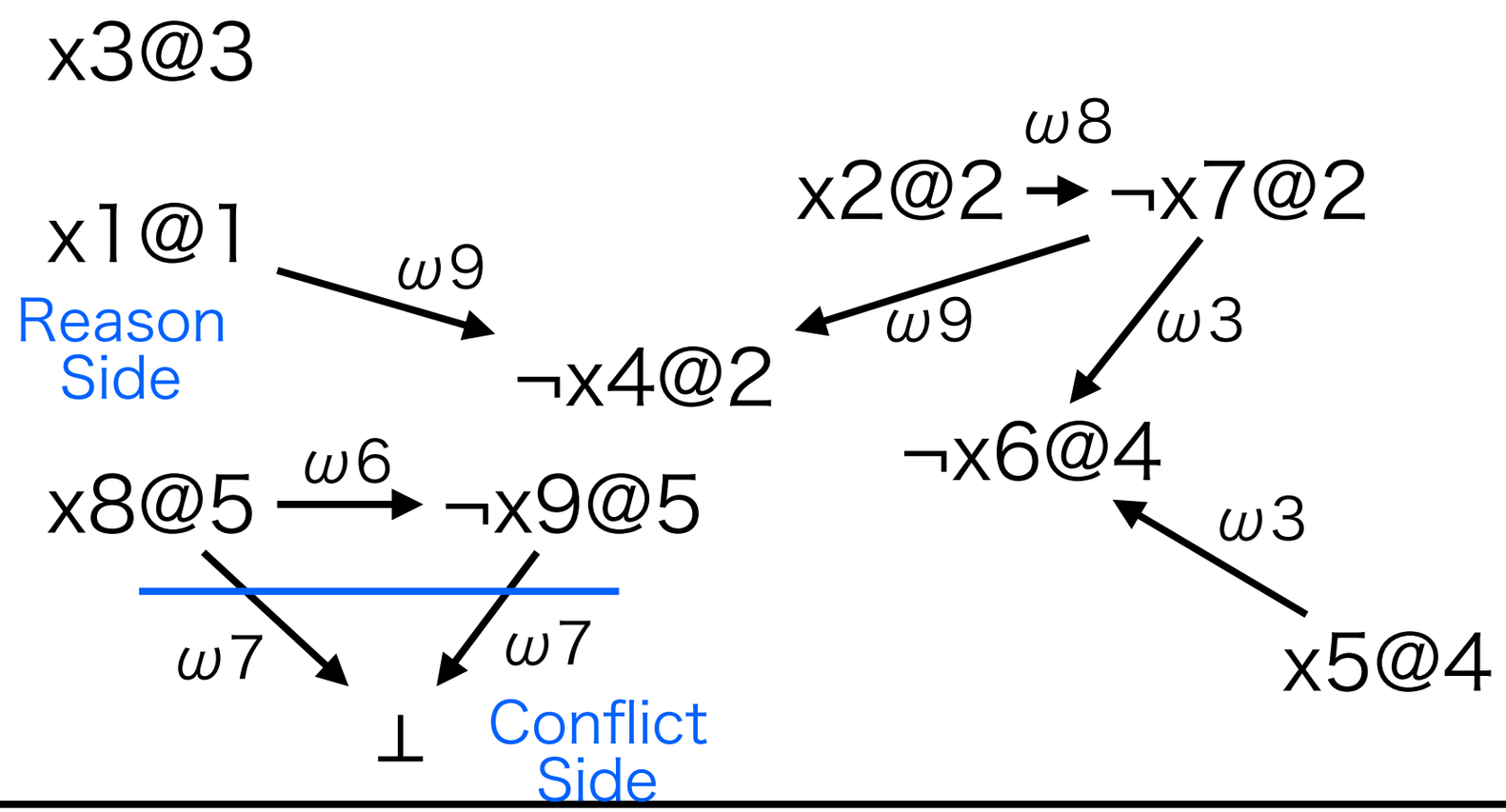
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7

Diagnose

$\omega 7$:
$\neg x8 \vee x9$

Implication Graph



Clause DB

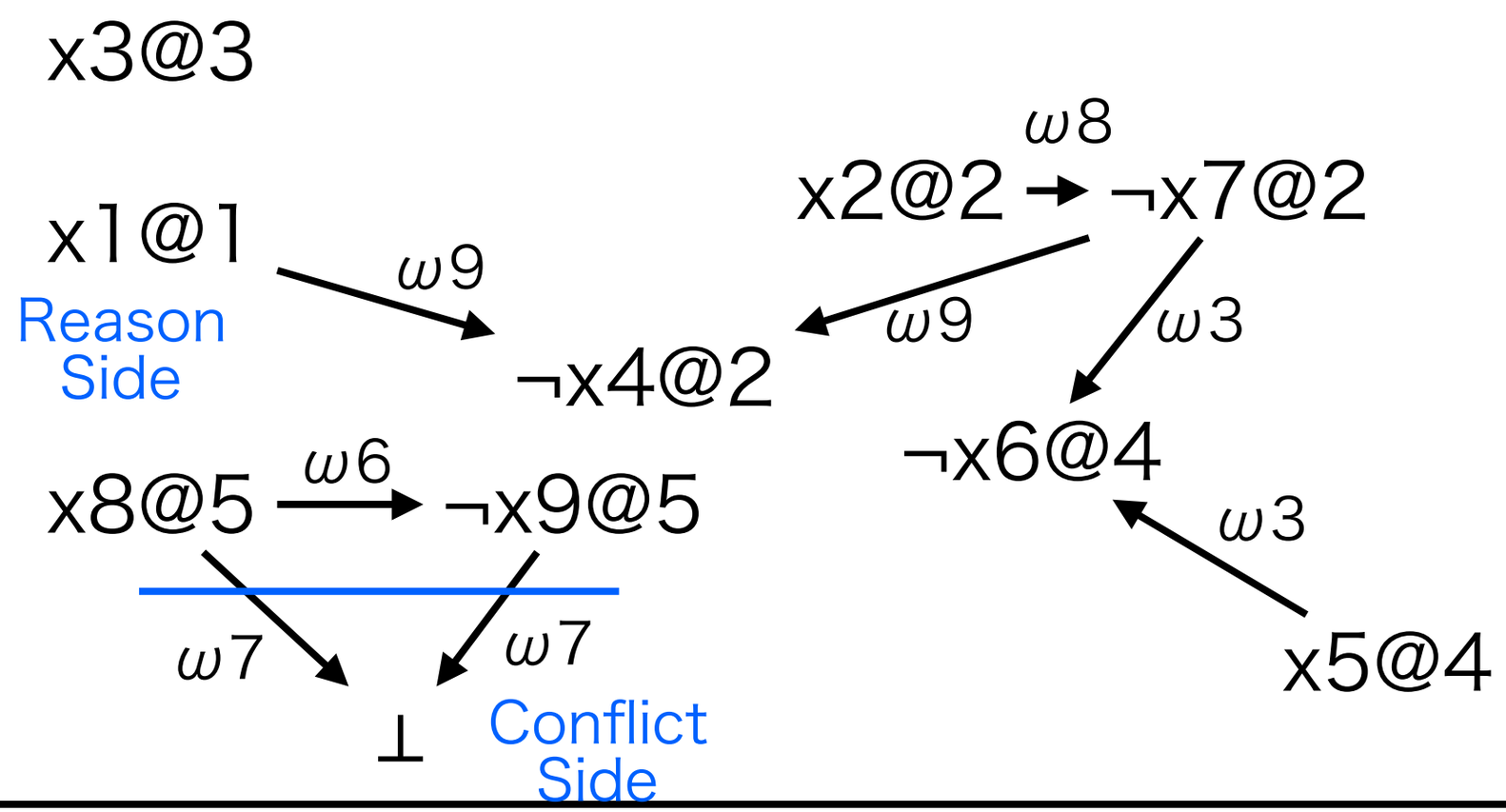
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$

Diagnose

$\omega 7$: $\neg x8 \vee x9$

$\omega 6$: $\neg x8 \vee \neg x9$

Implication Graph



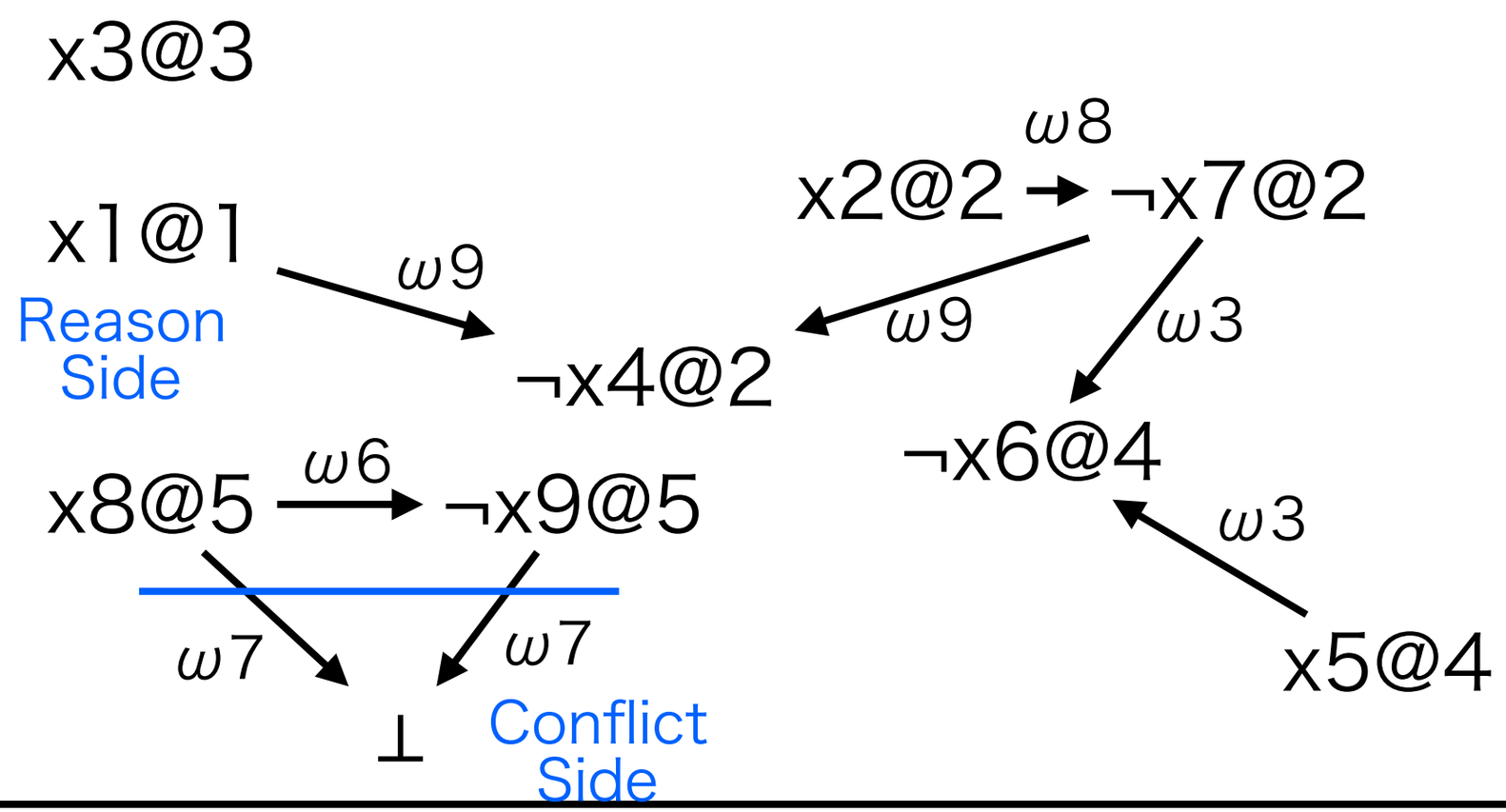
Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$

Diagnose

$\omega 7$: $\neg x8 \vee x9$ $\omega 6$: $\neg x8 \vee \neg x9$

Implication Graph



Clause DB

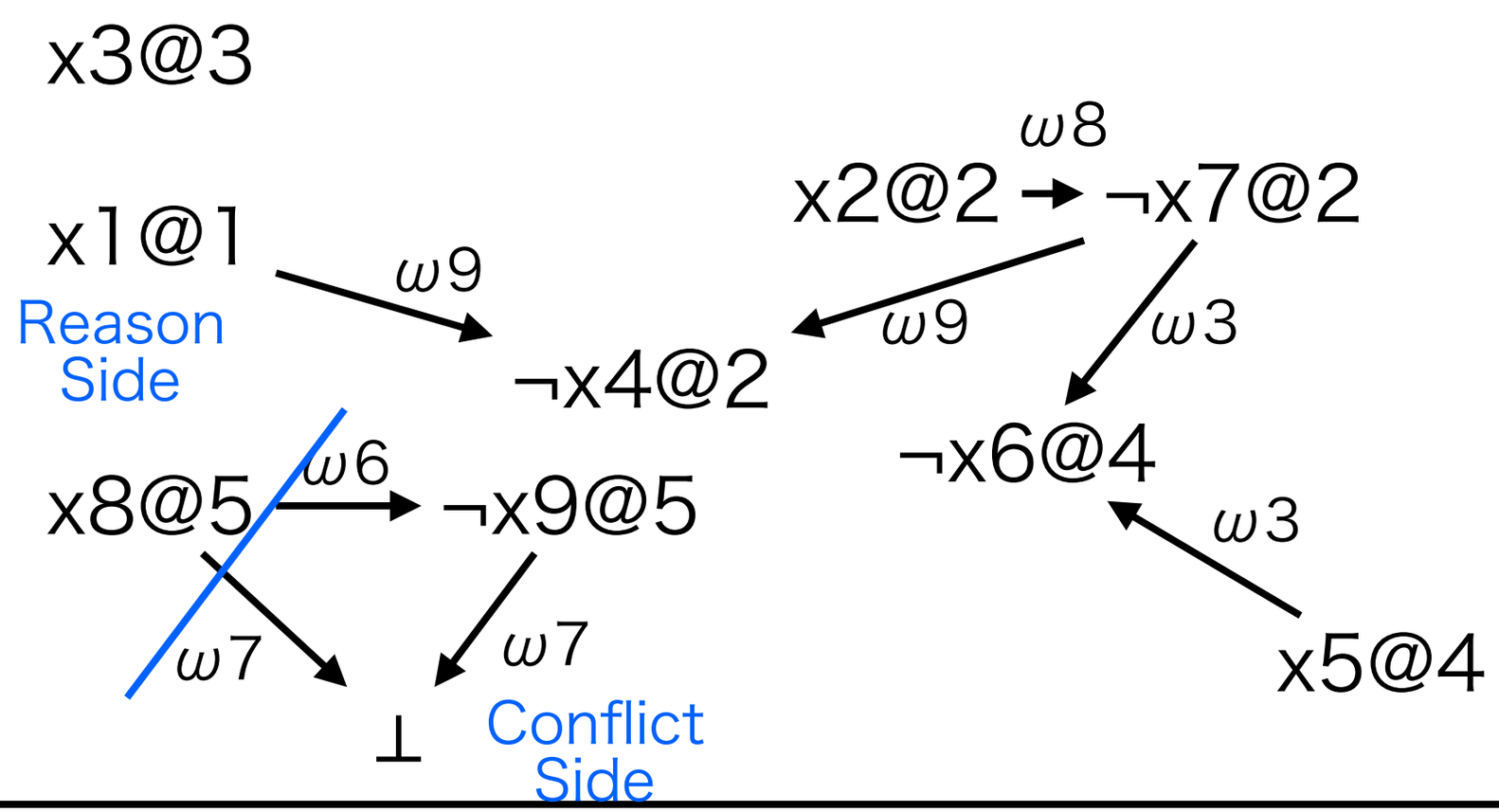
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$

