How a CDCL SAT Solver works



Masahiro Sakai

Twitter: @masahiro_sakai

Abstract

- Demonstrating how a modern SAT solver works on a small example.
- Target Algorithm
 - Conflict-Driven Clause Learning (CDCL)
 - 1-UIP learning + far-backtracking
 - details are omitted (decision heuristics, efficient data structure, etc.,)

An Example Problem

- ω 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5
- ω6: ¬x8 ∨ ¬x9

• $\omega 2: \neg x4 \lor x6$

• $\omega 7: \neg x8 \lor x9$

- ω 3: $\neg x5 \lor \neg x6 \lor x7$
- $\omega 4: \neg x7 \lor x8$
- ω 5: $\neg x2 \lor \neg x7 \lor x9$

1	eci		
	てし	IU	C

Implication Graph

Clause DB

• ω 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

• $\omega 2: \neg x4 \lor x6$

• ω 3: $\neg x5 \lor \neg x6 \lor x7$

• $\omega 4: \neg x7 \lor x8$

• $\omega 5: \neg x2 \lor \neg x7 \lor x9$

• ω 7: \neg x8 \vee x9

Implication Graph

x1

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

$$\vee \neg x4 \vee x5$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

$$\neg x \mid \lor \neg x 4 \lor x 5$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

Implication Graph

x4@4

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph

x1 I
x2
x3
 x4

x4@4

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5$ • ω 6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph



$$x3@3$$
 $x1@1 \xrightarrow{w1} x5@4$

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

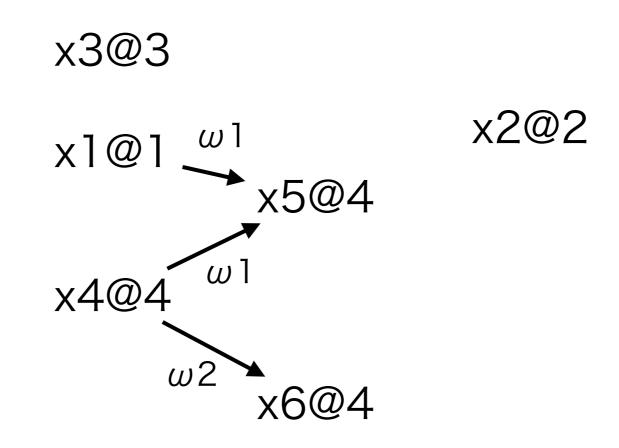
•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega 7: \neg x8 \lor x9$$

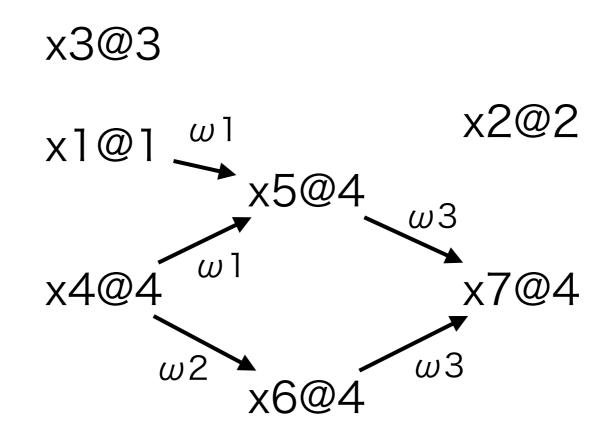
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega 7: \neg x8 \lor x9$$

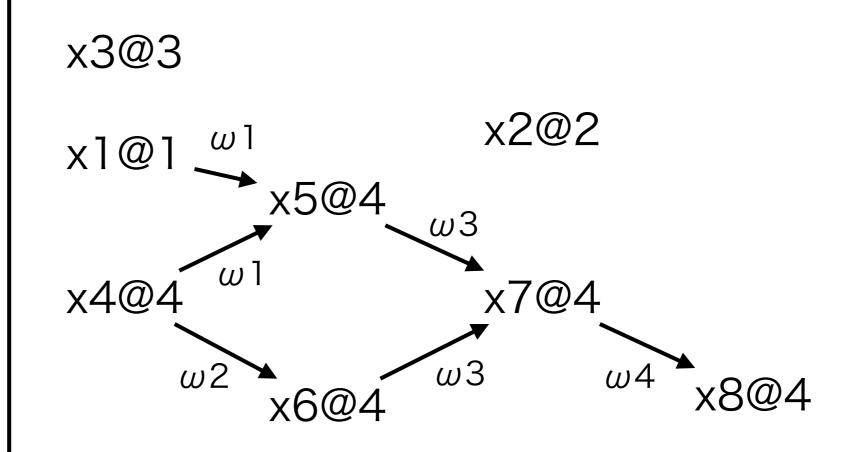
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: \neg x8 \vee \neg x9

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega 7: \neg x8 \lor x9$$

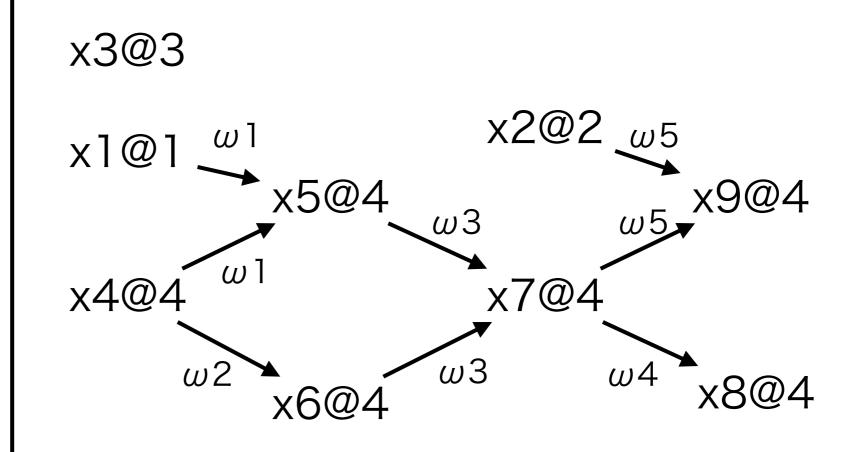
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

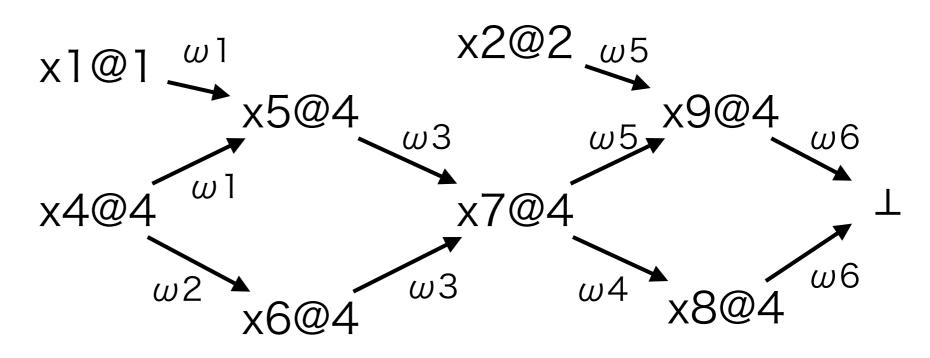
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: \neg x8 \vee \neg x9

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

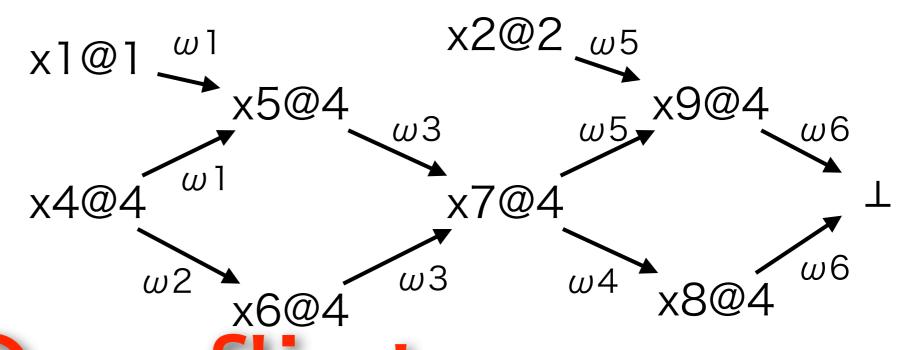
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

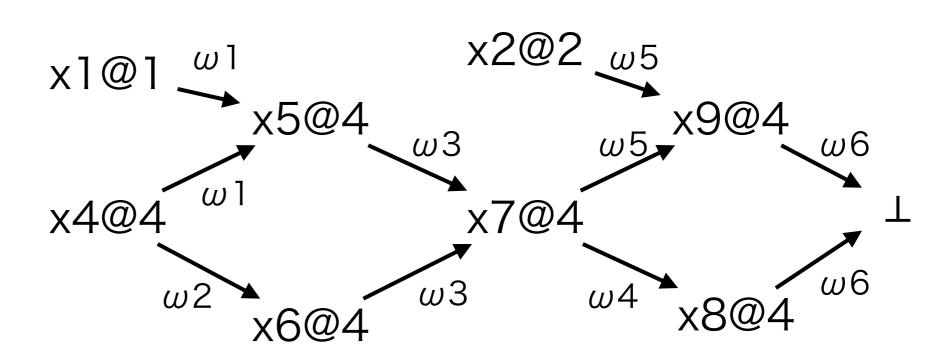
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
7: \neg x8 \vee x9

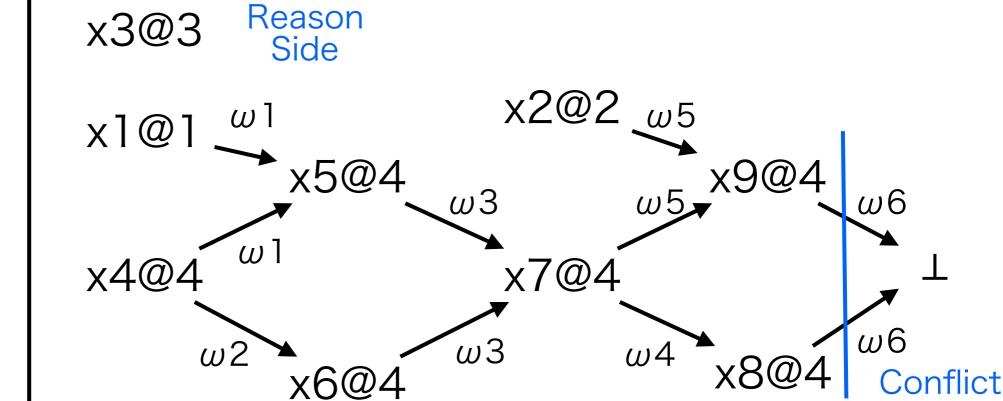
Implication Graph



- ω 1: $\neg x1 \lor \neg x4 \lor x5 <math>\omega$ 6: $\neg x8 \lor \neg x9$
- $\omega 2: \neg x4 \lor x6$
- ω 3: $\neg x5 \lor \neg x6 \lor x7$
- $\omega 4: \neg x7 \lor x8$
- $\omega 5: \neg x2 \lor \neg x7 \lor x9$