Diagnose

$\omega 7$: $\neg x8 \vee x9$ $\omega 6$: $\neg x8 \vee \neg x9$

$\neg x8$

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$

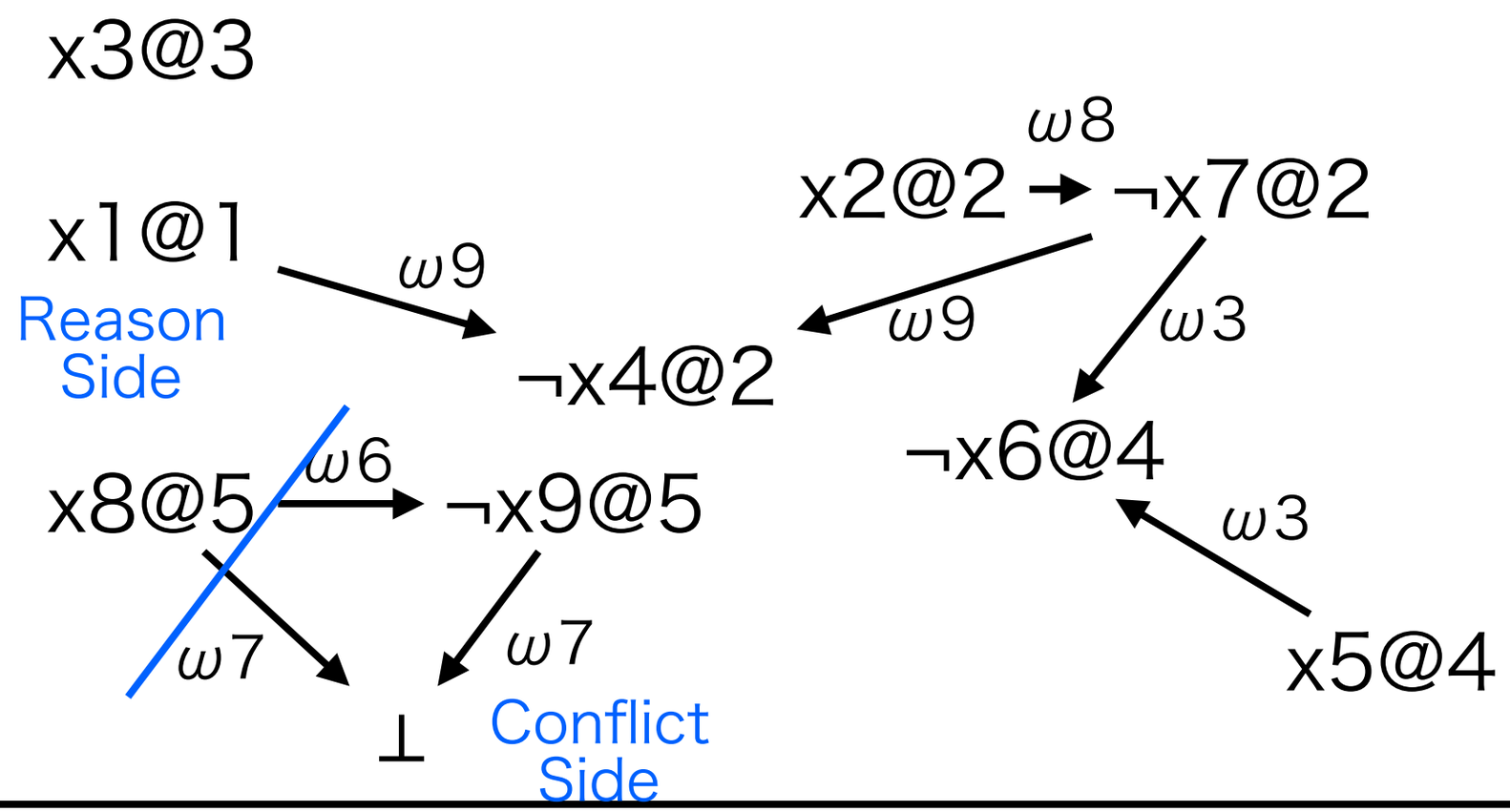
Diagnose

$\omega 7$: $\neg x8 \vee x9$ $\quad$ $\omega 6$: $\neg x8 \vee \neg x9$

$\neg x8$

It contains one literal in the current decision level (i.e. 5).

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$

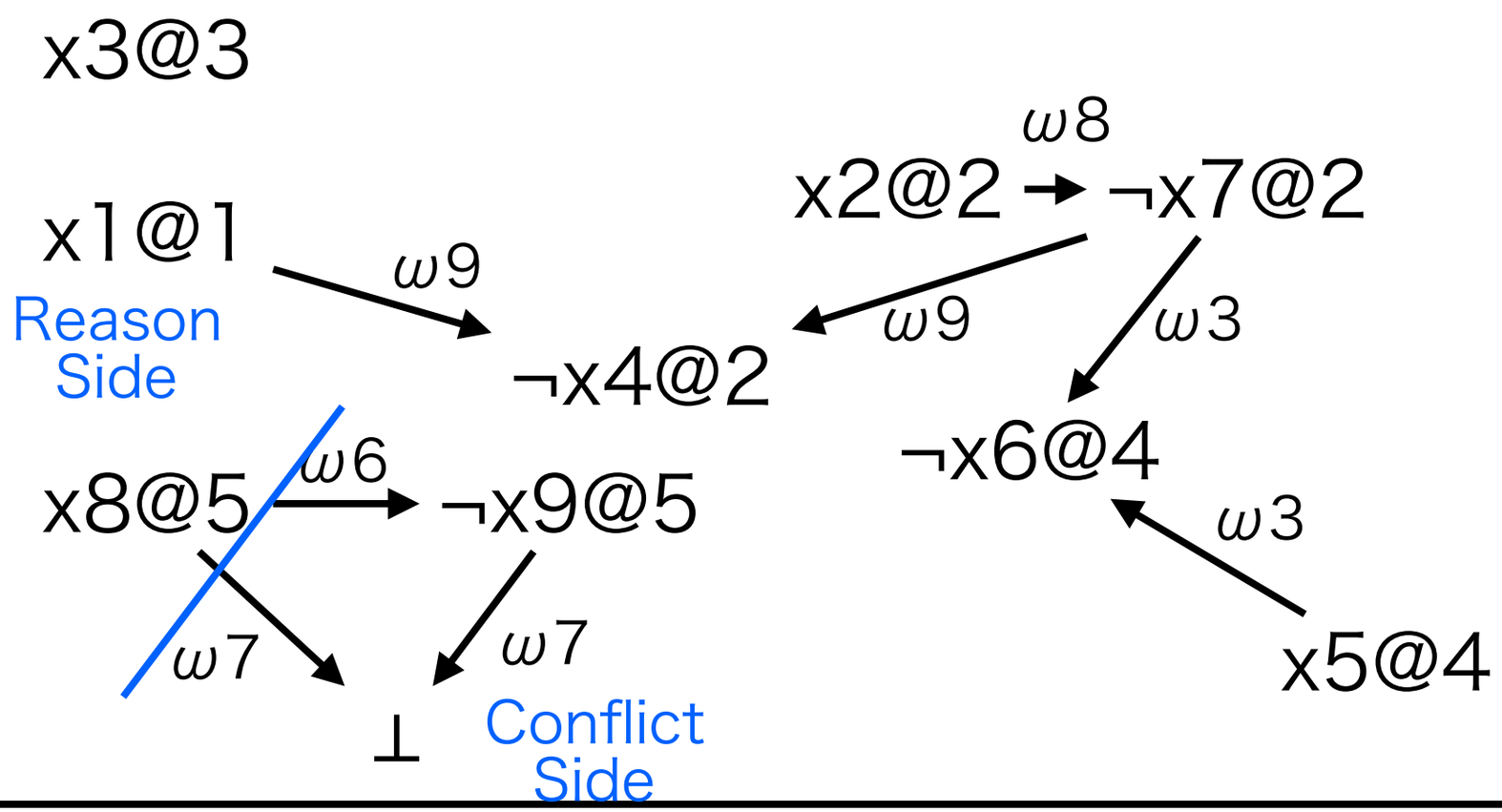
Diagnose

$\omega 7$: $\neg x8 \vee x9$ $\quad$ $\omega 6$: $\neg x8 \vee \neg x9$

$\neg x8$

It contains one literal in the current decision level (i.e. 5).

Implication Graph



Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- [illegible]
- $\omega 10$: $\neg x8$

New clause is learnt !

Backtrack

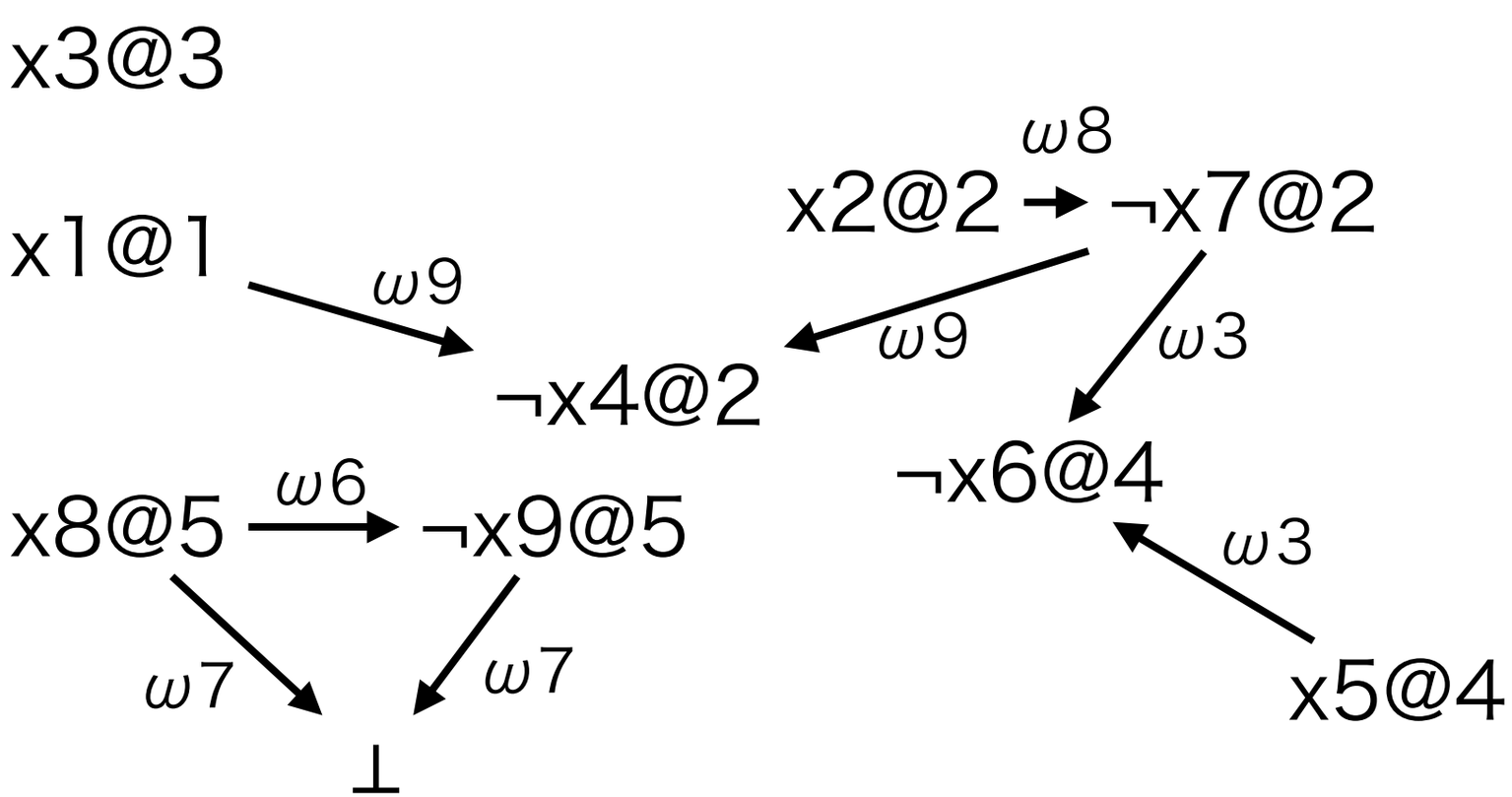
Learnt clause:
¬x8

x8 was set at level 5

Assertion level of this clause (which is the second highest level) is 0 by convention.

⇒ Backtrack to level 0 (root level)

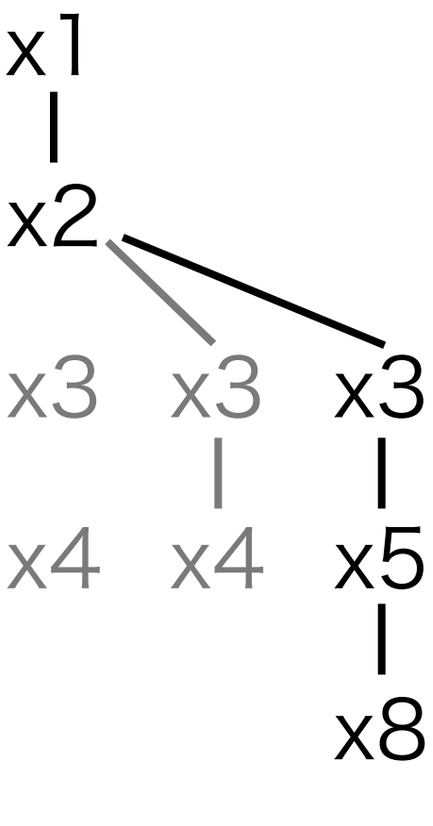
Implication Graph



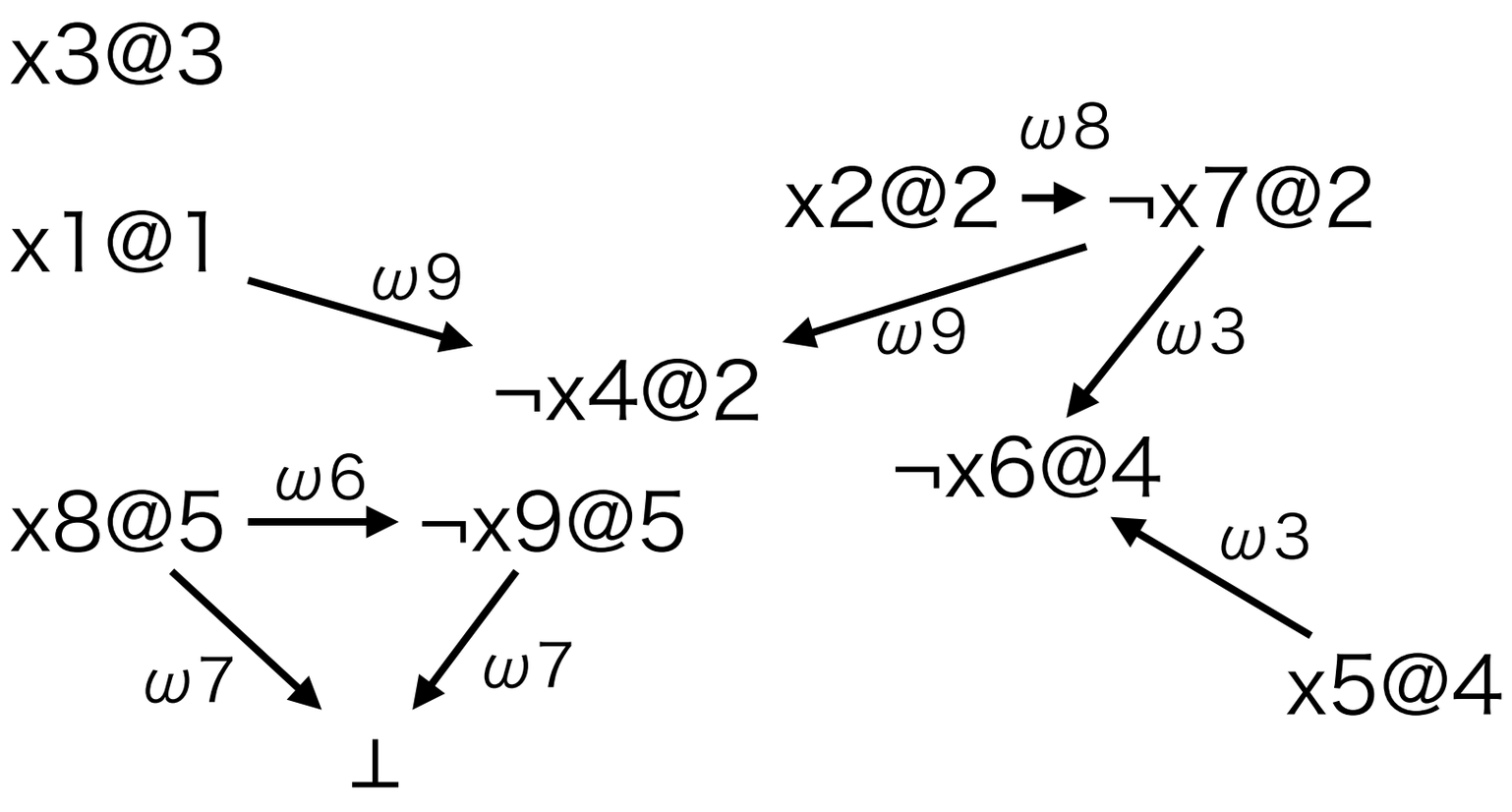
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Backtrack



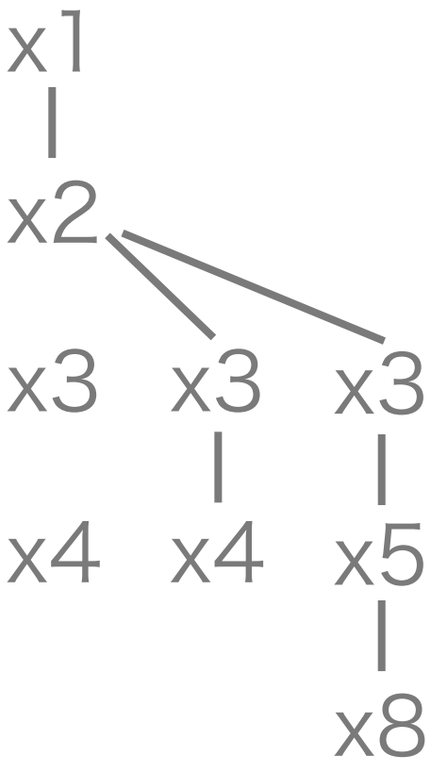
Implication Graph



Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Backtrack

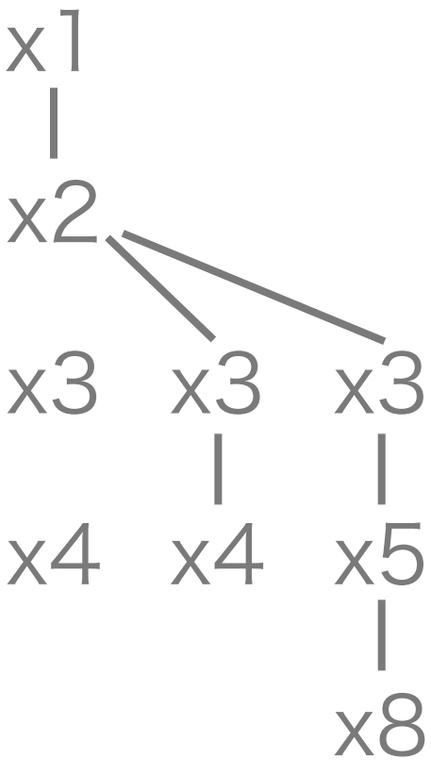


Implication Graph

Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Deduce

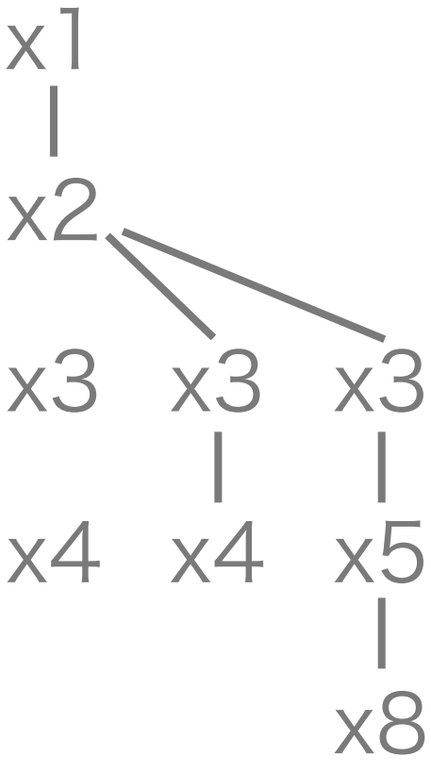


Implication Graph

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7
- ω10: ¬x8

Deduce



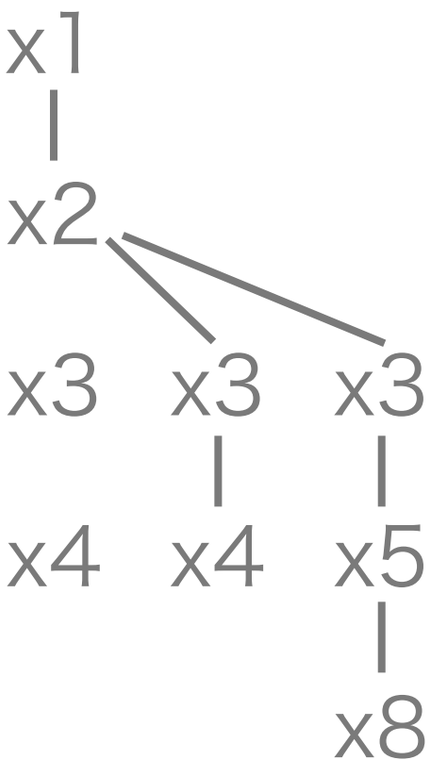
Implication Graph

¬x8@0

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7
- ω10: ¬x8

Decide



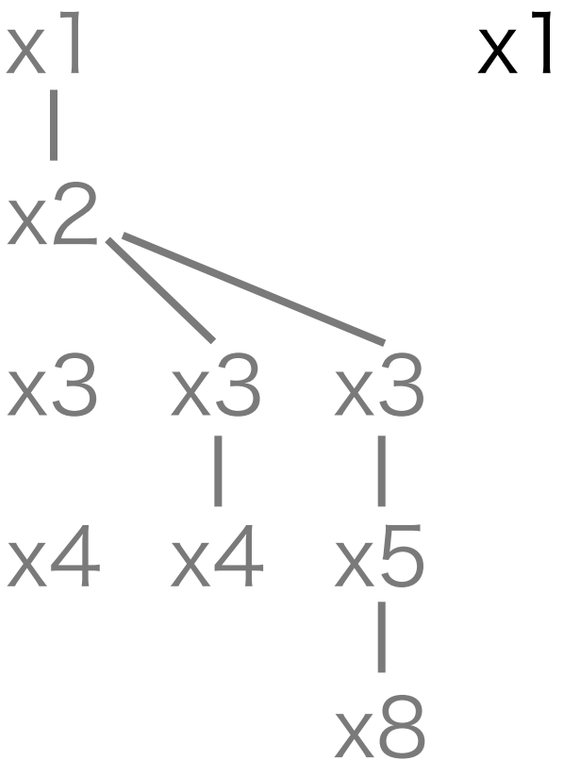
Implication Graph

¬x8@0

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7
- ω10: ¬x8

Decide



x1

Implication Graph

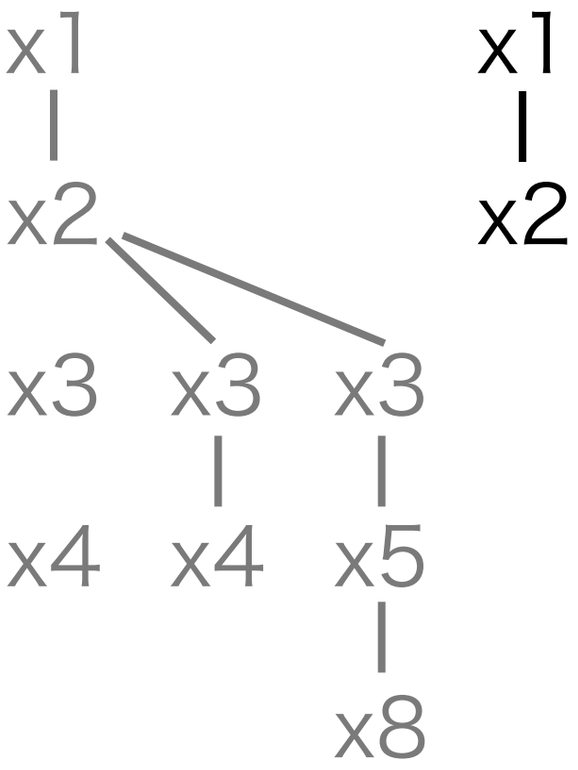
x1@1

¬x8@0

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7
- ω10: ¬x8

Decide



x1
x2

Implication Graph

x1@1

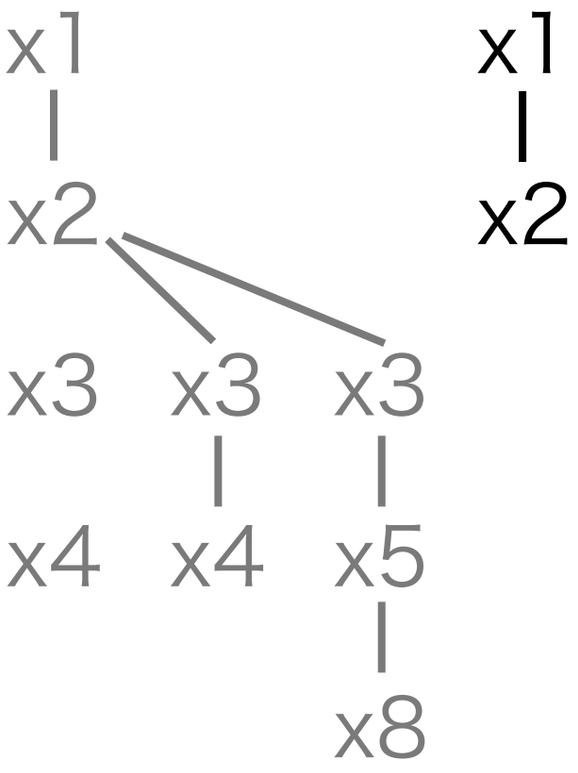
x2@2

¬x8@0

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7
- ω10: ¬x8

Deduce



Implication Graph

x1@1

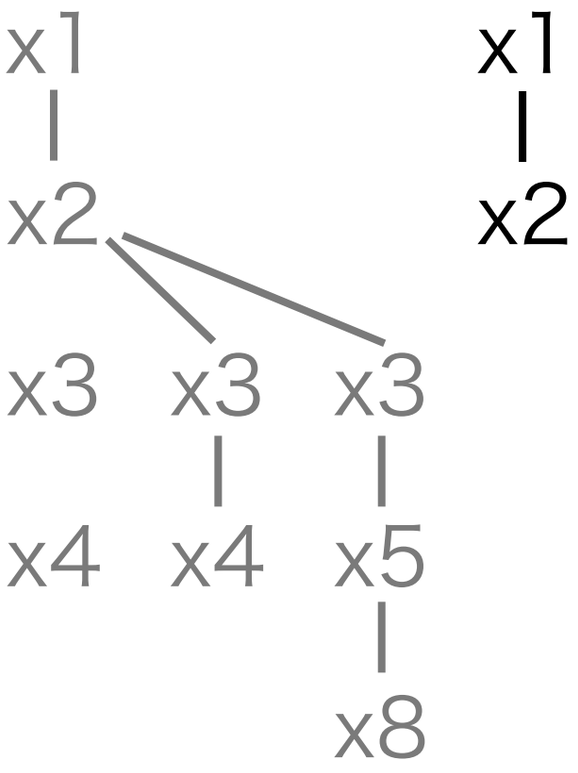
x2@2

¬x8@0

Clause DB

- ω1: ¬x1 ∨ ¬x4 ∨ x5
- ω2: ¬x4 ∨ x6
- ω3: ¬x5 ∨ ¬x6 ∨ x7
- ω4: ¬x7 ∨ x8
- ω5: ¬x2 ∨ ¬x7 ∨ x9
- ω6: ¬x8 ∨ ¬x9
- ω7: ¬x8 ∨ x9
- ω8: ¬x2 ∨ ¬x7
- ω9: ¬x1 ∨ ¬x4 ∨ x7
- ω10: ¬x8