- ω 7: \neg x8 \vee x9

Implication Graph



Clause DB

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

Side

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph

 ω 3

x6@4

 $\omega 4$

Clause DB

• ω 1: \neg x1 \vee \neg x4 \vee x5 • ω 6: \neg x8 \vee \neg x9

• ω 2: $\neg x4 \lor x6$

 ω 3: $\neg x5 \lor \neg x6 \lor x7$

• $\omega 4: \neg x7 \lor x8$

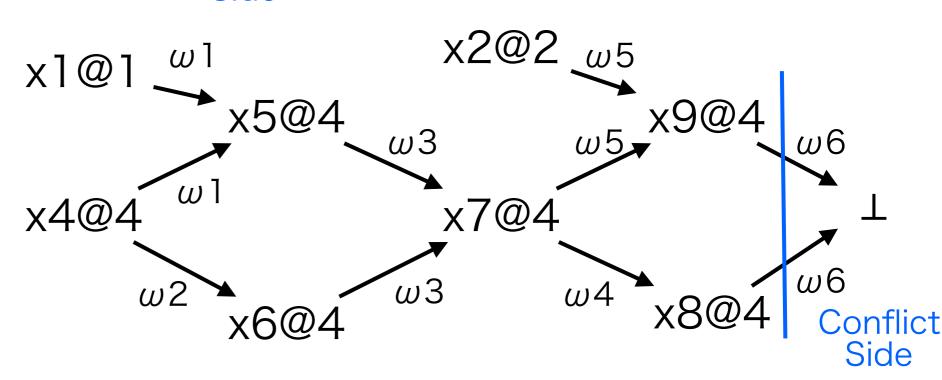
 ω 5: $\neg x2 \lor \neg x7 \lor x9$

ω6

Side

• ω 7: \neg x8 \vee x9

Implication Graph



•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

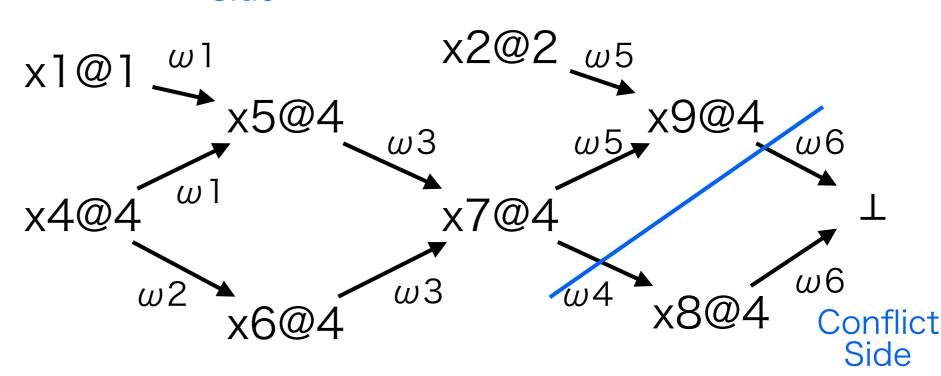
•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

$$\neg x7 \lor \neg x9$$

Implication Graph



•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

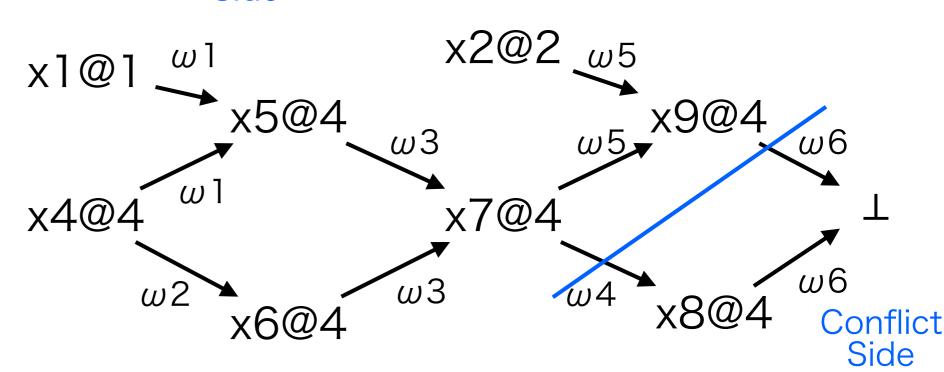
•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
7: ¬x8 ∨ x9

$$-x7\sqrt{-x9}$$
 $\omega 5:$ $-x2\sqrt{-x7}\sqrt{x9}$

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

Implication Graph

Side

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

Implication Graph

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

$$ω6: ω4:
¬x8∨¬x9 ¬x7∨x8$$
 $¬x7∨¬x9 ω5:
¬x2∨¬x7∨x9$

It contains one literal in the current decision level (i.e. 4).

Implication Graph

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega 2$$
: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

$$ω6: ω4:
¬x8∨¬x9 ¬x7∨x8$$
 $¬x7∨¬x9 ω5:
¬x2∨¬x7∨x9$

It contains one literal in the current decision level (i.e. 4).

Implication Graph

Clause DB

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega 8: \neg x2 \lor \neg x7$$

New clause is learnt!

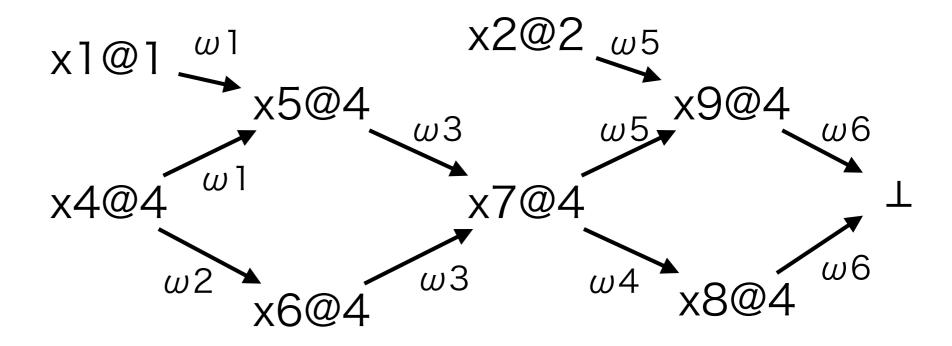
Learnt clause: $\neg x2 \lor \neg x7$

x2 was set at level 2 x7 was set at level 4

Assertion level of this clause (which is the second highest level) is 2.

⇒ Backtrack to level 2!

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
4: \neg x7 \vee x8

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

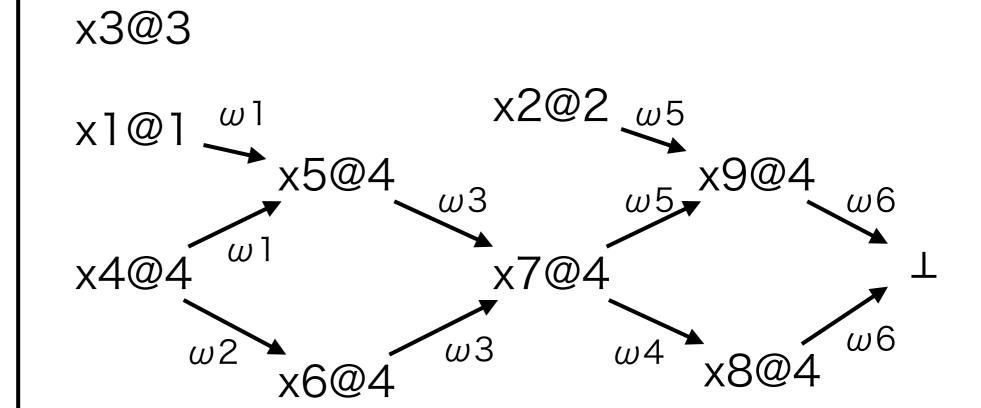
•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega 8: \neg x2 \lor \neg x7$$

Implication Graph





•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5 • ω 6: \neg x8 \vee \neg x9

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

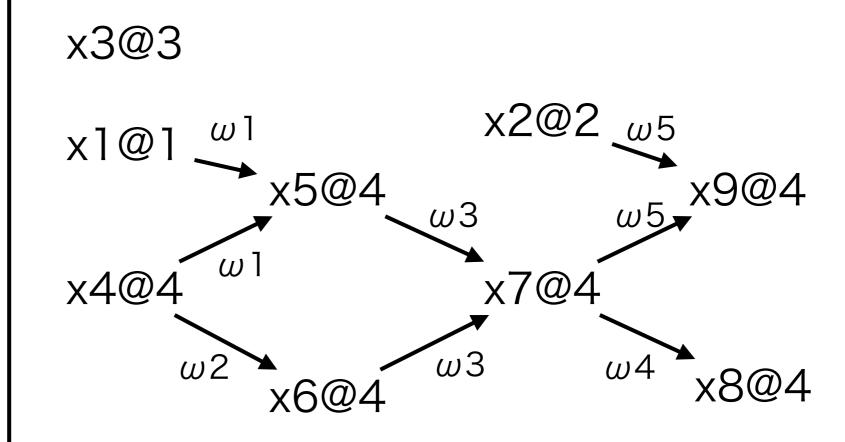
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