Deduce



Implication Graph

x1@1

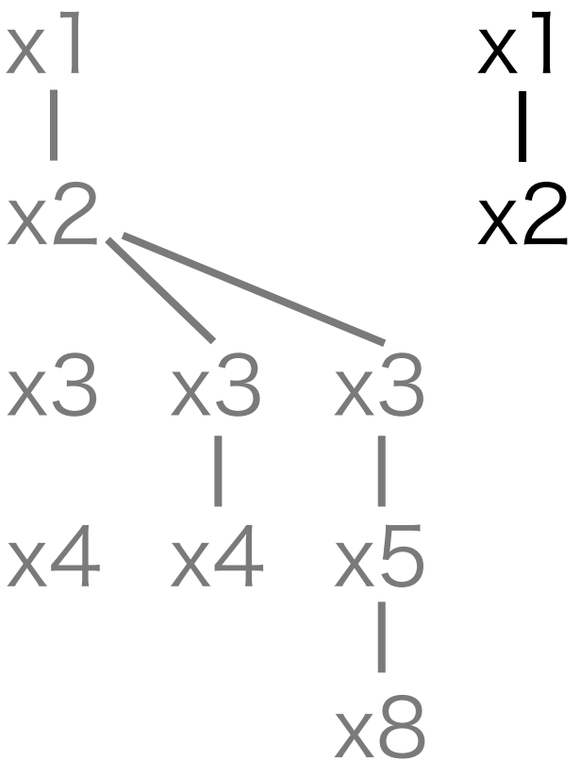
¬x8@0

$\overset{\omega 8}{\text{x2@2} \rightarrow \neg\text{x7@2}}$

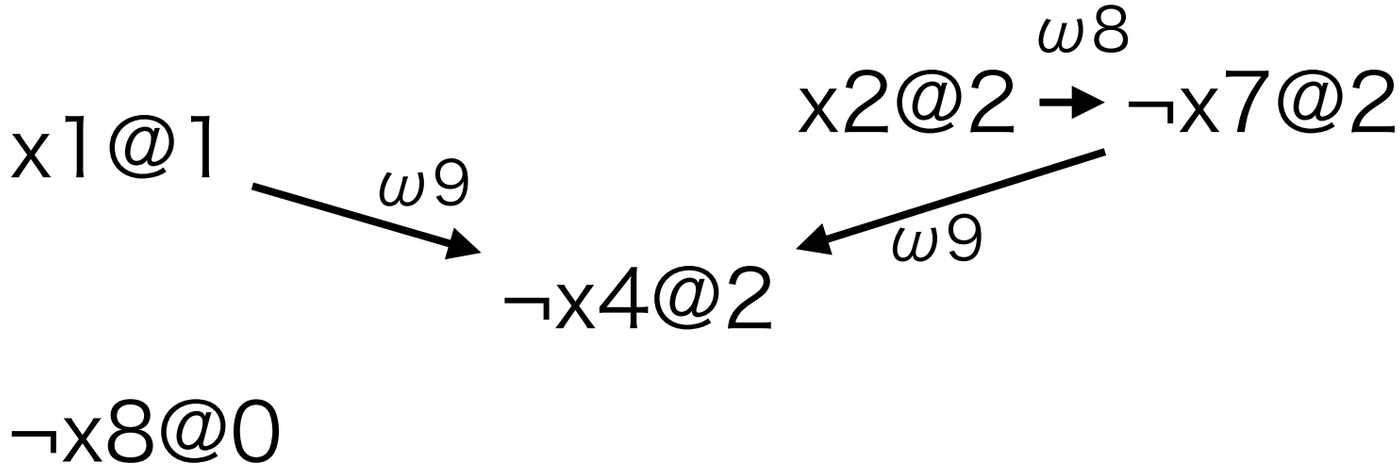
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Deduce



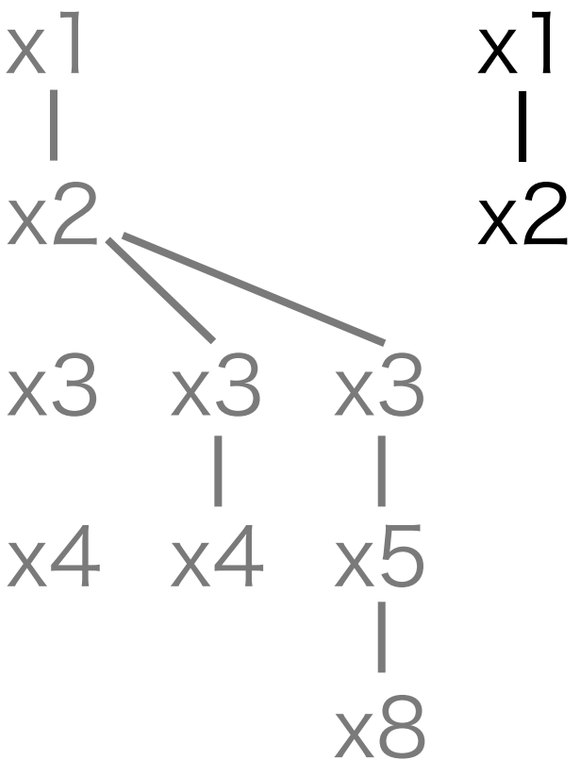
Implication Graph



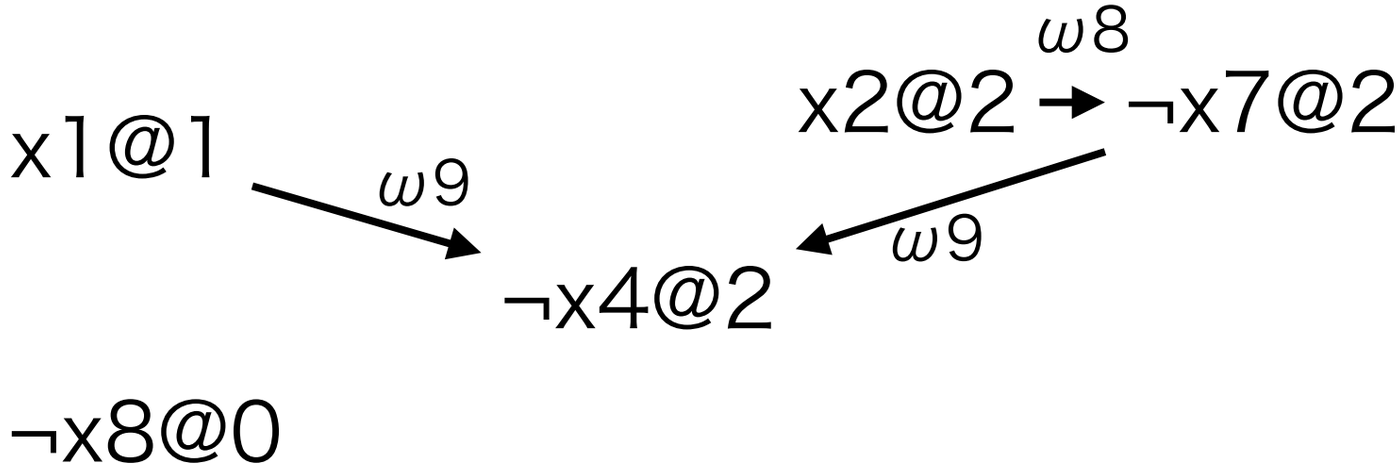
Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$
- $\omega 10$: $\neg x8$

Decide



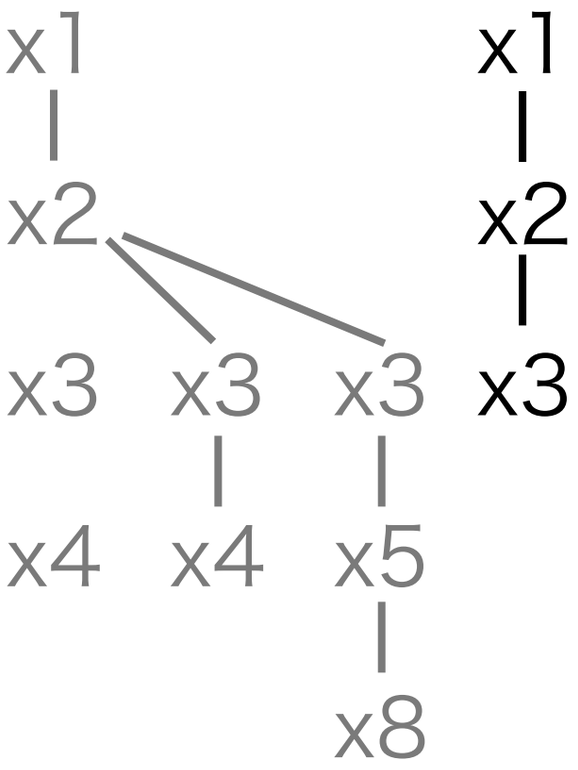
Implication Graph



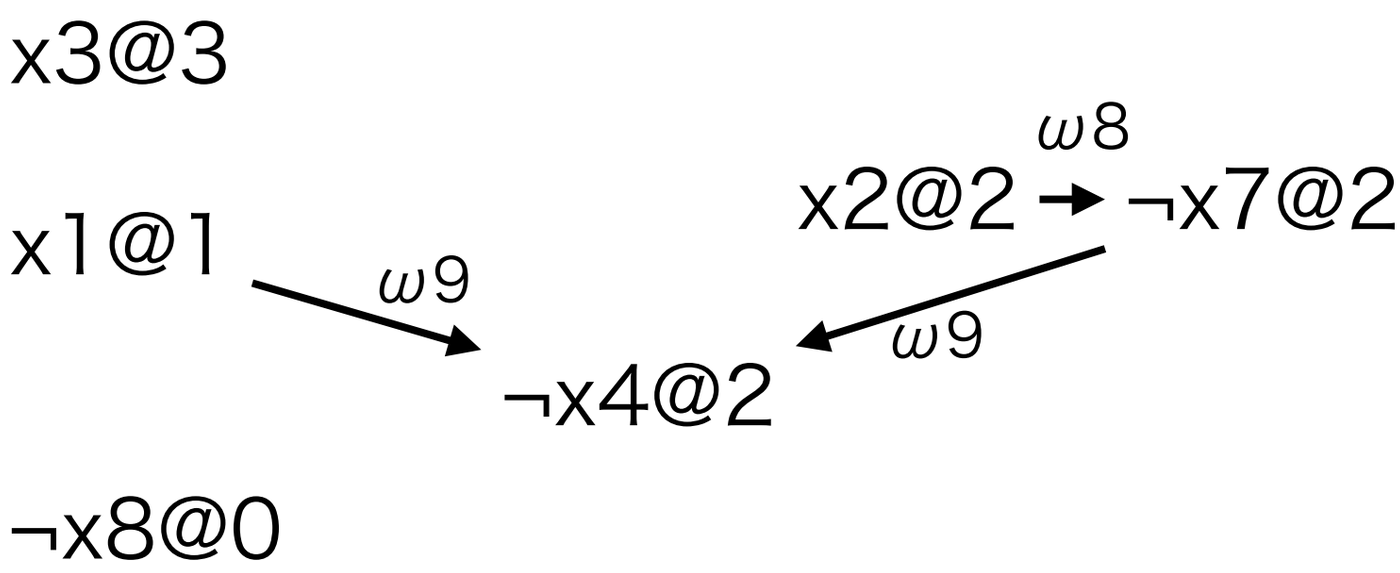
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Decide



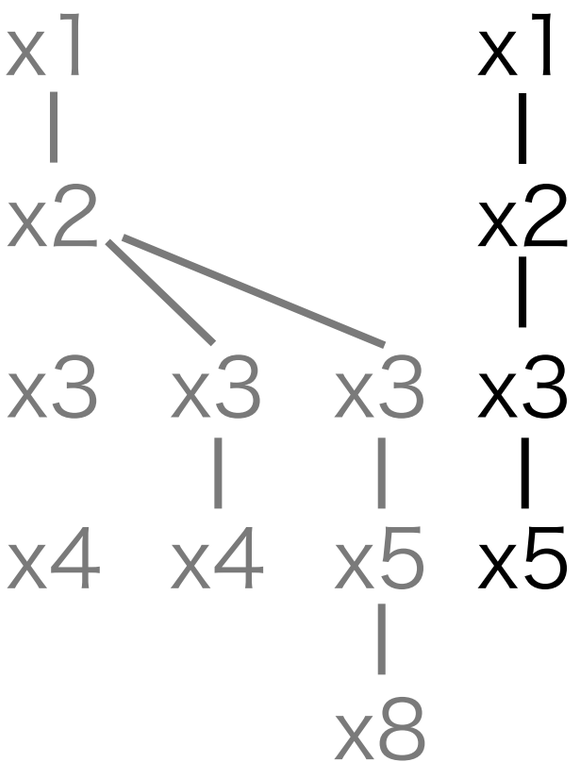
Implication Graph



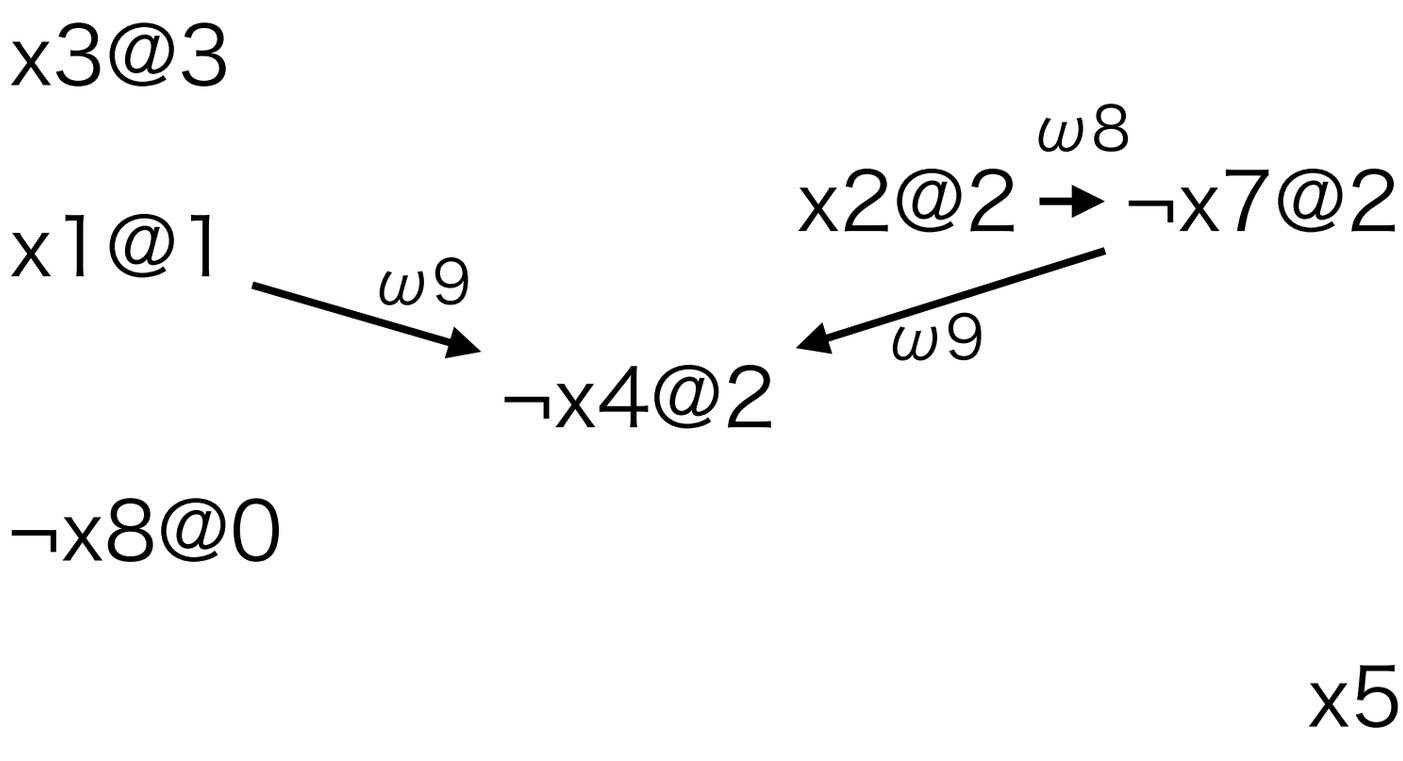
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Decide



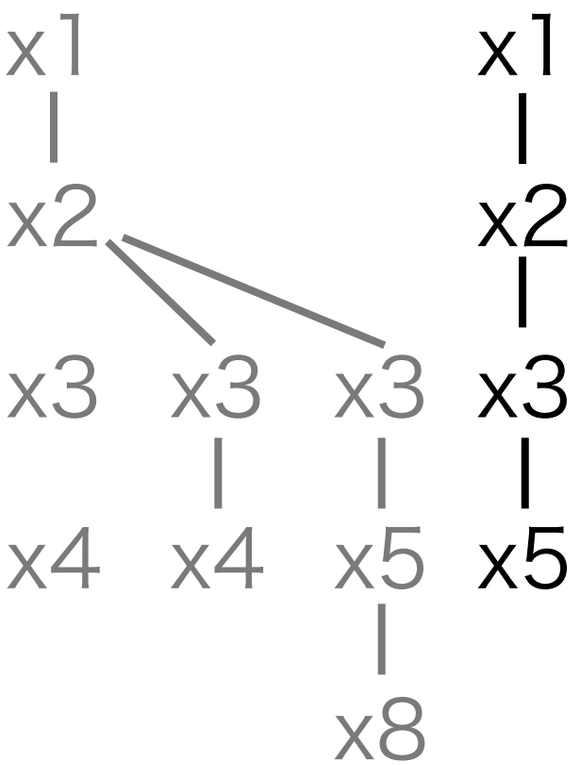
Implication Graph



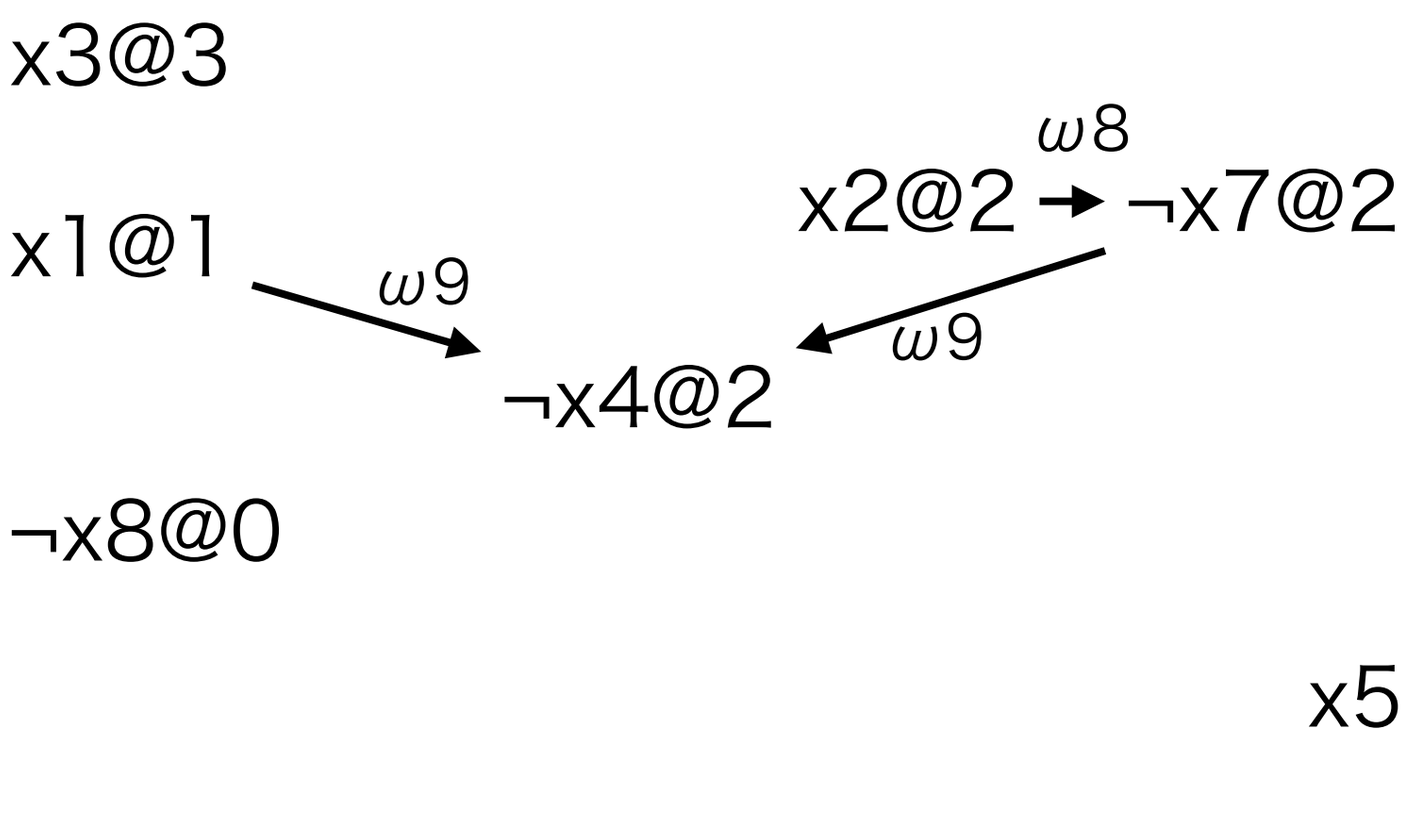
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Deduce



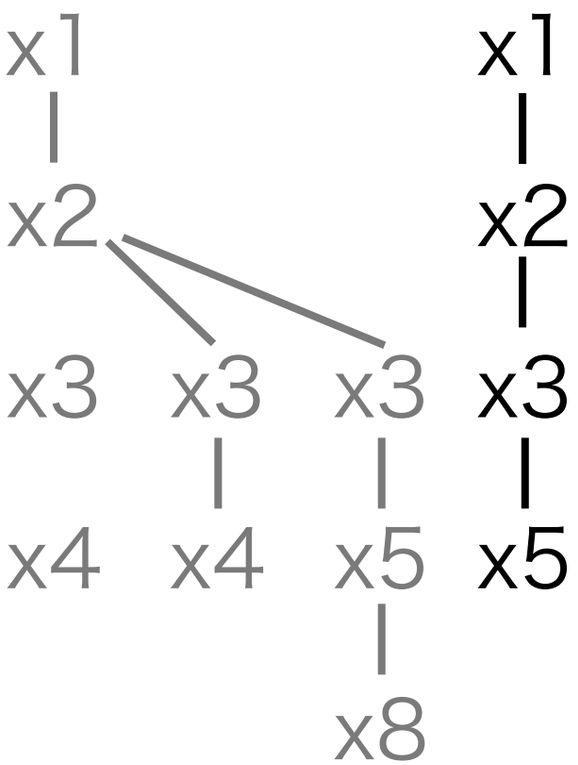
Implication Graph



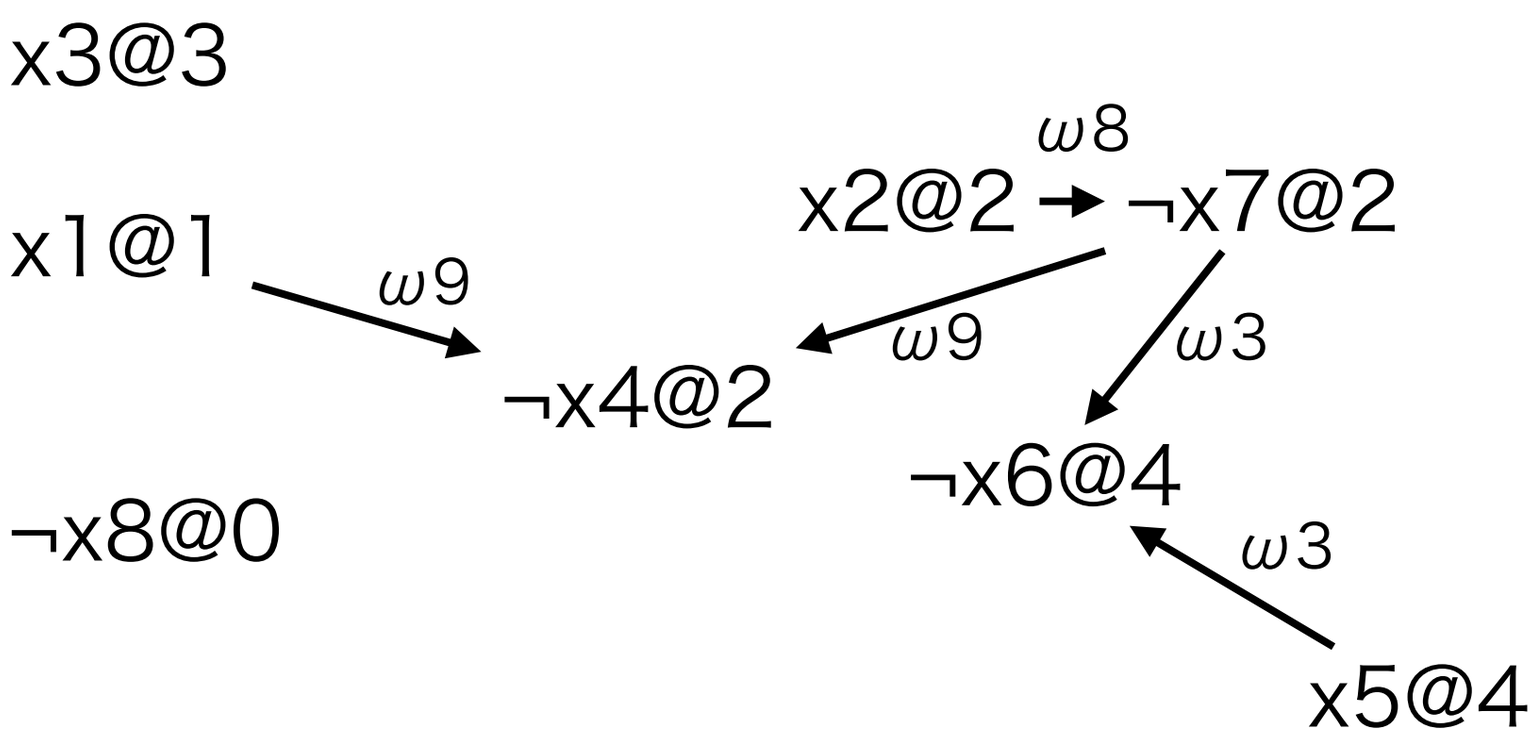
Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$
- $\omega 10$: $\neg x8$