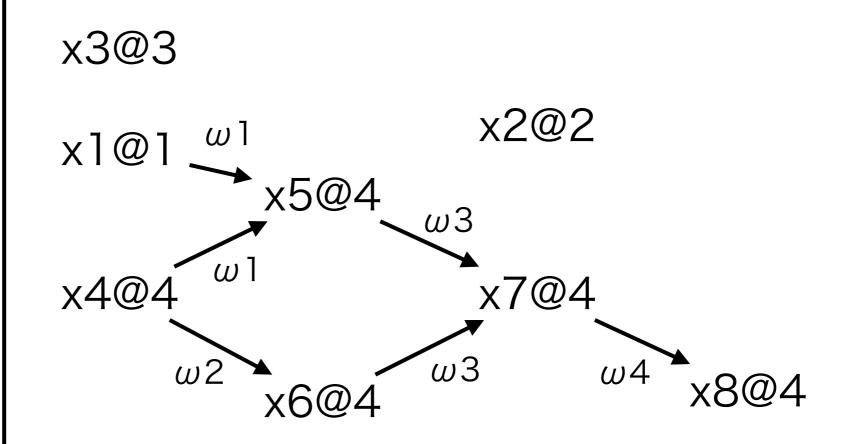
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega 3: \neg x5 \lor \neg x6 \lor x7$$
 • $\omega 8: \neg x2 \lor \neg x7$

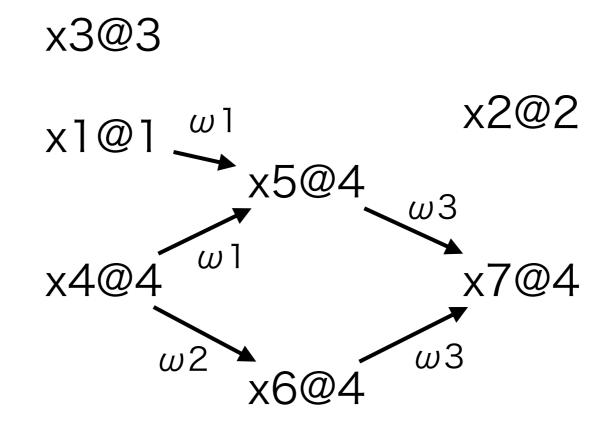
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

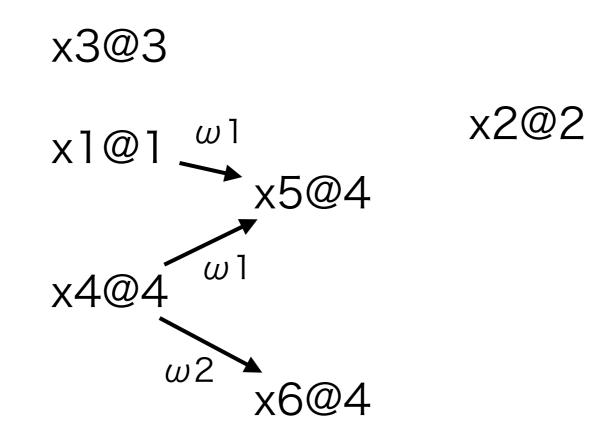
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
8: $\neg x^2 \vee \neg x^7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph



$$x3@3$$
 $x1@1 \xrightarrow{\omega 1} x5@4$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

•
$$\omega$$
 1. \times 1 \times \times \times

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
4: \neg x7 \vee x8

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

Implication Graph

x]
x2
x3 I
x4

x4@4

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
8: $\neg x^2 \vee \neg x^7$

•
$$\omega 4$$
: $\neg x7 \lor x8$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph

x4

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
8: $\neg x^2 \vee \neg x^7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

•
$$\omega$$
 1: $\neg x$ 1 \vee $\neg x$ 4 \vee x 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
 I: $\neg x$ I \vee $\neg x$ 4 \vee x 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
 1. $\neg x$ 1 \vee $\neg x$ \leftrightarrow $\vee x$:

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4$$
: $\neg x7 \lor x8$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5$ • ω 6: $\neg x8 \lor \neg x9$

•
$$\omega$$
 1. $\neg x$ 1 \vee $\neg x$ 4 \vee x 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

$$\omega$$
 1. ∇ ∇ ∇

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

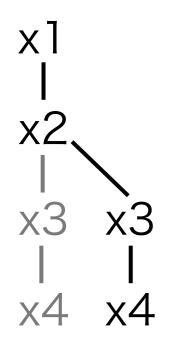
•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

Implication Graph



$$x3@3$$
 $x1@1 \xrightarrow{\omega 1} x5@4$
 $x4@4$

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

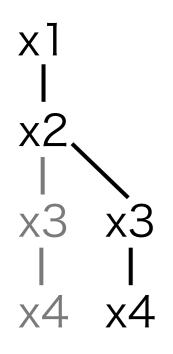
•
$$\omega 4$$
: $\neg x7 \lor x8$

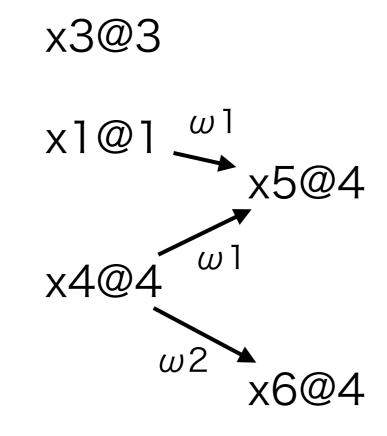
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5$ • ω 6: $\neg x8 \lor \neg x9$

•
$$\omega$$
 1. $\neg x$ 1 \vee $\neg x$ 4 \vee x 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

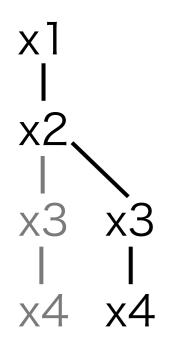
•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

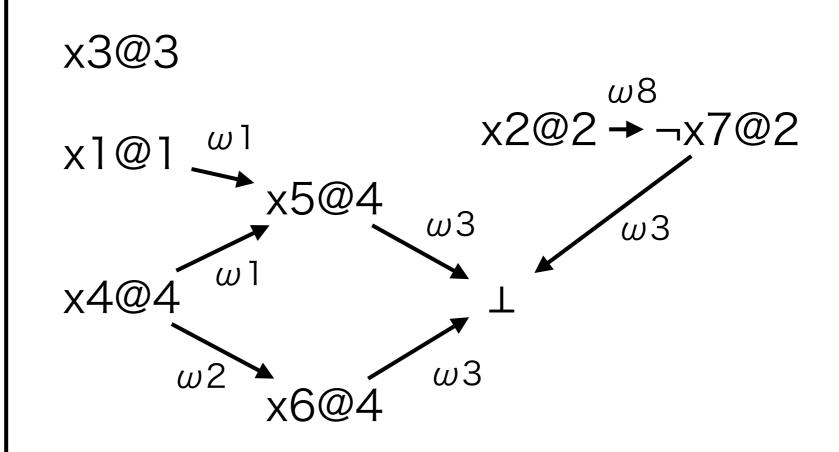
•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

Implication Graph





•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: ¬x8 ∨ x9

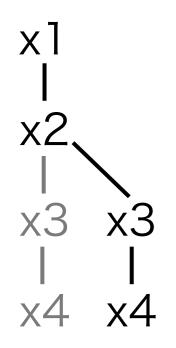
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

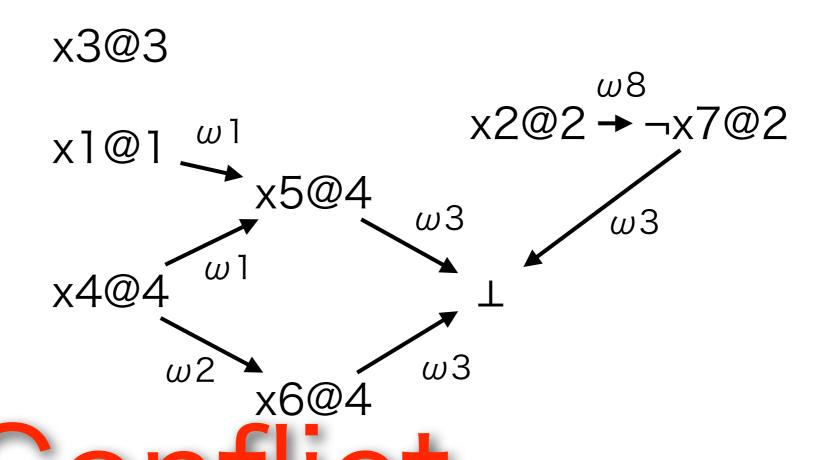
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4$$
: $\neg x7 \lor x8$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: ¬x8 ∨ x9

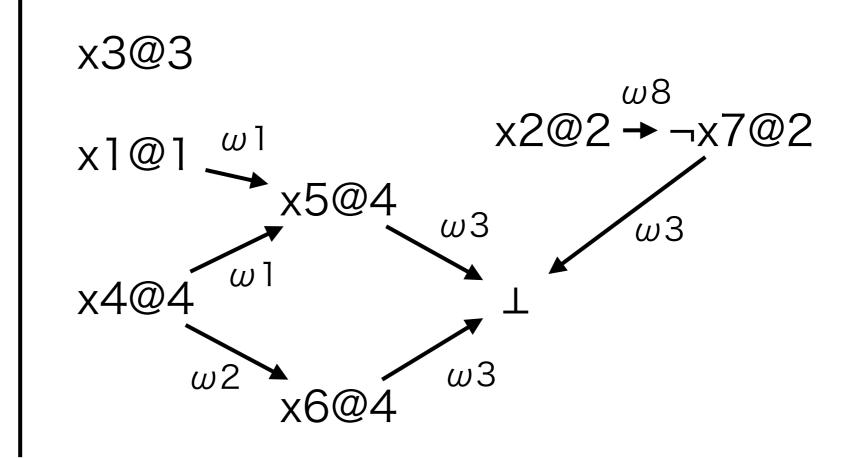
•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph



•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5 • ω 6: \neg x8 \vee \neg x9

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

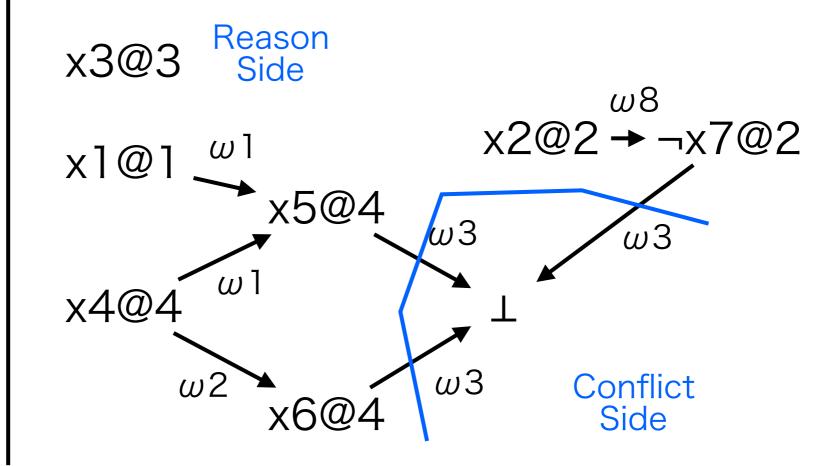
•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