Deduce



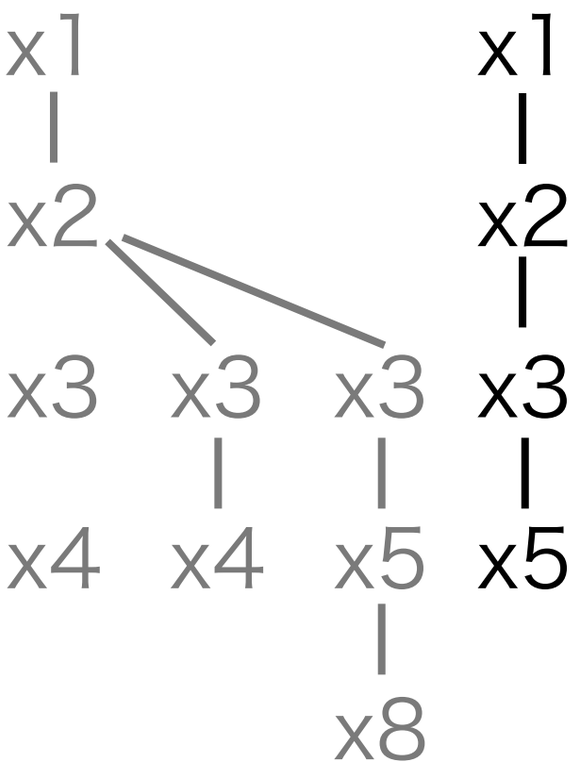
Implication Graph



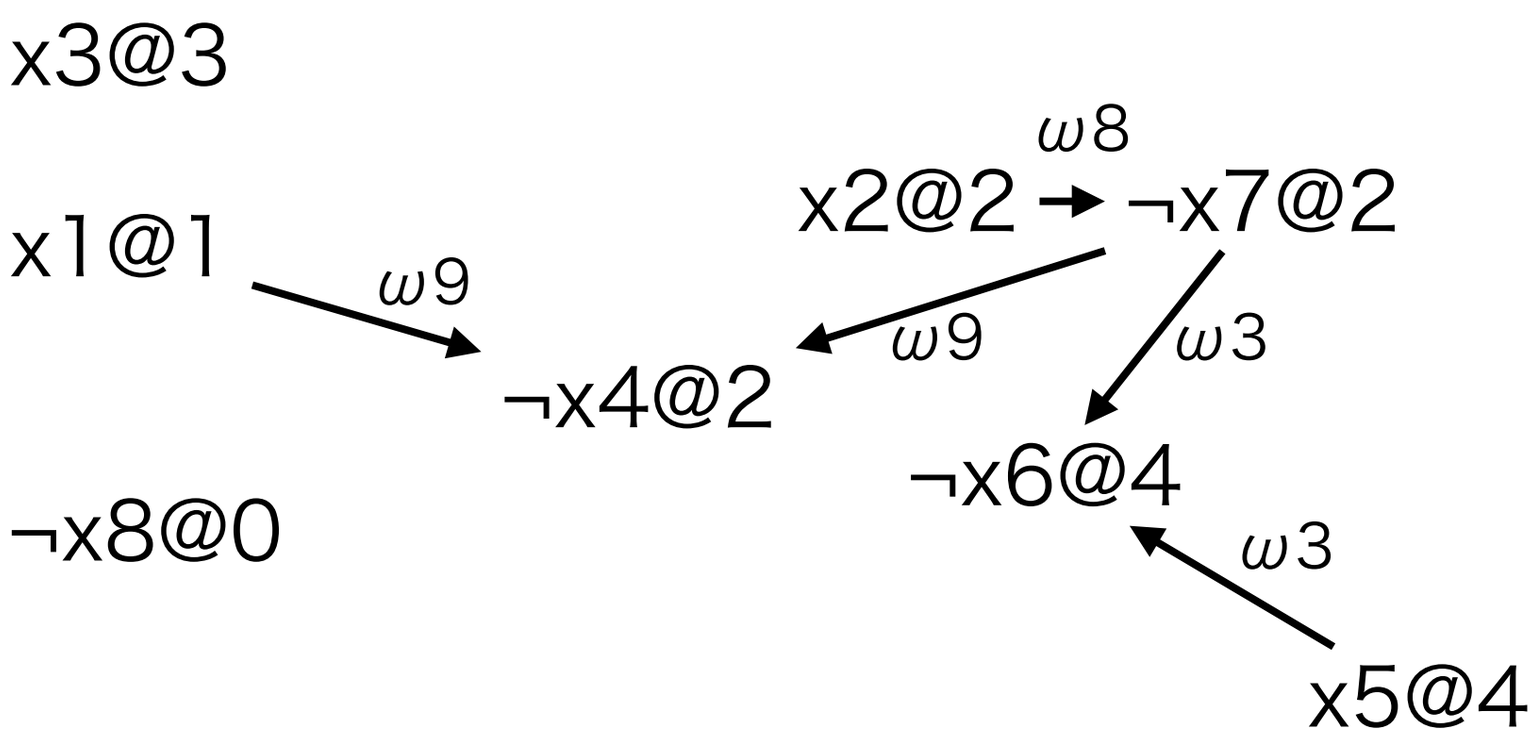
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Decide



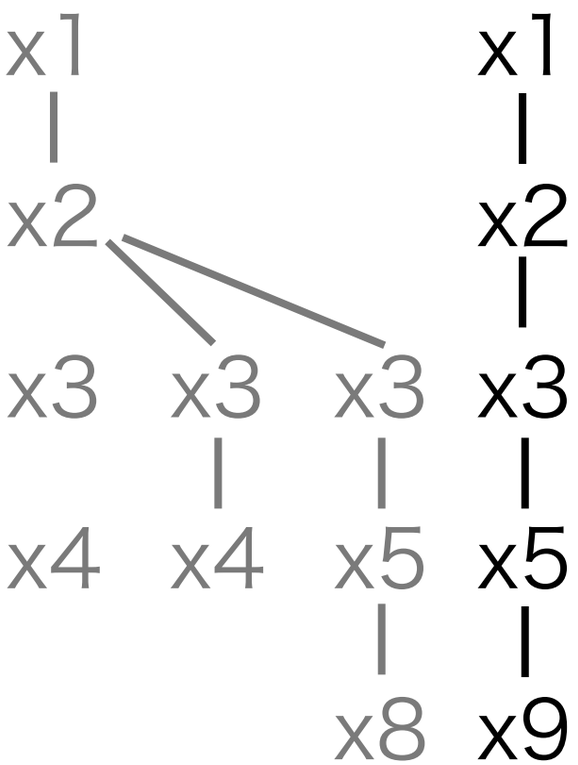
Implication Graph



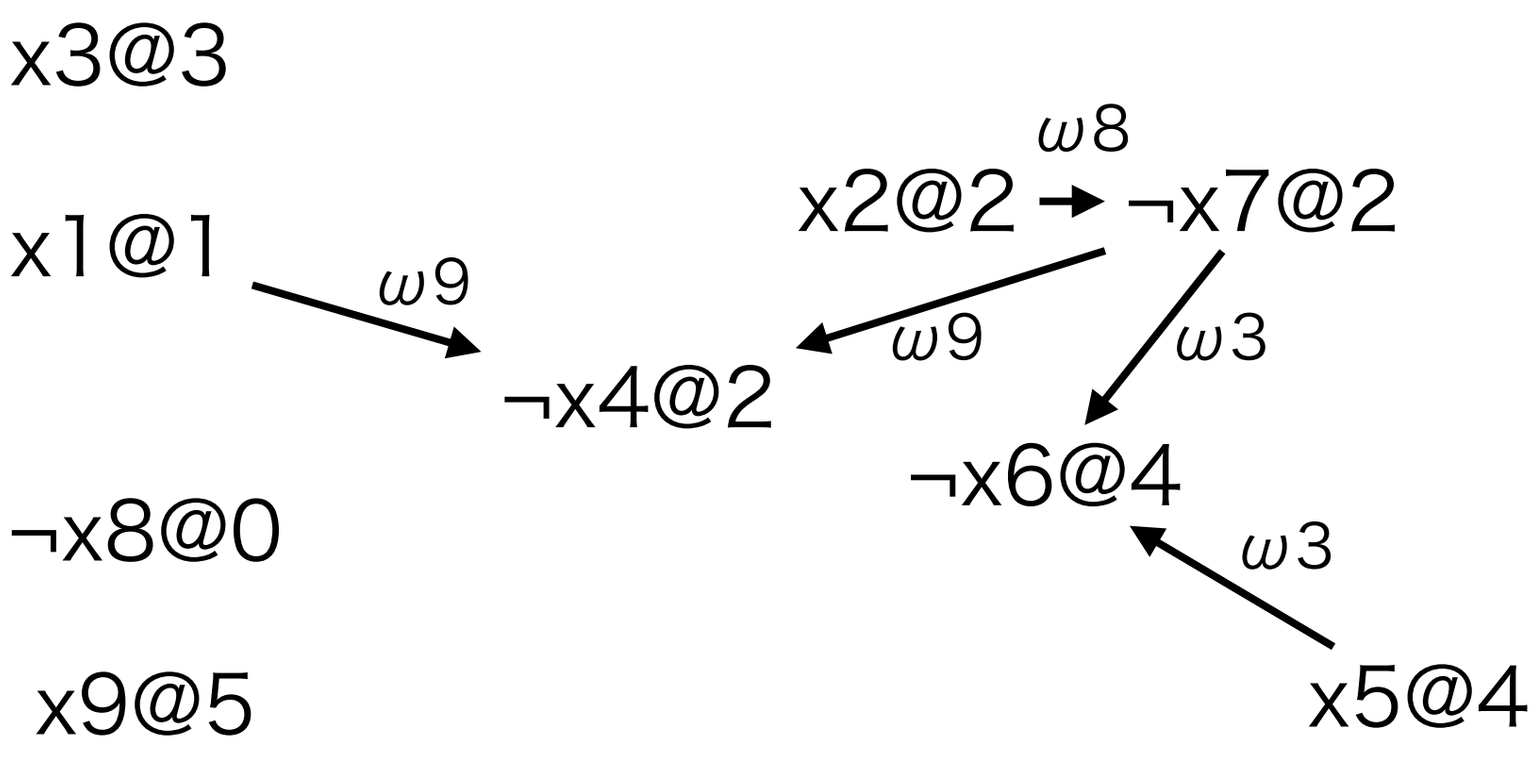
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

Decide

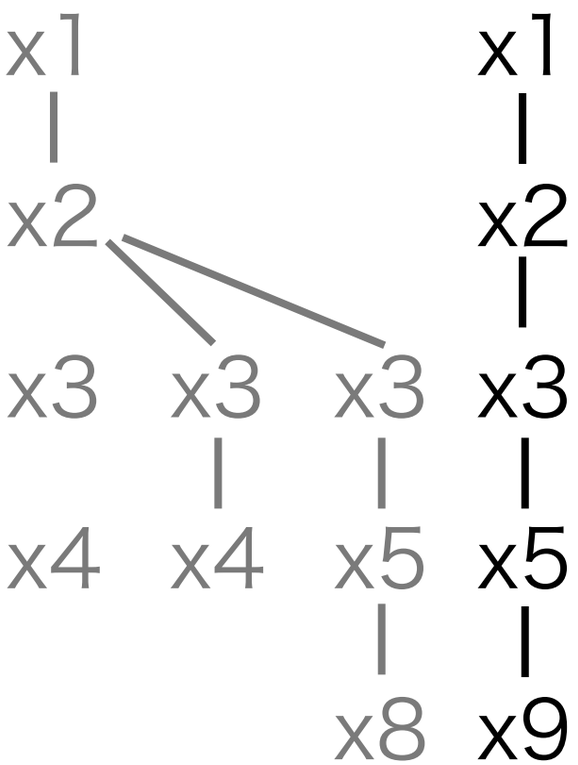


Implication Graph

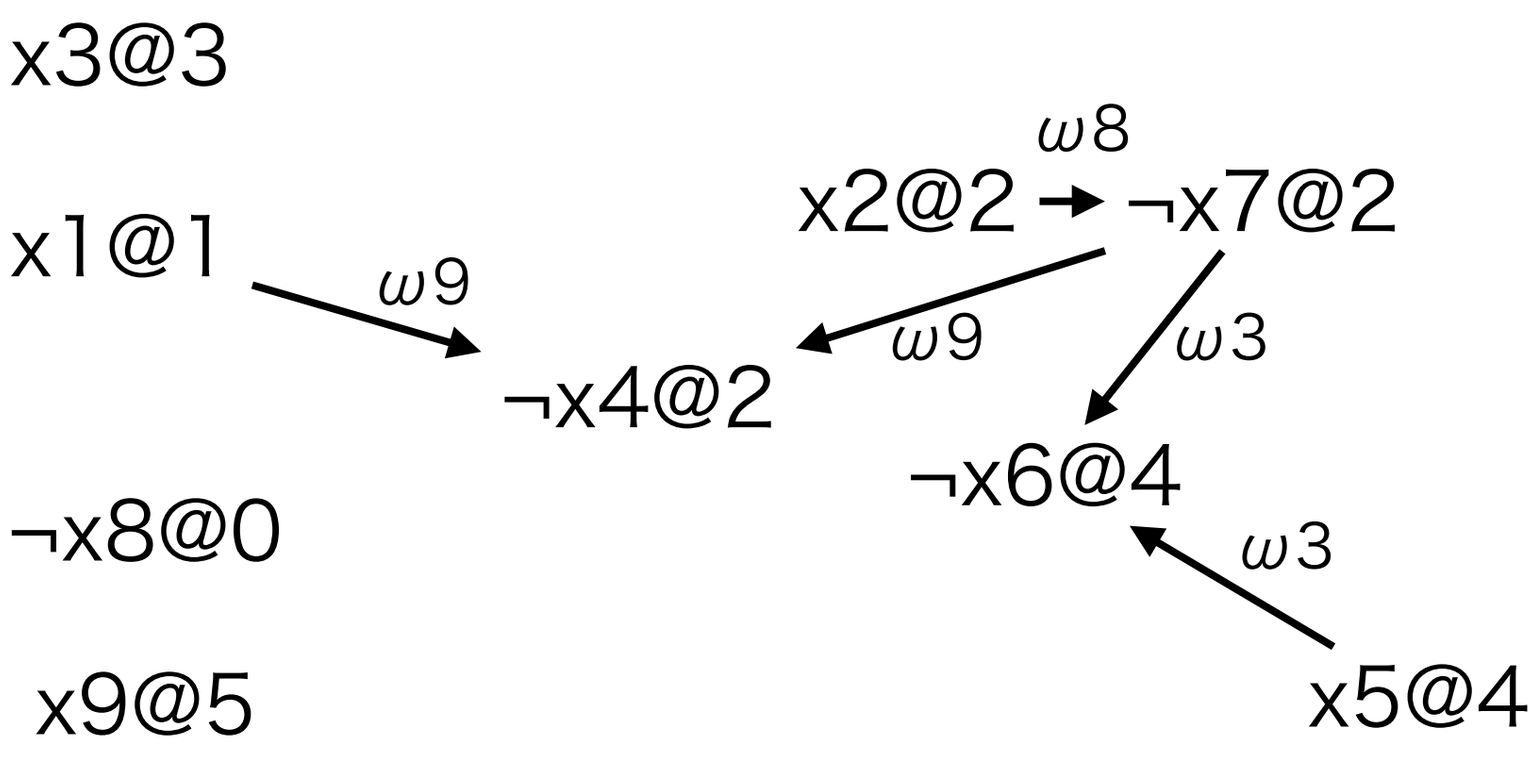


Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

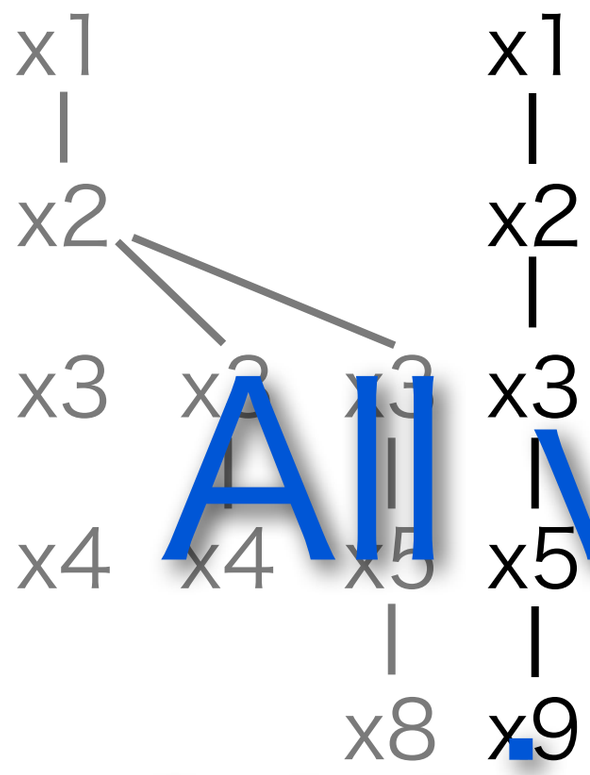


Implication Graph

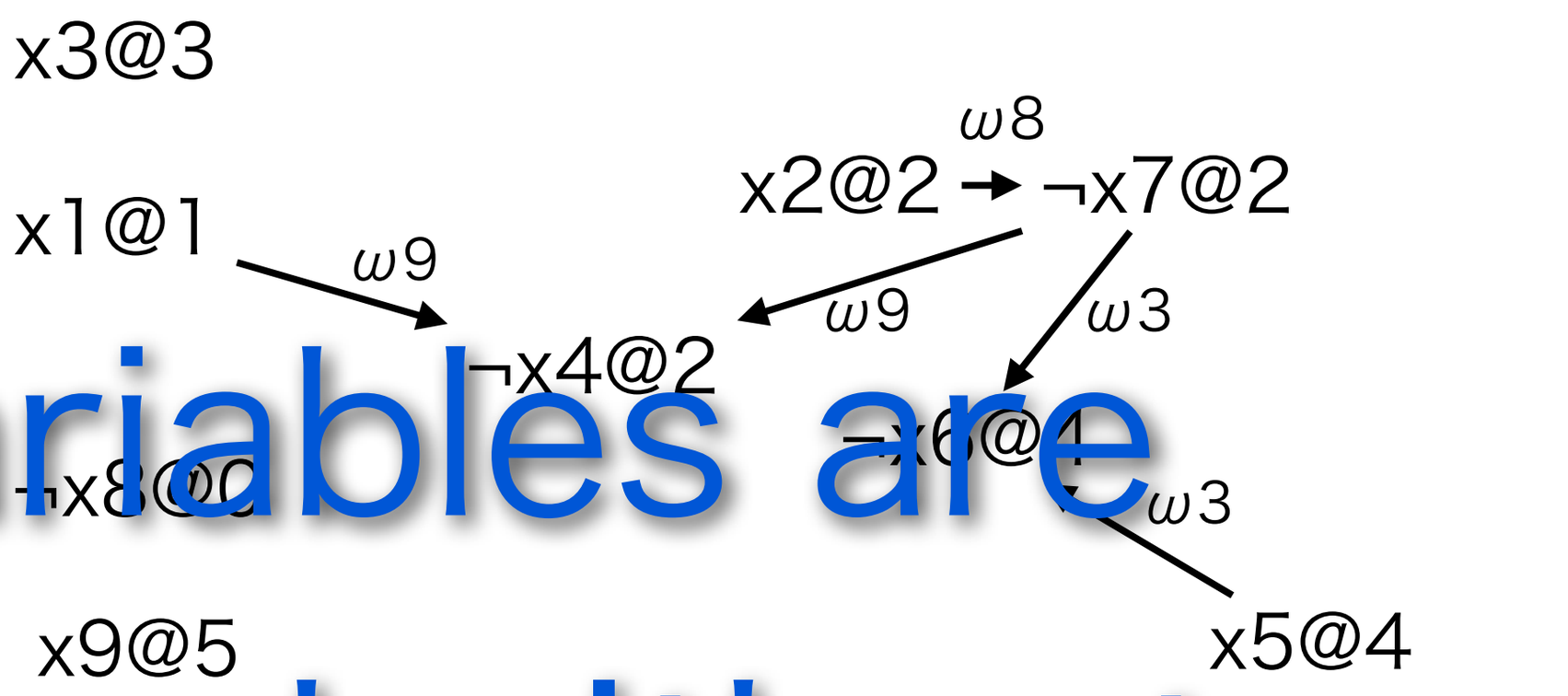


Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$
- $\omega 10$: $\neg x8$



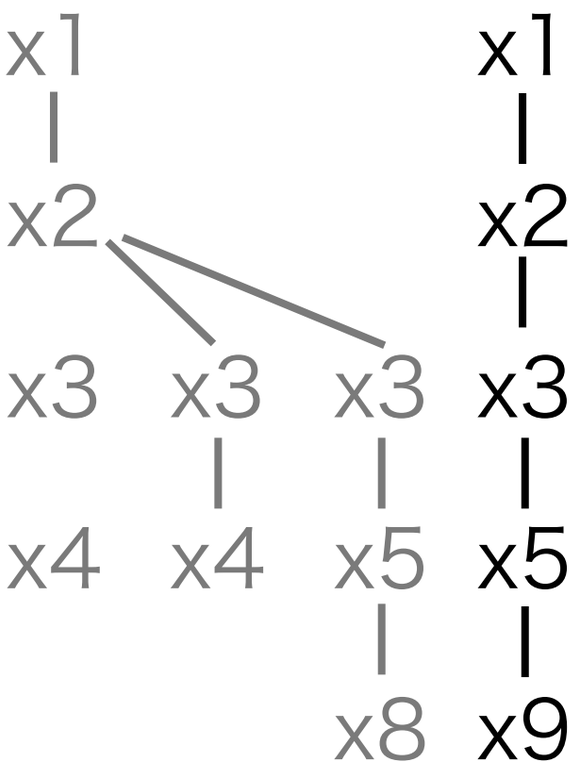
Implication Graph



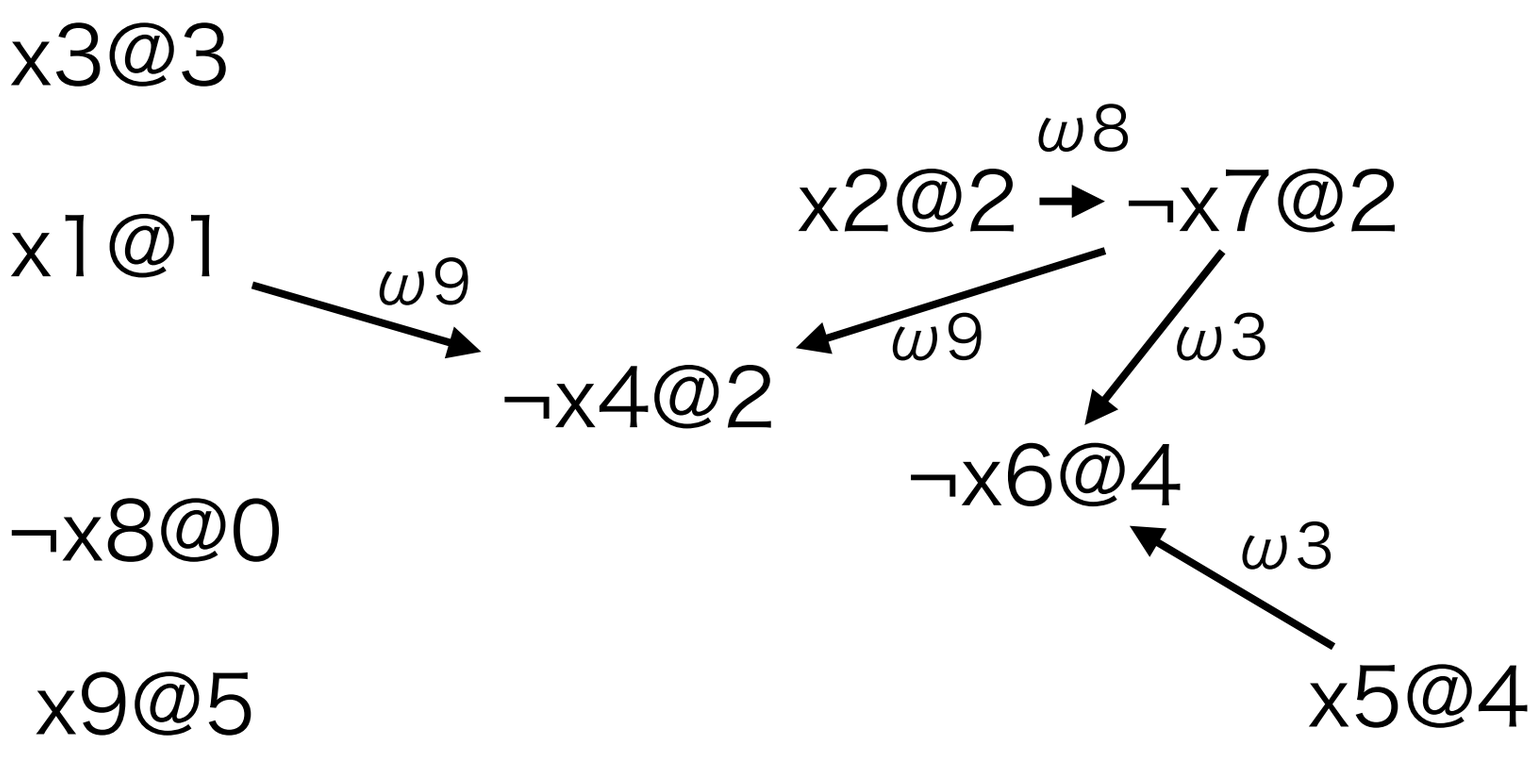
Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