Implication Graph



•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

Implication Graph

x3@3 Reason Side
$$x1@1 \xrightarrow{w1} x5@4 \xrightarrow{w3} x4@4 \xrightarrow{w2} x6@4$$
 Conflict Side

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

•
$$\omega$$
 1. \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
4: \neg x7 \vee x8

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

ω 3: $\omega 2$: $\neg x5 \lor \neg x6 \lor x7 \ \neg x4 \lor x6$

Implication Graph

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5 • ω 6: $\neg x$ 8 $\vee \neg x$ 9

•
$$\omega$$
 1. $\neg x$ 1 \vee $\neg x$ 4 \vee x 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega 3: \neg x5 \lor \neg x6 \lor x7$$
 • $\omega 8: \neg x2 \lor \neg x7$

•
$$\omega 4$$
: $\neg x7 \lor x8$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

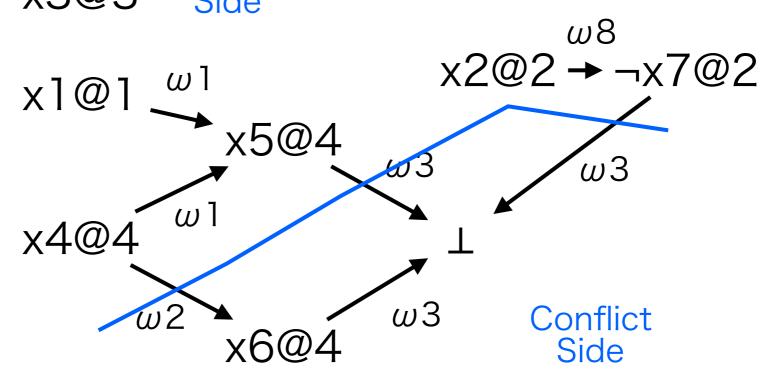
•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

ω 3: $\omega 2$: $\neg x5 \lor \neg x6 \lor x7 \ \neg x4 \lor x6$

$$\neg x4 \lor \neg x5 \lor x7$$

Implication Graph



•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
4: \neg x7 \vee x8

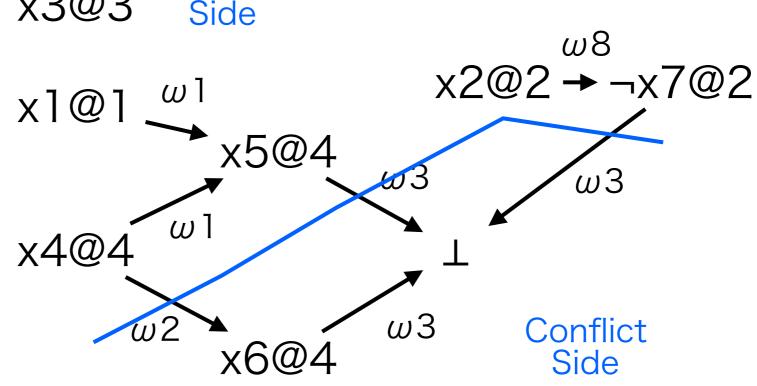
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

$$\neg x4 \lor \neg x5 \lor x7$$
 ω 1: $\neg x1 \lor \neg x4 \lor x5$

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega : \neg x \mid \vee \neg x + \vee x$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega$$
4: \neg x7 \vee x8

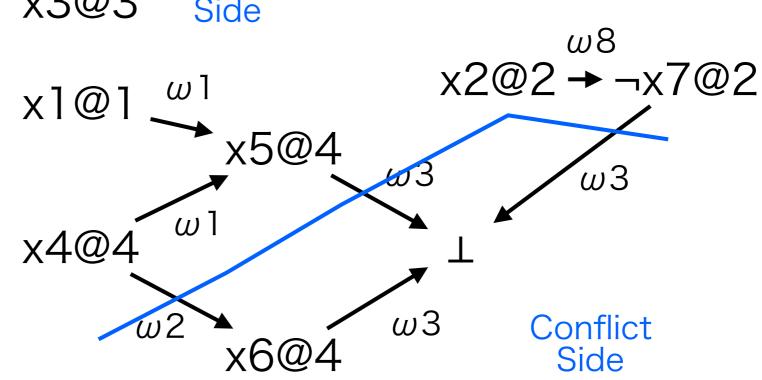
•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

$$\neg x4 \lor \neg x5 \lor x7$$
 ω 1: $\neg x1 \lor \neg x4 \lor x5$

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 1. $\neg x$ 1 $\vee \neg x$ 4 $\vee x$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

Implication Graph

x3@3 Reason Side
$$x2@2 \rightarrow \neg x7@2$$
 $x1@1$ $w1$ $x5@4$ $w3$ $w3$ $x4@4$ $w3$ $w3$ Conflict Side

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$ • ω 8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

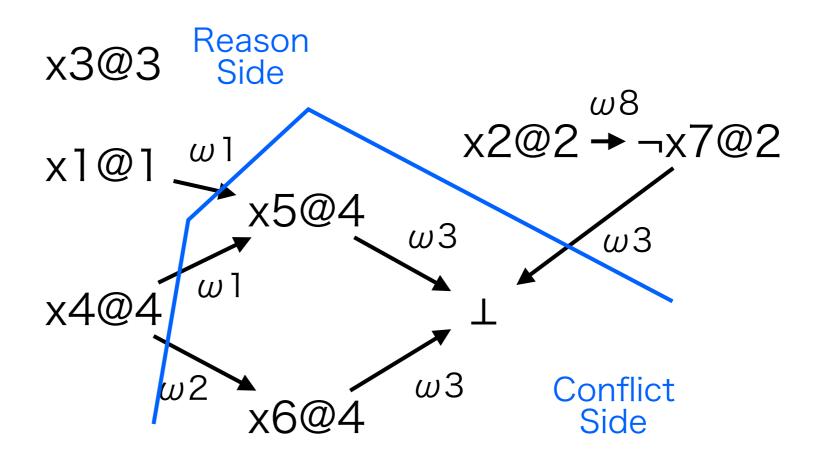
$$\omega$$
3: ω 2: $\neg x5 \lor \neg x6 \lor x7 \ \neg x4 \lor x6$

$$\neg x4 \lor \neg x5 \lor x7$$
 ω 1: $\neg x1 \lor \neg x4 \lor x5$

$$\neg x1 \lor \neg x4 \lor x7$$

It contains one literal in the current decision level (i.e. 4).

Implication Graph



•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

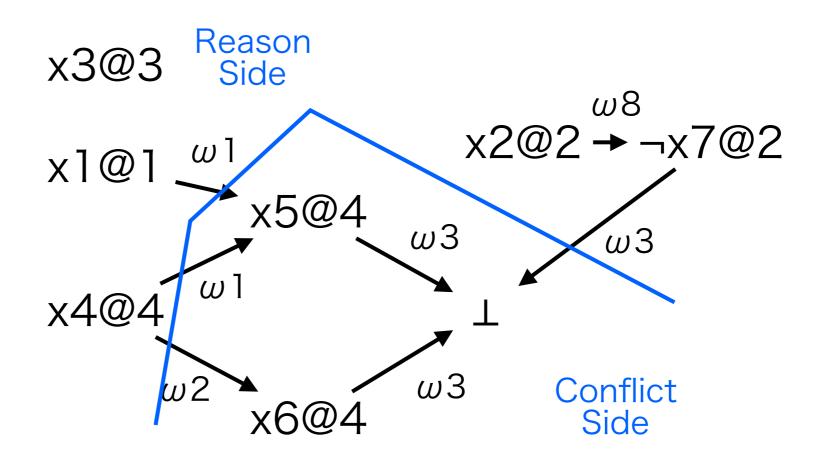
$$\omega$$
3: ω 2: $\neg x5 \lor \neg x6 \lor x7 \ \neg x4 \lor x6$

$$\neg x4 \lor \neg x5 \lor x7$$
 ω 1: $\neg x1 \lor \neg x4 \lor x5$

$$\neg x1 \lor \neg x4 \lor x7$$

It contains one literal in the current decision level (i.e. 4).

Implication Graph



Clause DB

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega 8: \neg x2 \lor \neg x7$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

New clause is learnt!

Backtrack

Learnt clause: $\neg x1 \lor \neg x4 \lor x7$

x1 was set at level 1 x4 was set at level 4 x7 was set at level 2

Assertion level of this clause is 2.

⇒ Backtrack to level 2!

Implication Graph

$$x3@3$$
 $x1@1 \xrightarrow{w1} x5@4 \xrightarrow{w3} x4@4 \xrightarrow{w2} x6@4$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

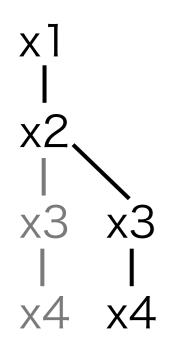
•
$$\omega$$
7: ¬x8 ∨ x9

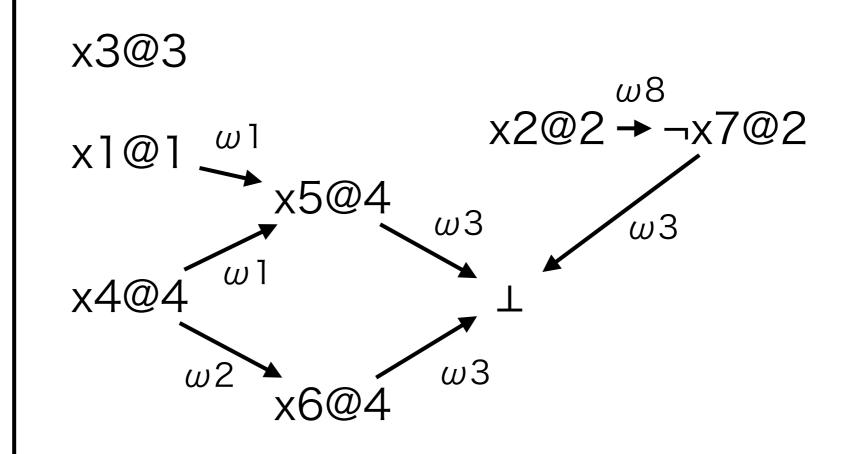
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Backtrack

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Backtrack

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Implication Graph

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

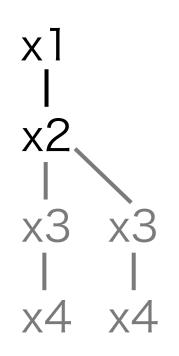
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

Implication Graph



$$x1@1 \xrightarrow{\omega 9} x2@2 \xrightarrow{\times} \neg x7@2$$

$$\neg x4@2$$

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5$ • ω 6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

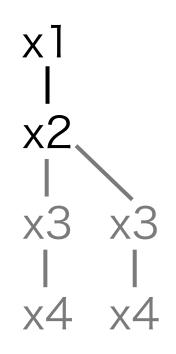
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

Implication Graph



$$x1@1 \xrightarrow{\omega 9} x2@2 \xrightarrow{\times 2} \neg x7@2$$

$$\neg x4@2$$

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

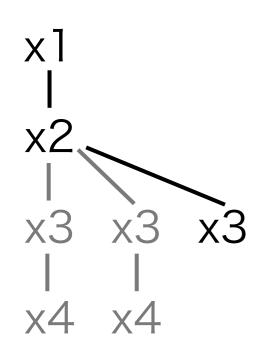
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

Implication Graph



$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x3@3$

•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

x1 | x2 | x3 x3 x3 | | | | | | x4 x4 x5

Implication Graph

$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x3@3$

x5@4

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

x1 | x2 | x3 x3 x3 | | | | | | x4 x4 x5

Implication Graph

$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x3@3$

x5@4

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

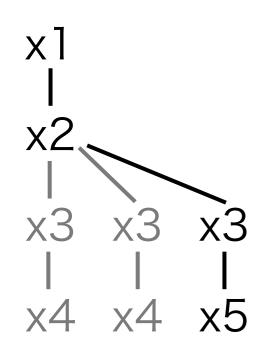
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

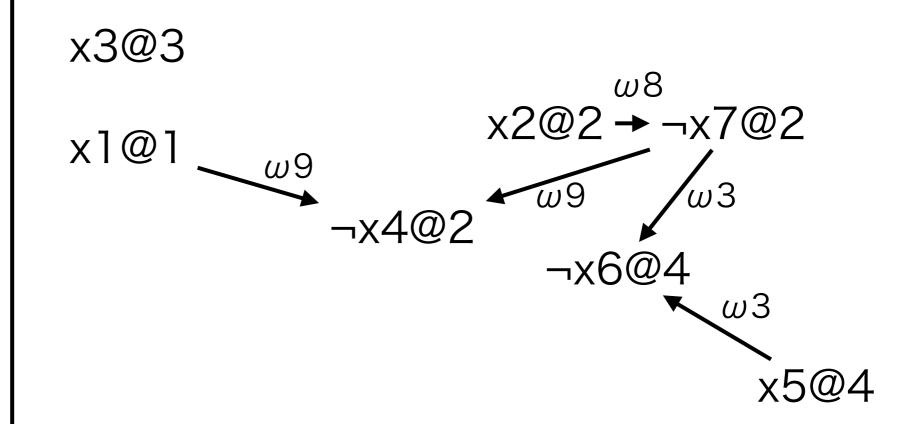
•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

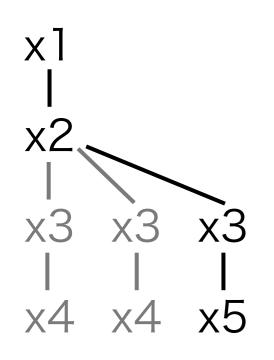
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

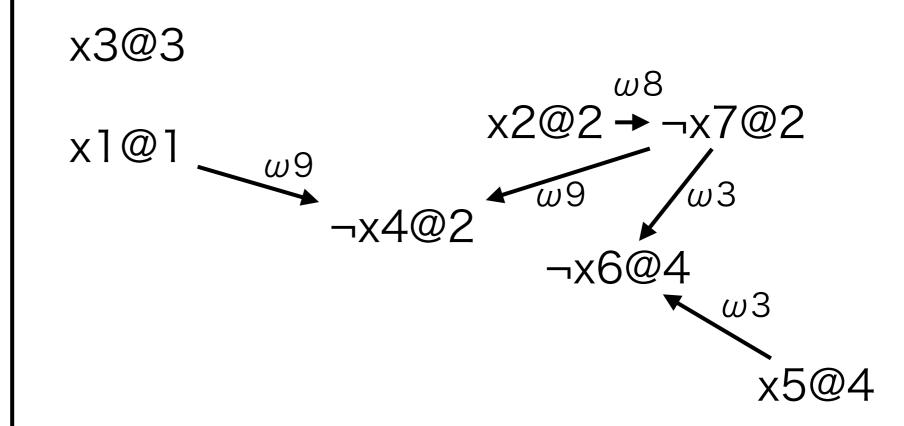
•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

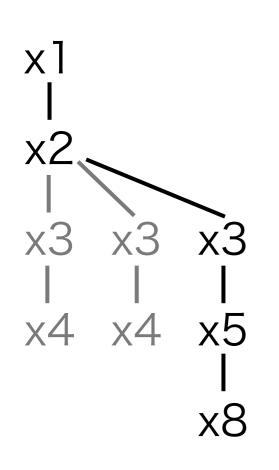
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

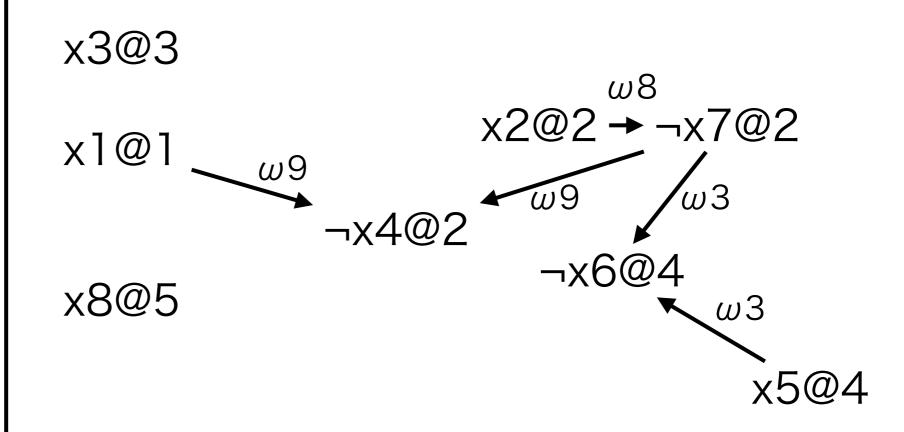
•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Implication Graph





•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ $\leftrightarrow x$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

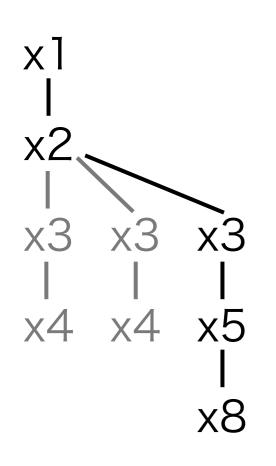
•
$$\omega$$
6: $\neg x8 \lor \neg x9$

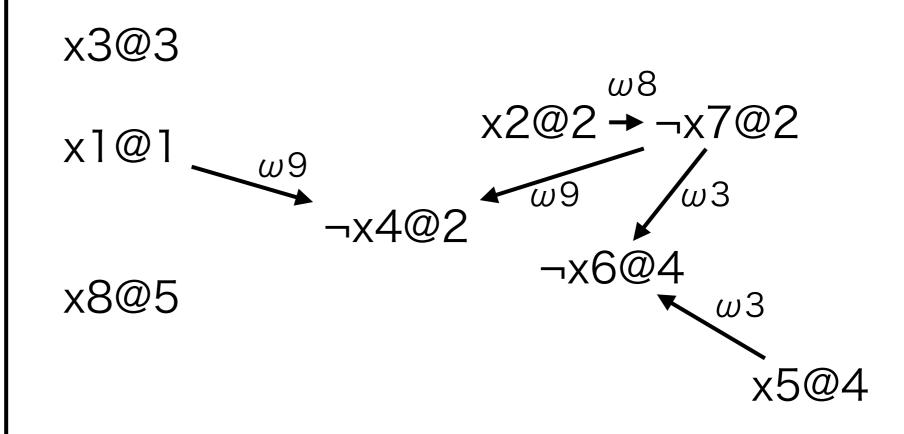
•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Implication Graph





•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
4: $\neg x7 \lor x8$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

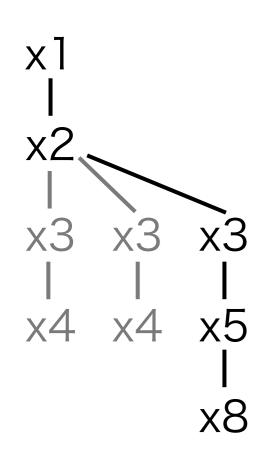
•
$$\omega$$
6: $\neg x8 \lor \neg x9$

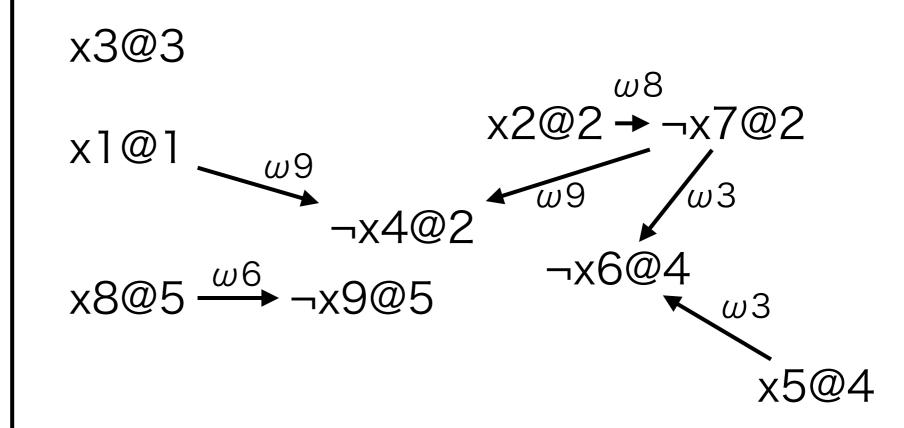
•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