All variables are assigned without conflict

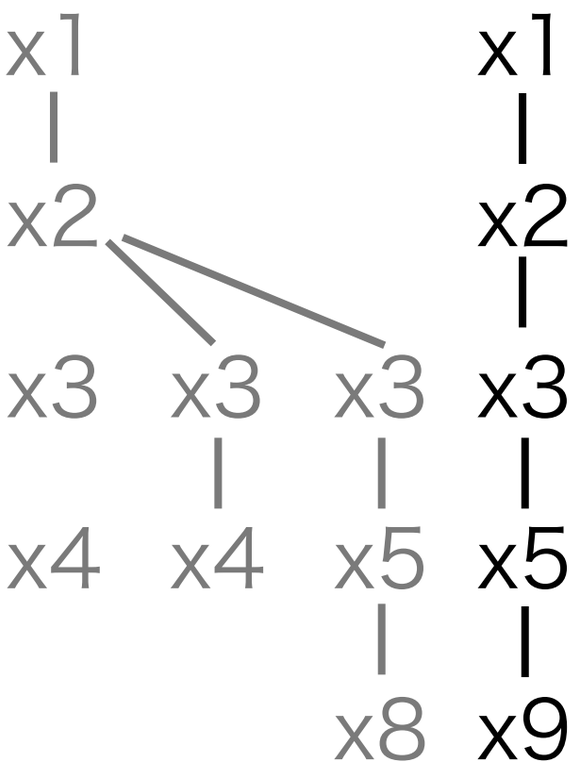


Implication Graph

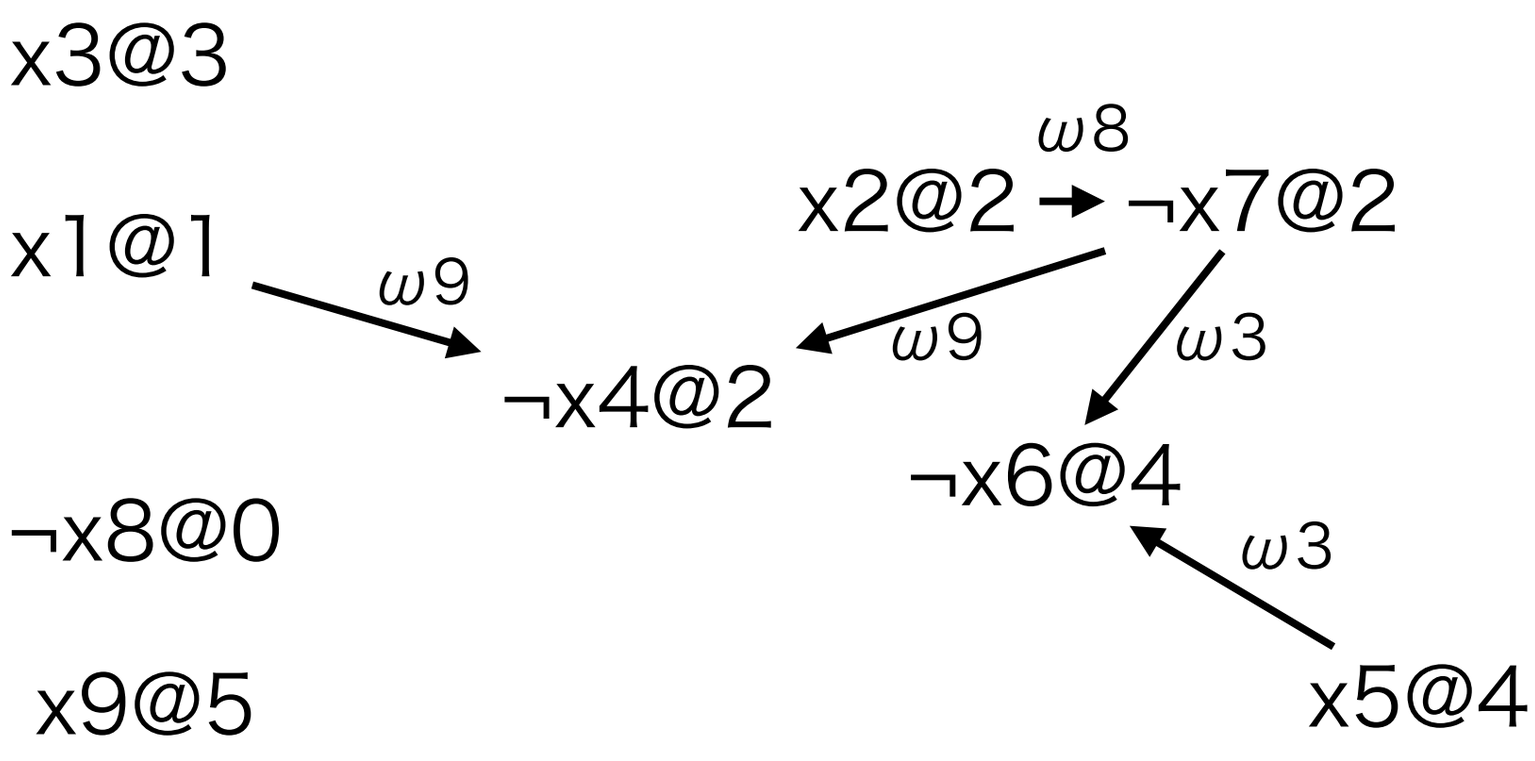


Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$
- $\omega 10$: $\neg x8$



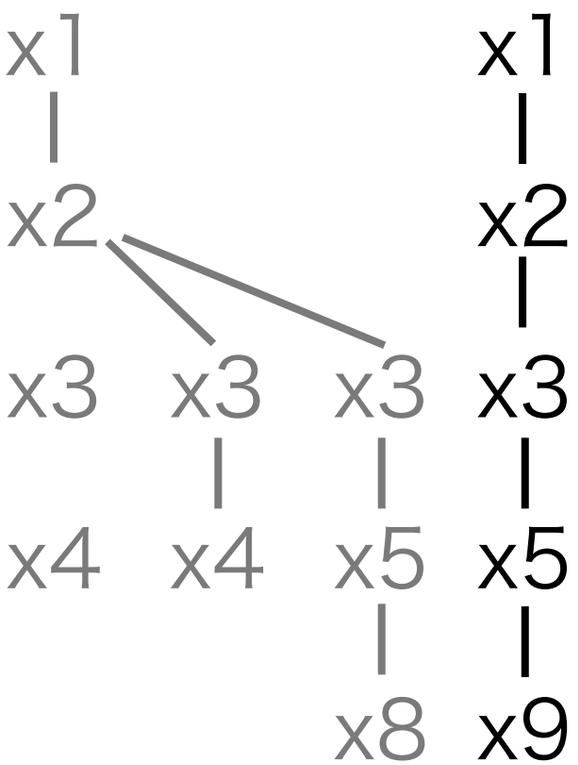
Implication Graph



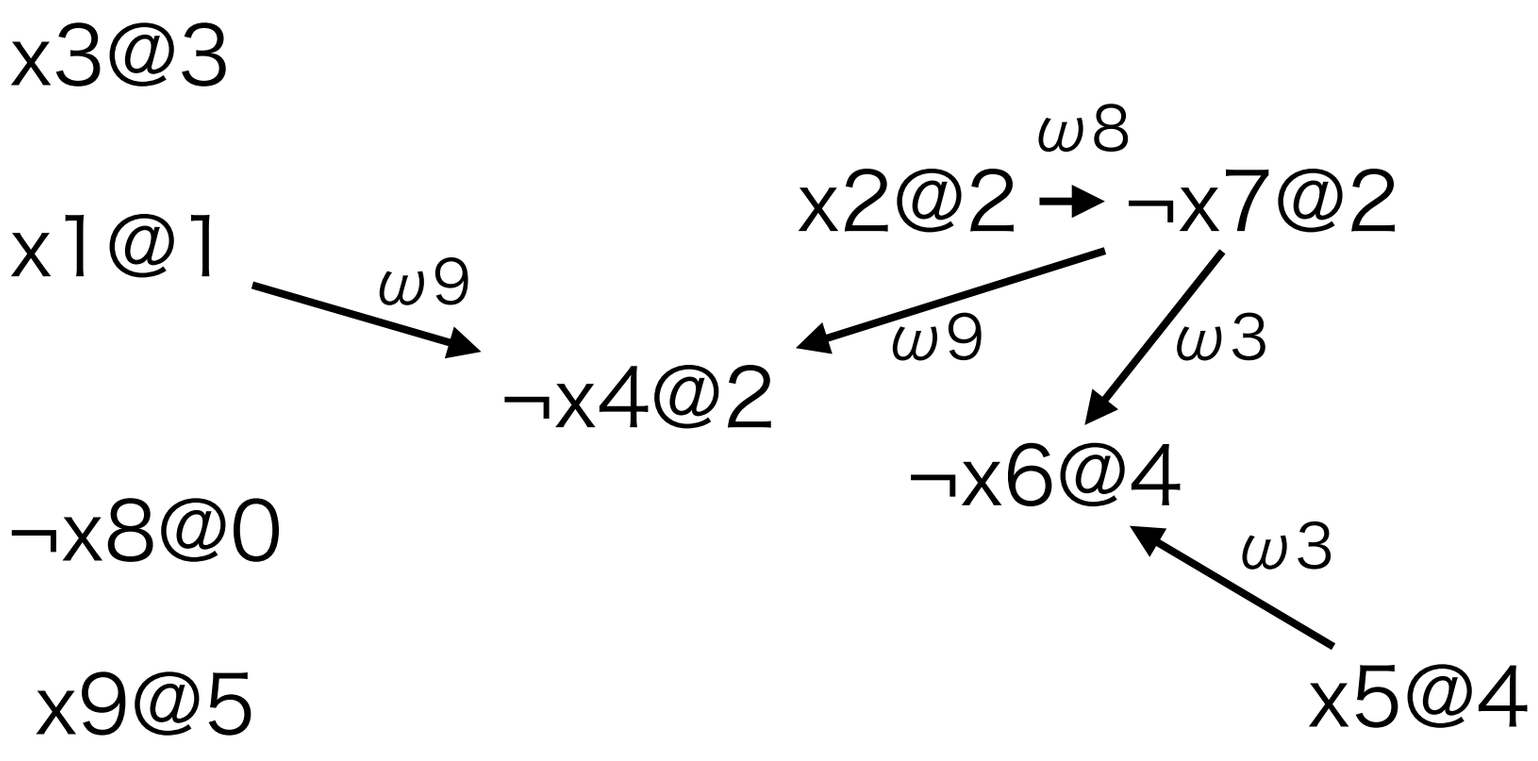
Satisfiable

Clause DB

- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$
- $\omega 10$: $\neg x8$



Implication Graph



Clause DB

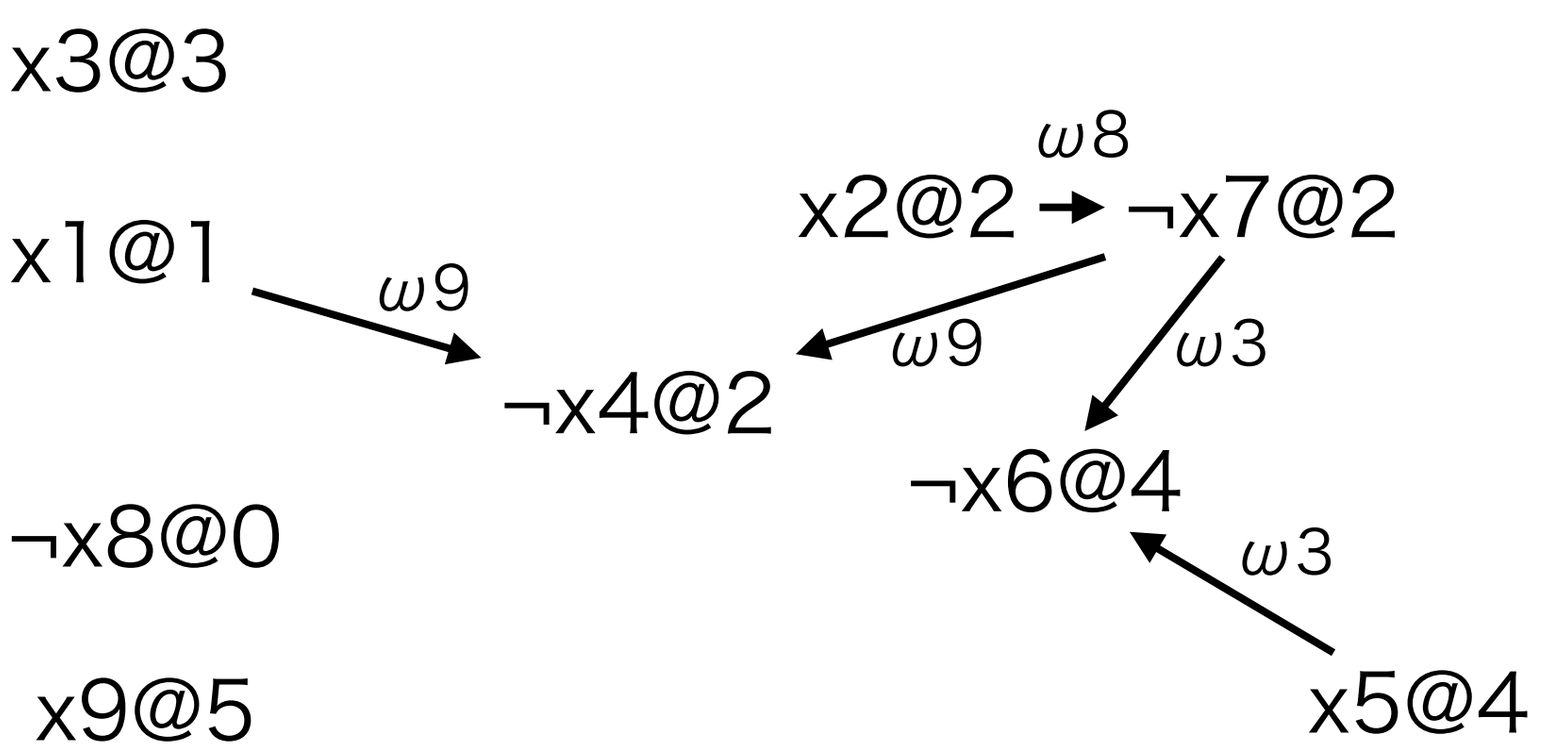
- $\omega 1$: $\neg x1 \vee \neg x4 \vee x5$
- $\omega 2$: $\neg x4 \vee x6$
- $\omega 3$: $\neg x5 \vee \neg x6 \vee x7$
- $\omega 4$: $\neg x7 \vee x8$
- $\omega 5$: $\neg x2 \vee \neg x7 \vee x9$
- $\omega 6$: $\neg x8 \vee \neg x9$
- $\omega 7$: $\neg x8 \vee x9$
- $\omega 8$: $\neg x2 \vee \neg x7$
- $\omega 9$: $\neg x1 \vee \neg x4 \vee x7$
- $\omega 10$: $\neg x8$

Found a model

M =

{ x1, x2, x3,
¬x4, x5, ¬x6
¬x7, ¬x8, x9 }

Implication Graph



Clause DB

- $\omega 1$: ¬x1 ∨ ¬x4 ∨ x5
- $\omega 2$: ¬x4 ∨ x6
- $\omega 3$: ¬x5 ∨ ¬x6 ∨ x7
- $\omega 4$: ¬x7 ∨ x8
- $\omega 5$: ¬x2 ∨ ¬x7 ∨ x9
- $\omega 6$: ¬x8 ∨ ¬x9
- $\omega 7$: ¬x8 ∨ x9
- $\omega 8$: ¬x2 ∨ ¬x7
- $\omega 9$: ¬x1 ∨ ¬x4 ∨ x7
- $\omega 10$: ¬x8

fin.

References

- J. P. Marques-Silva and K. A. Sakallah, "GRASP: a search algorithm for propositional satisfiability," Computers, IEEE Transactions on, vol. 48, no. 5, pp. 506-521, May 1999.
- Handbook of Satisfiability, A. Biere, M. Heule, H. Van Maaren, and T. Walsh, Eds. IOS Press, Feb. 2009.