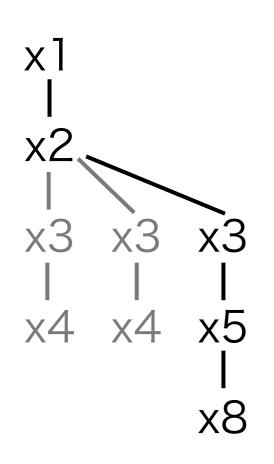
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

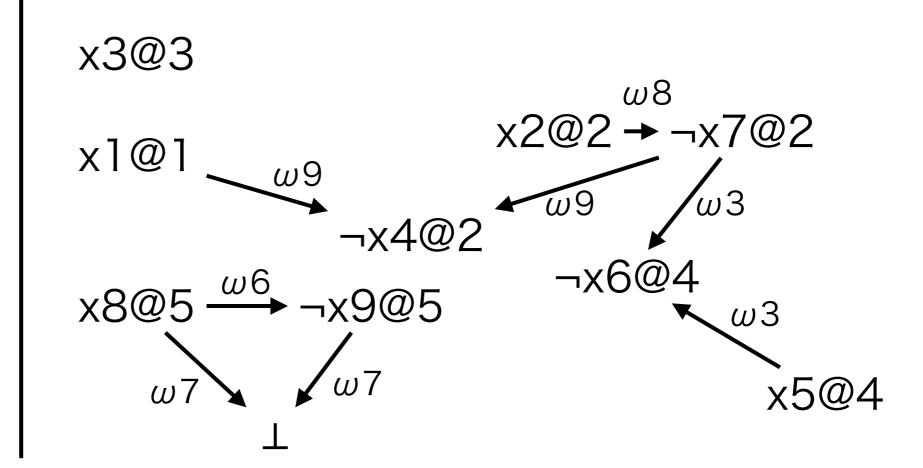
•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

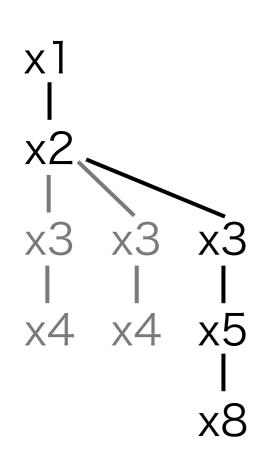
•
$$\omega$$
6: $\neg x8 \lor \neg x9$

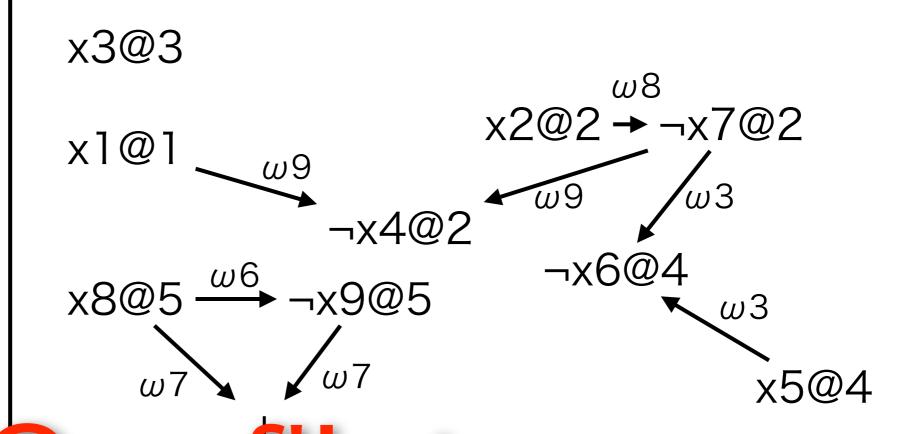
•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Implication Graph





Clause DB CONTIC

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph

$$x3@3$$

$$x1@1$$

$$w9$$

$$-x4@2$$

$$x8@5 \xrightarrow{\omega6} -x9@5$$

$$w7$$

$$w7$$

$$x5@4$$

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5 • ω 6: \neg x8 \vee \neg x9

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

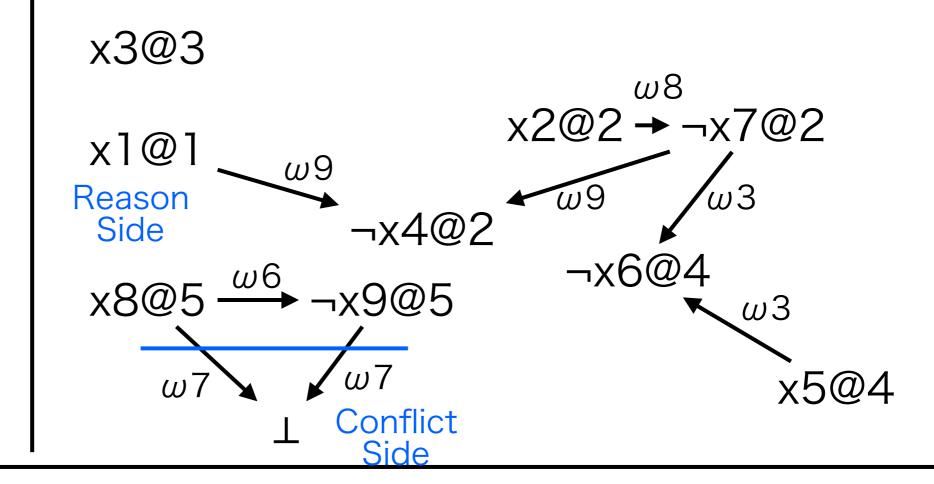
•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

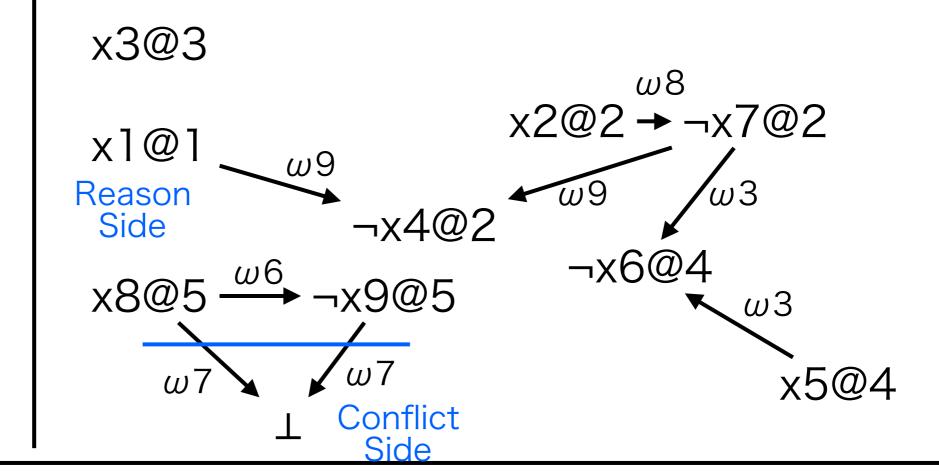
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

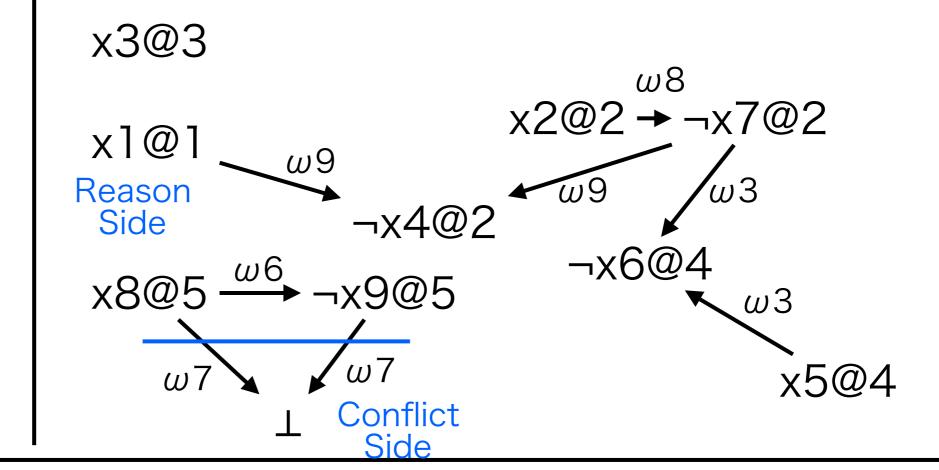
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

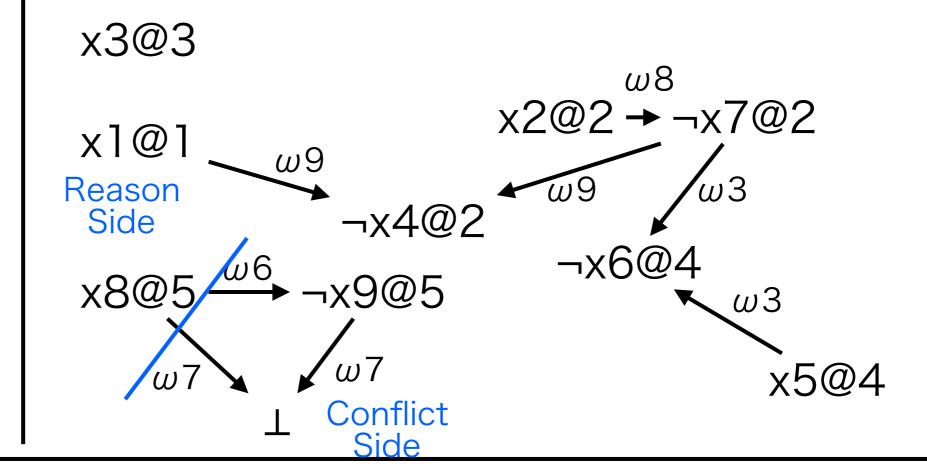
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 $\vee \neg$ x4 \vee x7

•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

ω7: ω6: ¬x8∨x9 ¬x8∨¬x9 ¬x8

It contains one literal in the current decision level (i.e. 5).

Implication Graph

$$x3@3$$
 $x1@1$
 $w9$
 $x8@5$
 $w9$
 $x4@2$
 $w9$
 $x8@5$
 $w9$
 $w9$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

ω7: ω6: ¬x8∨x9 ¬x8∨¬x9 ¬x8

It contains one literal in the current decision level (i.e. 5).

Implication Graph

x3@3

x1@1

Reason
Side

$$\neg x4@2$$
 $w9$
 $x8@5$
 $w6$
 $\neg x9@5$
 $w7$
 $w7$

Clause DB

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

New clause is learnt!

•
$$\omega$$
 10: $\neg x8$

Backtrack

Learnt clause: ¬x8

x8 was set at level 5

Assertion level of this clause (which is the second highest level) is 0 by convention.

⇒ Backtrack to level 0 (root level)

Implication Graph

$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x8@5$
 $w6$
 $-x9@5$
 $w7$
 $w7$
 $x5@4$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega 7: \neg x8 \lor x9$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

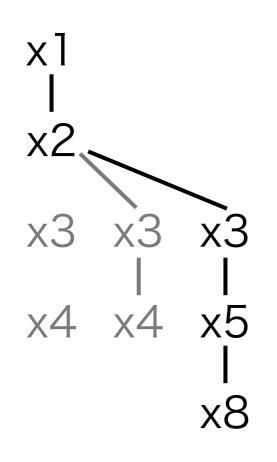
•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

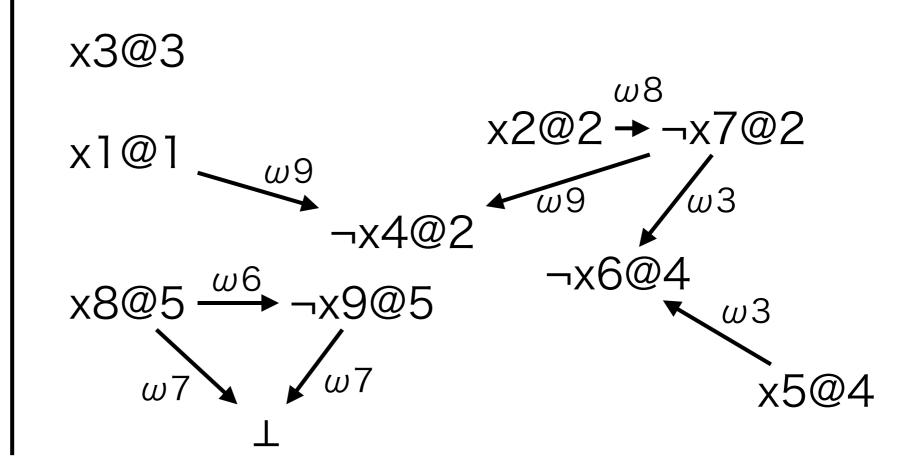
•
$$\omega 5: \neg x2 \lor \neg x7 \lor x9$$

•
$$\omega$$
 10: $\neg x8$

Backtrack

Implication Graph





•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

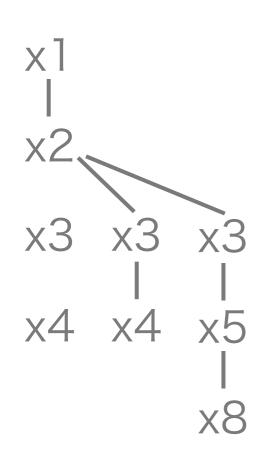
•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
 10: $\neg x8$

Backtrack

Implication Graph



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

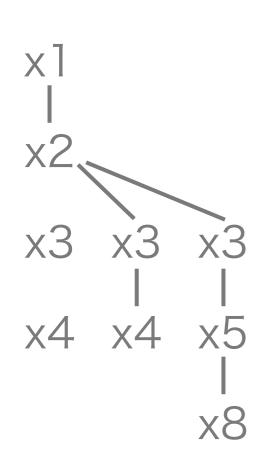
•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



•
$$\omega$$
1: $\neg x1 \lor \neg x4 \lor x5 • ω 6: $\neg x8 \lor \neg x9$$

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph

 $\neg x8@0$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph

 $\neg x8@0$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

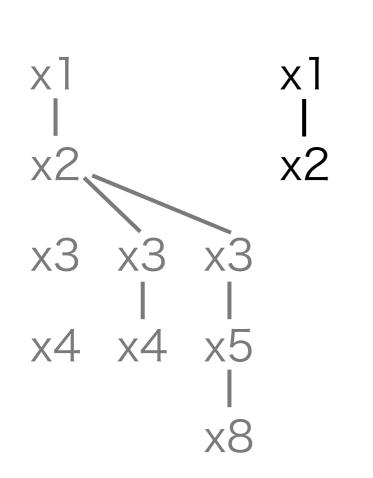
•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



 $\neg x8@0$

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

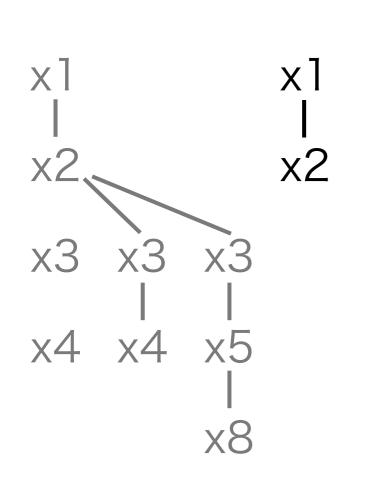
•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



¬x8@0

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega 8: \neg x2 \lor \neg x7$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

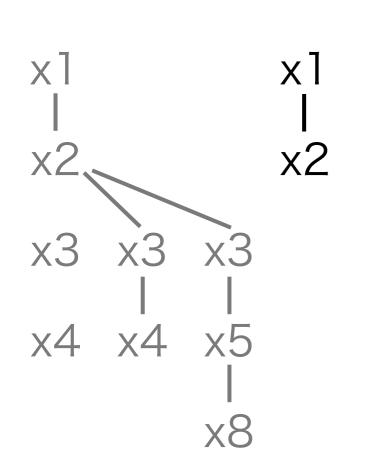
•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega 8: \neg x2 \lor \neg x7$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



$$x1@1$$
 $w9$
 $\neg x4@2$
 $x2@2 \xrightarrow{w8} \neg x7@2$
 $\neg x8@0$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

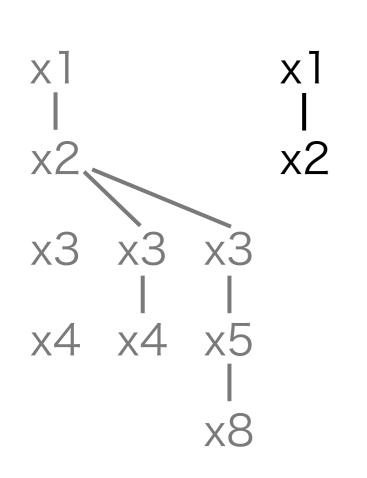
•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega 8: \neg x2 \lor \neg x7$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



$$x1@1$$
 $w9$
 $-x4@2$
 $x2@2 \xrightarrow{w8} -x7@2$
 $-x8@0$

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

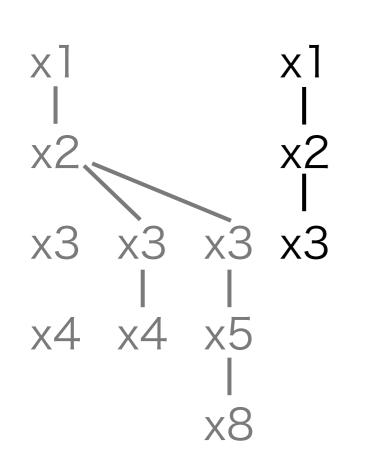
•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x2@2 \xrightarrow{w8} -x7@2$
 $x4@2$

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

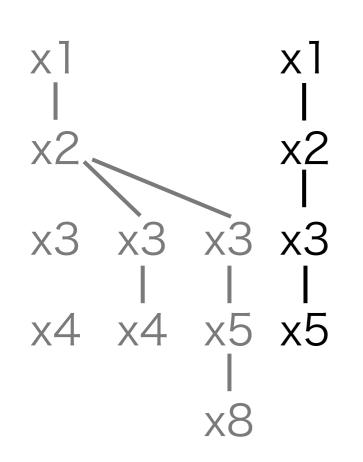
•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Implication Graph



$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x2@2 \xrightarrow{w8} -x7@2$
 $x4@2$

Clause DB

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
7: ¬x8 ∨ x9

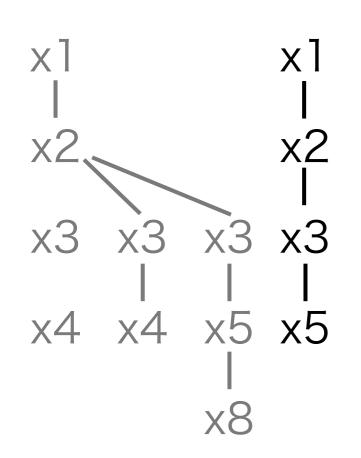
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

x5@4

•
$$\omega$$
 10: $\neg x8$

Implication Graph



$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x2@2 \xrightarrow{\omega 8} -x7@2$
 $x2@2 \xrightarrow{\omega 9} -x7@2$

Clause DB

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega$$
 1. $\neg x$ 1 \vee $\neg x$ $+$ \vee x

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega 7: \neg x8 \lor x9$$

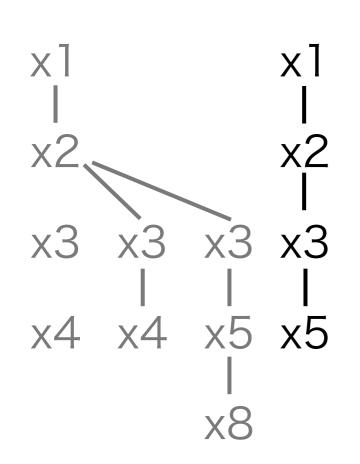
•
$$\omega$$
8: $\neg x2 \lor \neg x7$

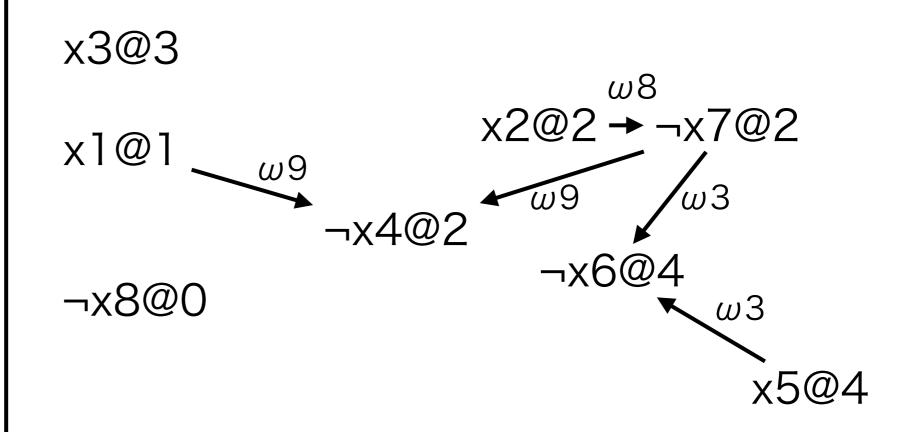
•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

x5@4

•
$$\omega$$
 10: $\neg x8$

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: \neg x8 \vee x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 8: \neg x2 \lor \neg x7$$

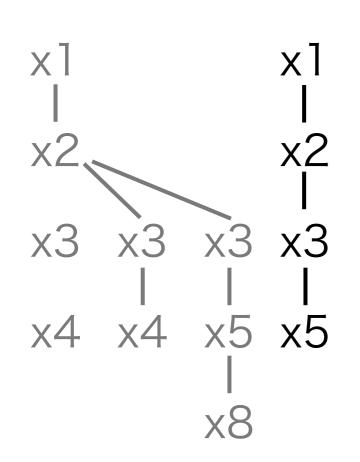
•
$$\omega 4: \neg x7 \lor x8$$

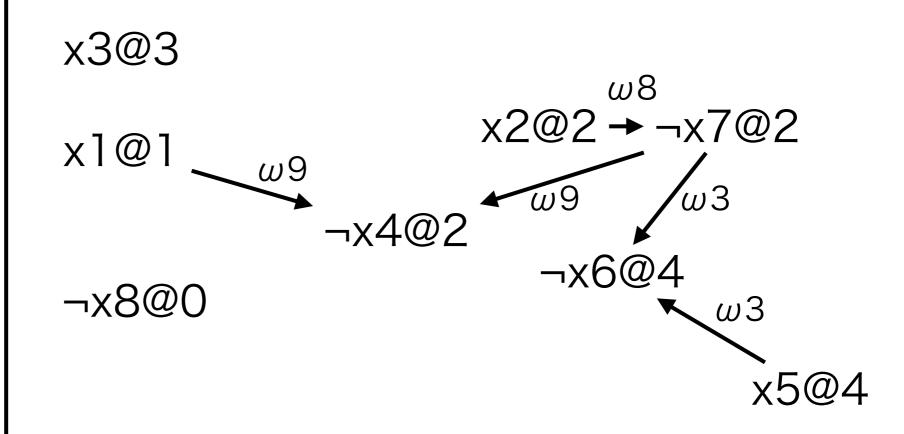
•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
 10: $\neg x8$

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

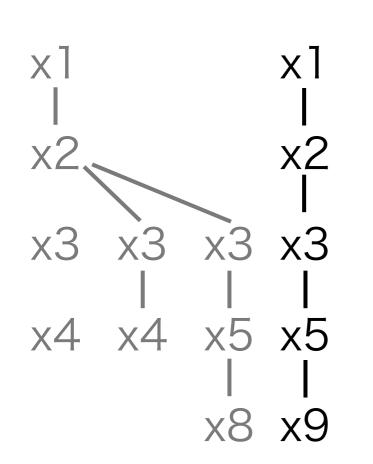
•
$$\omega 4: \neg x7 \lor x8$$

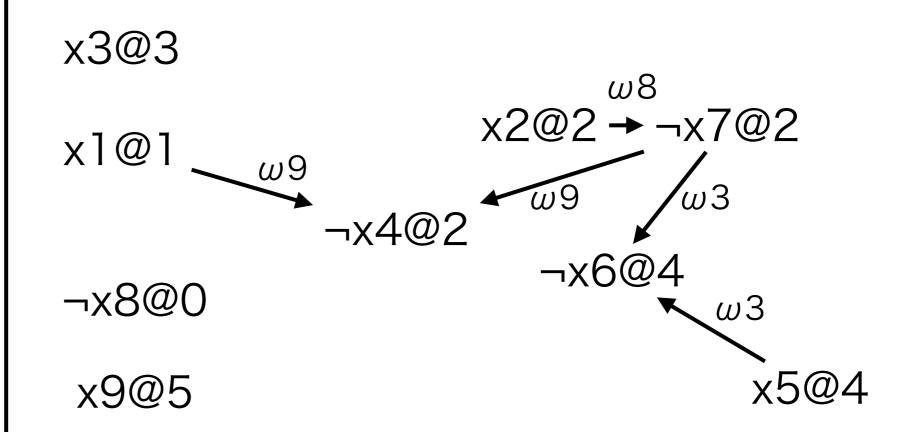
•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
 10: $\neg x8$

Implication Graph





•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
 10: $\neg x8$

$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x2@2 \rightarrow -x7@2$
 $w3$
 $-x6@4$
 $x9@5$
 $x5@4$

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$



- ω 1: $\neg x1 \lor \neg x4 \lor x5$ ω 6: $\neg x8 \lor \neg x9$
- $\omega 2: \neg x + \sqrt{x6}$ $\omega 7: \neg x8 \vee x9$
- $\omega 3: \neg x5 \lor \neg x6 \lor x7$ $\omega 8: \neg x2 \lor \neg x7$
- $\omega 4: \neg x7 \lor x8$
- $\omega 5: \neg x2 \lor \neg x7 \lor x9$
- ω 9: \neg x1 \vee \neg x4 \vee x7
- ω 10: $\neg x8$

$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x2@2 \rightarrow -x7@2$
 $w3$
 $-x6@4$
 $x9@5$
 $x5@4$

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

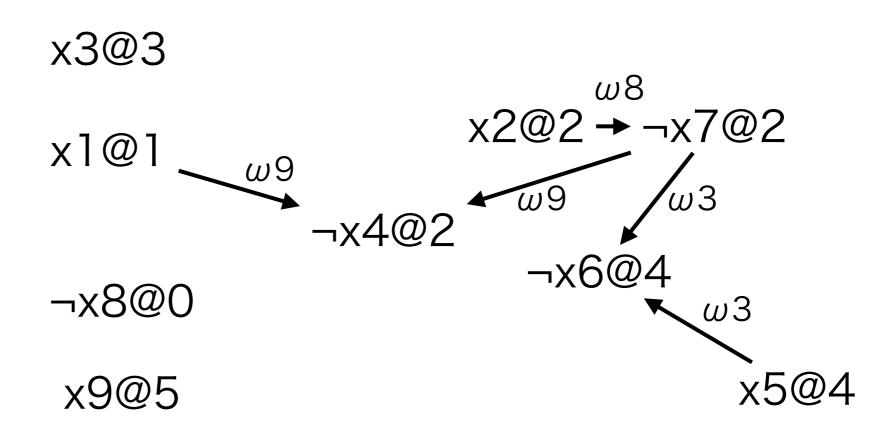
•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$



- ω 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5
- ω 2: $\neg x4 \lor x6$
- ω 3: $\neg x5 \lor \neg x6 \lor x7$
- $\omega 4: \neg x7 \lor x8$
- ω 5: $\neg x2 \lor \neg x7 \lor x9$

- ω 6: $\neg x8 \lor \neg x9$
- ω 7: \neg x8 \vee x9
- $\omega 8: \neg x2 \lor \neg x7$
- ω 9: \neg x1 $\vee \neg$ x4 \vee x7
- ω 10: $\neg x8$

$$x3@3$$
 $x1@1$
 $w9$
 $-x4@2$
 $x2@2 \rightarrow -x7@2$
 $w3$
 $-x6@4$
 $x9@5$
 $x5@4$

•
$$\omega$$
1: \neg x1 \vee \neg x4 \vee x5

•
$$\omega 2: \neg x4 \lor x6$$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega 4: \neg x7 \lor x8$$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

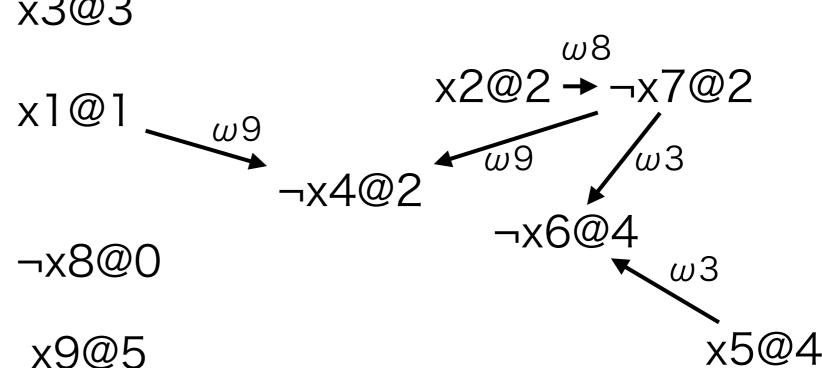
•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

Found a model

$$M = \{ x1, x2, x3, \\ \neg x4, x5, \neg x6, \\ \neg x7, \neg x8, x9 \}$$



•
$$\omega$$
 1: $\neg x$ 1 $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
 I: $\neg x$ I $\vee \neg x$ 4 $\vee x$ 5

•
$$\omega$$
2: $\neg x4 \lor x6$

•
$$\omega$$
3: $\neg x5 \lor \neg x6 \lor x7$

•
$$\omega$$
4: $\neg x7 \lor x8$

•
$$\omega$$
5: $\neg x2 \lor \neg x7 \lor x9$

•
$$\omega$$
6: $\neg x8 \lor \neg x9$

•
$$\omega$$
7: ¬x8 ∨ x9

•
$$\omega$$
8: $\neg x2 \lor \neg x7$

•
$$\omega$$
9: \neg x1 \vee \neg x4 \vee x7

•
$$\omega$$
 10: $\neg x8$

References

- J. P. Marques-Silva and K. A. Sakallah, "GRASP: a search algorithm for propositional satisfiability," Computers, IEEE Transactions on, vol. 48, no. 5, pp. 506-521, May 1999.
- Handbook of Satisfiability, A. Biere,
 M. Heule, H. Van Maaren, and T. Walsh,
 Eds. IOS Press, Feb. 2009.